ECE 656: Electronic Transport in Semiconductors Fall 2017

Landauer Approach

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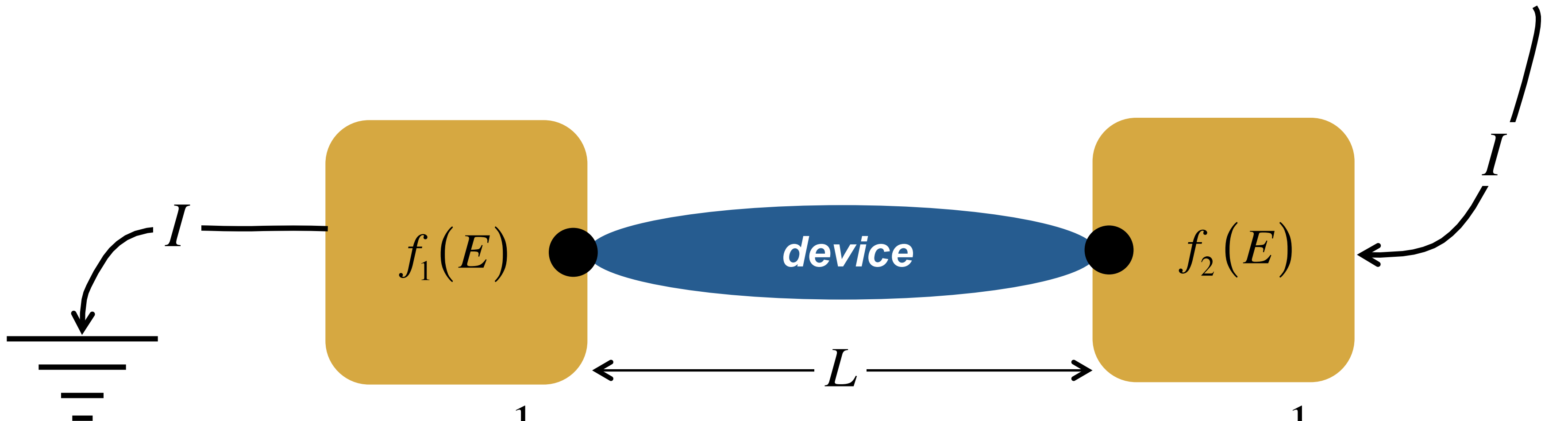
Current in a nano device



$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}} \qquad f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) (f_1 - f_2) dE$$

transmission, modes (channels), differences in Fermi functions

Outline

1) BTE to McKelvey-Shockley

2) McKelvey-Shockley to Landauer

Begin with the BTE

$$\frac{\partial f}{\partial t} + \vec{\upsilon} \bullet \nabla_r f + \vec{F}_e \bullet \nabla_p f = \left.\frac{\partial f}{\partial t}\right)_{coll}$$

Assume steady-state, no applied force (E- or B-field), spatial variation in 1D only, and the RTA:

$$\upsilon_x \frac{df\left(x, k_x\right)}{dx} = -\frac{f - f_S}{\tau_m} = -\frac{f_A\left(x, k_x\right)}{\tau_m}$$

