ECE 656: Electronic Transport in Semiconductors Fall 2017

Transmission

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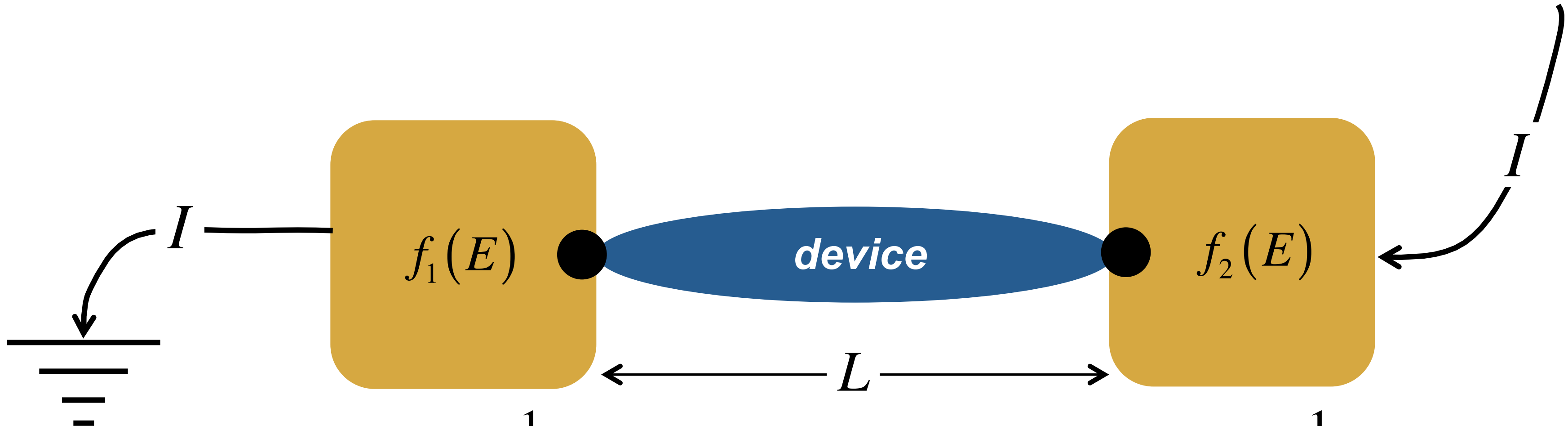
Landauer Approach



