Fermi window

and Current

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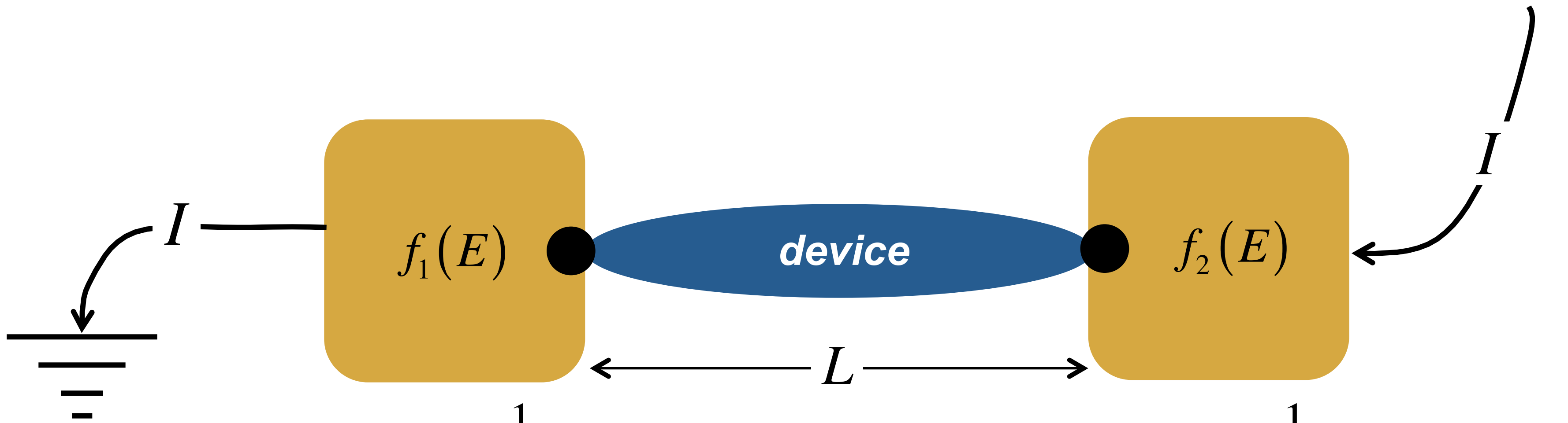
Landauer Approach



$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}} \qquad f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) \left(f_1 - f_2\right) dE$$

transmission, modes (channels), differences in Fermi functions

Channels (modes)

$$M_{1D}(E) = \frac{h}{4}\langle v_x^+ \rangle D_{1D}(E) \qquad \langle v_x^+ \rangle = v(E)$$

$$M_{2D}(E)W = W\frac{h}{4}\langle v_x^+ \rangle D_{2D}(E) \qquad \langle v_x^+ \rangle = \frac{2}{\pi}v(E)$$

$$M_{3D}(E)A = A\frac{h}{4}\langle v_x^+ \rangle D_{3D}(E) \qquad \langle v_x^+ \rangle = \frac{v(E)}{2}$$

Transmission

$$\mathcal{T}(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

λ is the "mean-free-path for backscattering"

1) Diffusive: $L >> \lambda \quad \mathcal{T} = \frac{\lambda}{L} << 1$

2) Ballistic: $L << \lambda \quad \mathcal{T} = 1$

3) Quasi-ballistic: $L \approx \lambda \quad \mathcal{T} < 1$

Outline

1) **Fermi window and current flow**
2) Conductance and quantized conductance
3) Voltage drop and power dissipation
4) Holes and bipolar conduction
5) Current in the bulk
6) Questions

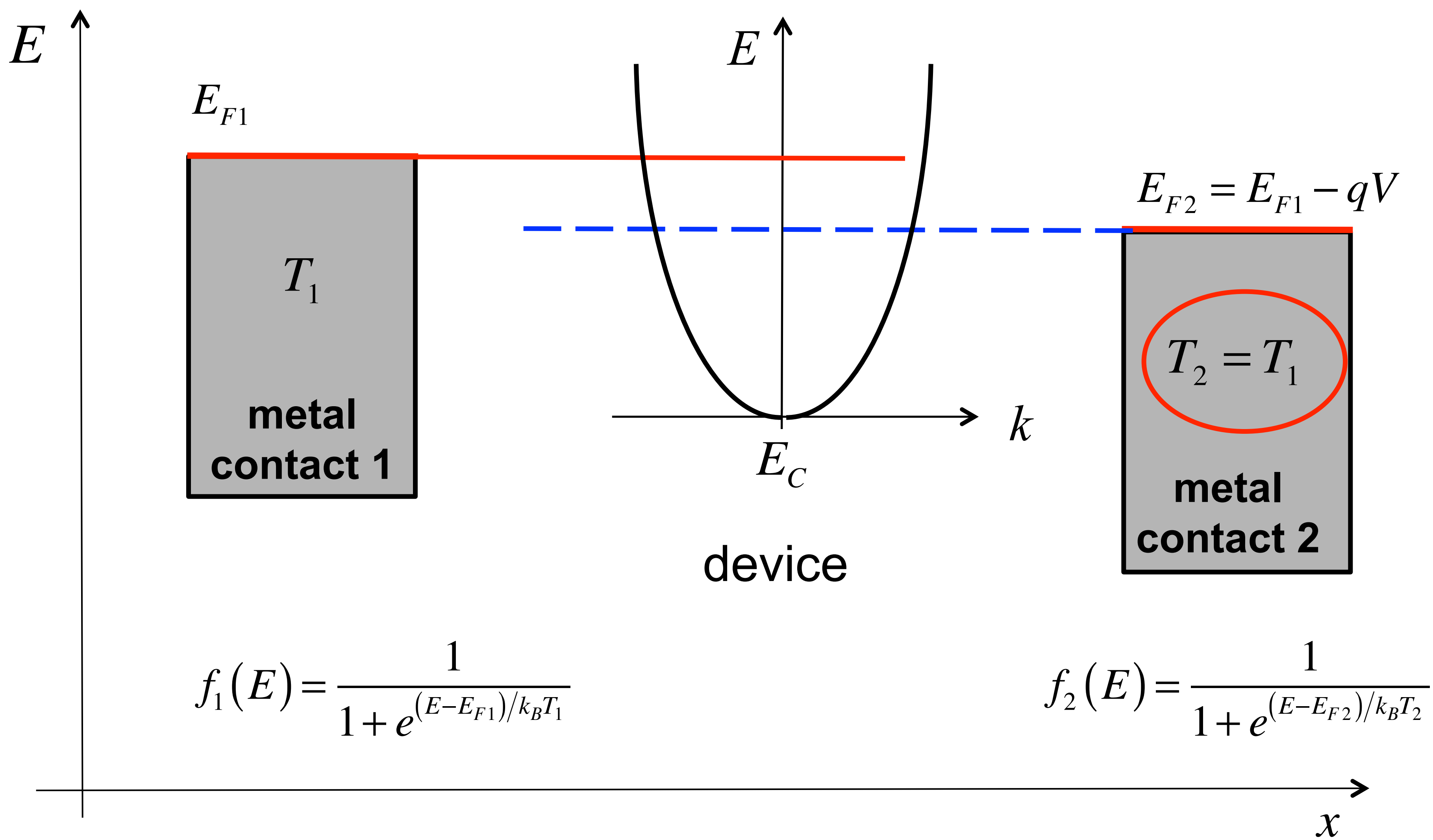
Fermi window

$$I = \frac{2q}{h}\int \mathcal{T}\left(E\right)M\left(E\right)\left(f_1 - f_2\right)dE$$

