Boltzmann Transport Equation

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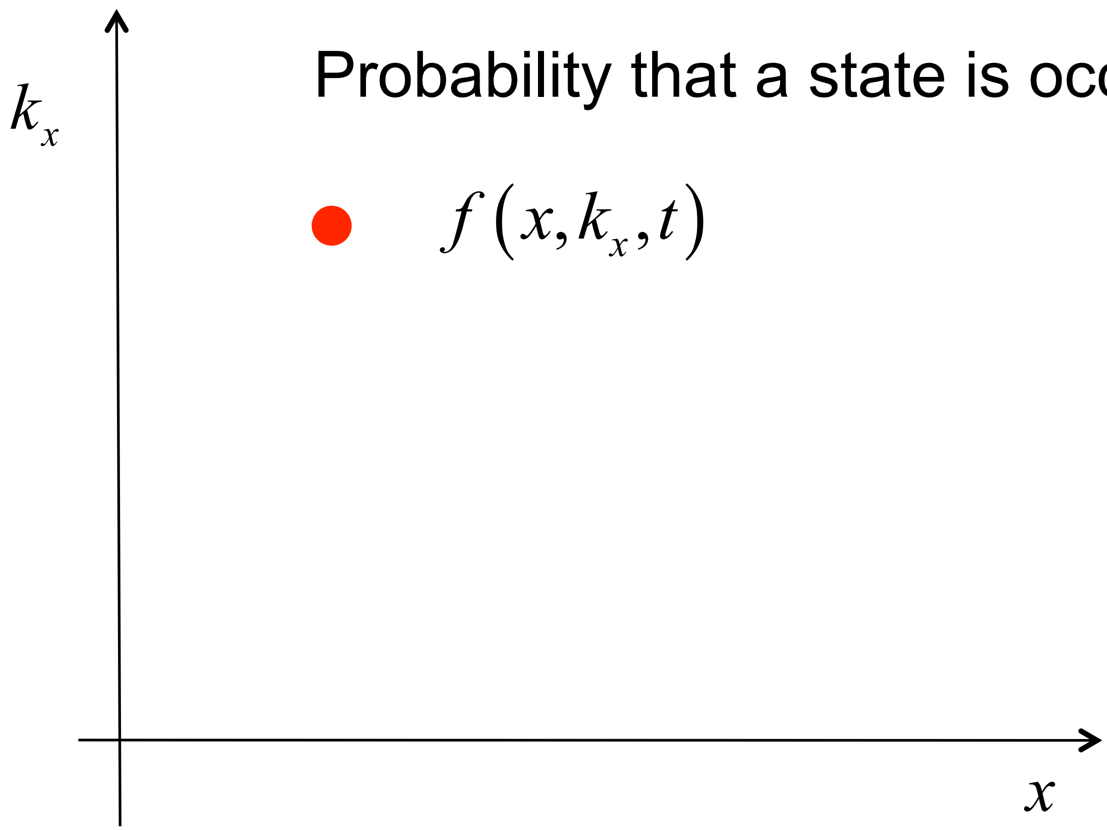
Outline

1) The distribution function

2) The BTE

$f(r, k, t)$

Probability that a state is occupied.



Distribution function

What is the probability that a state is occupied?

Answer: in equilibrium: $$f_0(x,k_x) = \frac{1}{1+e^{(E-E_F)/k_BT}}$$

ECE 606 Answer: $$f(x,k_x,t) = \frac{1}{1+e^{(E-F_n(x,t))/k_BT}}$$

Generally: $$f(x,k_x,t)$$

From the distribution function

Electron density: $n(x,t) = \frac{1}{\Omega} \sum_{\vec{k}} f(x,\vec{k},t)$

Electron current density: $J_{nx}(x,t) = \frac{1}{\Omega} \sum_{\vec{k}} (-q) \upsilon_x f(x,\vec{k},t)$

Kinetic energy per electron: $u(x,t) = \dfrac{\frac{1}{\Omega} \sum_{\vec{k}} E_k f(x,\vec{k},t)}{\frac{1}{\Omega} \sum_{\vec{k}} f(x,\vec{k},t)}$

etc….