$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}}$$

$$f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) (f_1 - f_2) dE$$

transmission, modes (channels), differences in Fermi functions

Outline

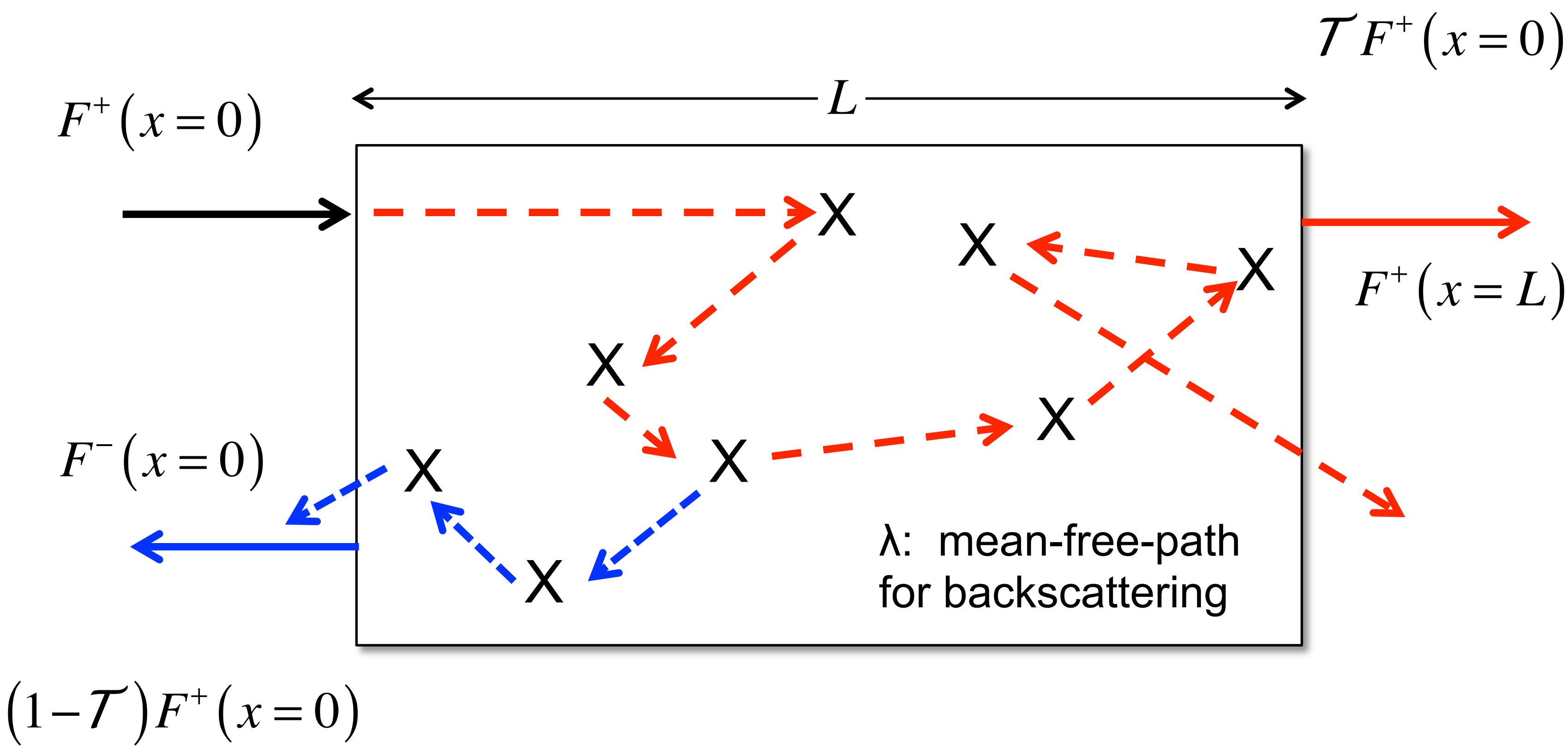
1) Diffusive vs. ballistic transport

2) Transmission and MFP

3) Transmission and diffusion coefficient from the BTE

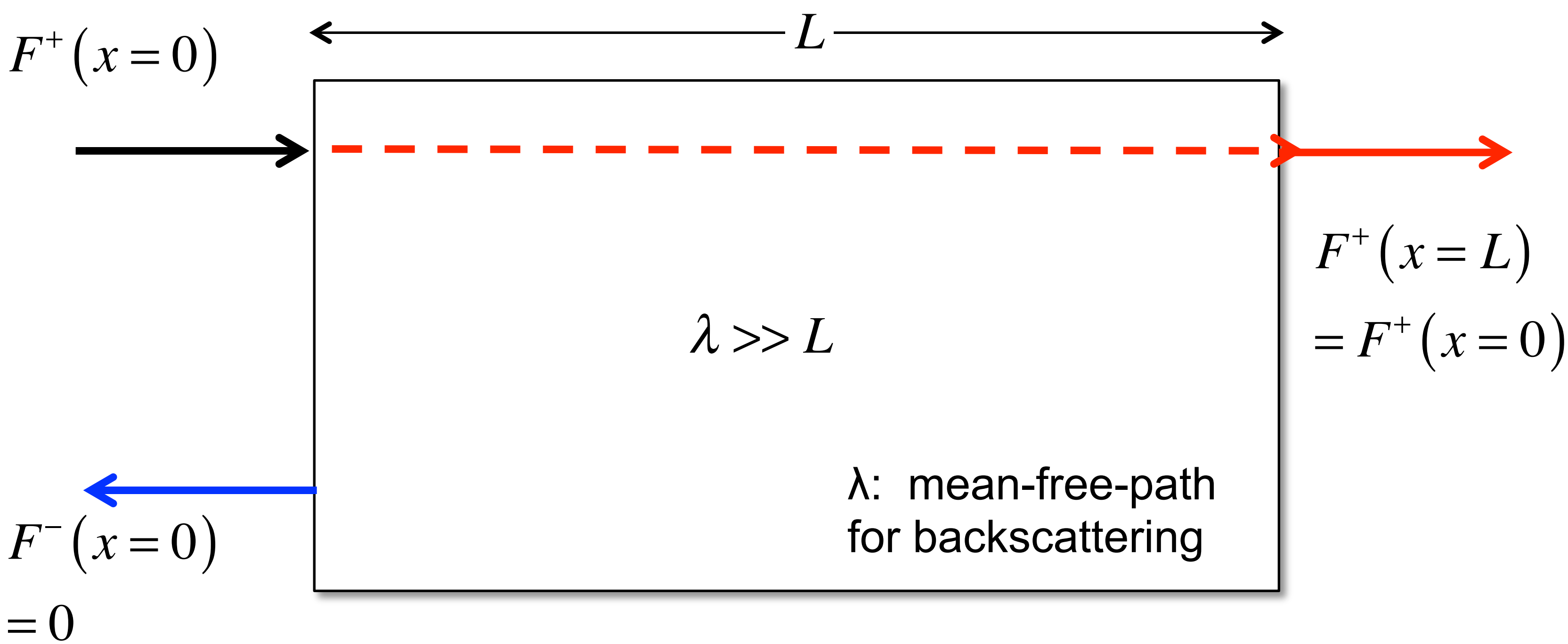
4) The MFP for backscattering

Transmission



"diffusive transport" $\mathcal{T} << 1$

Transmission (ballistic)



ballistic transport: $\mathcal{T} = 1$

Diffusive vs. ballistic transport

- Electrons undergo a **random walk** as they go from left to right contact.
- Some terminate at contact 1, and some at contact 2.
- The average distance between collisions is the "mfp for backscattering", λ
- "Diffusive" transport means $L >> \lambda$
- Ballistic transport means L << λ
- The diffusive transit time will be much longer than the ballistic transit time.

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Diffusive transmission

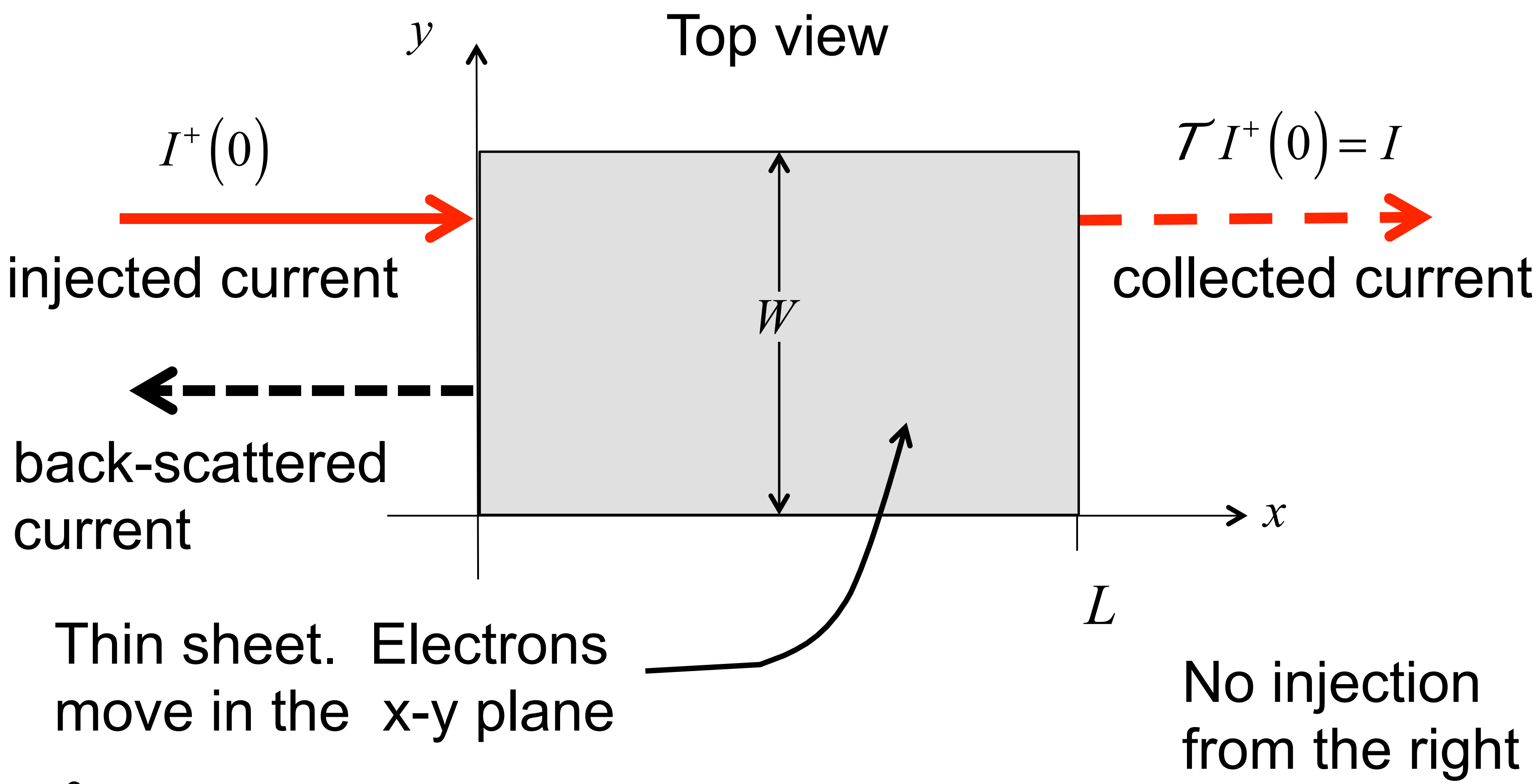
Assume a channel that is **much longer** than the mean-free-path for backscattering,