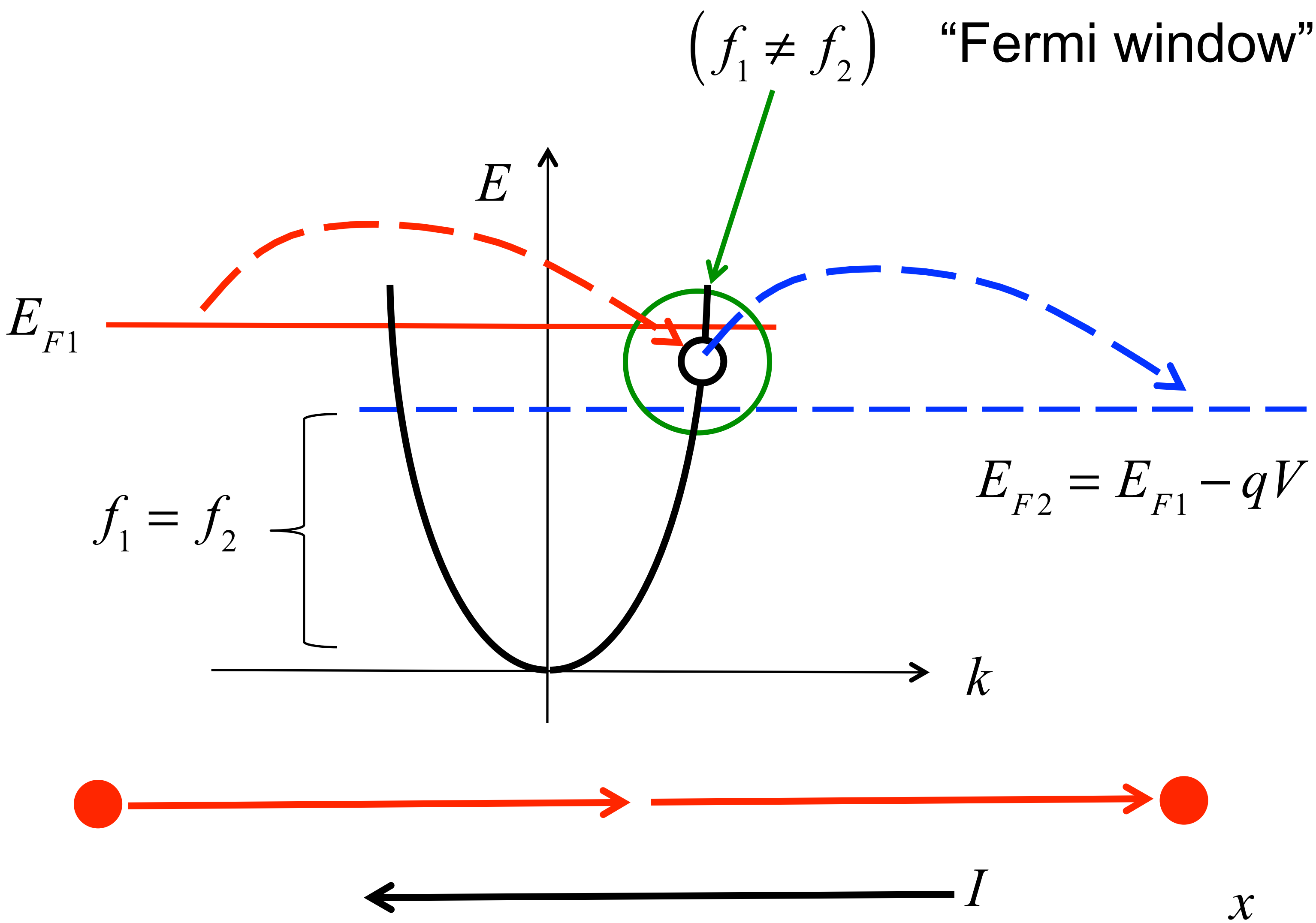
Fermi window:

The range of energies over which $\left(f_1 - f_2\right) \neq 0$

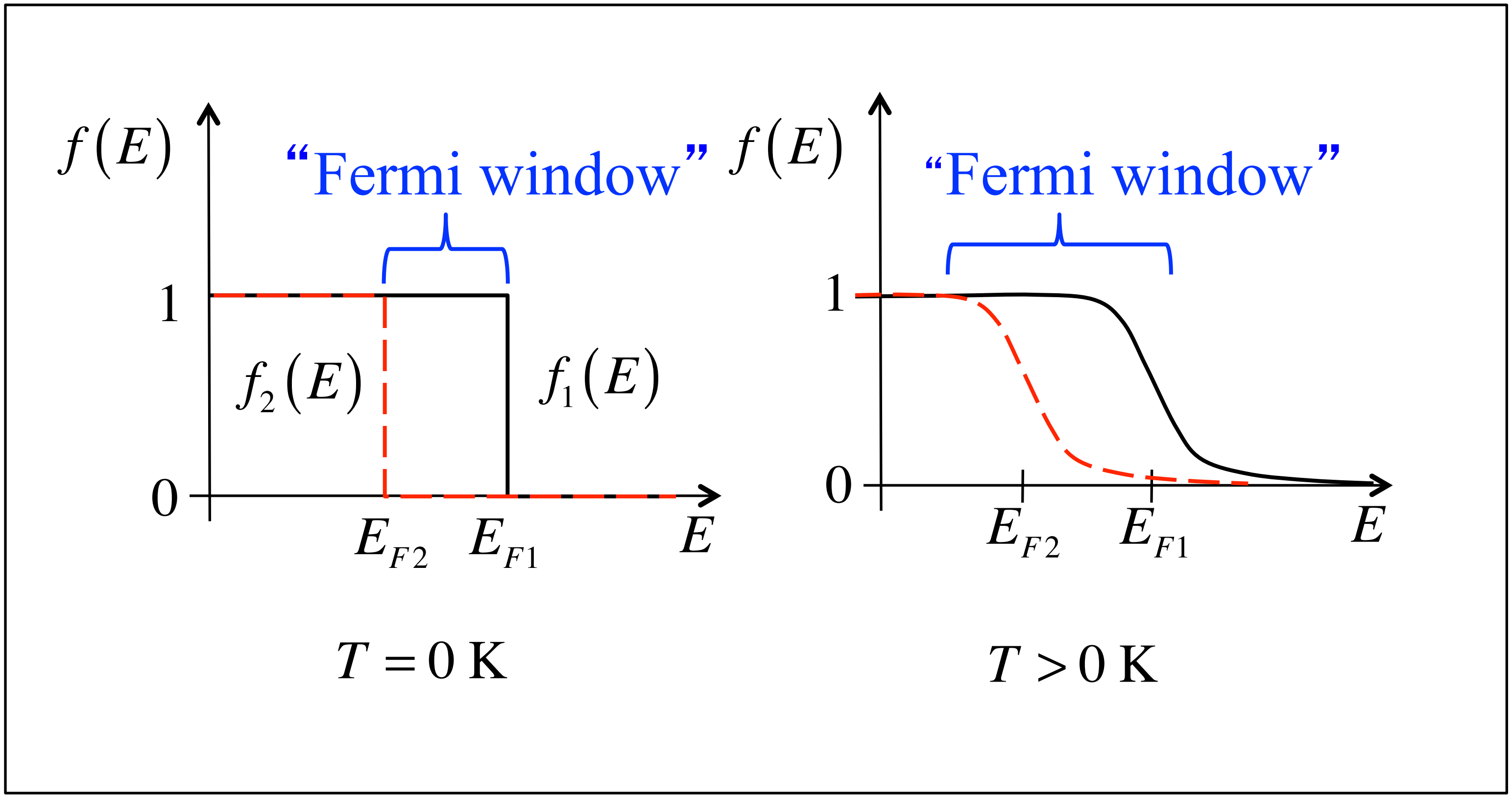
Differences in the Fermi levels drive current



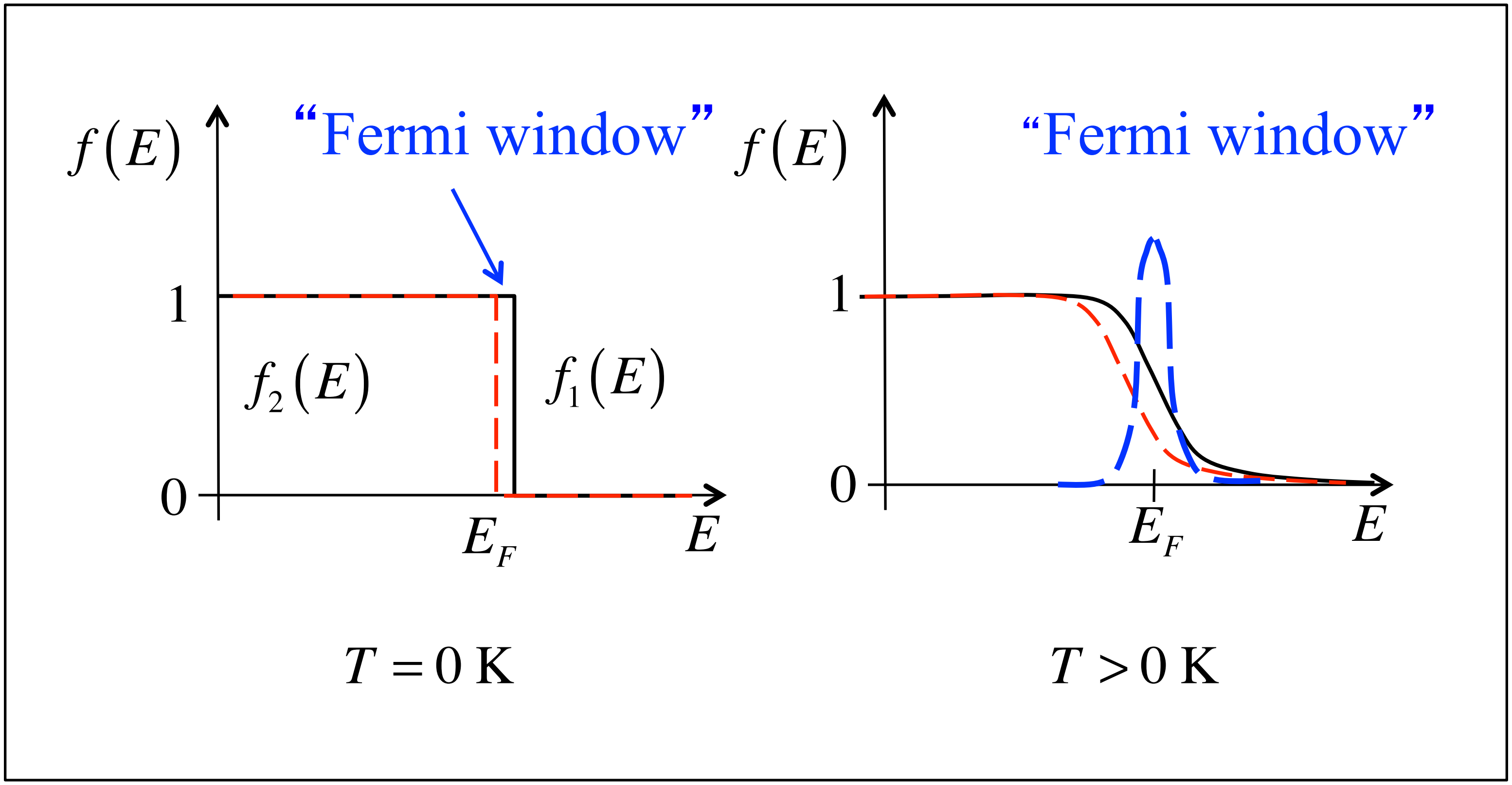
How current flows



Fermi window: Large bias



Fermi window: small bias



Small voltage (linear response)