Goals for the next few lectures

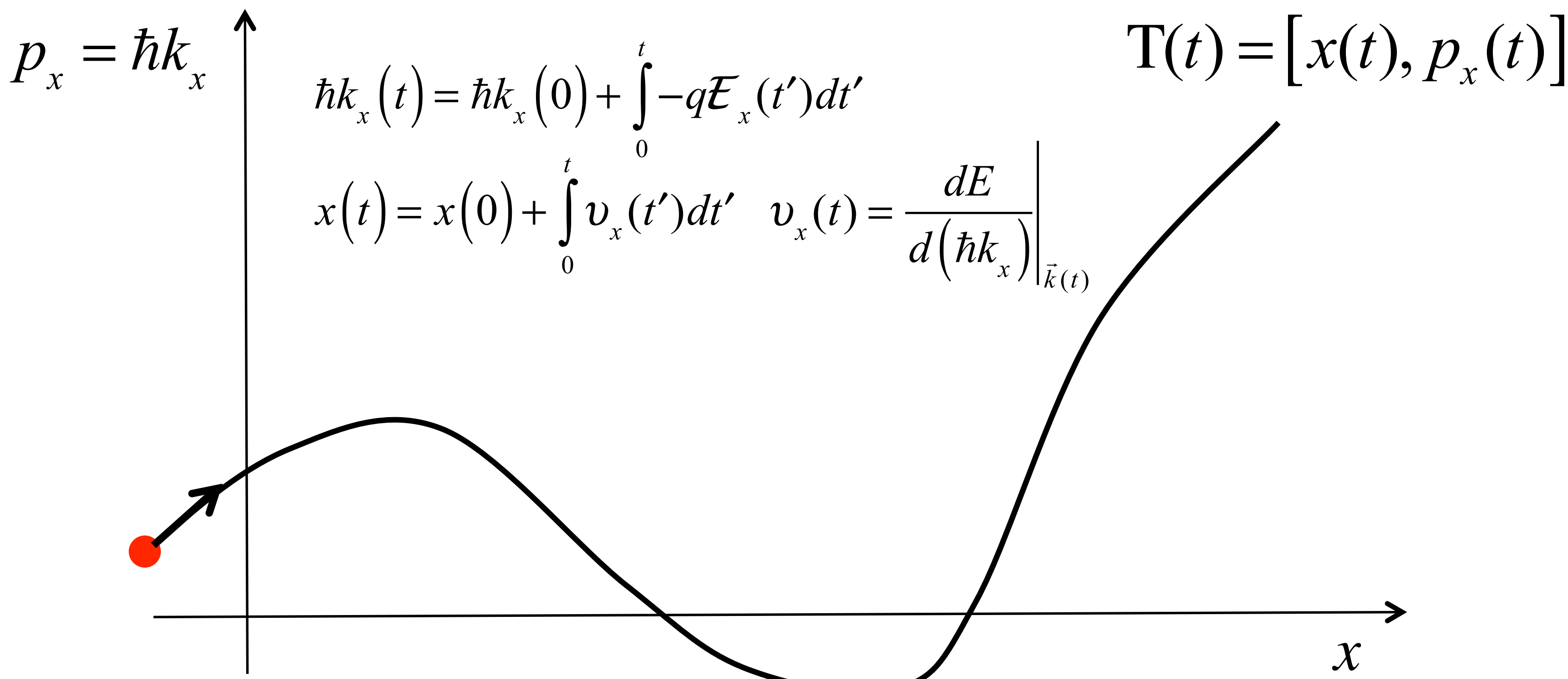
1) Find an equation for $f(r, p, t)$ out of equilibrium
2) Learn how to solve it near equilibrium

Later on in the course, we will learn to solve the equation under far-from-equilibrium conditions.