then, injected carriers diffuse to the other contact. Fick's Law of diffusion should apply.

$$L >> \lambda \quad J = -qD_n \frac{dn_S}{dx} \ \ \mathrm{A/cm} \quad \mathrm{(2D)}$$

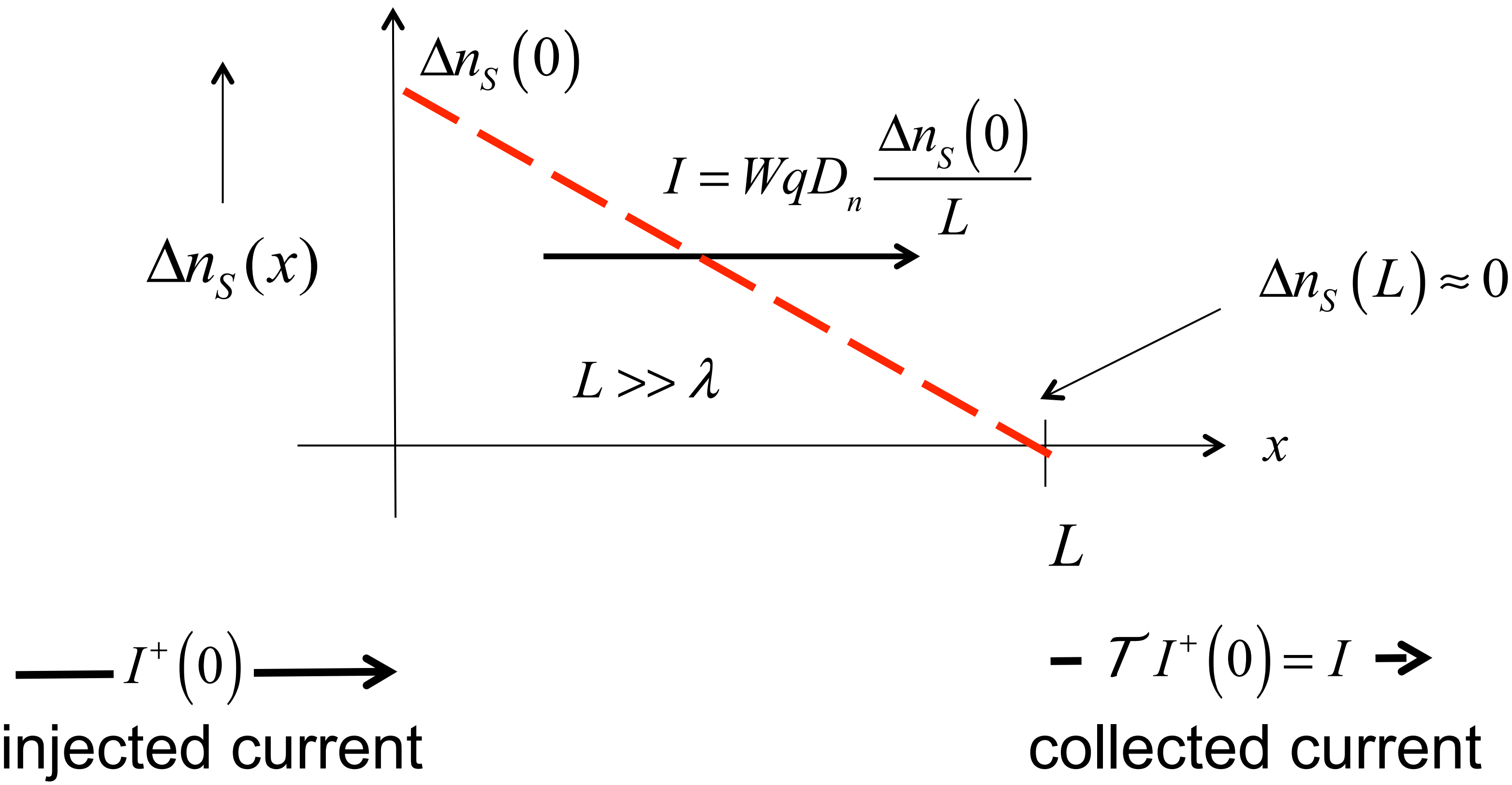
2D Diffusive transport

Inject from the left contact, and collect at the right contact.