$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) \big(f_1(E) - f_2(E) \big) dE$$

$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}}$$

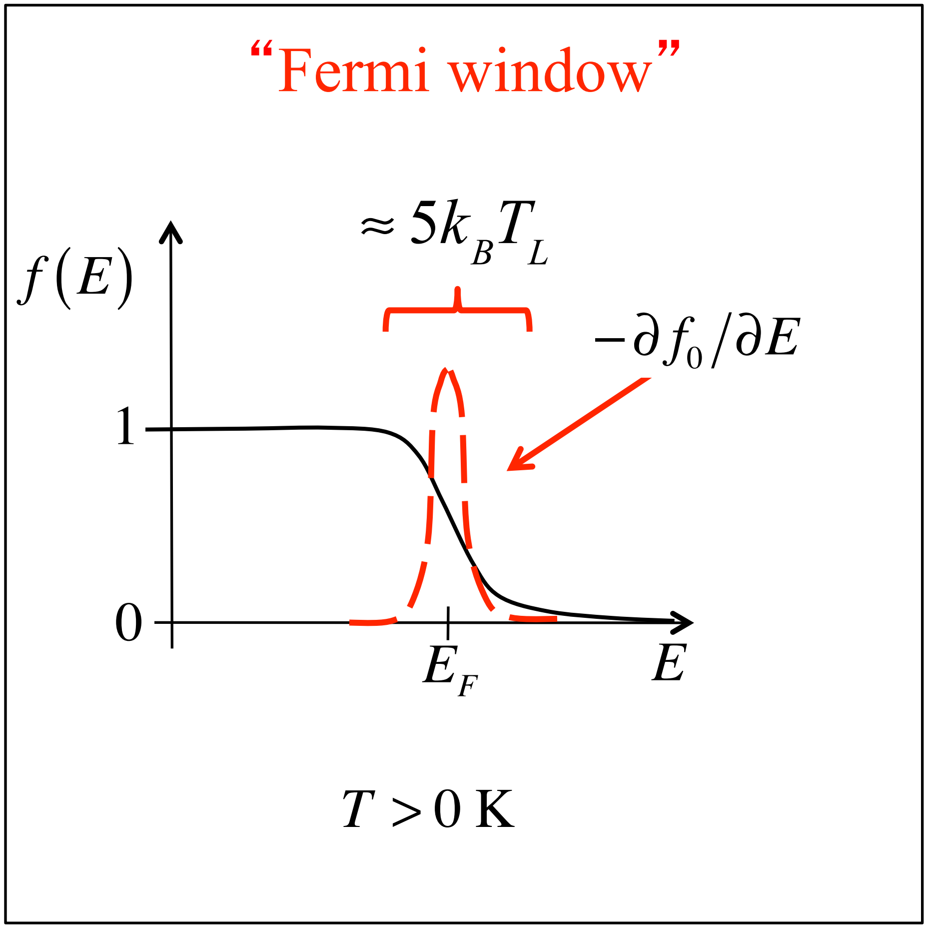
$$f_2(E) \approx f_1(E) + \frac{\partial f_1}{\partial E_F} \delta E_F$$

$$f_2(E) \approx f_1(E) + \left(-\frac{\partial f_1}{\partial E} \right) \delta E_F$$

$$f_2(E) \approx f_1(E) + \left(-\frac{\partial f_1}{\partial E} \right) (-qV)$$

$$f_1(E) - f_2(E) = \left(-\frac{\partial f_1}{\partial E} \right) (qV)$$

Fermi window: small bias



$$W_F(E) = \left(-\frac{\partial f_0}{\partial E}\right)$$

$$\int W_F(E)\,dE = 1$$

$$f_1(E) - f_2(E) = W_F(E)(qV)$$

Outline

1) Fermi window and current flow
2) **Conductance and quantized conductance**
3) Voltage drop and power dissipation
4) Holes and bipolar conduction
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Near-equilibrium conductance

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)\left(f_1(E) - f_2(E)\right)dE$$

$$f_1(E) - f_2(E) = \left(-\frac{\partial f_1}{\partial E}\right)(qV)$$

$$I = GV \qquad \text{A}$$

$$G = \frac{2q^2}{h}\int \mathcal{T}(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \qquad \text{S}$$

T = 0 K 2D conductance

$$G = \frac{2q^2}{h}\int \mathcal{T}(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \quad \text{S}$$

$T = 0\ \text{K}$:

$$\left(-\frac{\partial f_0}{\partial E}\right) = \delta(E_F)$$

$$G(T = 0\ \text{K}) = \frac{2q^2}{h}\mathcal{T}(E_F)M(E_F)$$

$$M(E) = WM_{2D}(E) = Wg_v\frac{\sqrt{2m^*E}}{\pi\hbar}$$

For large *W*, *M* is ~ W

For small *W*, *M* comes in discrete units (modes).

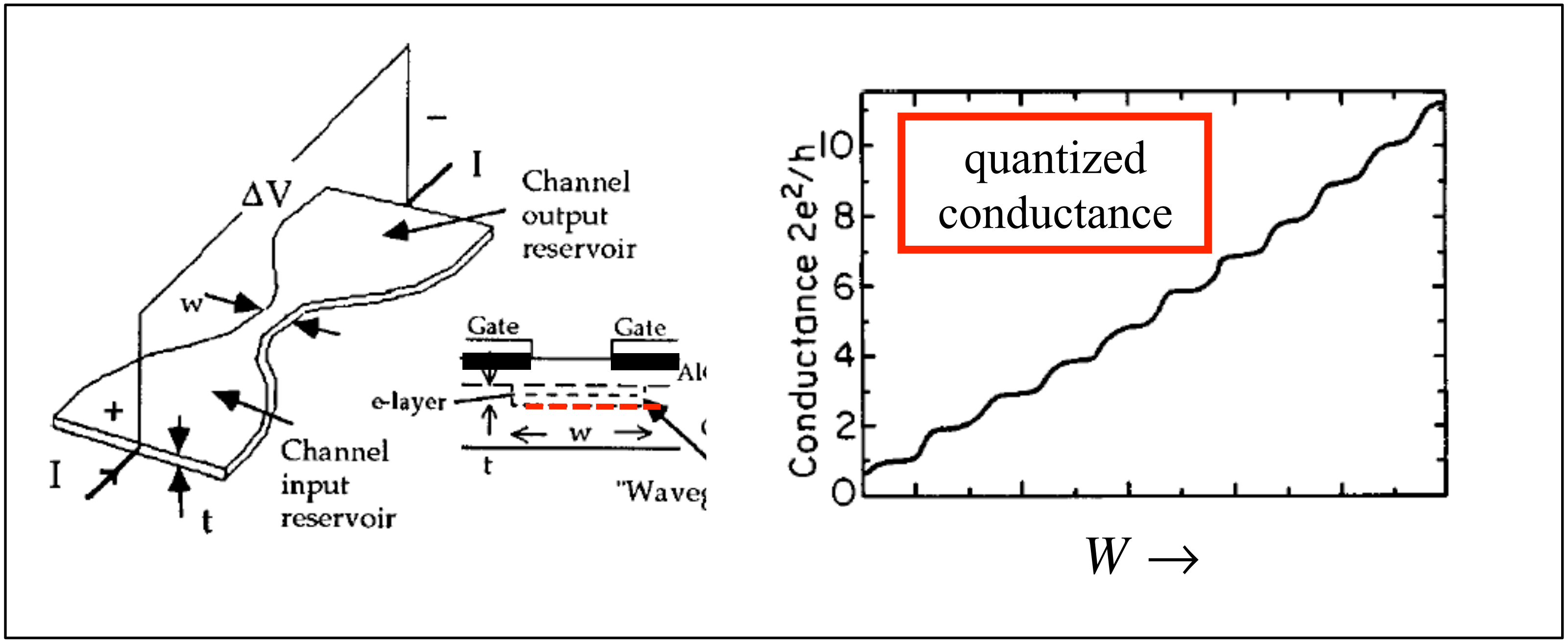
T = 0 K ballistic conductance

$$G(T = 0\text{ K}) = \frac{2q^2}{h}\mathcal{T}(E_F)M(E_F) \qquad \mathcal{T}(E_F) = 1$$

$$G_B = \frac{2q^2}{h}M(E_F) = \frac{1}{R_B}$$

For small *W*, *M* comes in discrete units (modes).