Outline

1) The distribution function
2) The BTE

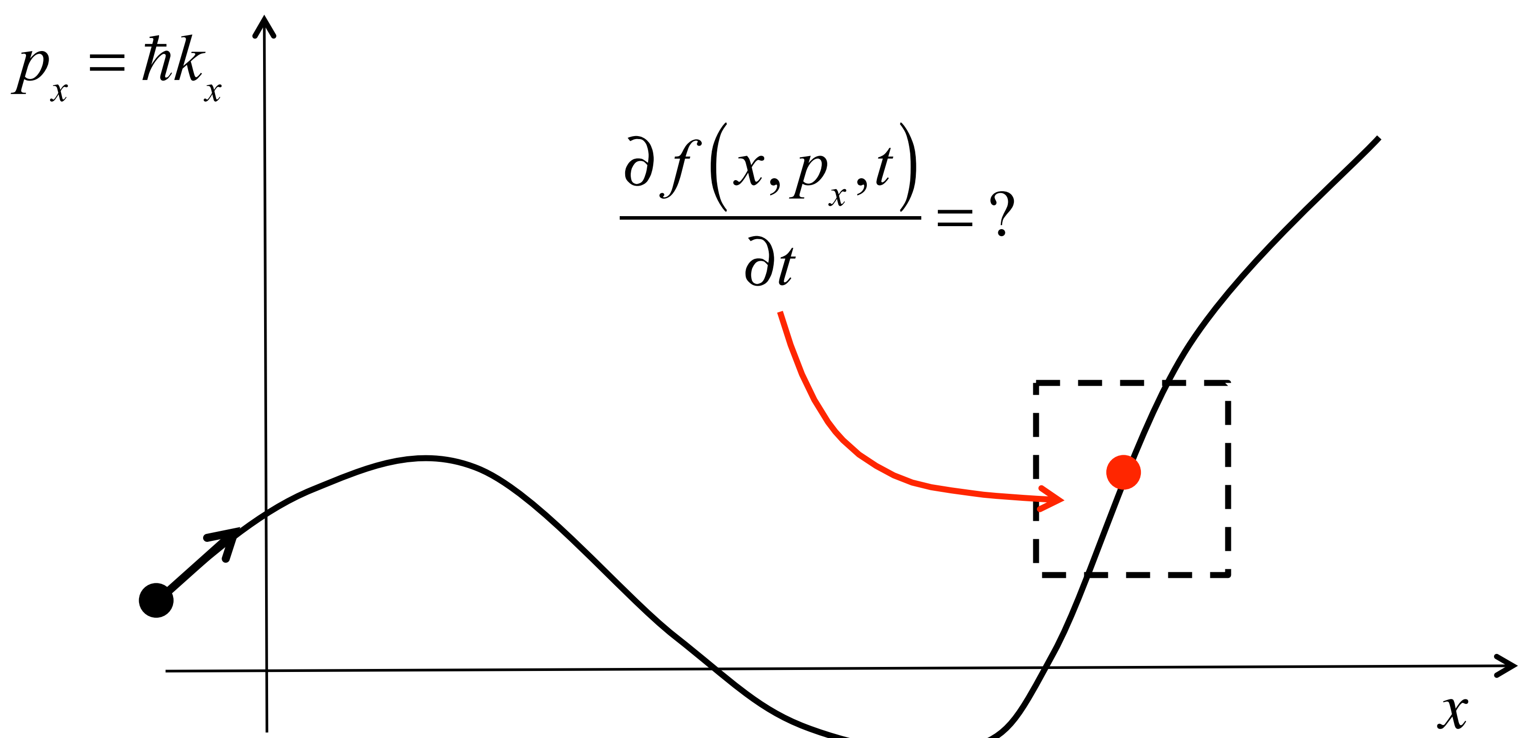
Trajectories in phase space



$$\hbar k_x\left(t\right) = \hbar k_x\left(0\right) + \int_0^t -q\mathcal{E}_x(t')dt'$$

$$x\left(t\right) = x\left(0\right) + \int_0^t \upsilon_x(t')dt' \quad \upsilon_x(t) = \left.\frac{dE}{d\left(\hbar k_x\right)}\right|_{\vec{k}(t)}$$