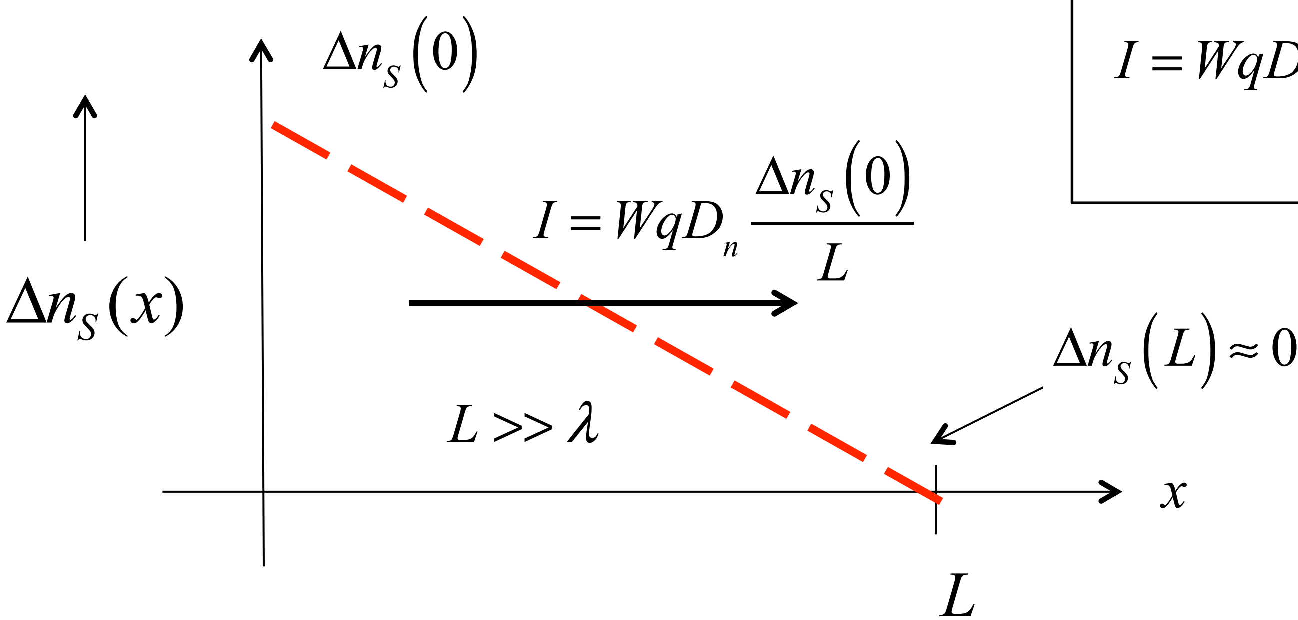
Diffusion across a thick base

Inject from the left contact, and collect at the right contact.



Transmission and current

$$I = WqD_n \frac{\Delta n_S(0)}{L} = \mathcal{T} I^+(0)$$

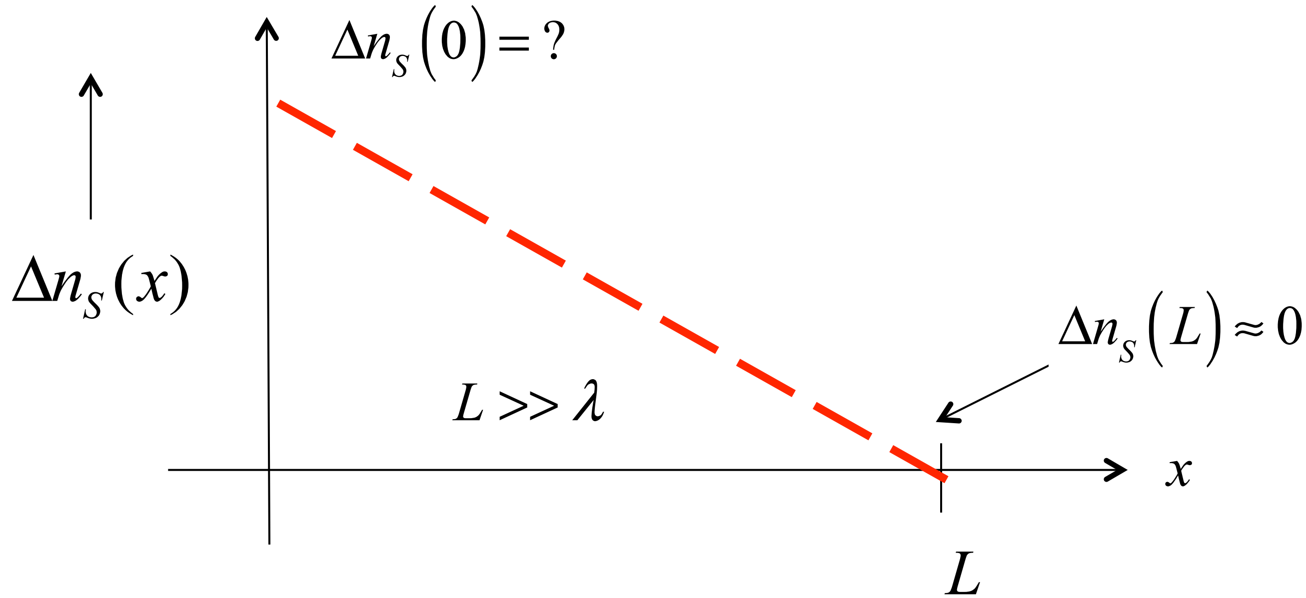


$I^+(0)$ →
injected current

$\mathcal{T} I^+(0) = I$ →
collected current

Carrier density at *x* = 0 and *x* = *L*

Inject from the left contact, and collect at the right contact.