Re-write the RTA

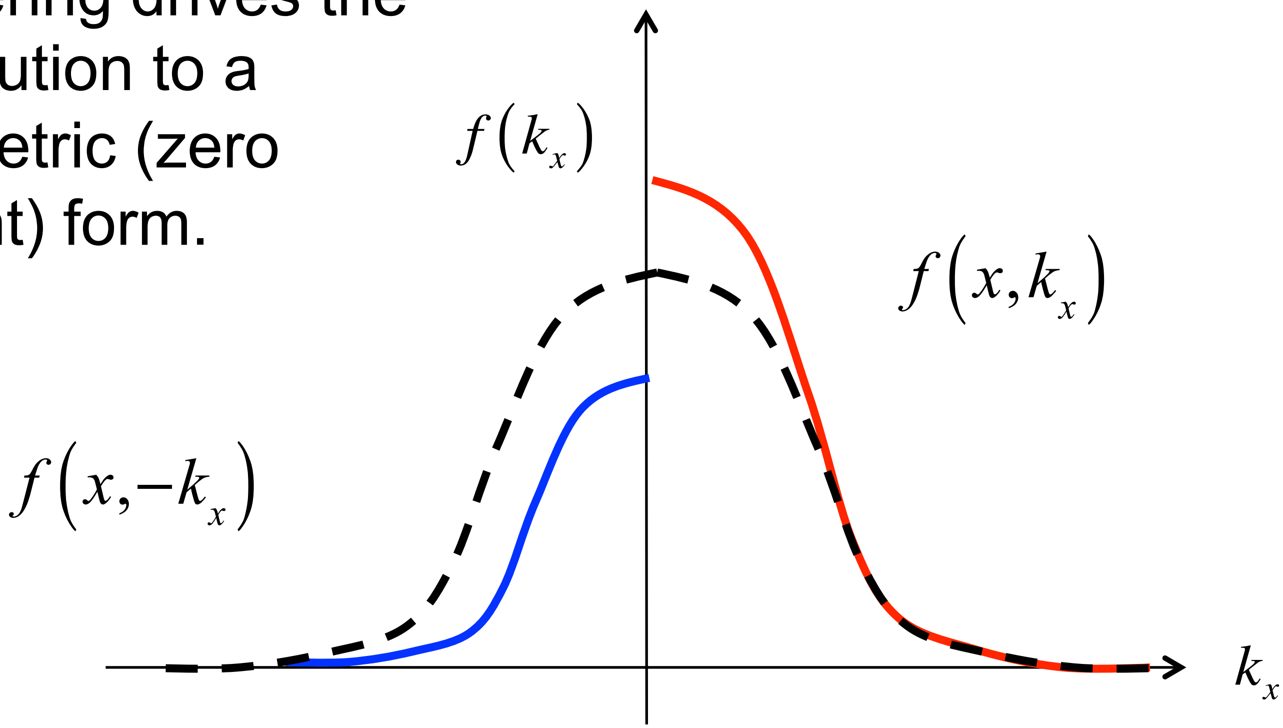
$$\upsilon_x \frac{df(x,k_x)}{dx} = -\frac{f_A(x,k_x)}{\tau_m}$$

$$f_A(x,k_x) \equiv \frac{f(x,k_x) - f(x,-k_x)}{2}$$

$$\upsilon_x \frac{df(x,k_x)}{dx} = -\frac{f(x,k_x) - f(x,-k_x)}{2\tau_m}$$

Physical picture of the RTA

Scattering drives the distribution to a symmetric (zero current) form.



$$\upsilon_x \frac{df(x,k_x)}{dx} = -\frac{f(x,k_x) - f(x,-k_x)}{2\tau_m}$$

Derivation of the "flux equations"

$$\upsilon_x \frac{df(x,k_x)}{dx} = -\frac{f(x,k_x) - f(x,-k_x)}{2\tau_m}$$

Multiply through by $2\tau_m \upsilon_x$

$$2\tau_m \upsilon_x^2 \frac{df(x,k_x)}{dx} = -\upsilon_x f(x,k_x) + \upsilon_x f(x,-k_x)$$

Next, sum over the $+k_x$ states assuming 3D k-states.

Derivation of the "flux equations"

$$2\tau_m \upsilon_x^2 \frac{df(x,k_x)}{dx} = -\upsilon_x f(x,k_x) + \upsilon_x f(x,-k_x)$$

$$\frac{\partial}{\partial x}\left[\frac{1}{\Omega}\sum_{\vec{k},k_x>0} 2\upsilon_x^2 \tau_m f\right] = -\frac{1}{\Omega}\sum_{\vec{k},k_x>0} \upsilon_x f(k_x) + \frac{1}{\Omega}\sum_{\vec{k},k_x>0} \upsilon_x f(-k_x)$$

$$F^+(x) = \frac{1}{\Omega}\sum_{\vec{k},k_x>0} \upsilon_x(k_x) f(x,k_x)$$

?