Quantized conductance



D. Holcomb, *American J. Physics*, **67**, pp. 278-297 1999.
Data from: B. J. van Wees, et al., *Phys. Rev. Lett*. **60**, 848851, 1988.

Outline

1) Fermi window and current flow
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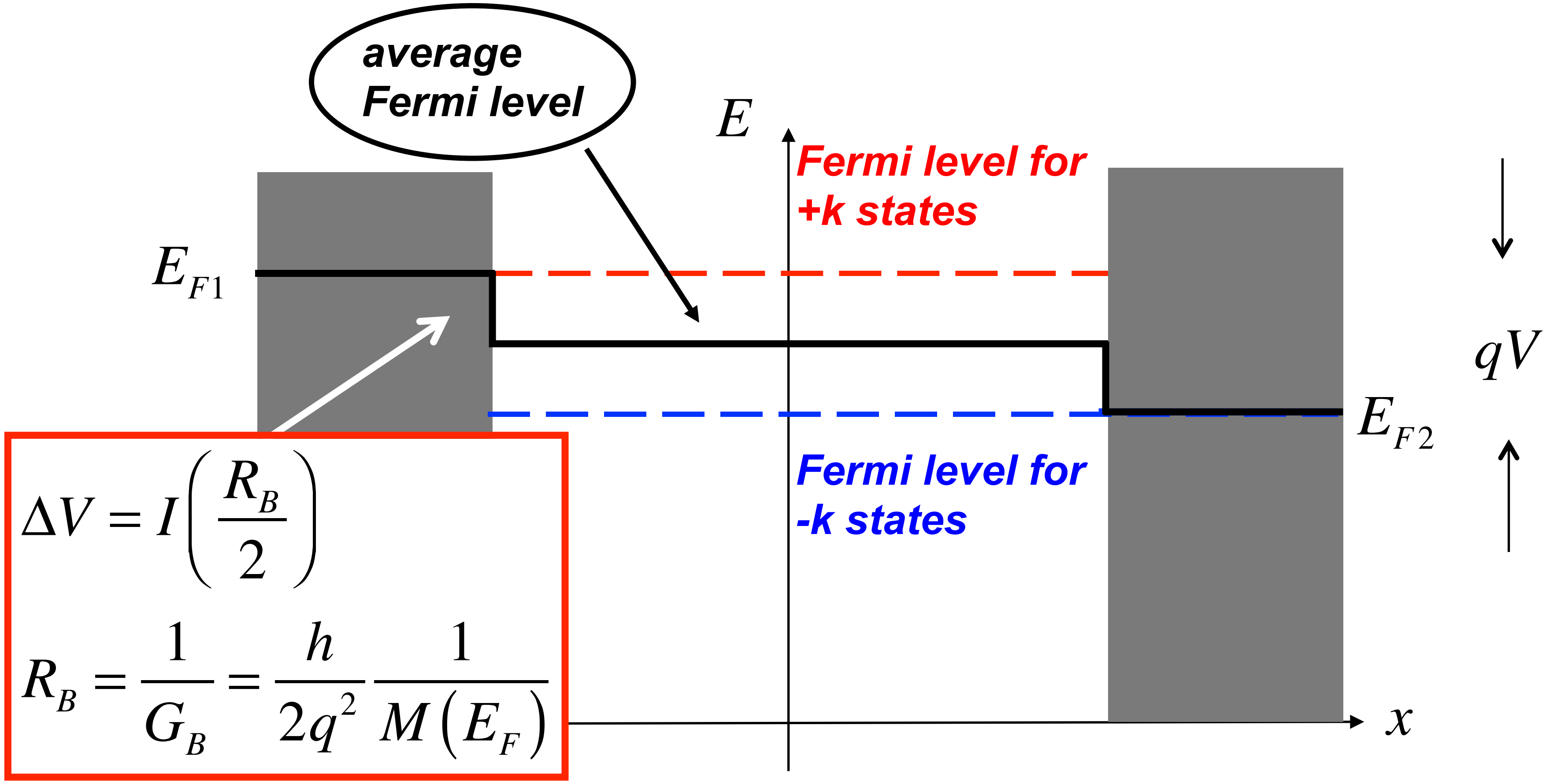
Where does the voltage drop in a ballistic resistor?

$$G_B = \frac{2q^2}{h} M\left(E_F\right) = \frac{1}{R_B}$$

$$I = G_B V$$

Where does the voltage drop?



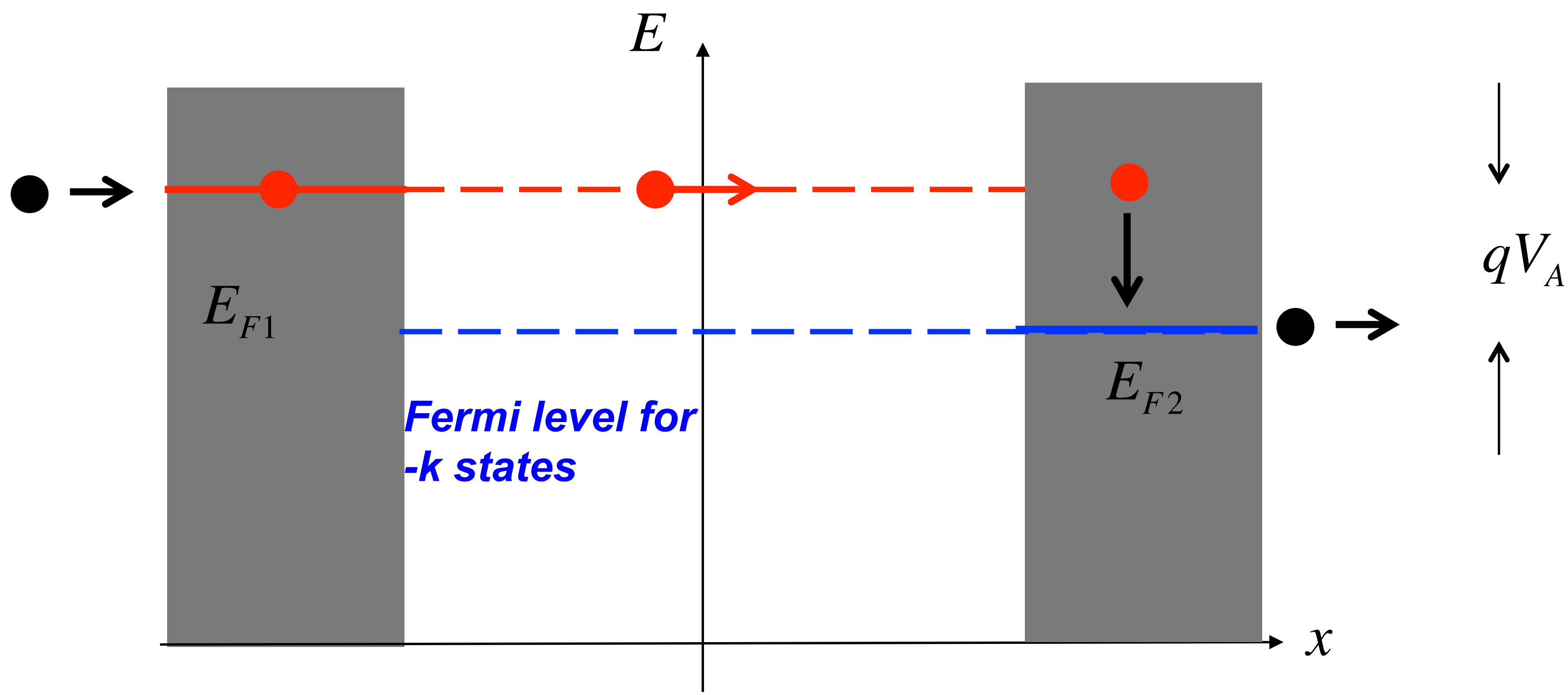
$$\Delta V = I\left(\frac{R_B}{2}\right)$$

$$R_B = \frac{1}{G_B} = \frac{h}{2q^2}\frac{1}{M(E_F)}$$

"quantum contact resistance"

