ECE 656: Electronic Transport in Semiconductors Fall 2017

Channels

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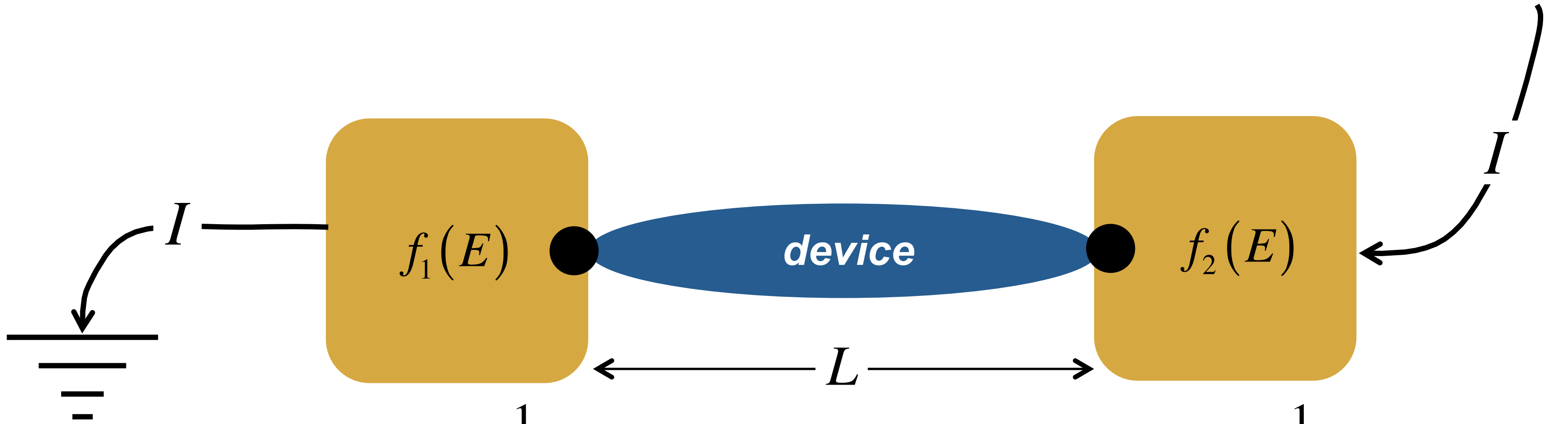
Landauer Approach



$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}} \qquad f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) (f_1 - f_2)\, dE$$

transmission, modes (channels), differences in Fermi functions

Definition of channels (2D)

Define: $M(E) \equiv W \frac{h}{4} \langle \upsilon_x^+(E) \rangle D_{2D}(E)$

Units: $M(E) = (\mathrm{m})(\mathrm{J\text{-}s})\left(\frac{\mathrm{m}}{\mathrm{s}}\right)\left(\frac{\#}{\mathrm{J\text{-}m^2}}\right) = \#$

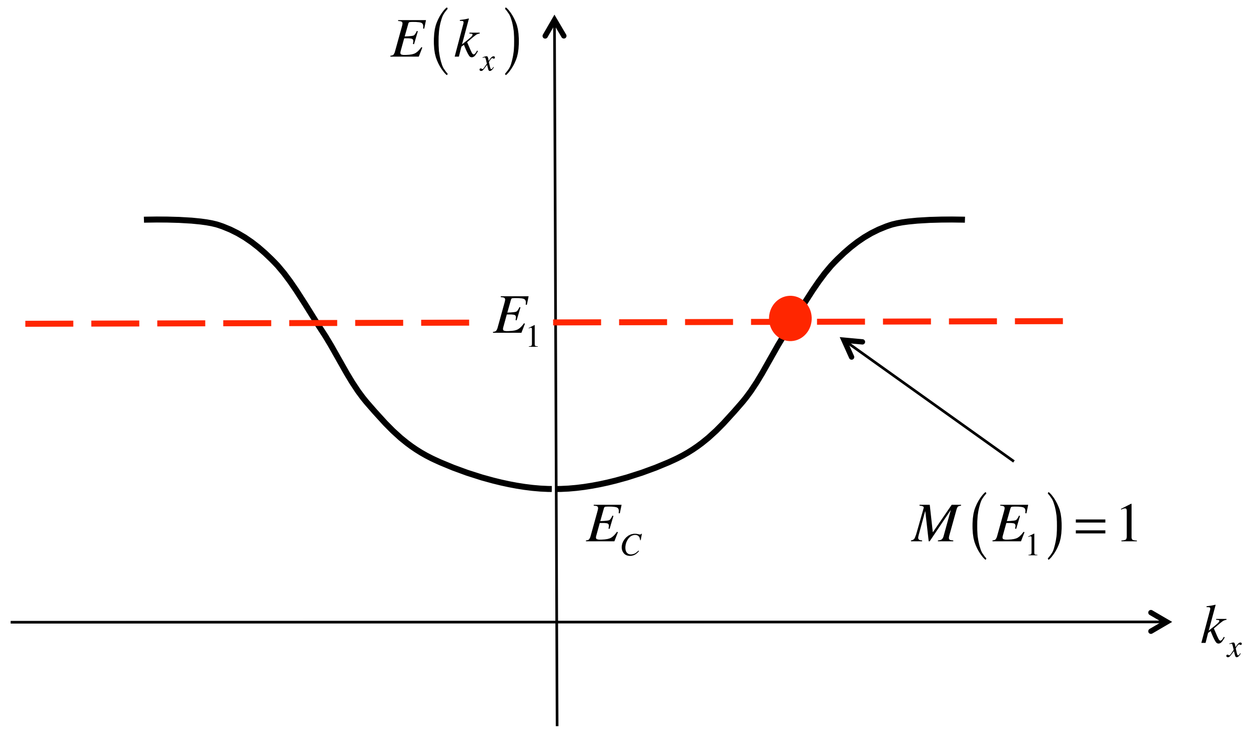
A channel (or mode) is a state with a velocity in the direction of transport.

