ECE 656: Electronic Transport in Semiconductors Fall 2017

# Solving the BTE:

# Metals

Mark Lundstrom

Electrical and Computer Engineering
Purdue University
West Lafayette, IN USA

nanoHUB.org

9/19/17

PURDUE UNIVERSITY

# Steady-state BTE for small E-fields

$$\frac{\partial f}{\partial t} + \vec{\upsilon} \bullet \nabla_r f + \vec{F}_e \bullet \nabla_p f = -\frac{f - f_0}{\tau_m}$$

$n \approx n_0$

$F_n \approx E_F$

Steady-state, 1D, spatially uniform, no B-field:

$$-q\mathcal{E}_x \frac{\partial f}{\partial p_x} \approx -q\mathcal{E}_x \frac{\partial f_0}{\partial p_x} = -\frac{f - f_0}{\tau_m}$$

$$f = f_0 + q\tau_m \mathcal{E}_x \frac{\partial f_0}{\partial p_x}$$

# Steady-state BTE for small E-fields

$$J_{nx} = \frac{1}{\Omega}\sum_{\vec{k}}(-q)\upsilon_x f$$

$$J_{nx} = \sigma_n \mathcal{E}_x$$

$$\sigma_n = \frac{1}{\Omega}\sum_{\vec{k}} q^2 \upsilon_x^2 \tau_m \left(-\frac{\partial f_0}{\partial E}\right)$$

# Conductivity of a metal

Our solution to the BTE gave: $\sigma_n = \frac{1}{\Omega}\sum_{\vec{k}} q^2 \upsilon_x^2 \tau_m \left( -\frac{\partial f_0}{\partial E} \right)$

For a non-degenerate semiconductor, we simplified this to:

$$\sigma_n = nq\mu_n$$

Metals have a very high density of electrons in the conduction band. What is the conductivity of a metal?

# Conductivity of a metal

Our solution to the BTE gave: $\sigma_n = \frac{1}{\Omega}\sum_{\vec{k}} q^2 \upsilon_x^2 \tau_m \left(-\frac{\partial f_0}{\partial E}\right)$

Make the replacement (valid near equilibrium): $\upsilon_x^2 \rightarrow \upsilon^2/3$

Convert the sum over k to an integral over k, then convert the integral over k to an integral over energy assuming parabolic bands. $E = \hbar^2 k^2 / 2m^*$

Result...

# Conductivity of a metal

Result...

$$\sigma_n = q^2 \int_0^\infty \frac{\upsilon^2 \tau_m}{3} \left( \frac{m^* \sqrt{2m^* E}}{\pi^2 \hbar^3} \right) \left( -\frac{\partial f_0}{\partial E} \right) dE$$

Recall:

$$D_{3D}(E) = \left( \frac{m^* \sqrt{2m^* E}}{\pi^2 \hbar^3} \right)$$

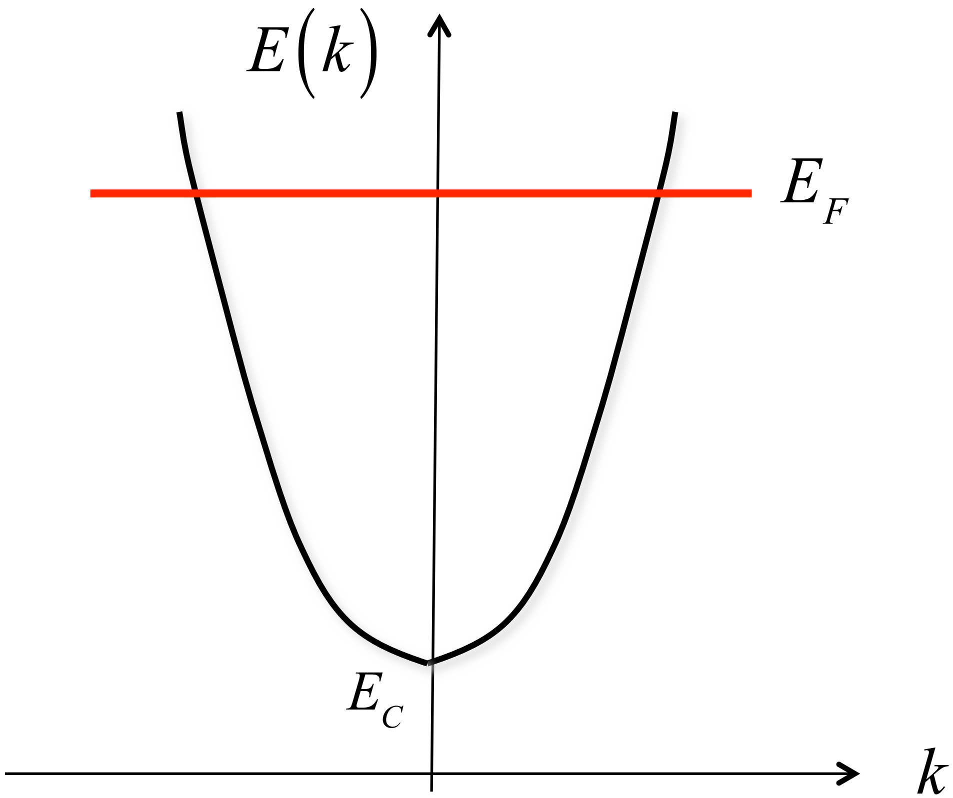
Recognize:

$$D_n(E) = \frac{\upsilon^2 \tau_m}{3}$$

$$\sigma_n = q^2 \int_0^\infty D_n(E) D_{3D}(E) \left( -\frac{\partial f_0}{\partial E} \right) dE$$