Derivation of the "flux equations"

$$\frac{\partial}{\partial x}\left[\frac{1}{\Omega}\sum_{\vec{k},k_x>0} 2\upsilon_x^2 \tau_m f\right] = -F^+ + \frac{1}{\Omega}\sum_{\vec{k},k_x>0} \upsilon_x f(-k_x) \qquad F^+(x) = \frac{1}{\Omega}\sum_{\vec{k},k_x>0} \upsilon_x(k_x) f(x,k_x)$$

$$F^-(x) = \frac{1}{\Omega}\sum_{\vec{k},k_x>0} \upsilon_x(k_x) f(x,-k_x) = \frac{1}{\Omega}\sum_{\vec{k},k_x<0} \upsilon_x(k_x) f(x,k_x)$$

$$\frac{\partial}{\partial x}\left[\frac{1}{\Omega}\sum_{\vec{k},k_x>0} 2\upsilon_x^2 \tau_m f\right] = -F^+(x) + F^-(x)$$

Derivation of the "flux equations"

$$\frac{\partial}{\partial x}\left[\frac{1}{\Omega}\sum_{\vec{k},k_x>0} 2\upsilon_x^2 \tau_m f\right] = -F^+(x) + F^-(x)$$

Now work on the LHS:

$$\frac{\partial}{\partial x}\left(\left\{\frac{\sum_{\vec{k},k_x>0} 2\upsilon_x^2 \tau_m f}{\sum_{\vec{k},k_x>0}\left(\upsilon_x f\right)}\right\}\sum_{\vec{k},k_x>0}\left(\upsilon_x f\right)\right) = -F^+(x) + F^-(x)$$

$$\frac{\partial}{\partial x}\left(\lambda F^+\right) = -F^+(x) + F^-(x)$$

$$\lambda \equiv \frac{\sum_{\vec{k},k_x>0} 2\upsilon_x^2 \tau_m f}{\sum_{\vec{k},k_x>0}\left(\upsilon_x f\right)}$$

"MFP for backscattering"

$$\frac{\partial}{\partial x}\left(\lambda F^{+}\right) = -F^{+}(x) + F^{-}(x) \qquad \lambda \equiv \left\{ \frac{\sum_{\vec{k},k_x>0} 2\upsilon_x^2 \tau_m f}{\sum_{\vec{k},k_x>0} \left(\upsilon_x f\right)} \right\}$$

$$\lambda \equiv \frac{\sum_{\vec{k},k_x>0} 2\upsilon_x^2 \tau_m f}{\sum_{\vec{k},k_x>0} \upsilon_x f} = \frac{\sum_{\vec{k}} 2\upsilon_x^2 \tau_m f}{\sum_{\vec{k}} \left|\upsilon_x\right| f} = 2\frac{\left\langle \upsilon_x^2 \tau_m \right\rangle}{\left\langle \left|\upsilon_x\right| \right\rangle}$$

Conventional MFP: $\Lambda \equiv \upsilon \tau_m$

The **MFP for backscattering** will play a central role in our theory:

Flux equations

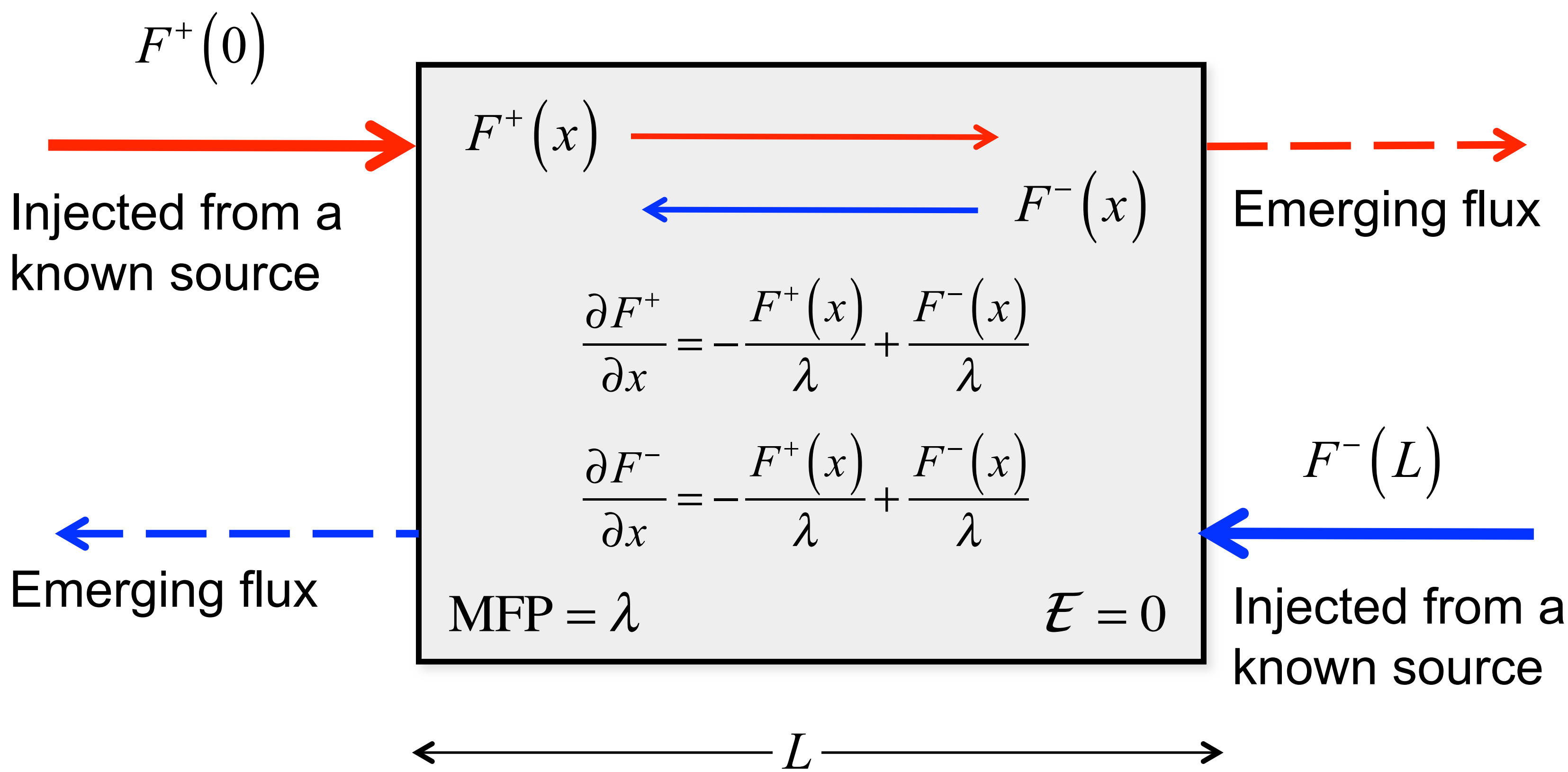
$$\frac{\partial}{\partial x}\left(\lambda F^{+}\right) = -F^{+}(x) + F^{-}(x) \qquad \lambda = 2\frac{\left\langle \upsilon_x^2 \tau_m \right\rangle}{\left\langle \left|\upsilon_x\right| \right\rangle}$$

Assume that the MFP for backscattering is spatially uniform.

$$\frac{\partial F^{+}}{\partial x} = -\frac{F^{+}(x)}{\lambda} + \frac{F^{-}(x)}{\lambda}$$

$$\frac{\partial F^{-}}{\partial x} = -\frac{F^{+}(x)}{\lambda} + \frac{F^{-}(x)}{\lambda}$$

Flux equations + boundary conditions



Flux equations

The flux equations have a long history:

J. P. McKelvey, R. L. Longini, and T. P. Brody, "Alternative approach to the solution of added carrier transport problems in semiconductors," *Phys. Rev*, **123**, pp. 51–57, 1961.

W. Shockley, "Diffusion and drift of minority carriers in semiconductors for comparable capture and scattering mean free paths," *Phys. Rev*., **125**, no. 5, 1570–1576, 1962.