Average velocity in the +x (transport) direction =

$$\langle \upsilon_x^+(E) \rangle = \langle |\upsilon_x(E)| \rangle$$

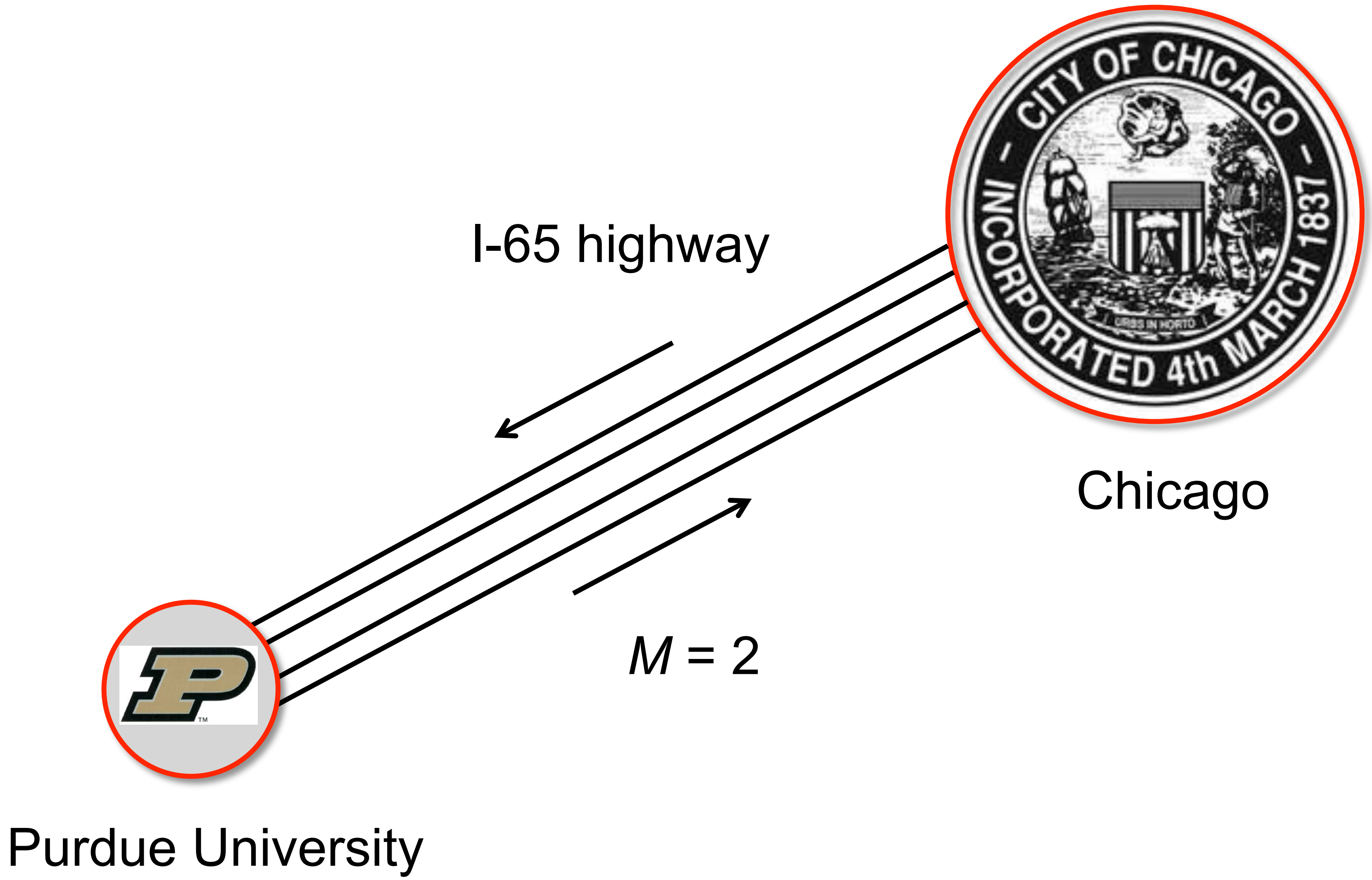
Channels (modes) in 1D

$$M_{1D}(E) = \frac{h}{4}\left\langle \upsilon_x^+(E) \right\rangle D_{1D}(E)$$



(Easily generalized to arbitrary bandstructures in 2D and 3D.)

Channels



Example: $M(E)$ in 2D

$$M(E) \equiv W \frac{h}{4} \left\langle \upsilon_x^+(E) \right\rangle D_{2D}(E)$$

(number)

$$M_{2D}(E) \equiv M(E)/W = \frac{h}{4} \left\langle \upsilon_x^+(E) \right\rangle D_{2D}(E)$$

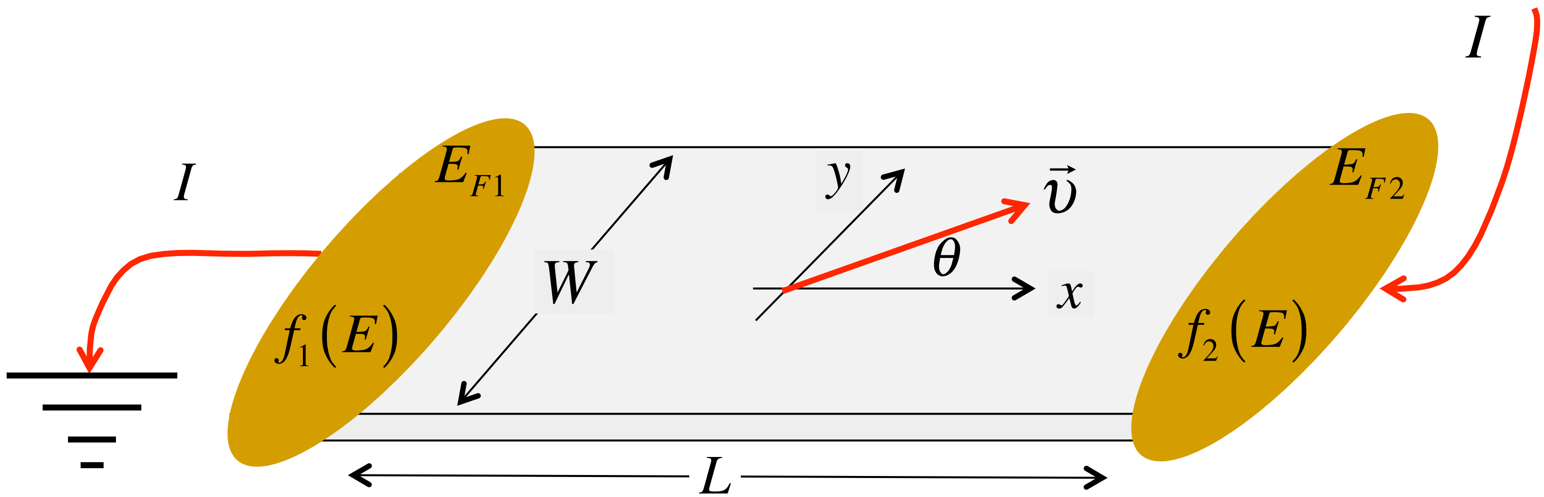
(number per unit width)

$$D_{2D}(E) = g_v \left(\frac{m^*}{\pi \hbar^2} \right) \ \mathrm{J^{-1} m^{-2}}$$

(parabolic energy bands)

$$\left\langle \upsilon_x^+(E) \right\rangle = ?$$

Average velocity in the direction of transport



$$E = E_C + \frac{1}{2} m^* \upsilon^2$$ (parabolic energy bands)

$$\upsilon(E) = \sqrt{\frac{2(E - E_C)}{m^*}}$$

$$\left\langle \upsilon_x^+(E) \right\rangle = \frac{\int_{-\pi/2}^{+\pi/2} \upsilon(E)\cos(\theta)\, d\theta}{\pi} = \frac{2}{\pi}\upsilon(E)$$ ✓

$$\upsilon_x(E) = \upsilon(E)\cos(\theta)$$

$M(E)$ in 2D

$$M(E) \equiv W \frac{h}{4} \langle \upsilon_x^+(E) \rangle D_{2D}(E) \qquad M_{2D}(E) \equiv M(E)/W = \frac{h}{4} \langle \upsilon_x^+(E) \rangle D_{2D}(E)$$

$$D_{2D}(E) = g_v \left(\frac{m^*}{\pi \hbar^2} \right) \ \mathrm{J^{-1} m^{-2}}$$

$$\langle \upsilon_x^+(E) \rangle = \frac{2}{\pi} \upsilon(E)$$

$$\upsilon(E) = \sqrt{\frac{2(E - E_C)}{m^*}}$$

$$M_{2D}(E) = g_v \frac{\sqrt{2m^*(E - E_C)}}{\pi \hbar} \ \mathrm{m^{-1}}$$

How do we interpret this result?

