Resistors at $T = 0$ K

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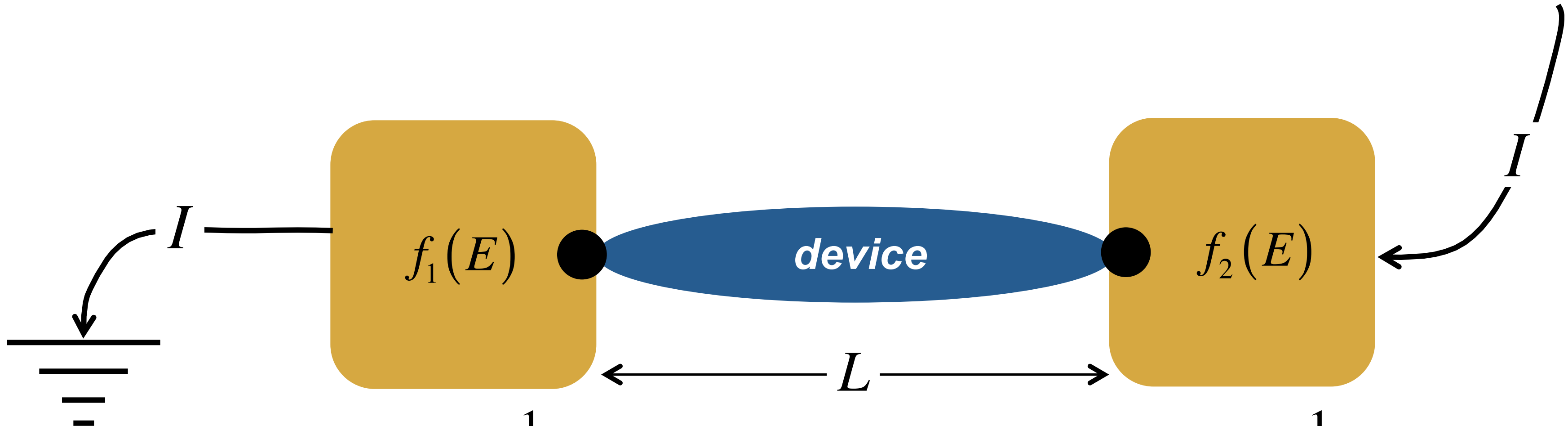
Landauer Approach



$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}} \qquad f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) \left(f_1 - f_2\right) dE$$

transmission, modes (channels), differences in Fermi functions

Near-equilibrium (constant temperature)

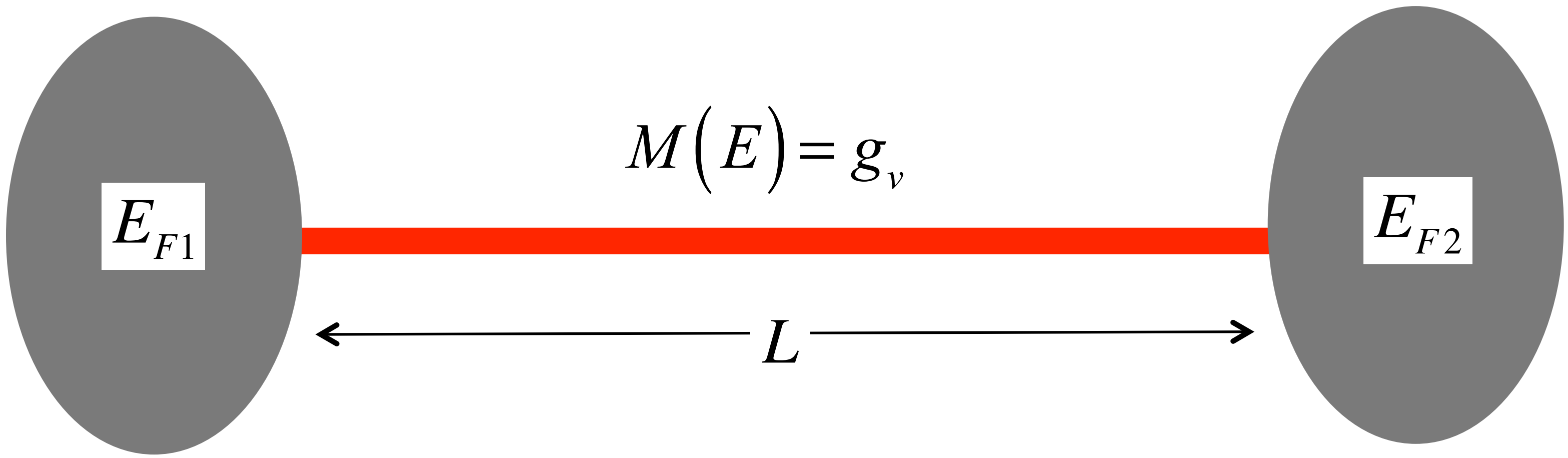
$$I = \frac{2q}{h}\int \mathcal{T}(E) M(E)\left(f_1 - f_2\right) dE$$

$$\left(f_1 - f_2\right) \rightarrow \left(-\frac{\partial f_0}{\partial E}\right) qV$$

$$I = GV \qquad G = \frac{2q^2}{h}\int \mathcal{T}(E) M(E)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

Linear (near-equilibrium) transport.