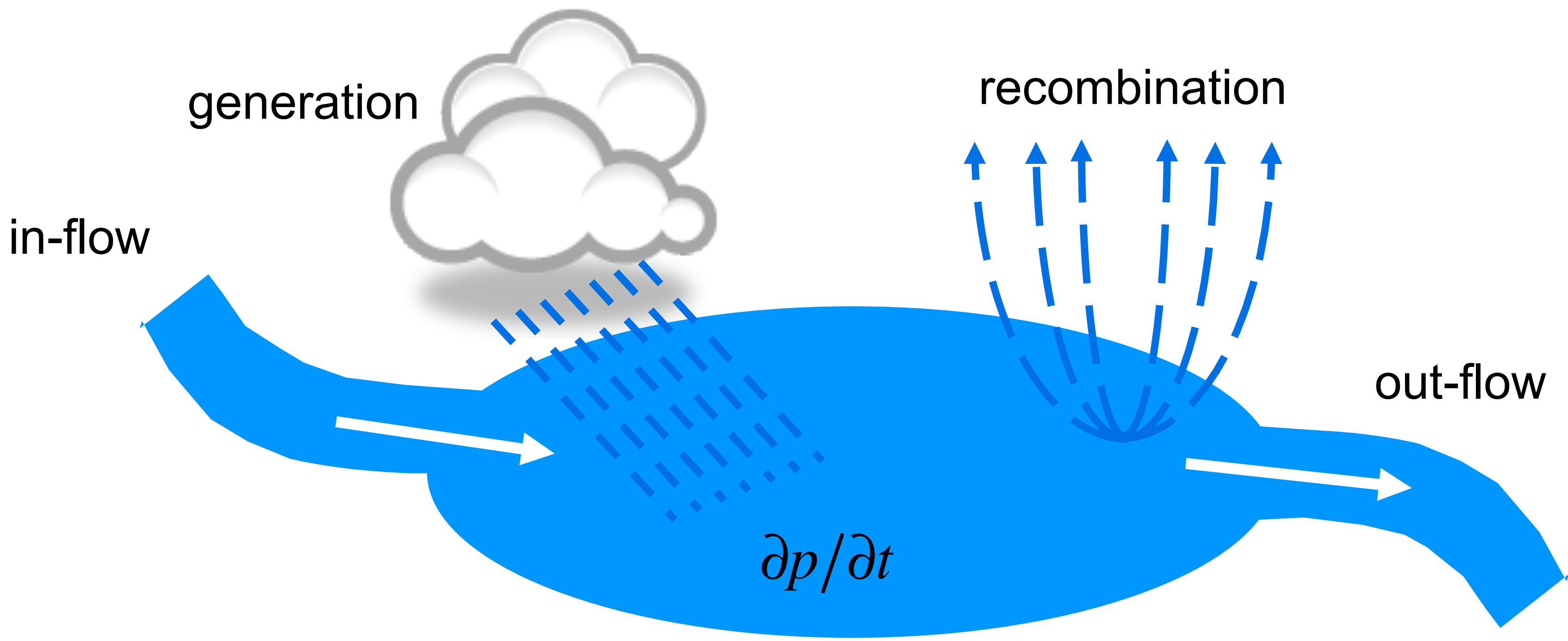
Continuity (bookkeeping) equation



Another continuity equation

$$\frac{\partial p}{\partial t} = \text{in-flow - out-flow} + G - R$$

$$\frac{\partial p}{\partial t} = -\nabla \bullet \frac{\vec{J}_p}{q} + G_p - R_p$$