Interpretation

$$M(E) = W\frac{h}{4}\langle \upsilon_x^+ \rangle D_{2D}(E)$$

$$M(E) = g_V W\frac{\sqrt{2m^*(E-E_C)}}{\pi\hbar}$$

$$k = \frac{2\pi}{\lambda_B} = \frac{\sqrt{2m^*(E-E_C)}}{\hbar}$$

λ_B is the de Broglie wavelength of electrons.

$$M(E) = g_V\frac{Wk}{\pi} = g_V\frac{W}{\lambda_B(E)/2}$$

$$\langle \upsilon_x^+ \rangle = \frac{2}{\pi}\upsilon$$

$$\upsilon = \sqrt{\frac{2(E-E_C)}{m^*}}$$

$$D_{2D}(E) = g_V\frac{m^*}{\pi\hbar^2}$$

$$E - E_C + \frac{\hbar^2k^2}{2m^*}$$

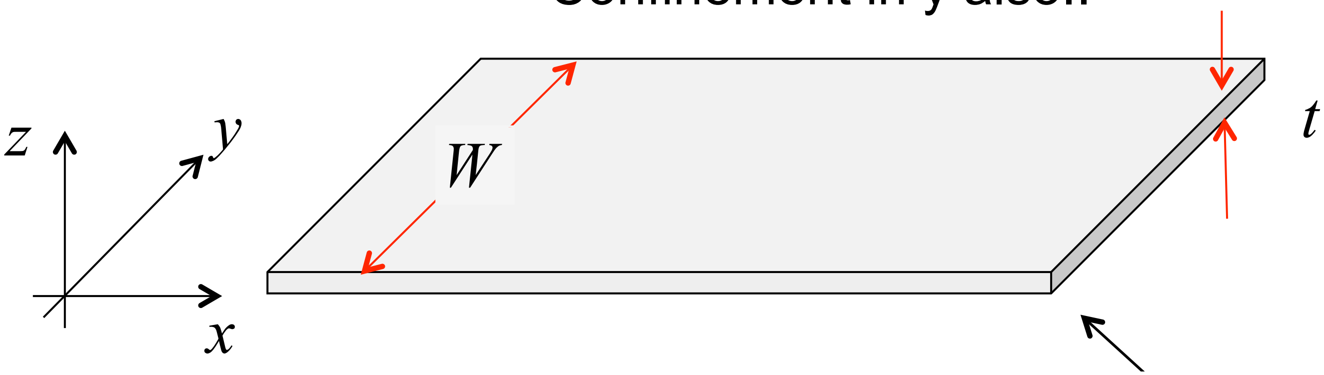
Interpretation

$$M\left(E\right) = g_V \frac{W\,k}{\pi} = g_V \frac{W}{\lambda_B\left(E\right)/2}$$

At any energy, *E*, the electron has a de Broglie wavelength. The number of half wavelengths that fit into the width of the 2D conductor is the number of channels *M*(*E*).

Waveguide modes

Confinement in y also..



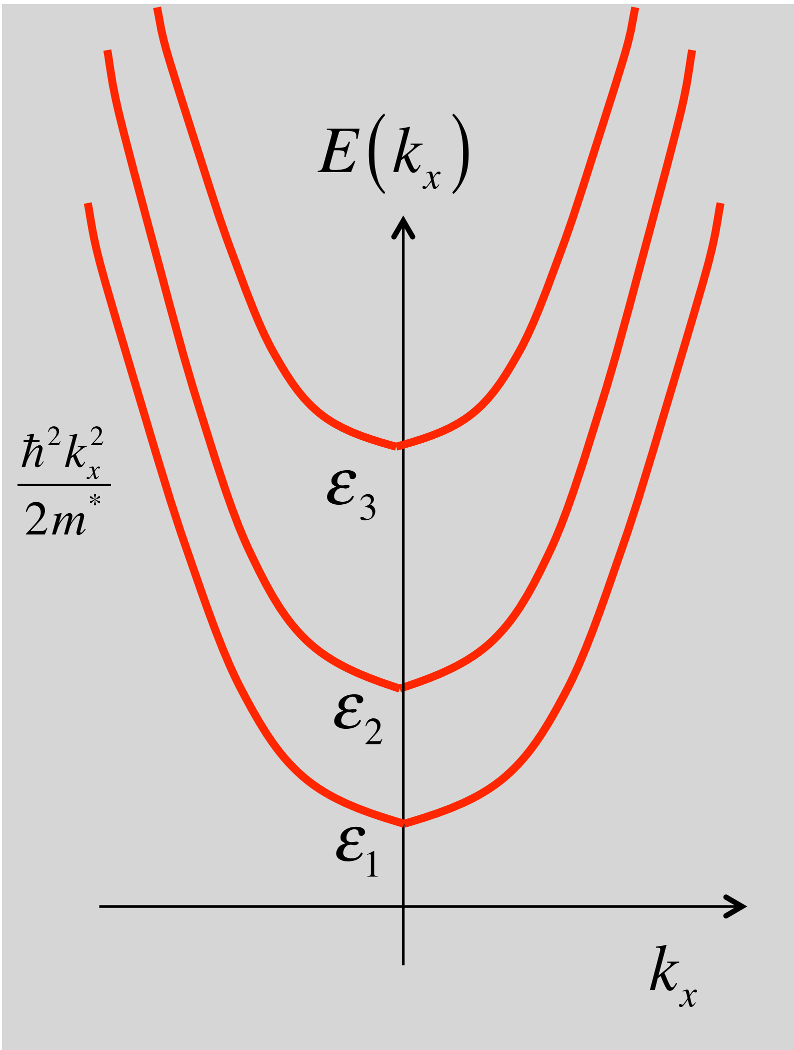
Confinement in z produces widely spaced subbands. We will consider only the first.

$$\psi(x,y) \propto e^{ik_x x} \sin k_y y$$

$$k_y = j\pi / W \quad j = 1, 2, \ldots$$

$$E = \varepsilon_{z1} + \frac{\hbar^2 k_y^2}{2m^*} + \frac{\hbar^2 k_x^2}{2m^*} = \varepsilon_{z1} + \frac{\hbar^2 j^2 \pi^2}{2m^* W^2} + \frac{\hbar^2 k_x^2}{2m^*}$$