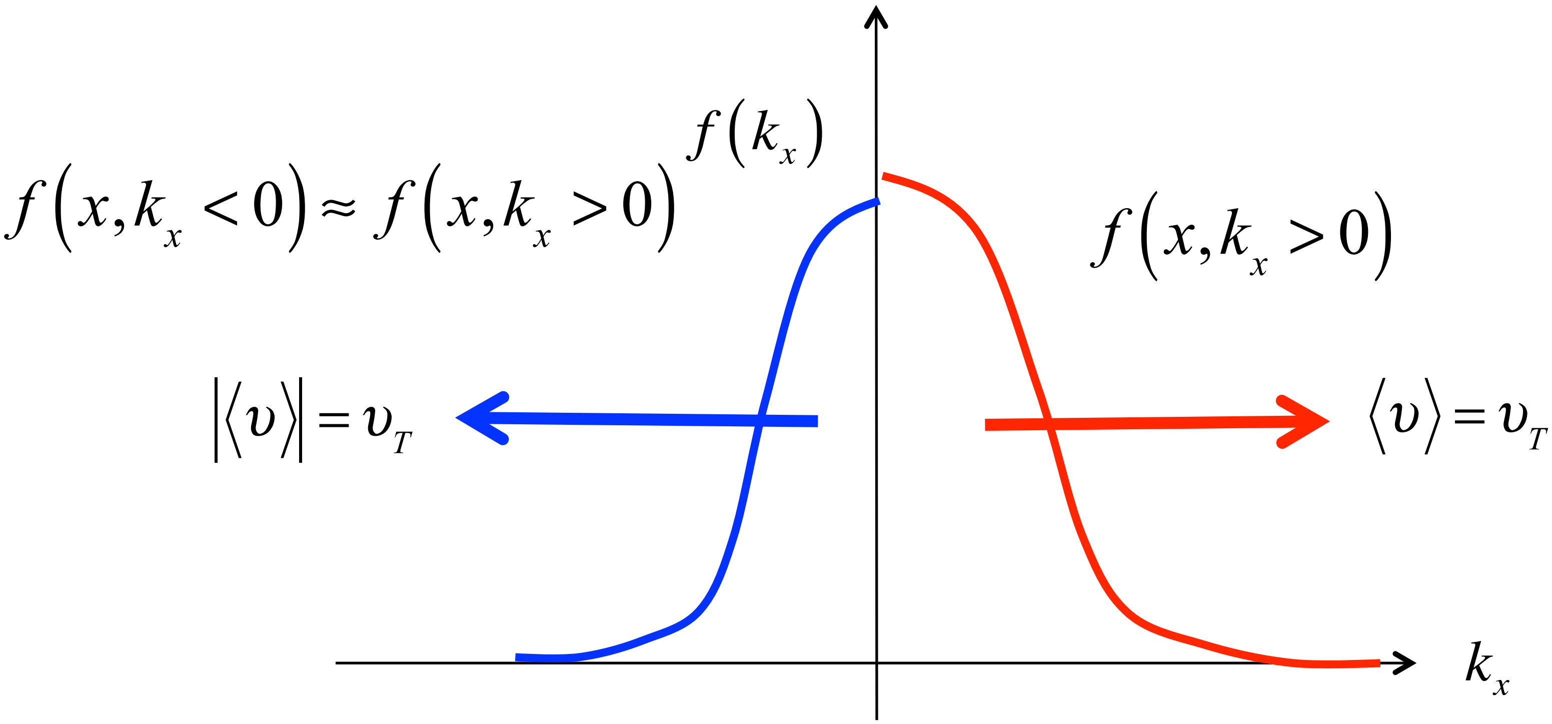


$I^+(0)$ → $I^+(0) = qWq\upsilon_T \Delta n_S^+(0)$ $\qquad$ $\Delta n_S^+(0) = \dfrac{I^+(0)}{Wq\upsilon_T} \approx \Delta n_S^-(0)$

injected current

diffusive transport

Carrier distribution



Both the +x and −x-directed fluxes move at the same velocity – the unidirectional thermal velocity.

Transmission in the diffusive limit

$$\Delta n_S^+(0) = \frac{I^+(0)}{Wq\upsilon_T} \approx \Delta n_S^-(0) \qquad \Delta n_S(0) = \Delta n_S^+(0) + \Delta n_S^-(0) \qquad \Delta n_S(0) = 2 \times \frac{I^+(0)}{Wq\upsilon_T}$$

$$I = \mathcal{T} I^+(0) = \mathcal{T}\left\{qWq\upsilon_T \Delta n_S^+(0)\right\} \qquad I = WqD_n \frac{\Delta n_S(0)}{L} \qquad D_n = \frac{\upsilon_T \lambda}{2}$$

$$\mathcal{T} = \frac{\lambda}{L} << 1$$

diffusive limit

Can prove from the flux equations.

Transmission

1) Diffusive: $L >> \lambda \quad \mathcal{T} = \frac{\lambda}{L} << 1$

2) Ballistic: $L << \lambda \quad \mathcal{T} = 1$

3) Quasi-ballistic: $L \approx \lambda \quad \mathcal{T} < 1$

$$T(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

λ is the "mean-free-path for backscattering"

This expression can be derived with relatively few assumptions.

$$\lambda(E) \neq \upsilon(E)\tau(E) = \Lambda$$

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Introduction

We have argued that there is a simple connection between the mean-free-path and transmission:

$$T(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

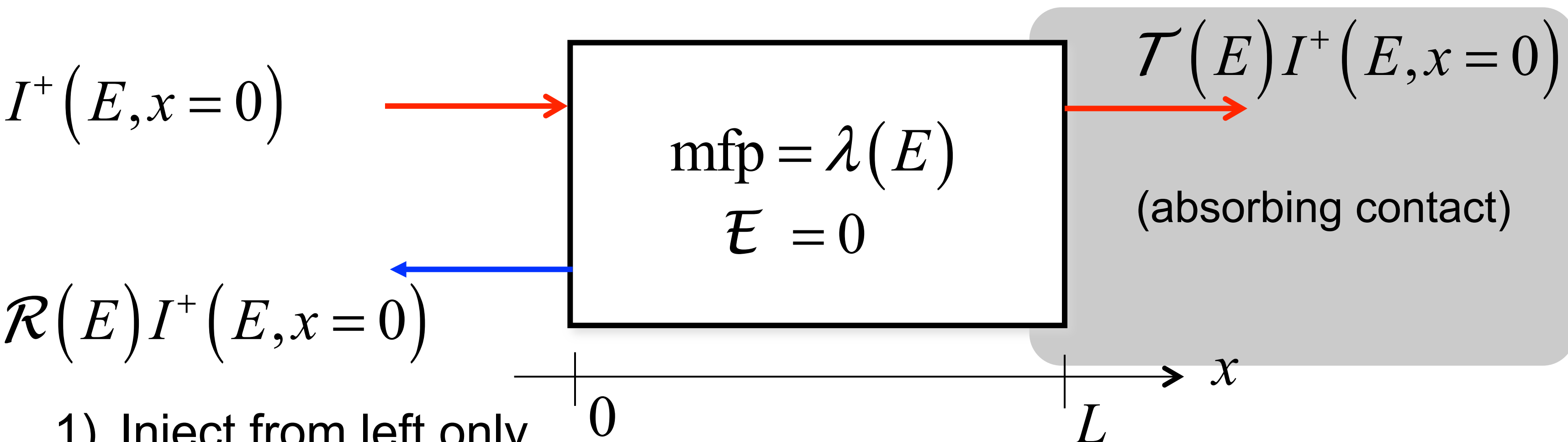
Where does this expression come from?

The mean-free-path is expected to be the "average distance" between scattering events:

$$\lambda(E) \propto \upsilon(E)\tau(E)$$

Exactly what is the relation?