S. Datta, *Electronic Conduction in Mesoscopic Systems*, Chpt. 2, Cambridge, 1995

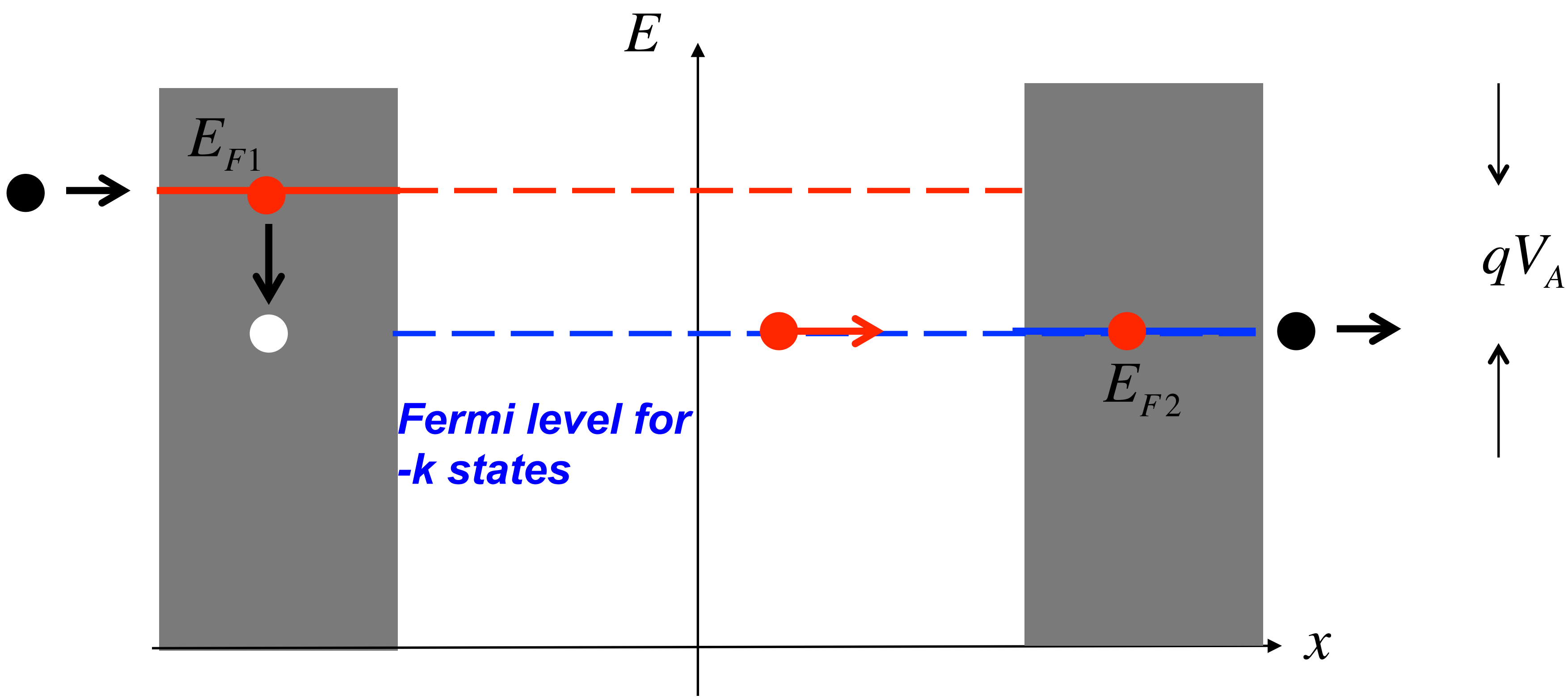
Power dissipation in a ballistic resistor



power dissipated in right contact

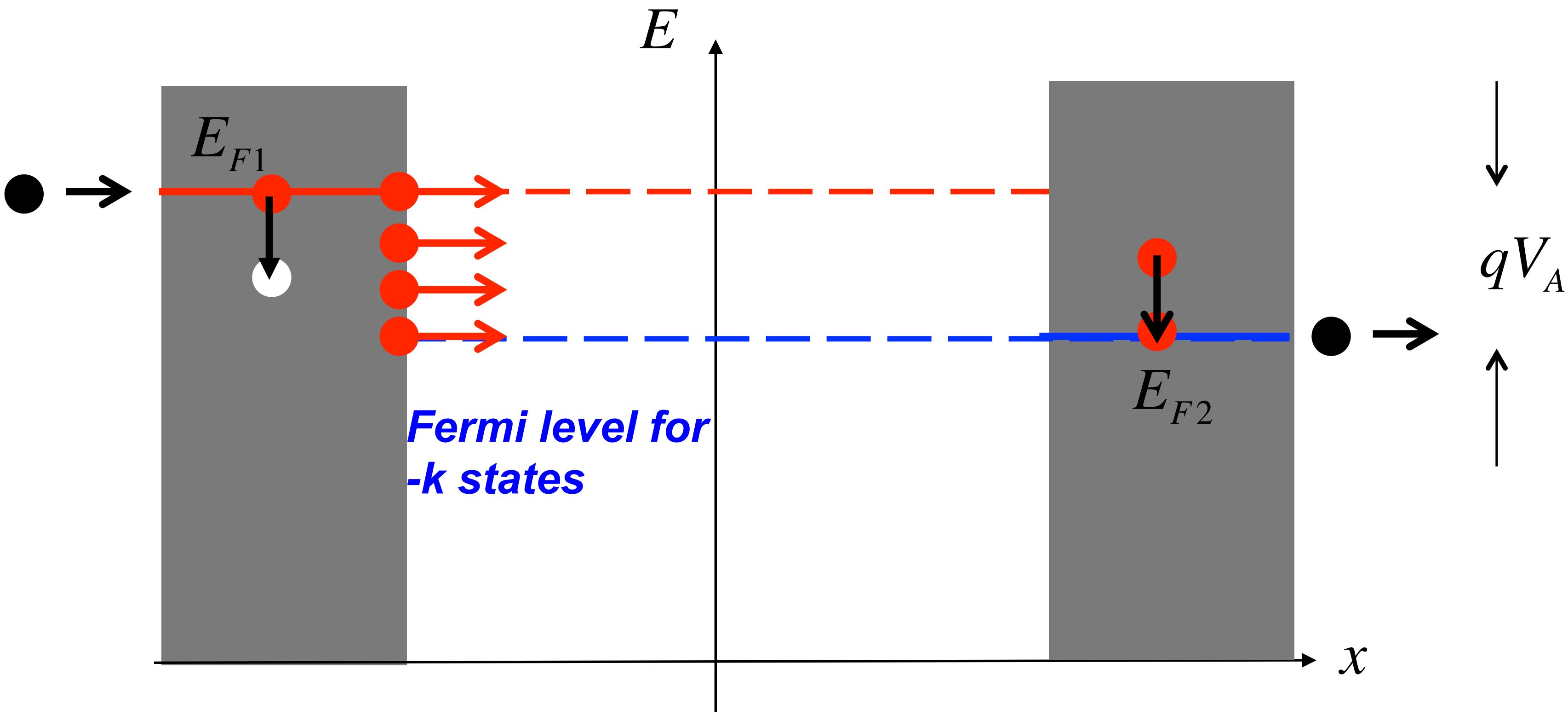
x

Power dissipation in a ballistic resistor



power dissipated in left contact

Power dissipation in a ballistic resistor



power dissipated equally in both contacts

x

Recap

In a ballistic resistor:

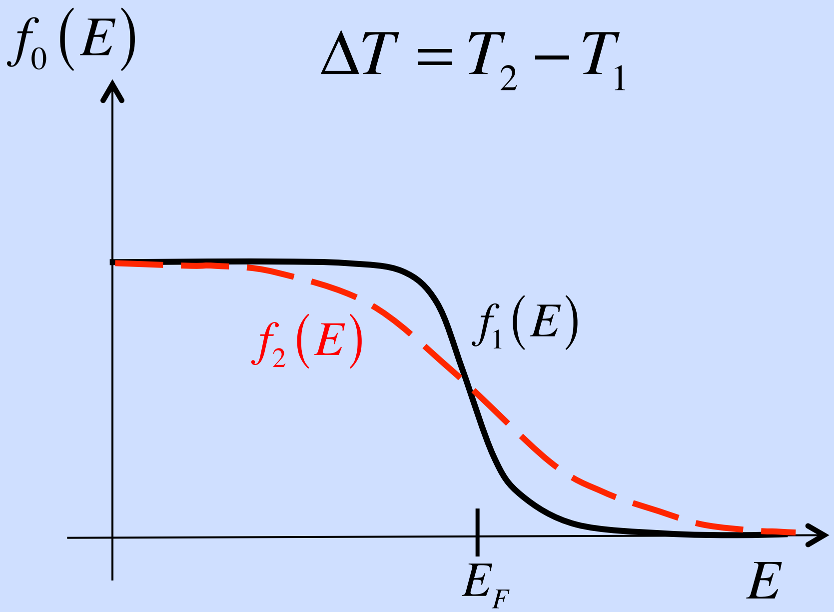
1) The voltage drops at the contact (quantum contact resistance)

2) The power dissipation occurs in the contacts.