Outline

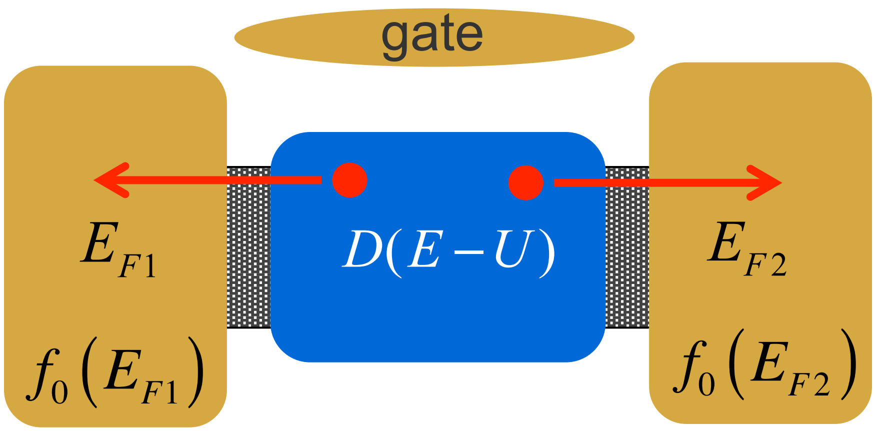
1) BTE to McKelvey-Shockley

2) McKelvey-Shockley to Landauer

Assumptions

2) **Contacts** are large with strong inelastic scattering, always near equilibrium

3) *U* is the self-consistent (mean-field) potential. (For "strongly correlated" transport, see Datta.)