Problem specification

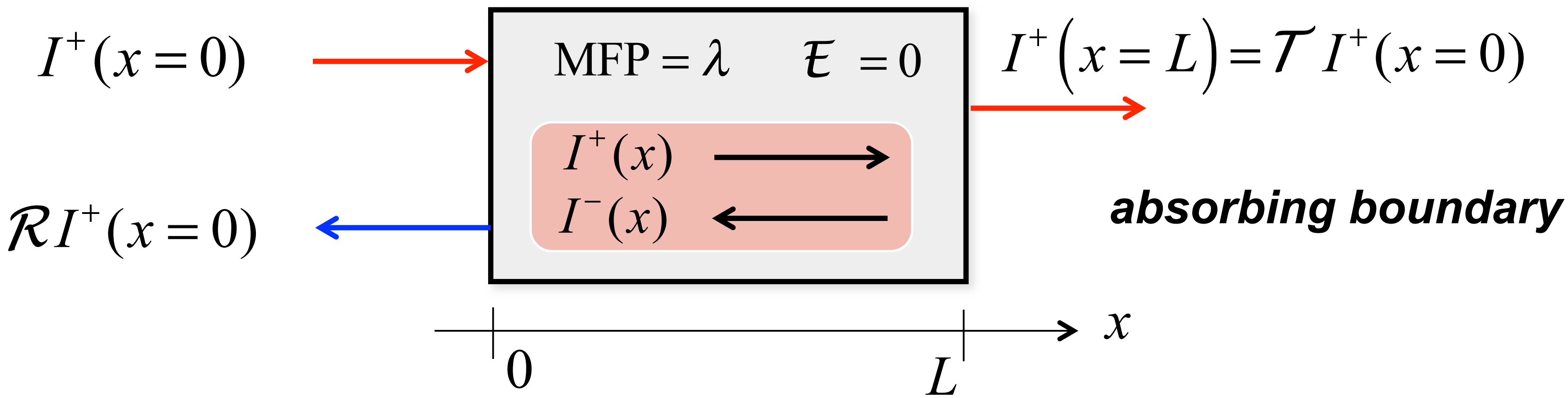


1) Inject from left only.

2) Ignore "vertical transport" (elastic scattering or near-equilibrium), so $T_{12}(E) = T_{21}(E) = T(E)$.

Then relate T to the mean-free-path for backscattering within the slab. (No assumption about whether the slab length, L, is long or short compared to the mfp, but we **do assume** that the mean-free-path is not position-dependent.)

Solving the flux equations



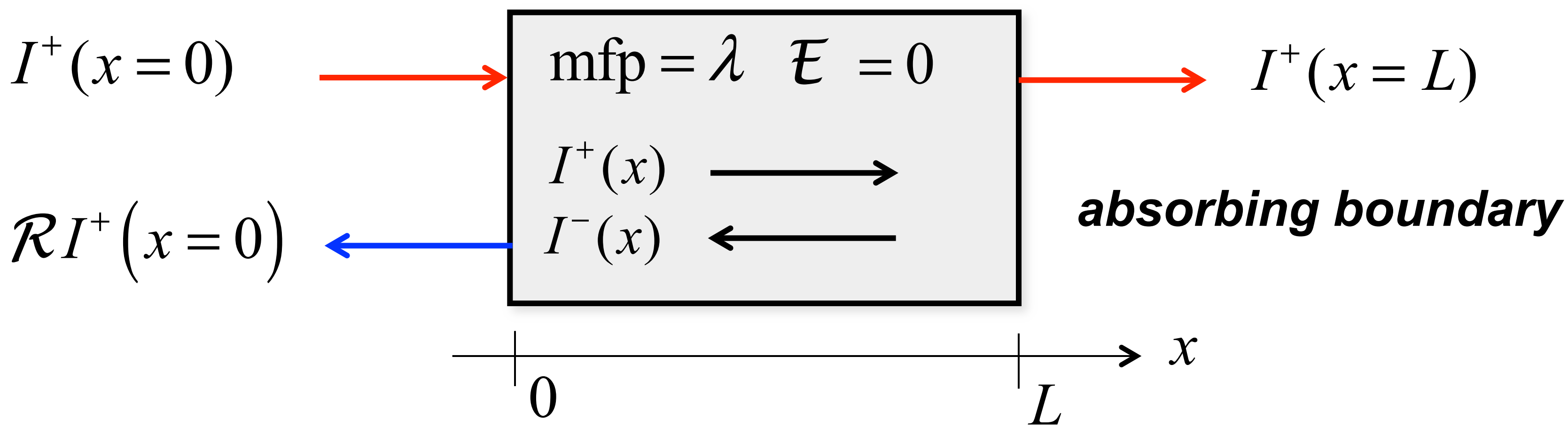
$$\frac{dI^+(x)}{dx} = -\frac{I^+(x)}{\lambda} + \frac{I^-(x)}{\lambda}$$