Next: we have seen that differences in Fermi levels cause current to flow, **but** current flows whenever $f_1 \neq f_2$.

x

Differences in the Fermi levels (at constant *V*)



$|f_1 - f_2| > 0$ so current flows, but the sign depends on whether the states are located above or below E_F (n-type or p-type).

near-equilibrium
(small temperature difference)

$$(f_1 - f_2) \approx -\left(-\frac{\partial f_0}{\partial E}\right)\frac{(E - E_F)}{T}\Delta T$$

$$(f_1 \approx f_2 \approx f_0)$$

"thermoelectric effects" which we will discuss later in the course.

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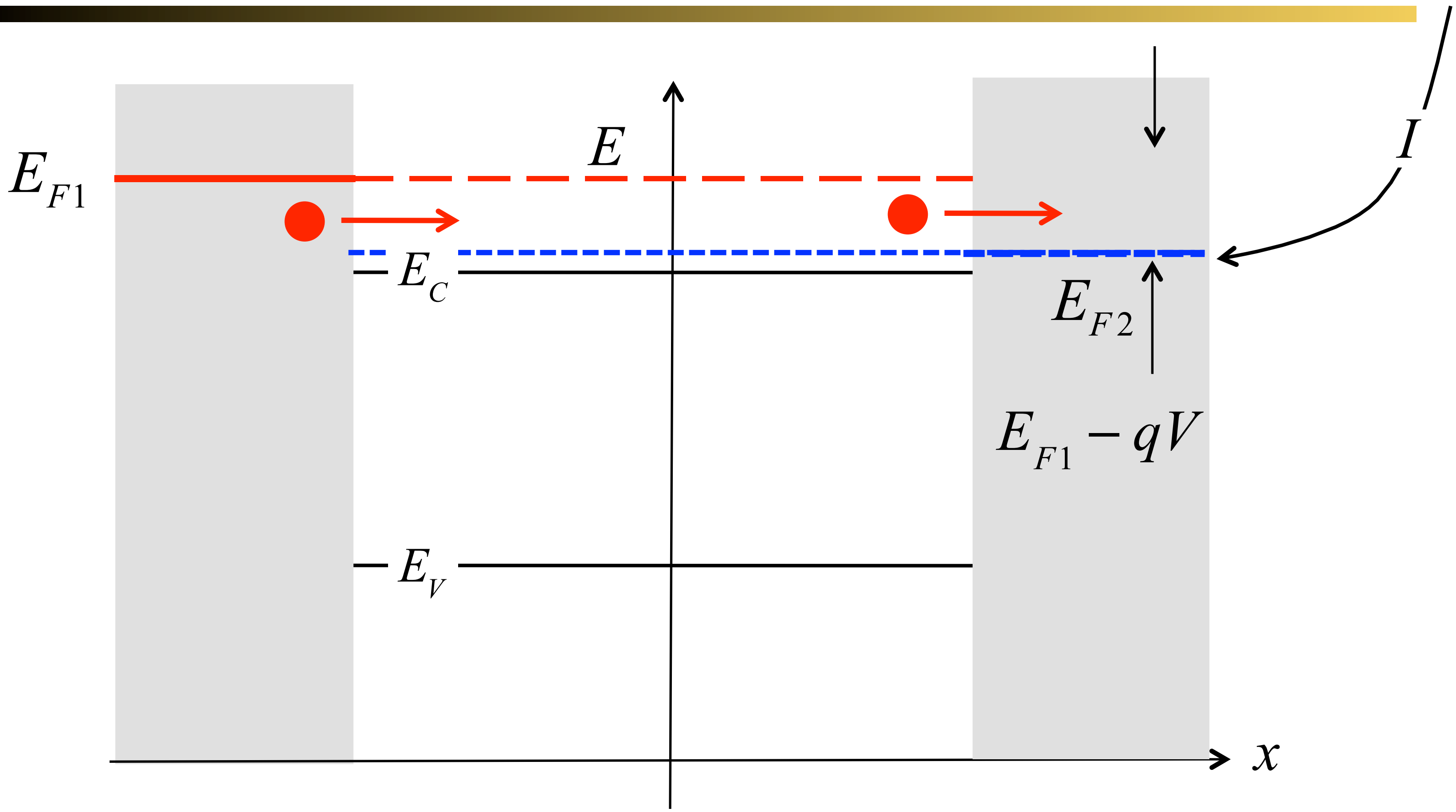
What about holes?

Landauer expression for electrons:

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)(f_1 - f_2)dE$$

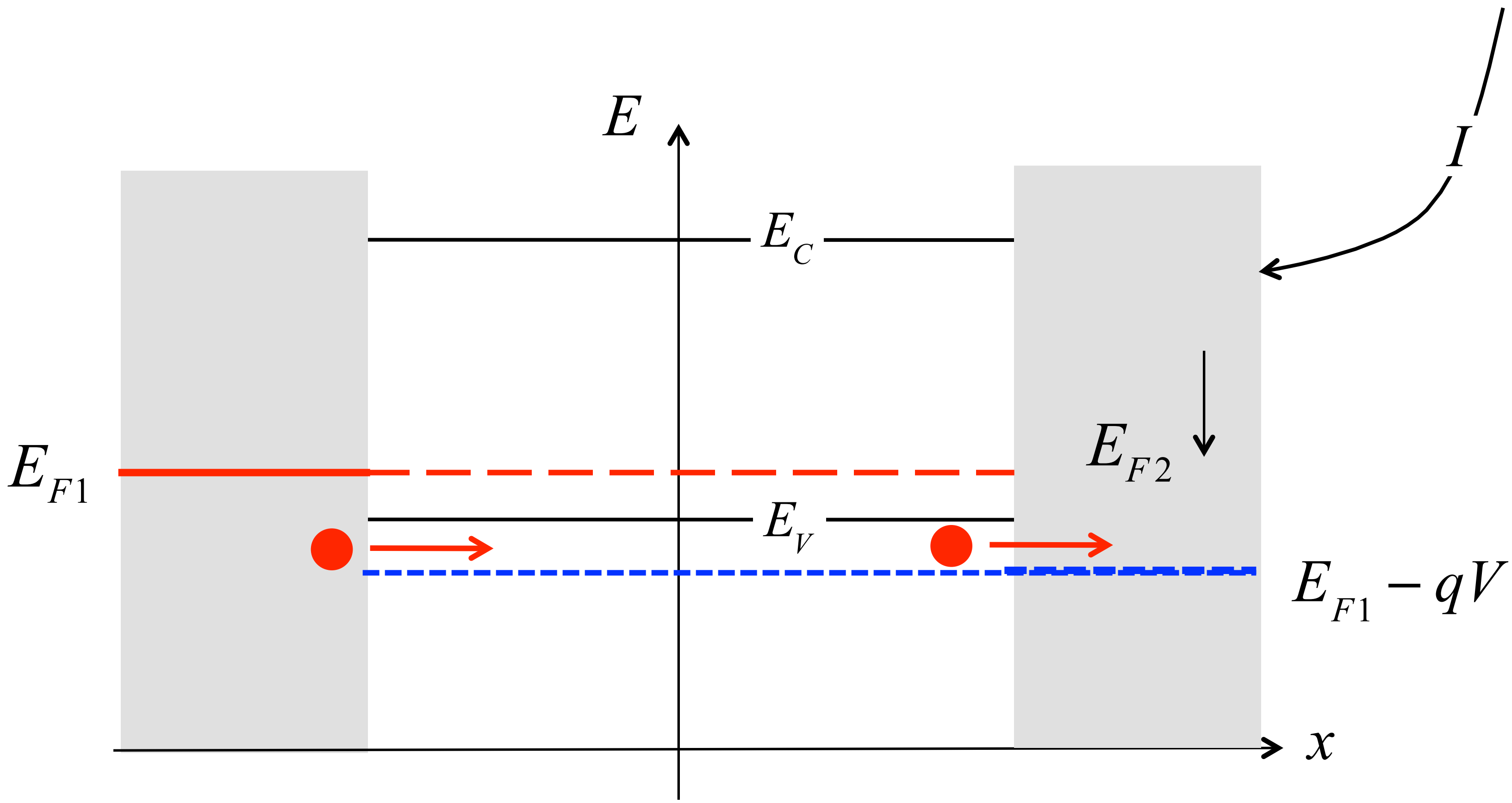
Do we need a Landauer expression for holes?

N-type conduction



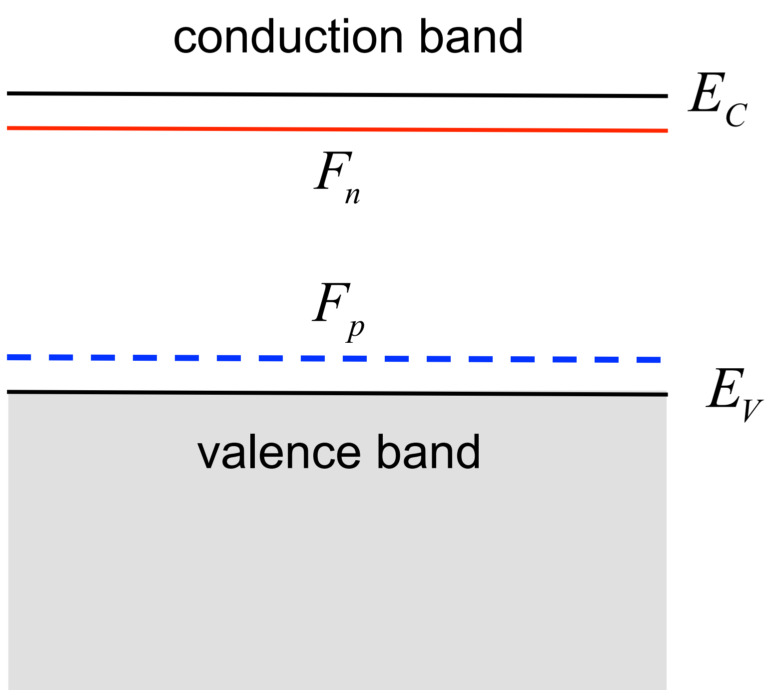
current is due to electrons flowing in the conduction band

P-type conduction



current is due to **electrons** flowing in the valence band

What about ~~holes~~?