Recall: Subbands



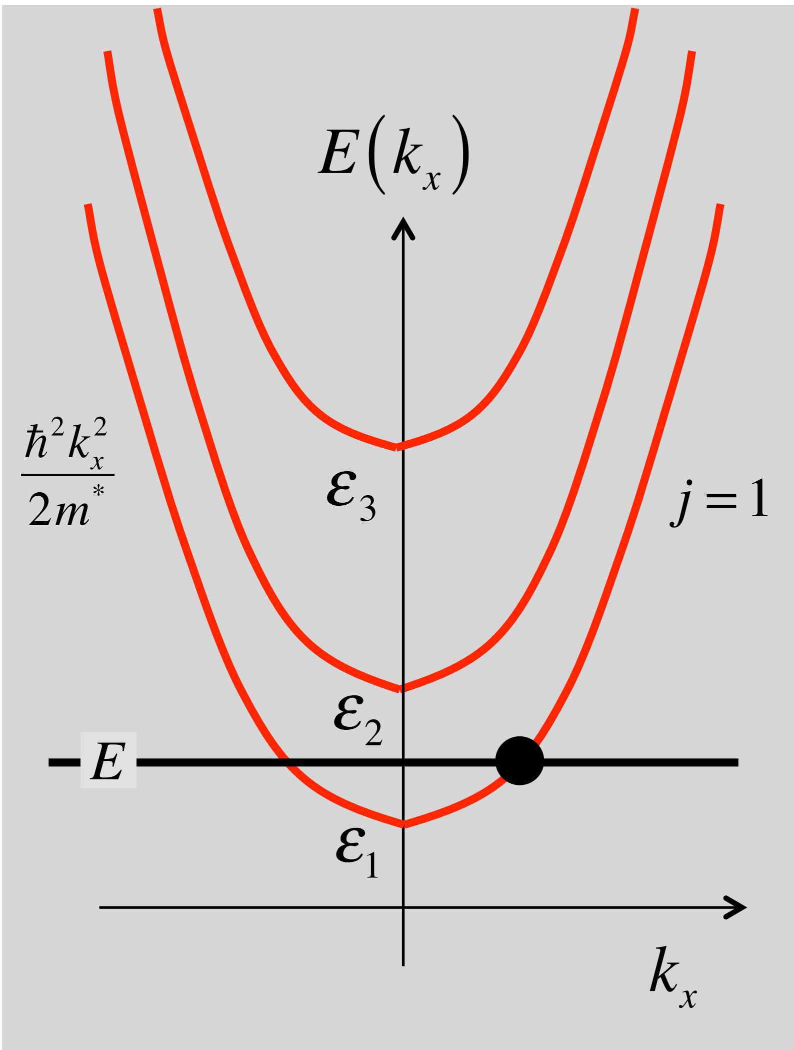
$$\varepsilon_j = \frac{\hbar^2 \pi^2 j^2}{2m^* W^2} \qquad j = 1,2,3\ldots$$

$$k_y = k_{yj} = \frac{\pi}{W} j \qquad j = 1,2,3\ldots$$

$$k_{yj} = \frac{2\pi}{\lambda_{Bj}} \qquad j = \frac{W}{\lambda_{Bj}/2}$$

For subband *j*, *j* half wavelengths fit into the width of the conductor, *W*.

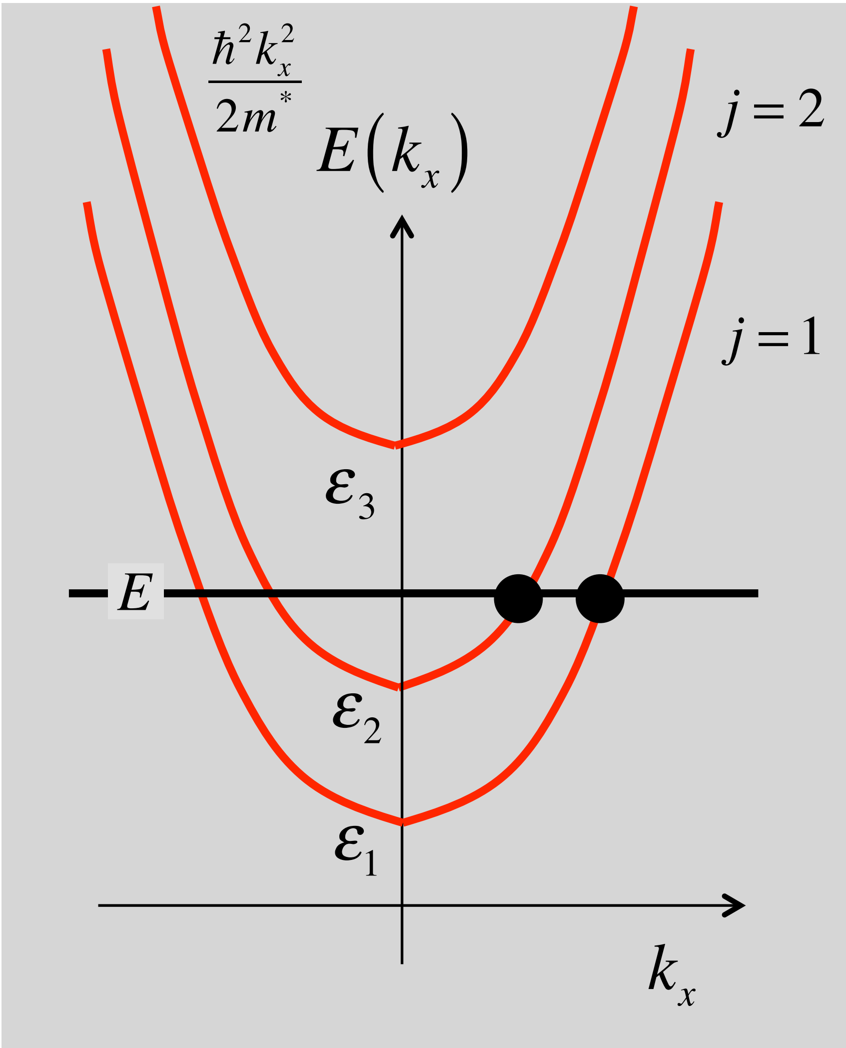
One mode



$$j = \frac{W}{\lambda_{Bj}/2} = 1 \qquad M(E) = 1$$

j = 1 corresponds to subband 1. At this energy, only subband 1 has channel.