$$I = I^+(x) - I^-(x) \quad \text{(constant)}$$

$$I^-(x) = I^+(x) - I$$

$$\boxed{\frac{dI^+(x)}{dx} = -\frac{I}{\lambda}}$$

Position-dependent flux

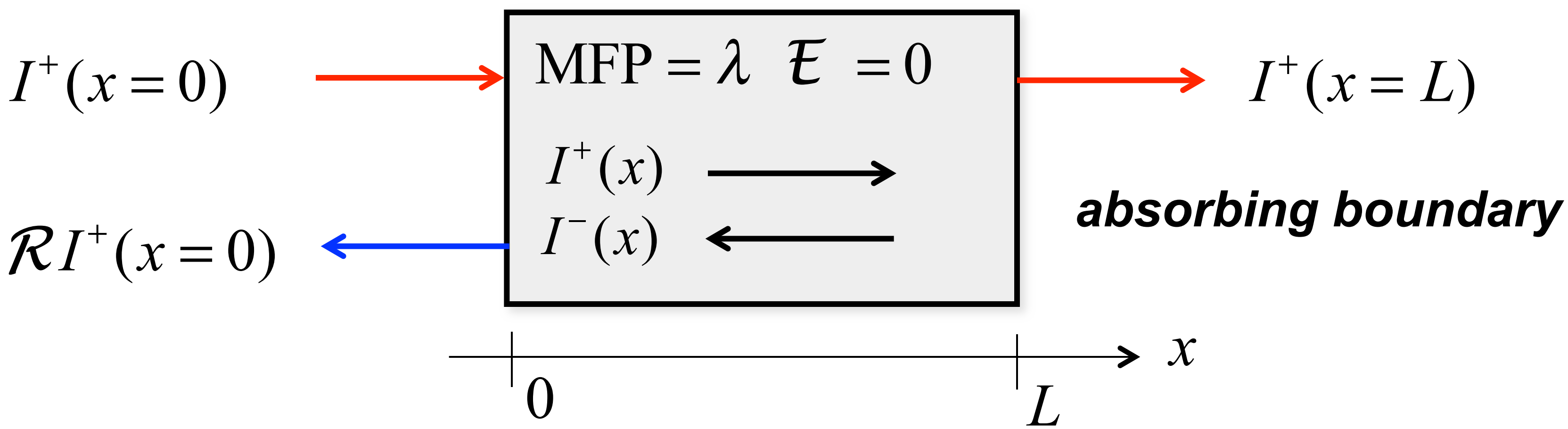


$$\frac{dI^+(x)}{dx} = -\frac{I}{\lambda}$$

$$\int_{I^+(0)}^{I^+(x)} dI^+ = -\frac{I}{\lambda}\int_0^x dx'$$

$$I^+(x) = I^+(0) - I\frac{x}{\lambda}$$

Flux that emerges from the right



$$I^+(x) = I^+(0) - I\frac{x}{\lambda}$$

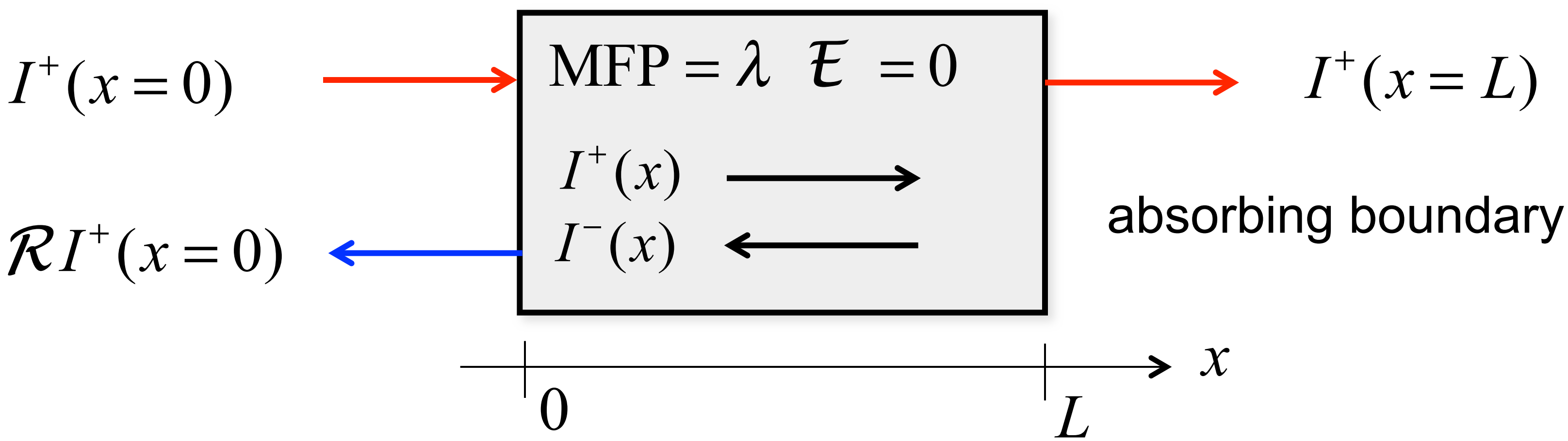
$$I^+(x) = I^+(0) - \left(I^+(x) - I^-(x)\right)\frac{x}{\lambda}$$

$$I^+(L) = I^+(0) - \left(I^+(L) - I^-(L)\right)\frac{L}{\lambda}$$

$$I^-(L) = 0$$

$$I^+(L) = I^+(0) - I^+(L)\frac{L}{\lambda}$$

Transmission



$$I^+(L) = I^+(0) - I^+(L)\frac{L}{\lambda}$$

$$I^+(L) = \frac{I^+(0)}{1 + L/\lambda}$$

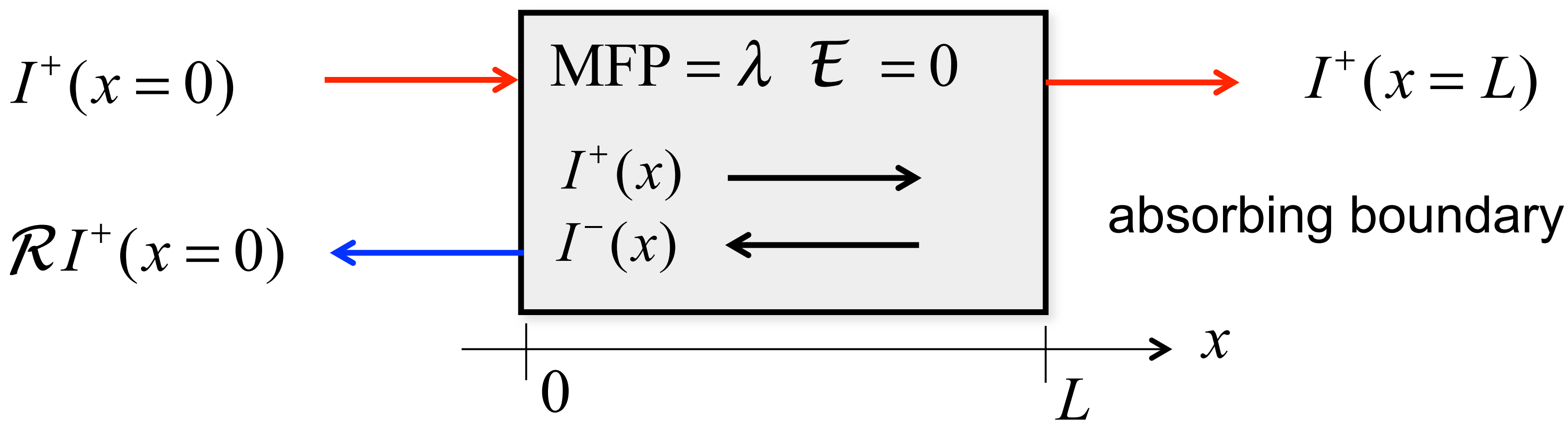
$$\frac{I^+(L)}{I^+(0)} = \mathcal{T} = \frac{\lambda}{\lambda + L}$$

$$\mathcal{T}(E) = \frac{\lambda(E)}{\lambda(E) + L} \qquad \mathcal{T}(E) + \mathcal{R}(E) = 1$$

$$\mathcal{T} \to 0 \quad L >> \lambda$$

$$\mathcal{T} \to 1 \quad L << \lambda$$

Diffusive limit



$$I = I^+(x=L) = \mathcal{T}\, I^+(x=0)$$

$$I^+(x=L) = \frac{\lambda}{L} I^+(x=0)$$

$$I^+(0) = qWq\upsilon_T \Delta n_S^+(0)$$

$$I = \mathcal{T}\, I^+(0) = \frac{\lambda}{L}\left\{qWq\upsilon_T \Delta n_S^+(0)\right\}$$