All of these expressions refer to ***electrons*** in the conduction and valence bands

n-type

$$I = G_n V$$

$$G_n = \frac{2q^2}{h} \int_{E_C}^{\infty} \mathcal{T}_C(E) M_C(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$f_0(E) = \frac{1}{1 + e^{(E - F_n)/k_B T}}$$

p-type

$$I = G_p V$$

$$G_p = \frac{2q^2}{h} \int_{-\infty}^{E_V} \mathcal{T}_V(E) M_V(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$f_0(E) = \frac{1}{1 + e^{(E - F_p)/k_B T}}$$

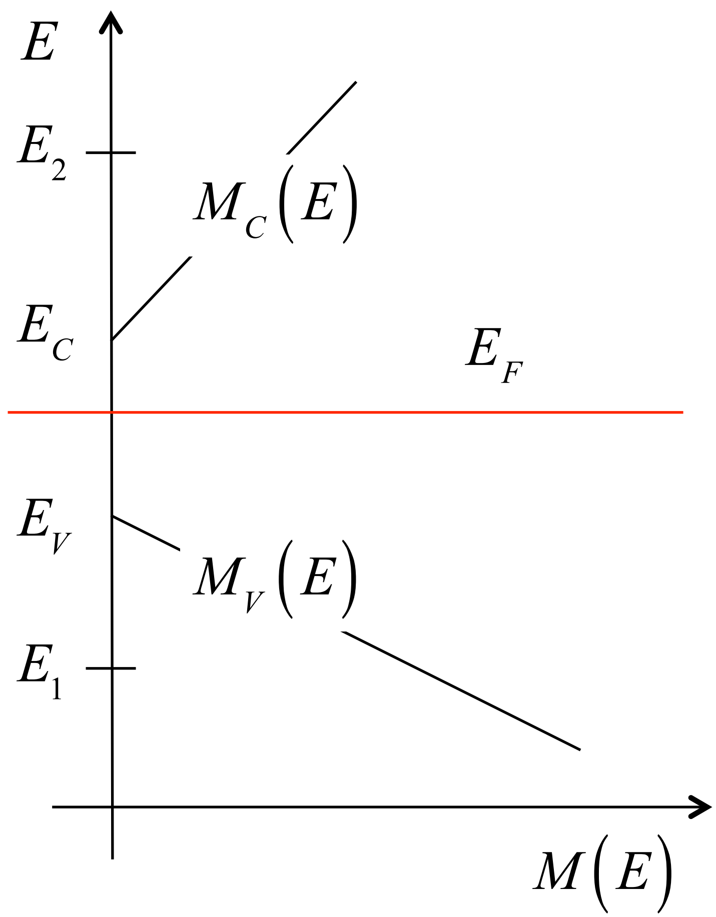
Bipolar conduction

$$I = \frac{2q}{h}\int_{E_1}^{E_2} \mathcal{T}(E)M(E)\left(f_1 - f_2\right)dE$$

$$M(E) = M_V\left(E_V - E\right) + M_C\left(E - E_C\right)$$

$$M_C(E) = A\frac{m_n^*}{2\pi\hbar^2}\left(E - E_C\right)$$

$$M_V(E) = A\frac{m_p^*}{2\pi\hbar^2}\left(E_V - E\right)$$



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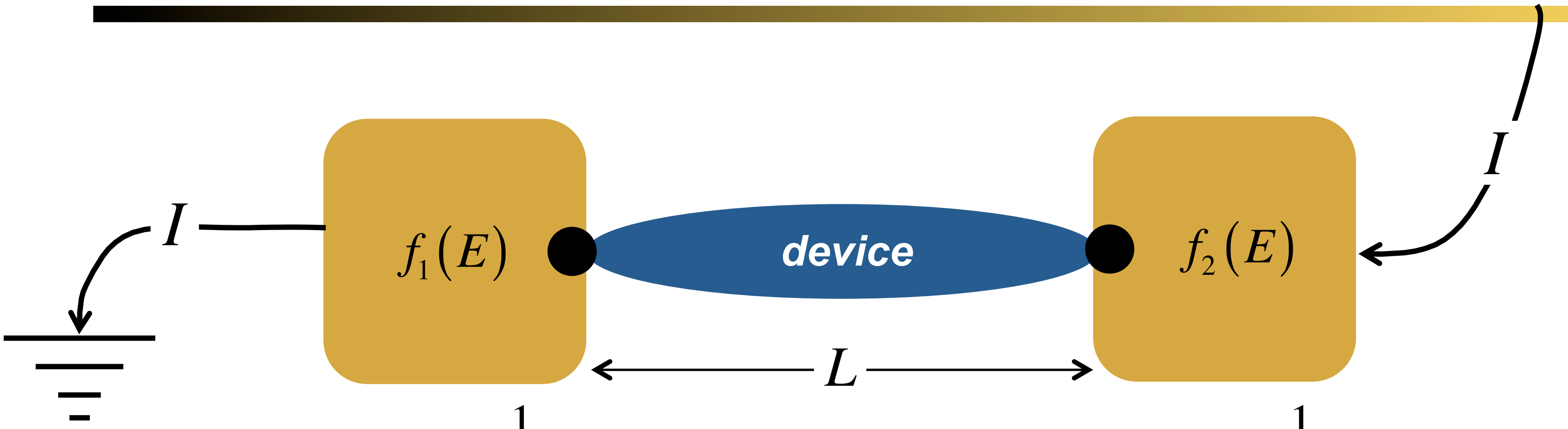
Landauer Approach to bulk transport



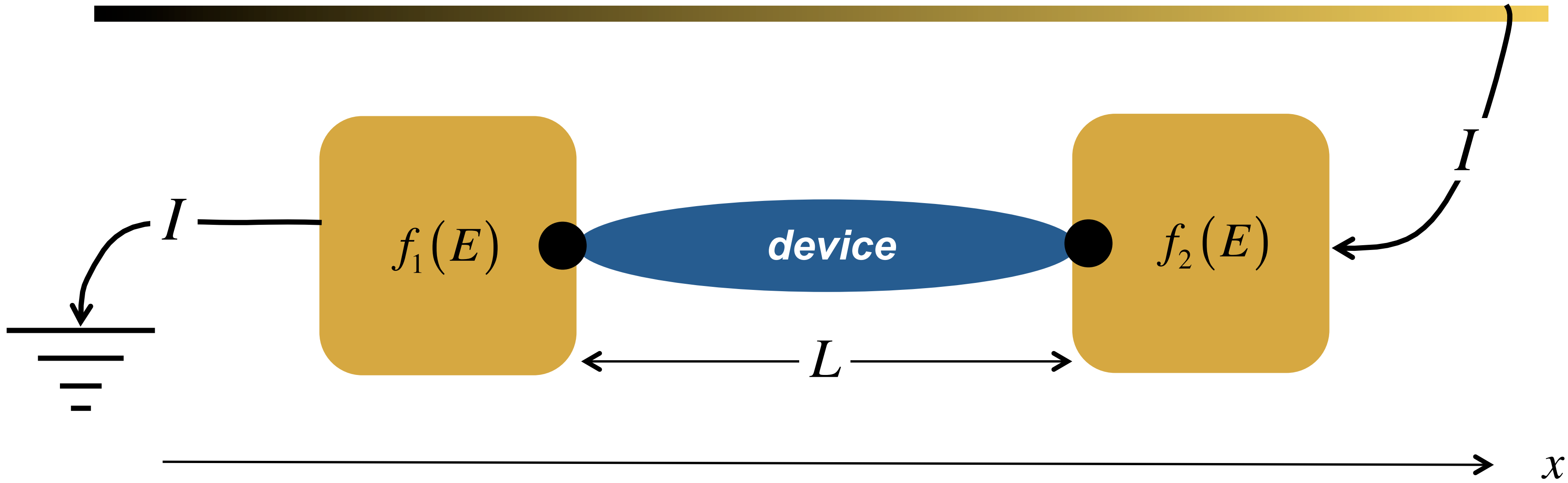
$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}}$$

$$f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

x

The Landauer Approach was developed for small devices. Can we use it for transport in the bulk where *L* >> MFP and the contacts don’t matter.