# Conductivity of a metal

Our solution to the BTE gave: $$\sigma_n = q^2 \int_0^\infty D_n(E) D_{3D}(E) \left( -\frac{\partial f_0}{\partial E} \right) dE$$




In a metal, the Fermi level is very high in the band. The with of the Fermi window is small in comparison to the variation of the DOS.

$$\left( -\frac{\partial f_0}{\partial E} \right) \approx \delta(E_F)$$

# Conductivity of a metal

$$\sigma_n = q^2 \int_0^\infty D_n(E) D_{3D}(E) \delta(E_F) dE$$

$$J_{nx} = \sigma_n \mathcal{E}_x$$

$$\sigma_n = q^2 D_n(E_F) D_{3D}(E_F)$$

# Recap:

$$-q\mathcal{E}_x \frac{\partial f_0}{\partial p_x} = -\frac{f - f_0}{\tau_m} \rightarrow J_{nx} = \sigma_n \mathcal{E}_x$$

s.s. BTE with spatially uniform conditions.

non-degenerate semiconductor

$$\sigma_n = nq\mu_n$$

$$\mu_n = \frac{q\langle\langle\tau_m\rangle\rangle}{m^*}$$

$$\langle\langle\tau_m\rangle\rangle \equiv \frac{\langle E\tau_m\rangle}{\langle E\rangle}$$

degenerate metal or semiconductor

$$\sigma_n = q^2 D_n(E_F) D_{3D}(E_F)$$

$$D_n(E_F) = \frac{\upsilon_F^2 \tau_m(E_F)}{3}$$

$$D_{3D}(E_F) = \left(\frac{m^*\sqrt{2m^*E_F}}{\pi^2\hbar^3}\right)$$

# Now to confuse matters...

Our result for metals is: $\sigma_n = q^2 D_n(E_F) D_{3D}(E_F)$

Let's see if we can express this in terms of the electron density,

$$n = \int_0^{E_F} D_{3D}(E)\,dE = \int_0^{E_F} \left( \frac{m^* \sqrt{2m^* E}}{\pi^2 \hbar^3} \right) dE = \left( \frac{m^* \sqrt{2m^*}}{\pi^2 \hbar^3} \right) \int_0^{E_F} \sqrt{E}\, dE$$

$$n = \left( \frac{m^* \sqrt{2m^*}}{\pi^2 \hbar^3} \right) \frac{2}{3} E_F^{2/3} = D_{3D}(E_F) \frac{2}{3} E_F$$

$$D_{3D}(E_F) = \frac{3}{2} \frac{n}{E_F}$$

# Now to confuse matters...

$$\sigma_n = q^2 D_n(E_F) D_{3D}(E_F) \qquad D_{3D}(E_F) = \frac{3}{2}\frac{n}{E_F}$$

$$\sigma_n = q^2 D_n(E_F)\frac{n}{\frac{2}{3}E_F} = nq\left(\frac{D_n(E_F)}{\frac{2}{3}E_F/q}\right)$$

$$\mu_n = \left(\frac{D_n(E_F)}{\frac{2}{3}E_F/q}\right) = \left(\frac{\frac{\upsilon_F^2\tau_m(E_F)}{3}}{\frac{2}{3}E_F/q}\right) = \frac{q\tau_m(E)}{m^*}$$

$$\frac{D_n}{k_B T/q} = \mu_n$$

nondegenerate Einstein relation

$$\frac{1}{2}m^*\upsilon_F^2 = E_F$$

# Second Recap:

$$-q\mathcal{E}_x \frac{\partial f_0}{\partial p_x} = -\frac{f - f_0}{\tau_m} \rightarrow J_{nx} = \sigma_n \mathcal{E}_x$$

s.s. BTE with spatially uniform conditions.

**non-degenerate semiconductor**

$$\sigma_n = nq\mu_n$$

$$\mu_n = \frac{q\langle\langle\tau_m\rangle\rangle}{m^*}$$

$$\langle\langle\tau_m\rangle\rangle \equiv \frac{\langle E\tau_m\rangle}{\langle E\rangle}$$

**degenerate metal or semiconductor**

$$\sigma_n = nq\mu_n$$

$$\mu_n = \frac{q\tau_m(E_F)}{m^*}$$

But only a tiny fraction of the electrons contribute to conduction!

# Take away

**For conductivity, remember this:**

$$\sigma_n = \frac{1}{\Omega}\sum_{\vec{k}} q^2 \upsilon_x^2 \tau_m \left(-\frac{\partial f_0}{\partial E}\right)$$

Not this:

$$\sigma_n = nq\mu_n \qquad \mu_n = \frac{q\tau}{m^*}$$

# Questions?

$$\frac{\partial f}{\partial t} + \vec{\upsilon} \bullet \nabla_r f + \vec{F}_e \bullet \nabla_p f = -\frac{f - f_S}{\tau_m}$$

general case:

$$\sigma_n = \frac{1}{\Omega} \sum_{\vec{k}} q^2 \upsilon_x^2 \tau_m \left( -\frac{\partial f_0}{\partial E} \right)$$

for a metal:

$$\sigma_n = q^2 D_n \left( E_F \right) D_{3D} \left( E_F \right)$$