Two modes

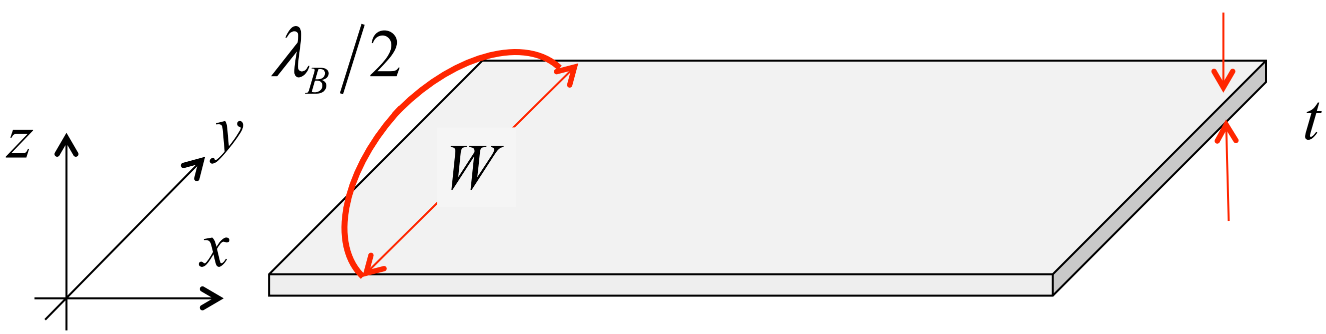


$$j = \frac{W}{\lambda_j/2} = 2 \qquad M(E) = 2$$

j = 2 corresponds to subband 2. At this energy, subbands 1 2 both provide a channel.

Waveguide modes

$$M(E) = \frac{W}{\lambda_B(E)/2}$$ When $$M(E) = 1 \rightarrow \lambda_B(E)/2 = W$$



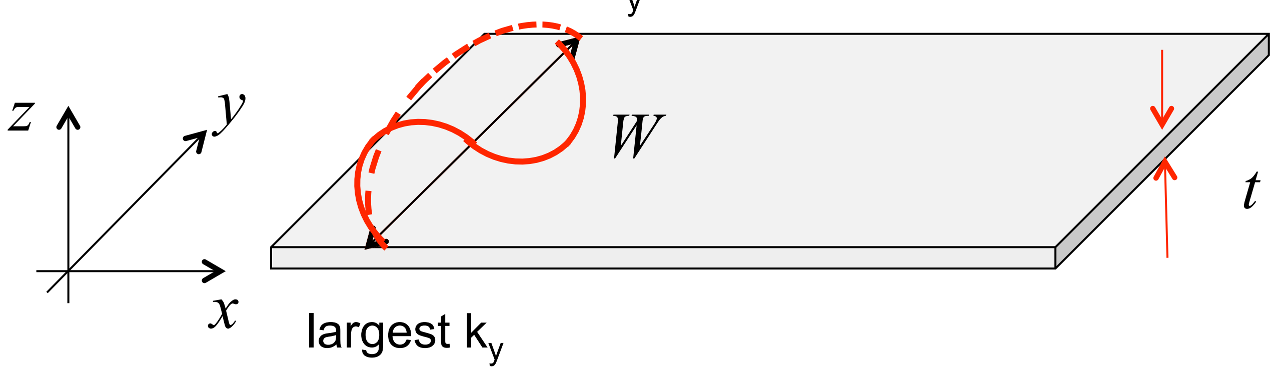
smallest k_y

$$M(E) = j = 1$$

Waveguide modes: M = 2

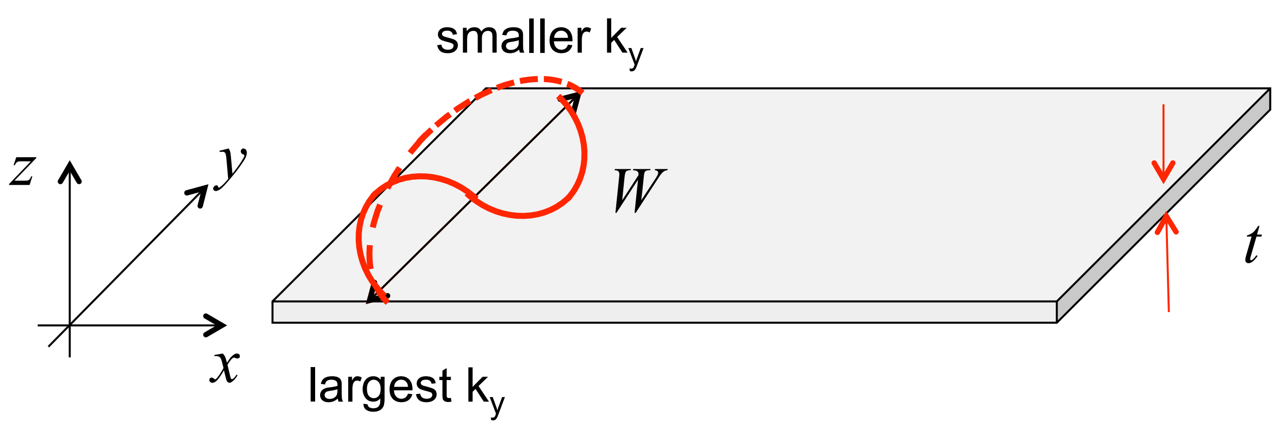
$$M(E) = \frac{W}{\lambda_B(E)/2}$$

When $M(E) = 2 \rightarrow \lambda_B(E) = W$



$$M(E) = j = 2$$

Waveguide modes



***M* = # of electron half wavelengths that fit into *W*.**

Most convenient when the number of modes is small and countable.

Modes from the DOS

1D (single subband):

$$M(E) \equiv \frac{h}{4}\left\langle \upsilon_x^+(E)\right\rangle D_{1D}(E) \qquad \left\langle \upsilon_x^+(E)\right\rangle = \upsilon(E)$$

2D:

$$M(E) = W\frac{h}{4}\left\langle \upsilon_x^+(E)\right\rangle D_{2D}(E) \qquad \left\langle \upsilon_x^+(E)\right\rangle = \frac{2}{\pi}\upsilon(E)$$

3D:

$$M(E) \equiv A\frac{h}{4}\left\langle \upsilon_x^+(E)\right\rangle D_{3D}(E) \qquad \left\langle \upsilon_x^+(E)\right\rangle = \frac{\upsilon(E)}{2}$$

Most convenient when the number of modes is large and NOT countable.

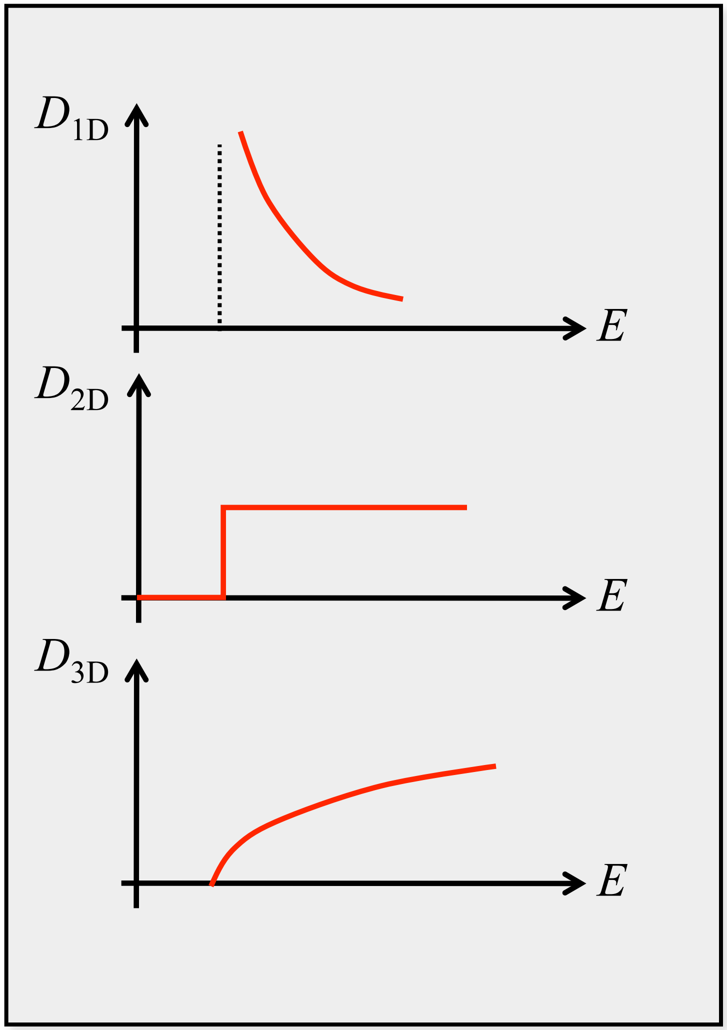
DOS (for parabolic energy bands)