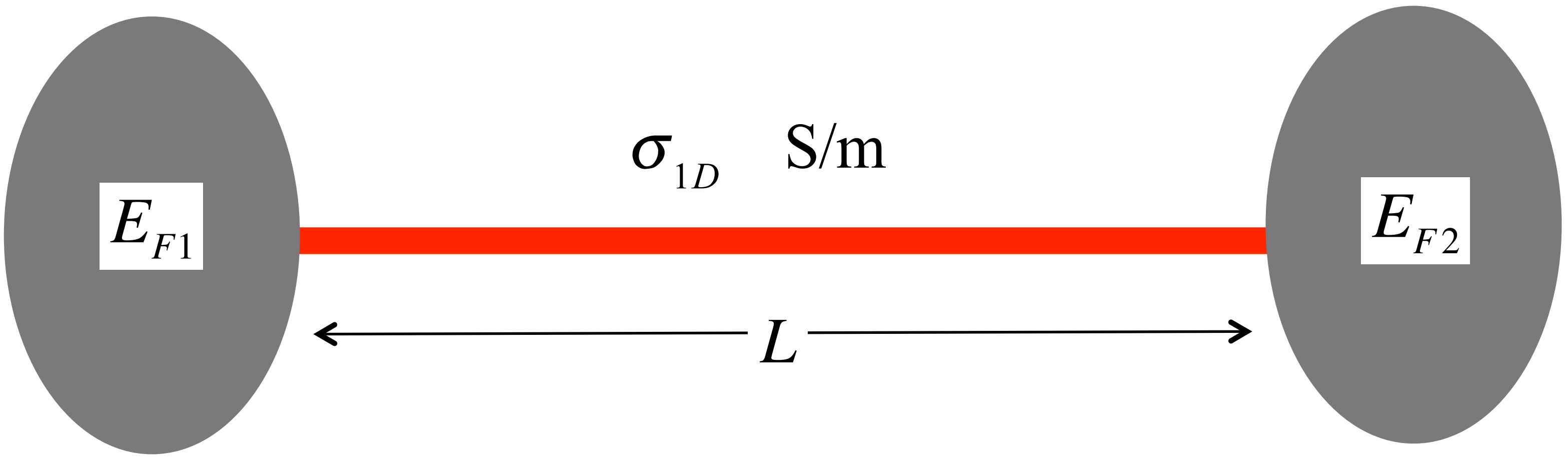
1D Resistor



Independent of bandstructure (except for valley degeneracy).

Because it simplifies the math, we will use a 1D resistor to illustrate several, more general points.

1D Resistor: conventional, diffusive



$$R = \sigma_{1D} L \qquad \sigma_{1D} = n_L q \mu_n \text{ S-m}$$

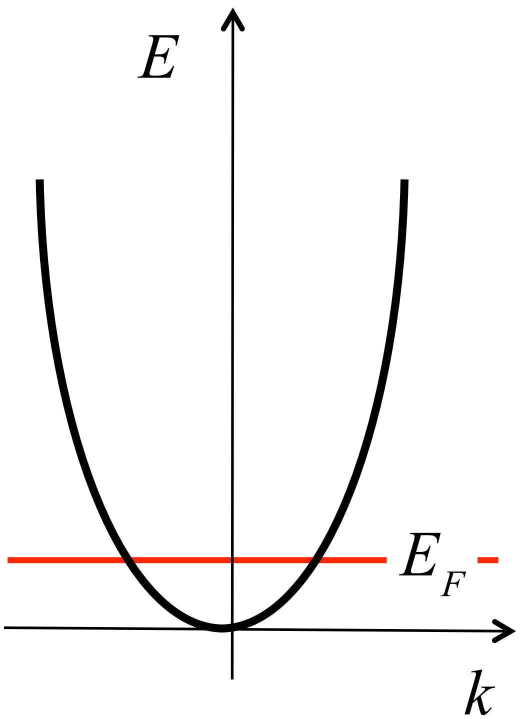
What answer does the Landauer Approach give?