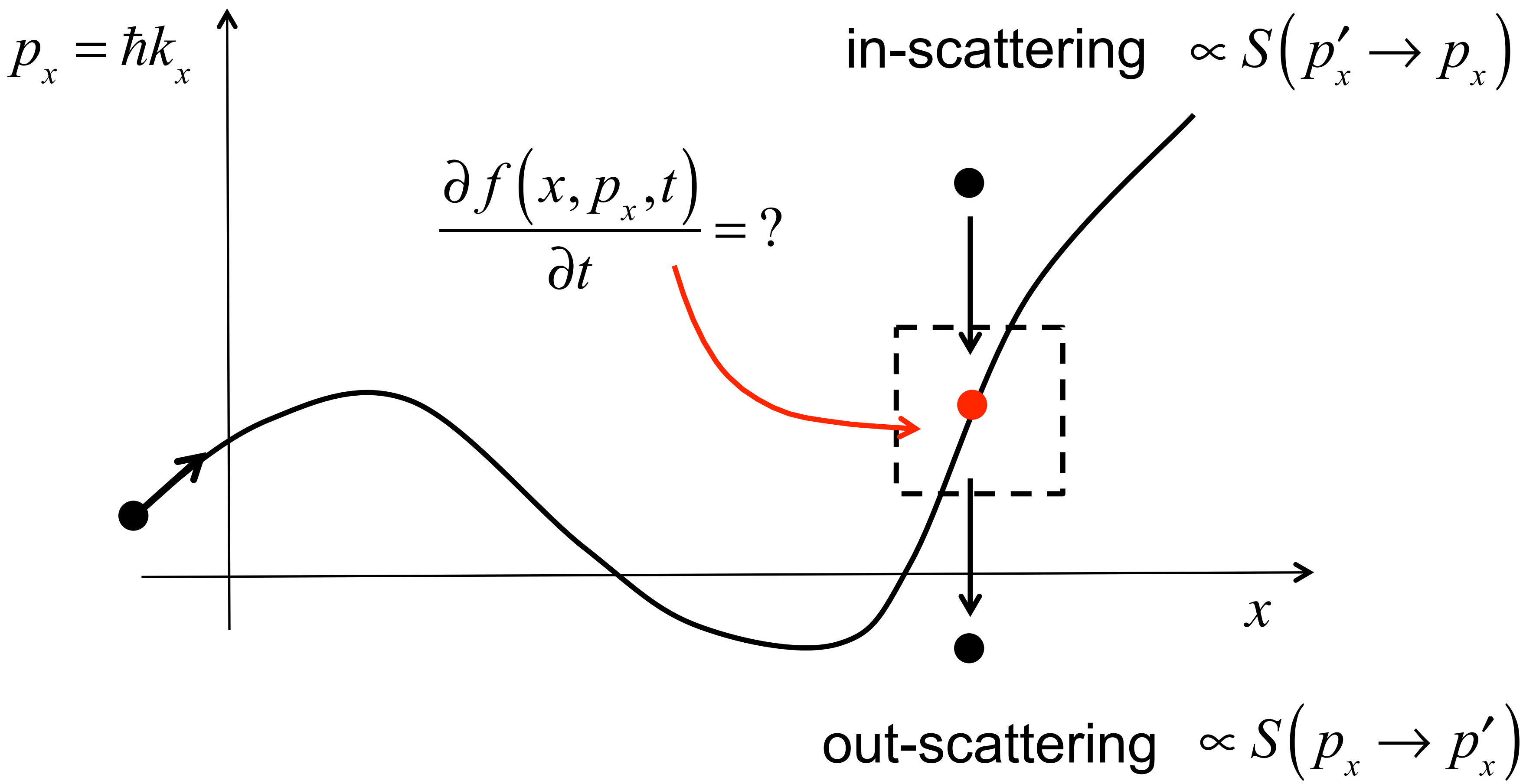
Continuity equation for *f*

$$\frac{\partial p}{\partial t} = -\nabla \bullet \frac{\vec{J}_p}{q} + G_p - R_p \qquad \frac{\partial p}{\partial t} = -\frac{\partial J_{px}}{\partial x} - \frac{\partial J_{py}}{\partial y} + \left(G_p - R_p\right)$$

$$\frac{\partial f\left(x, p_x, t\right)}{\partial t} = -\frac{\partial\left(\upsilon_x f\right)}{\partial x} - \frac{\partial\left(\frac{dp_x}{dt} f\right)}{\partial p_x} + \left(\frac{\partial f}{\partial t}\right)_{g-r} + \ldots$$

$$\frac{\partial f}{\partial t} = -\upsilon_x \frac{\partial f}{\partial x} - F_x \frac{\partial f}{\partial p_x} + \left(\frac{\partial f}{\partial t}\right)_{g-r} + \ldots \quad ?$$

Continuity (bookkeeping) equation



Continuity equation for *f*

$$\frac{\partial f}{\partial t} = -\upsilon_x \frac{\partial f}{\partial x} - F_x \frac{\partial f}{\partial p_x} + \left(\frac{\partial f}{\partial t}\right)_{g-r} + \left(\frac{\partial f}{\partial t}\right)_{coll}$$

"Slow" recombination-generation processes. Can be important, but we will ignore in this class.

$$\frac{\partial f}{\partial t} = -\upsilon_x \frac{\partial f}{\partial x} - F_x \frac{\partial f}{\partial p_x} + \left(\frac{\partial f}{\partial t}\right)_{coll}$$

Boltzmann Transport Equation (BTE)

$$\frac{\partial f}{\partial t} + \vec{\upsilon} \bullet \nabla_r f + \vec{F}_e \bullet \nabla_p f = \left.\frac{\partial f}{\partial t}\right)_{coll}$$

$$\vec{F}_e = -q\vec{\mathcal{E}} - q\vec{\upsilon} \times \vec{B}$$

$$\nabla_r f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

$$\nabla_p f = \frac{\partial f}{\partial p_x}\hat{p}_x + \frac{\partial f}{\partial p_y}\hat{p}_y + \frac{\partial f}{\partial p_z}\hat{p}_z$$

$$\vec{p} = \hbar\vec{k}$$

BTE

$$\frac{\partial f}{\partial t} + \vec{\upsilon} \bullet \nabla_r f + \vec{F}_e \bullet \nabla_p f = \hat{C} f$$

$$\left.\frac{\partial f}{\partial t}\right)_{coll} = \hat{C} f = \text{in-scattering - out-scattering}$$

"collision operator"

"collision integral"

Time-dependent and six dimensional. The collision integral also makes it nonlinear.

Questions?

$$\frac{\partial f}{\partial t} + \vec{\upsilon} \bullet \nabla_r f + \vec{F}_e \bullet \nabla_p f = \hat{C} f$$