Fick's Law:

$$I = WqD_n \frac{\Delta n_S(0)}{L}$$

$$D_n = \frac{\upsilon_T \lambda}{2}$$

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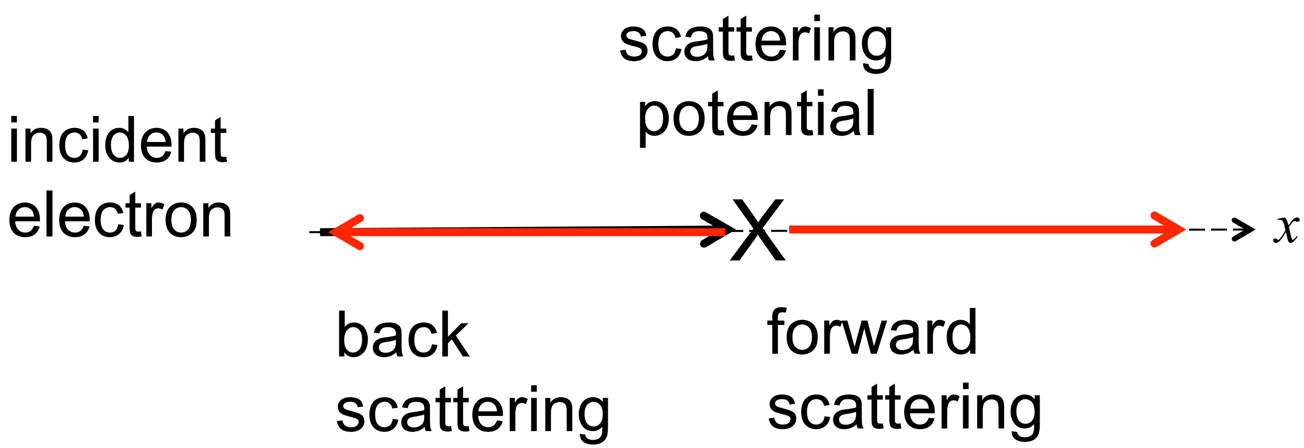
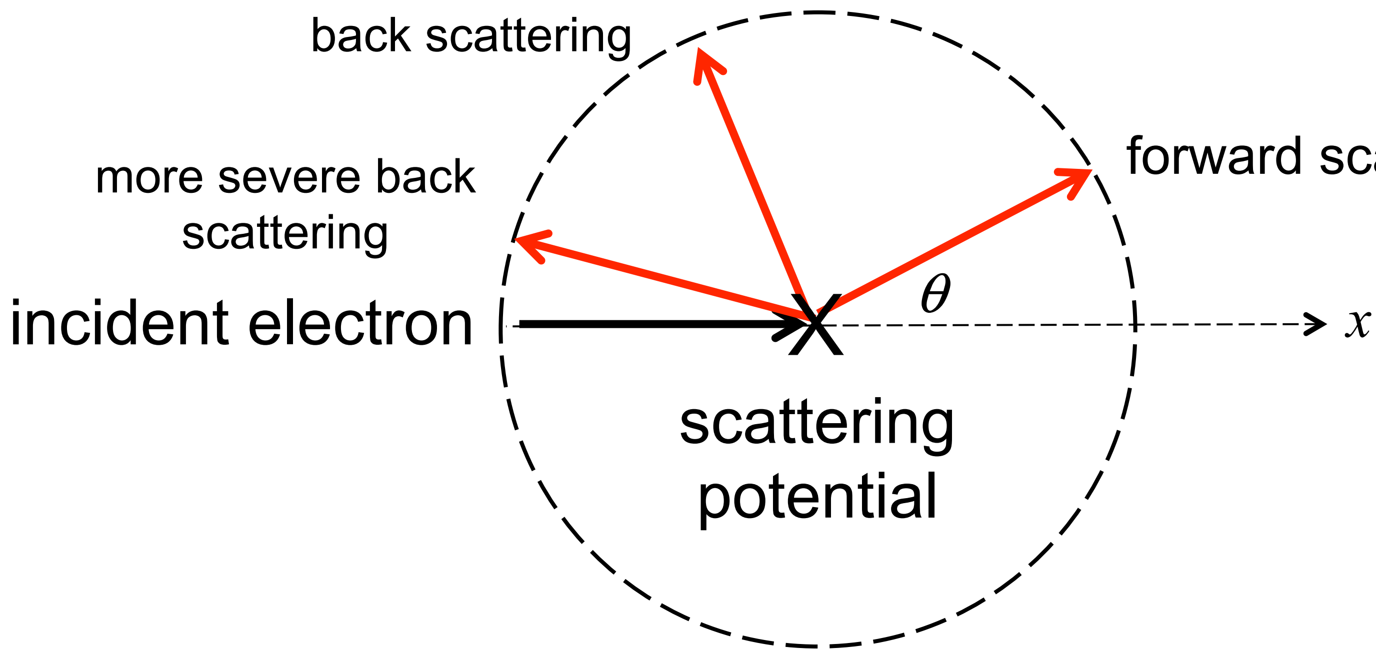
Backscattering in 1D



If we assume that the scattering is ***isotropic*** (equal probability of scattering forward or back) then average time between backscattering events is 2τ.

$$\lambda(E) = 2\upsilon(E)\tau_m(E) \qquad \lambda(E) \equiv 2\frac{\langle \upsilon_x^2 \tau_m \rangle}{\langle |\upsilon_x| \rangle}$$

Backscattering in 2D



If we assume that the scattering is ***isotropic***:

$$\lambda(E) = \frac{\pi}{2}\upsilon(E)\tau_m(E) \qquad \lambda(E) \equiv 2\frac{\langle \upsilon_x^2 \tau_m \rangle}{\langle |\upsilon_x| \rangle}$$

Mean-free-path for backscattering

$$\lambda(E) \equiv 2\frac{\langle \upsilon_x^2 \tau_m \rangle}{\langle |\upsilon_x| \rangle}$$

This is an average over angle at a specific energy, *E.*

$$\lambda(E) = 2\upsilon(E)\tau_m(E) \quad \text{1D}$$

$$\lambda(E) = \frac{\pi}{2}\upsilon(E)\tau_m(E) \quad \text{2D}$$

$$\lambda(E) = \frac{4}{3}\upsilon(E)\tau_m(E) \quad \text{3D}$$

Changwook Jeong, et al. "On Landauer vs. Boltzmann and Full Band vs. Effective Mass Evaluation of Thermoelectric Trans-port Coefficients," *J. Appl. Phys*., **107**, 023707, 2010.

Questions?

$$\mathcal{T}(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

$$\lambda(E) = 2\upsilon(E)\tau_m(E) \quad \text{1D}$$

$$\lambda(E) = \frac{\pi}{2}\upsilon(E)\tau_m(E) \quad \text{2D}$$

$$\lambda(E) = \frac{4}{3}\upsilon(E)\tau_m(E) \quad \text{3D}$$

$$D_n = \frac{\upsilon_T \lambda}{2}$$