$$D(E) = L\,D_{1D}(E) = \frac{L}{\pi\hbar}\sqrt{\frac{2m^*}{(E-\varepsilon_1)}}\,\Theta(E-\varepsilon_1)$$

$$D(E) = AD_{2D}(E) = A\frac{m^*}{\pi\hbar^2}\Theta(E-\varepsilon_1)$$

$$D(E) = \Omega\,D_{3D}(E) = \Omega\frac{m^*\sqrt{2m^*(E-E_C)}}{\pi^2\hbar^3}\Theta(E-E_C)$$

$$\left(E(k) = E_C + \hbar^2k^2/2m^*\right)$$



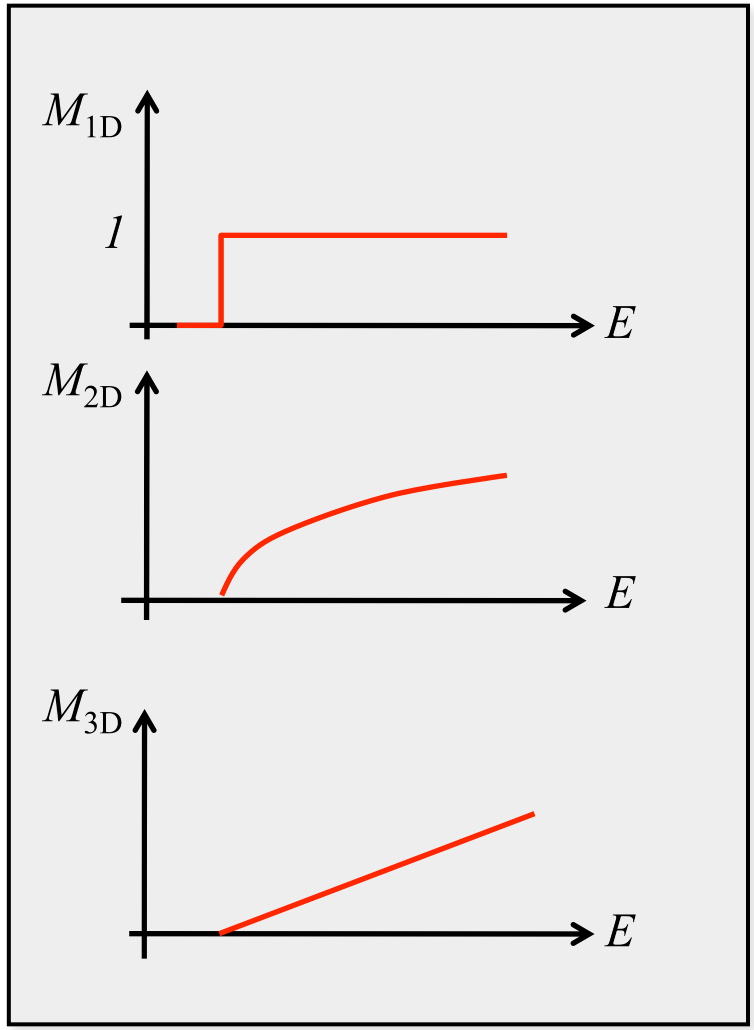
Number of modes (for parabolic energy bands)

$$M(E) = M_{1D}(E) = 1$$

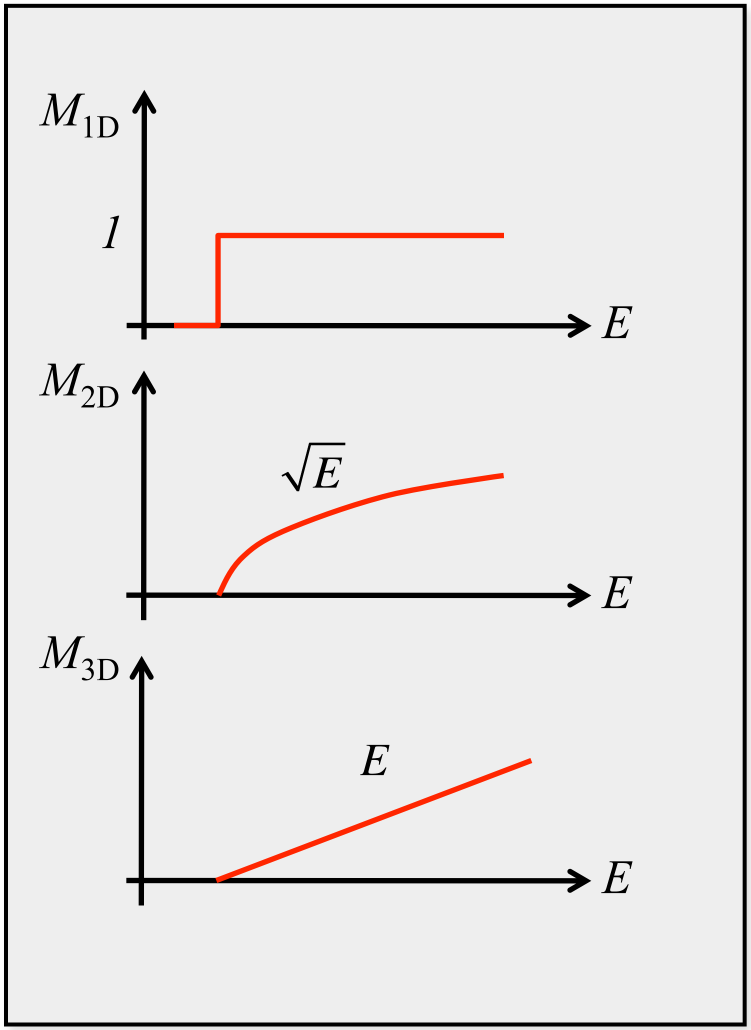
$$M(E) = W\, M_{2D}(E) = W \frac{\sqrt{2m^*(E - E_C)}}{\pi\hbar}$$