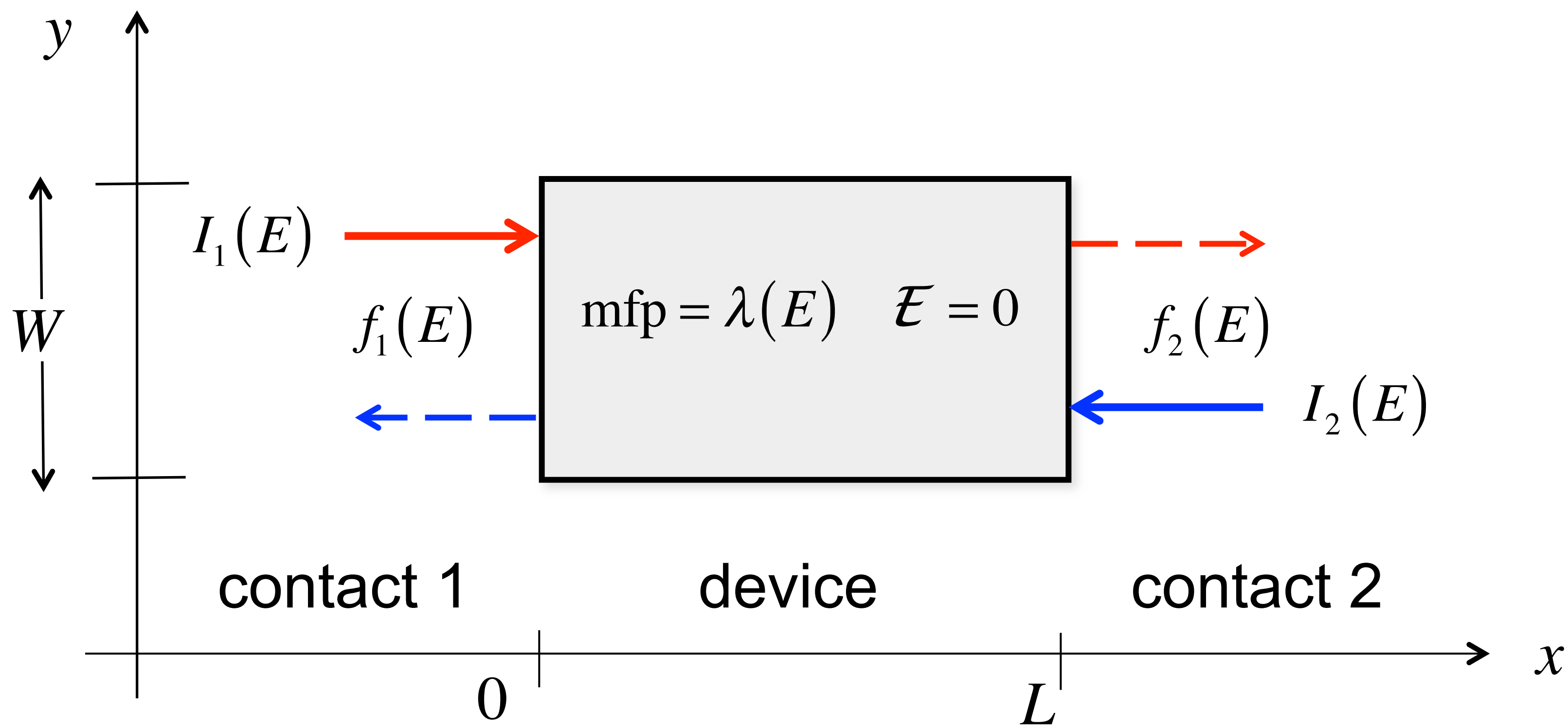


5) ***Reflectionless*** ("absorbing") contacts.

1) Device is described by an *E*(*k*). For the more general case, see Datta.

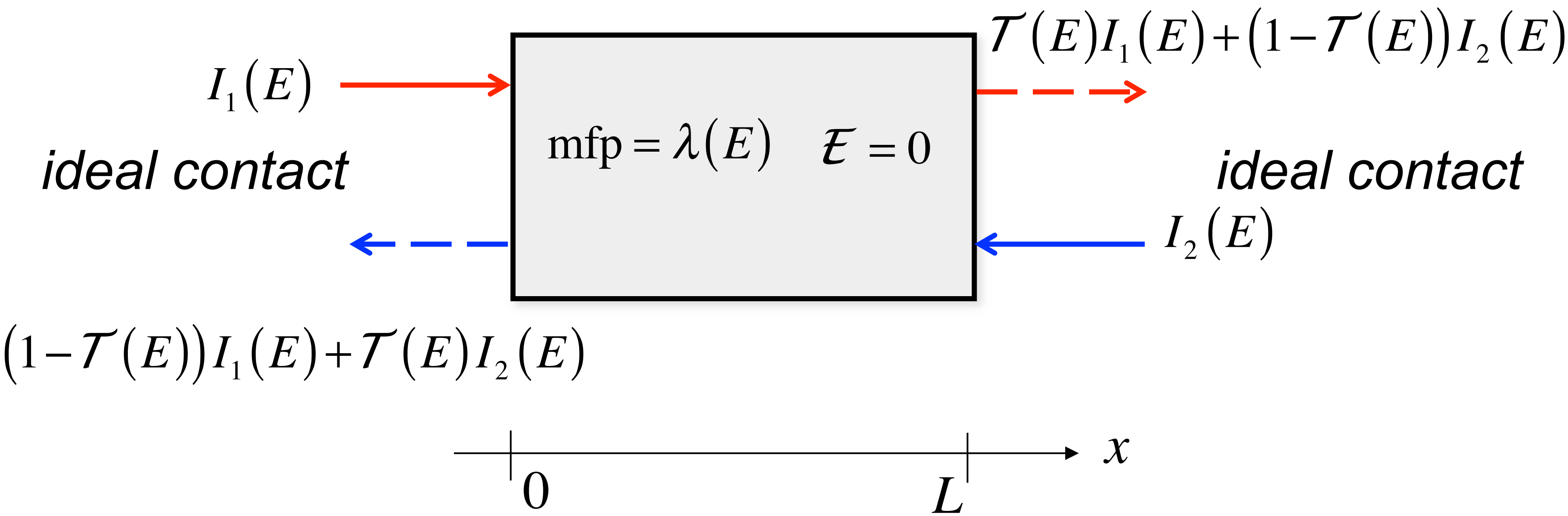
4) All **inelastic scattering** takes place in the contacts. Electrons flow from left to right (or right to left) in **independent** energy channels.