Sign of the current



For the device, we define a positive current as a current that flows into contact 2. In the bulk, we define a positive current as one that flows in the +x direction.

Also prefer to work with current density. So in 2D: $J_x = -I/W$

Current equation (2D)

$$G = \frac{2q^2}{h}\int \mathcal{T}(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \quad \text{Siemens} \qquad I = GV = -(J_x W)V$$

$$G = \frac{2q^2}{h}\int \frac{\lambda(E)}{L}M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \ \text{ siemens}$$

diffusive

$$J_x = -I/W = \left\{\frac{2q^2}{h}\int \frac{\lambda(E)}{L}\frac{M(E)}{W}\left(-\frac{\partial f_0}{\partial E}\right)dE\right\}V \qquad \frac{qV}{L} = -\frac{dF_n}{dx}$$

$$J_x = \sigma_n \frac{d(F_n/q)}{dx} \qquad \sigma_n = \frac{2q^2}{h}\int \lambda(E)M_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

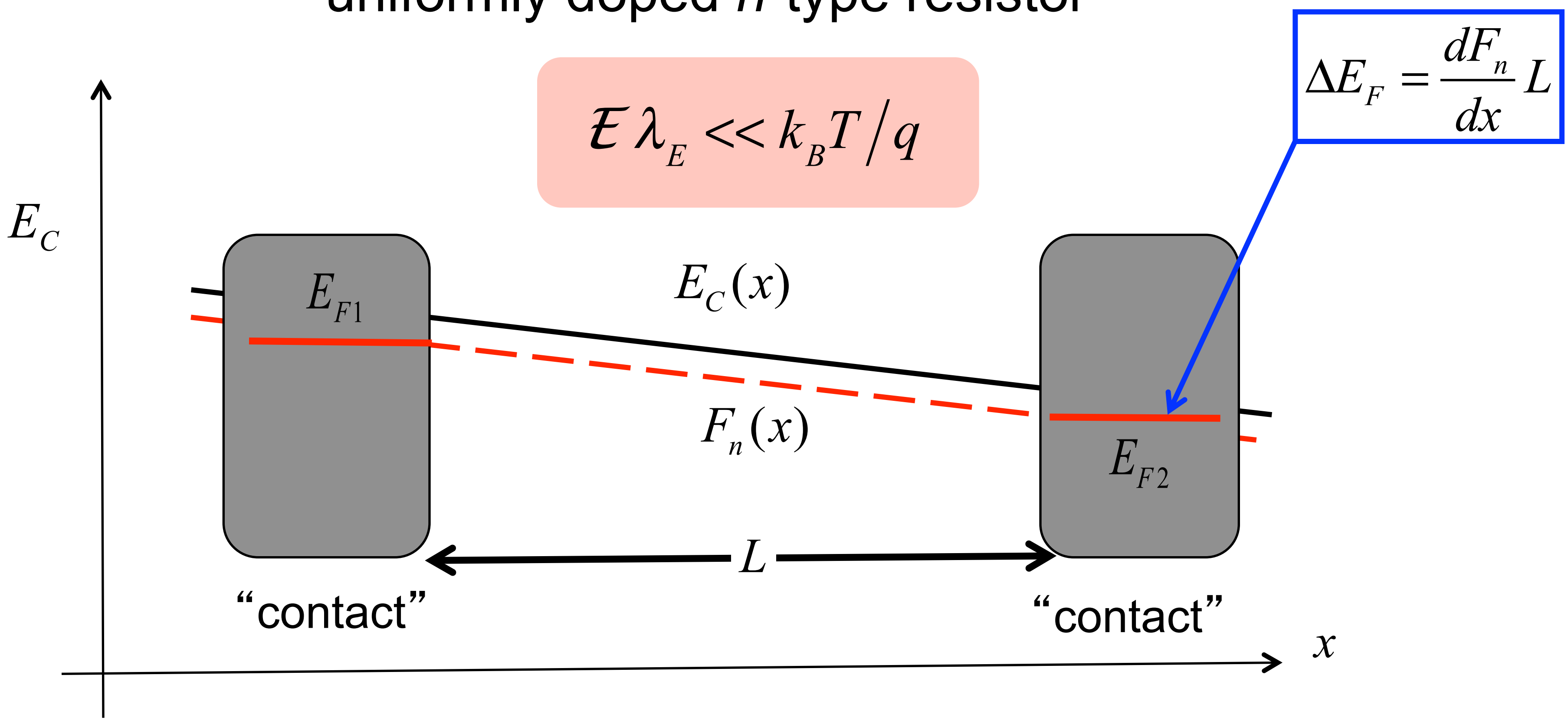
Exercise

$$J_x = \sigma_n \frac{d\left(F_n/q\right)}{dx} \qquad \sigma_n = \frac{2q^2}{h}\int \lambda(E) M_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

Show that a solution of the BTE in the Relaxation Approximation for a 2D conductor with a small electric field gives exactly the same answer.

Elastic and inelastic scattering in the bulk

uniformly doped *n*-type resistor



F_n(x) is the *electrochemical potential* (or "quasi-Fermi level") which we now regard as slowly varying across the sample.

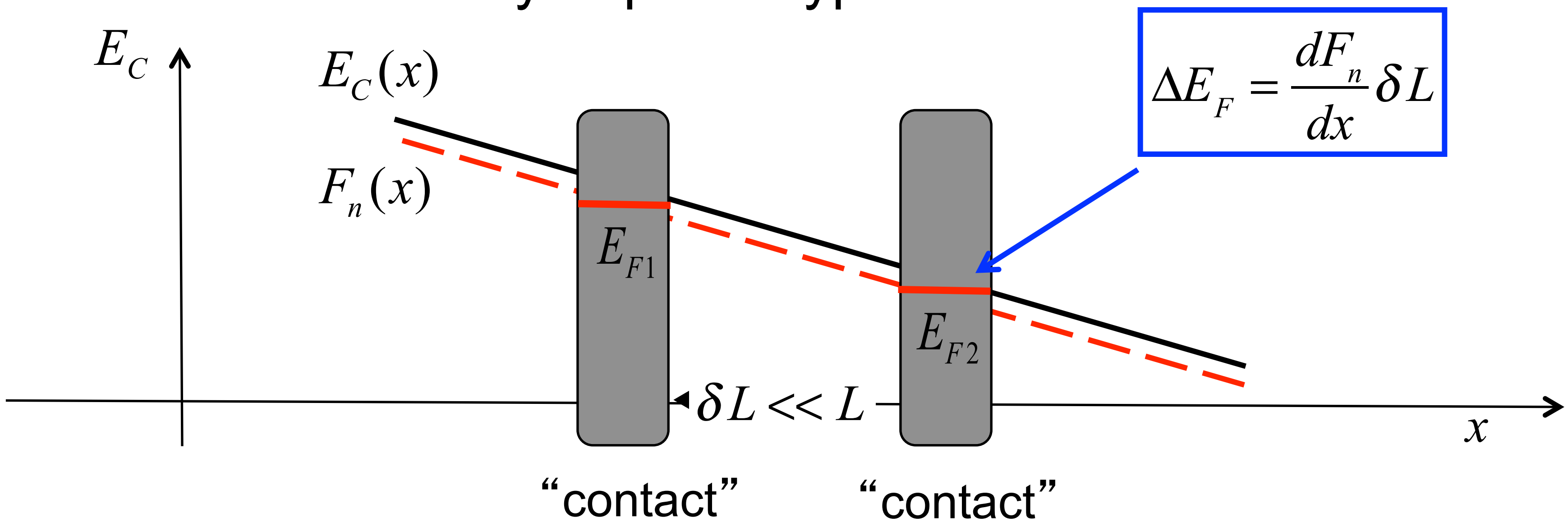
Discussion

The criterion, $\mathcal{E}\lambda_E << k_B T/q$ ensures that for small enough electric fields, even a long resistor will be near-equilibrium everywhere.

But a Landauer device separates elastic and in-elastic scattering. All inelastic scattering takes place in the contacts. Elastic scattering takes place in between the contacts. How do we think about transport in the bulk in terms of a Landauer device?

The bulk as a series of Landauer devices

uniformly doped *n*-type resistor



Think of one short section of the long resistor, and separate out the elastic and inelastic scattering. Put the inelastic scattering in two virtual contact. In this picture, a long device is a series of short, Landauer devices.

Exercise

$$J_x = \sigma_n \frac{d\left(F_n/q\right)}{dx}$$

From the general current equation above, derive a drift-diffusion equation:

$$J_{nx} = n_L q \mu_n \mathcal{E}_x + qD_n \frac{dn_L}{dx}$$

What is the generalized Einstein relation, $\frac{D_n}{\mu_n}$?

Questions?

1) Fermi window $W_F(E) = \left(-\frac{\partial f_0}{\partial E}\right)$

2) Conductance $I = \frac{2q}{h}\int \mathcal{T}(E)M(E)(f_1 - f_2)dE$

3) Voltage drops and power dissipation

Contacts

4) What about holes?

Use appropriate energy limits, MFP, and modes