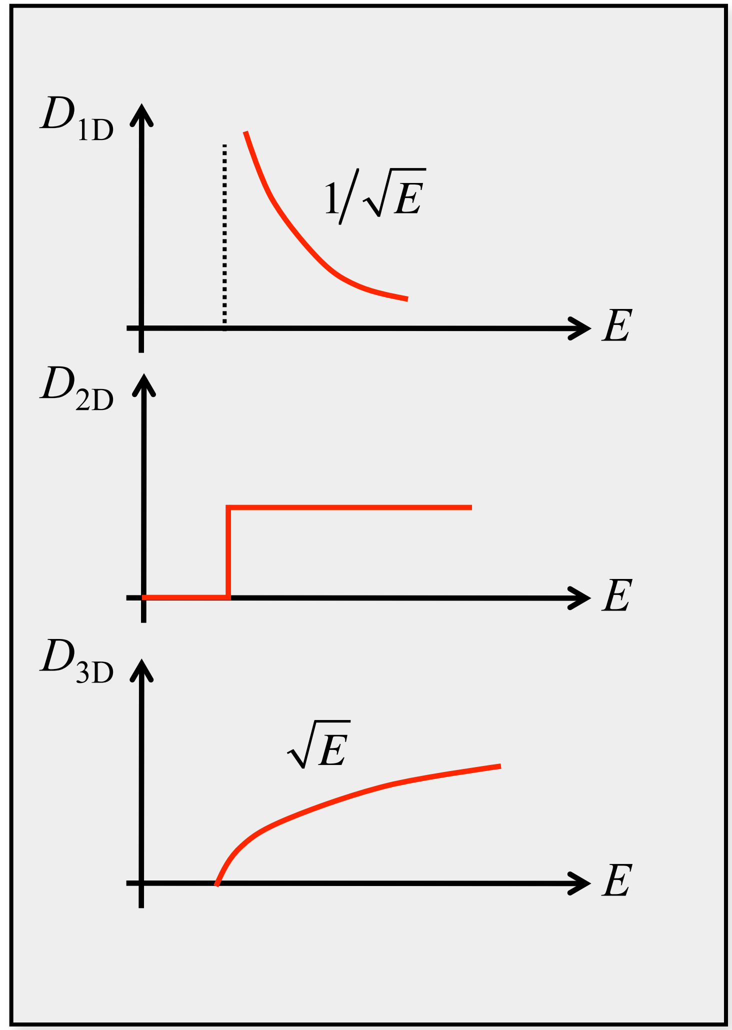
$$M(E) = A\, M_{3D}(E) = A \frac{m^*}{2\pi\hbar^2}(E - E_C)$$

$$\left(E(k) = E_C + \hbar^2 k^2 / 2m^*\right)$$

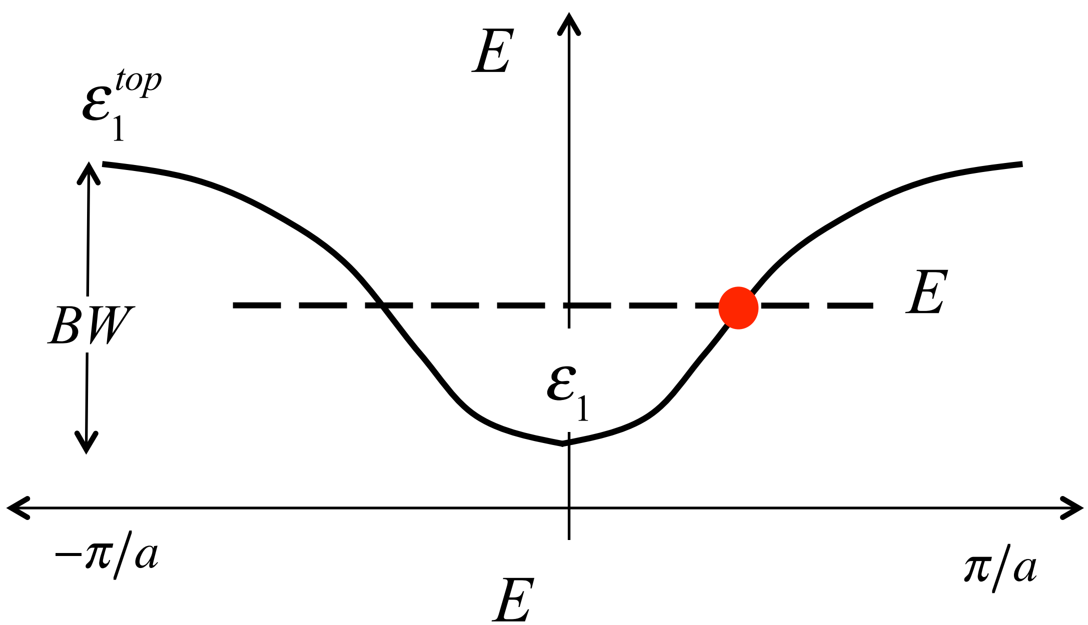


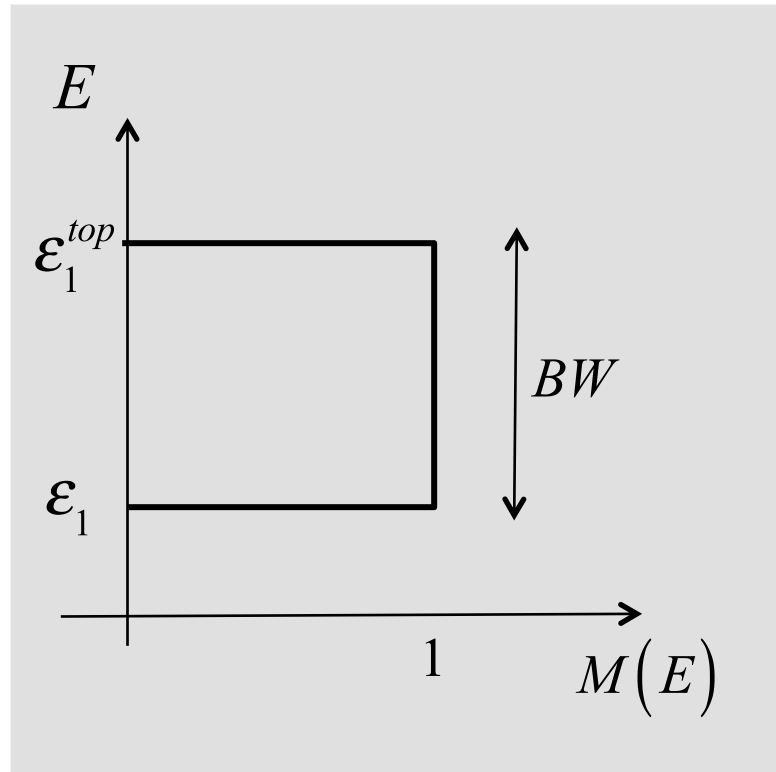
M(E) vs. DOS (parabolic bands)