Derivation (2D)



1) Ideal ("Landauer") contacts
2) Independent energy channels

Derivation (i)



$$I(E) = \mathcal{T}(E)\left(I_1(E) - I_2(E)\right)$$

$$I = \int I(E)\,dE$$

$$\mathcal{T}_{12}(E) = \mathcal{T}_{21}(E)$$

$$S(\vec{p},\vec{p}\,') = S(\vec{p}\,',\vec{p})$$

elastic scattering

Derivation (ii)

$$I(E) = \mathcal{T}(E)\left(I_1(E) - I_2(E)\right)$$

$$n_S^+(E)dE = \frac{D_{2D}(E)}{2} f_1(E)dE \text{ cm}^{-2}$$

$$I_1(E)dE = qW\left\langle \upsilon_x^+(E)\right\rangle n_S^+(E)dE$$

$$I_1(E) = qW\left\langle \upsilon_x^+(E)\right\rangle \frac{D_{2D}(E)}{2} f_1(E)$$

$$I_2(E) = qW\left\langle \upsilon_x^+(E)\right\rangle \frac{D_{2D}(E)}{2} f_2(E)$$

Derivation (iii)

$$I(E) = \mathcal{T}(E)\left(I_1(E) - I_2(E)\right) \qquad I_{1,2}(E) = qW\left\langle \upsilon_x^+(E)\right\rangle \frac{D_{2D}(E)}{2} f_{1,2}(E)$$

$$I(E) = \mathcal{T}(E)\left\{qW\left\langle \upsilon_x^+(E)\right\rangle \frac{D_{2D}(E)}{2}\right\}\left(f_1(E) - f_2(E)\right)$$

Define: $M(E) \equiv W\frac{h}{4}\left\langle \upsilon_x^+(E)\right\rangle D_{2D}(E)$

Units: $M(E) = (\mathrm{m})(\mathrm{J\text{-}s})\left(\frac{\mathrm{m}}{\mathrm{s}}\right)\left(\frac{\#}{\mathrm{J\text{-}m}^2}\right) = \#$

M(*E*) is the number of "channels" at energy, *E*.

Derivation (iv)

$$I(E) = \mathcal{T}(E)\left(I_1(E) - I_2(E)\right)$$

$$I(E) = \mathcal{T}(E)\left\{qW\left\langle \upsilon_x^+(E)\right\rangle \frac{D_{2D}(E)}{2}\right\}\left(f_1(E) - f_2(E)\right)$$

$$M(E) \equiv W\frac{h}{4}\left\langle \upsilon_x^+(E)\right\rangle D_{2D}(E)$$

$$I(E) = \frac{2q}{h}\mathcal{T}(E)M(E)\left(f_1(E) - f_2(E)\right)$$

$$I = \int I(E)dE = \frac{2q}{h}\int \mathcal{T}(E)M(E)\left(f_1(E) - f_2(E)\right)dE$$

Discussion

$$I = \frac{2q}{h}\int \mathcal{T}(E) M(E)\left(f_1(E) - f_2(E)\right) dE$$

Fundamental constants

Transmission:

$\mathcal{T}(E) = 1$ ballistic

$\mathcal{T}(E) << 1$ diffusive

Channels or Modes

Ideal "Landauer" contacts

Next lecture

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)\big(f_1(E) - f_2(E)\big)dE$$

Modes (channels)

Questions?

$$\upsilon_x \frac{df\left(x,k_x\right)}{dx} = -\frac{f\left(x,k_x\right) - f\left(x,-k_x\right)}{2\tau_m}$$

$$\frac{\partial F^+}{\partial x} = -\frac{F^+\left(x\right)}{\lambda} + \frac{F^-\left(x\right)}{\lambda}$$

$$\frac{\partial F^-}{\partial x} = -\frac{F^+\left(x\right)}{\lambda} + \frac{F^-\left(x\right)}{\lambda}$$

$$I = \frac{2q}{h}\int \mathcal{T}\left(E\right)M\left(E\right)\left(f_1\left(E\right) - f_2\left(E\right)\right)dE$$