M(E) in 1D: physical picture





M(E) for graphene

$$M(E) = W\frac{h}{4}\langle \upsilon_x^+ \rangle D(E)$$

for graphene:

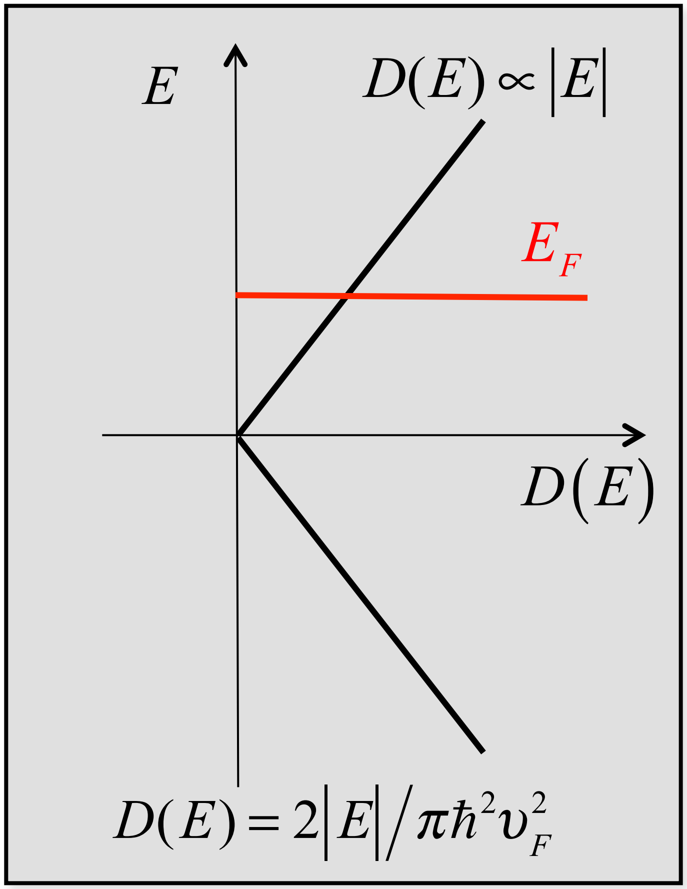
$$D(E) = 2|E|/\pi\hbar^2\upsilon_F^2$$

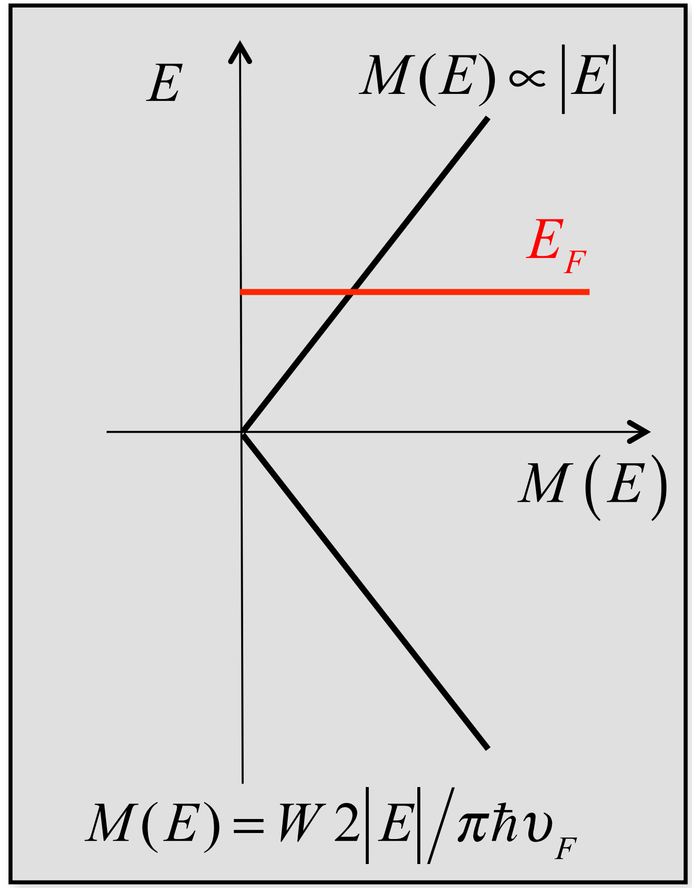
$$\upsilon(E) = \upsilon_F$$

$$\langle \upsilon_x^+ \rangle = \frac{2}{\pi}\upsilon_F$$

$$M(E) = W\frac{2|E|}{\pi\hbar\upsilon_F}$$

M(E) vs. D(E) for graphene





M(E) for graphene: two definitions

$$M(E) = W\frac{h}{4}\langle \upsilon_x^+ \rangle D(E)$$

DOS definition

$$M(E) = g_V \frac{W}{(\lambda_B/2)}$$

Waveguide mode definition

Both give the same answer:

$$M(E) = W\frac{2|E|}{\pi\hbar\upsilon_F}$$

Aside: DOS and conductivity effective masses

$$n = N_C \mathcal{F}_{1/2}(\eta_F)$$

$$\eta_F = \frac{E_F - E_C}{k_b T_L}$$

$$N_C = \frac{\left(2 m_D^* k_B T\right)^{3/2}}{4\pi^{3/2}\hbar^3}$$

$$m_D^* = 6^{2/3}\left(m_l^* m_t^*\right)^{1/3}$$

"density of states effective mass" (silicon)

$$\mu = \frac{q\tau}{m_c^*}$$

$$\frac{1}{m_C^*} = \frac{1}{3}\left[\frac{1}{m_l^*} + \frac{2}{m_t^*}\right]$$

"conductivity effective mass" (silicon)

DOM effective mass

To see how to compute M(E) for an arbitrary E(k), and how to compute the "distribution of modes" effective mass, see:

Changwook Jeong, Raseong Kim, Mathieu Luisier, Supriyo Datta, and Mark Lundstrom, "On Landauer vs. Boltzmann and Full Band vs. Effective Mass Evaluation of Thermoelectric Transport Coefficients," *J. Appl. Phys*., **107**, 023707, 2010.

Summary

$$I = \frac{2q}{h}\int T(E)M(E)(f_1 - f_2)dE$$

modes

$$M_{1D}(E) = \frac{h}{4}\langle v_x^+ \rangle D_{1D}(E)$$

$$M_{2D}(E)W = W\frac{h}{4}\langle v_x^+ \rangle D_{2D}(E)$$

$$M_{3D}(E)A = A\frac{h}{4}\langle v_x^+ \rangle D_{3D}(E)$$

Summary

1) The density of states is used to compute carrier densities.

2) The number of modes (channels) is used to compute the current.

3) The number of modes at energy, *E*, is proportional to the average velocity (in the direction of transport) at energy, *E* times the DOS(*E*).

4) *M*(*E*) depends on the bandstructure ***and*** on dimensionality.

Questions?

To compute the current, simple use the correct M(E) for the **material** and for the **dimension** (1D, 2D, or 3D).

$$I = \frac{2q}{h}\int \mathcal{T}\left(E\right)M\left(E\right)\left(f_1 - f_2\right)dE$$