Outline

1) **Fully degenerate, ballistic, 1D conductance**
2) Effect of scattering
3) Apparent conductivity and apparent MFP
4) Mobility and apparent mobility
5) Alternative expressions for conductivity
6) Other materials and dimensions

Degenerate conditions

Semiconductor (T = 0 K)

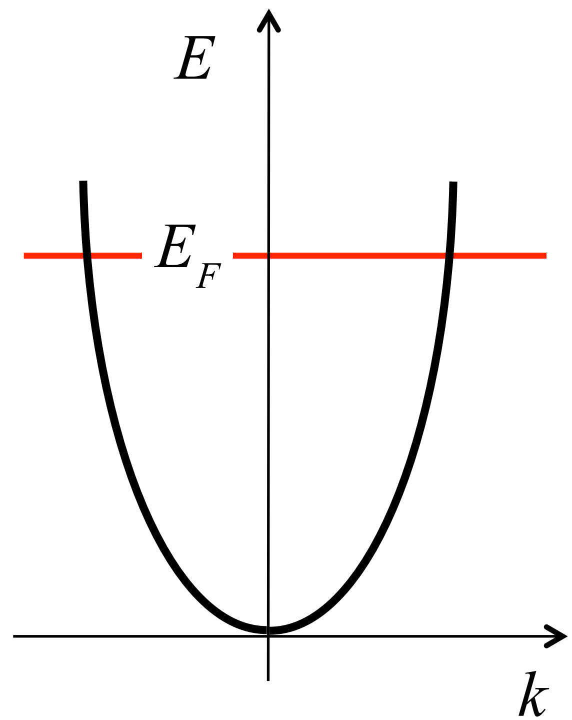


In both cases, we can assume:

$$\left(-\frac{\partial f_0}{\partial E}\right) = \delta\left(E = E_F\right)$$

$$f_0\left(E\right) = \frac{1}{1 + e^{(E-E_F)/k_BT}}$$

Metal (T = 300 K)



Ballistic current: degenerate

$$G = \frac{2q}{h}\int \mathcal{T}(E) M_{1D}(E)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$G = \frac{2q}{h}\mathcal{T}(E_F) M_{1D}(E_F)$$

$$G_B = \frac{2q}{h} g_v$$

ballistic conductance of a nanowire at T = 0 K.

Ballistic transport:

$$\mathcal{T}(E_F) = 1$$

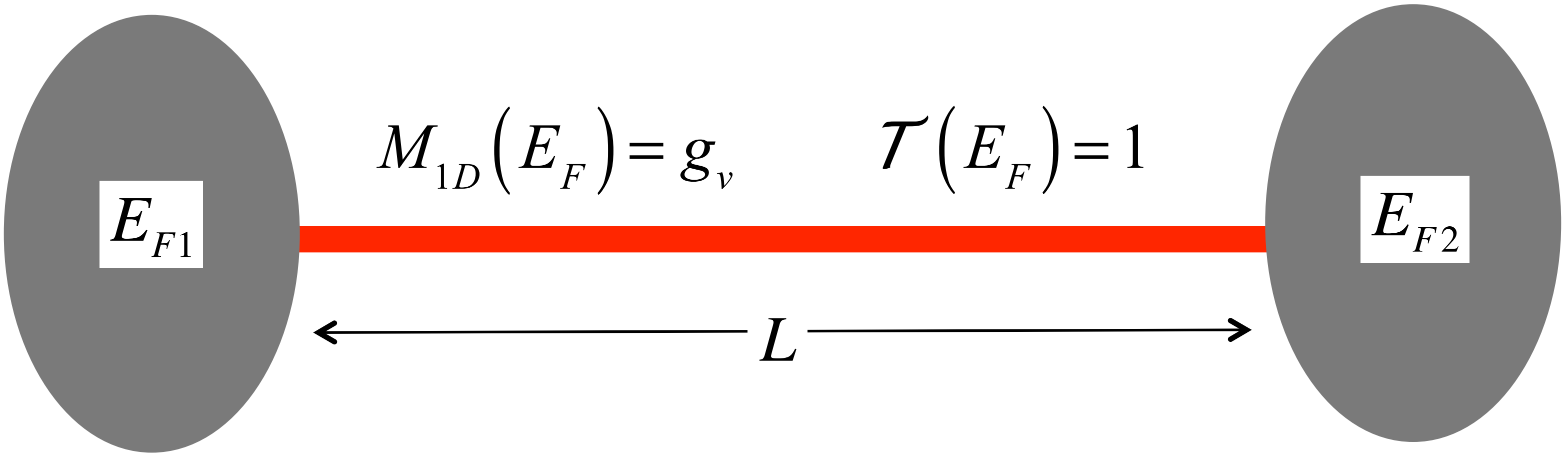
1D:

$$M_{1D}(E_F) = g_v$$

Degenerate:

$$\left(-\frac{\partial f_0}{\partial E}\right) = \delta(E = E_F)$$

1D Resistor: ballistic and fully degenerate



$$I = G_B V = V/R_B$$

$$G_B = \frac{2q^2}{h} M_{1D}(E_F) = 1/R_B$$

Resistance is independent of length!

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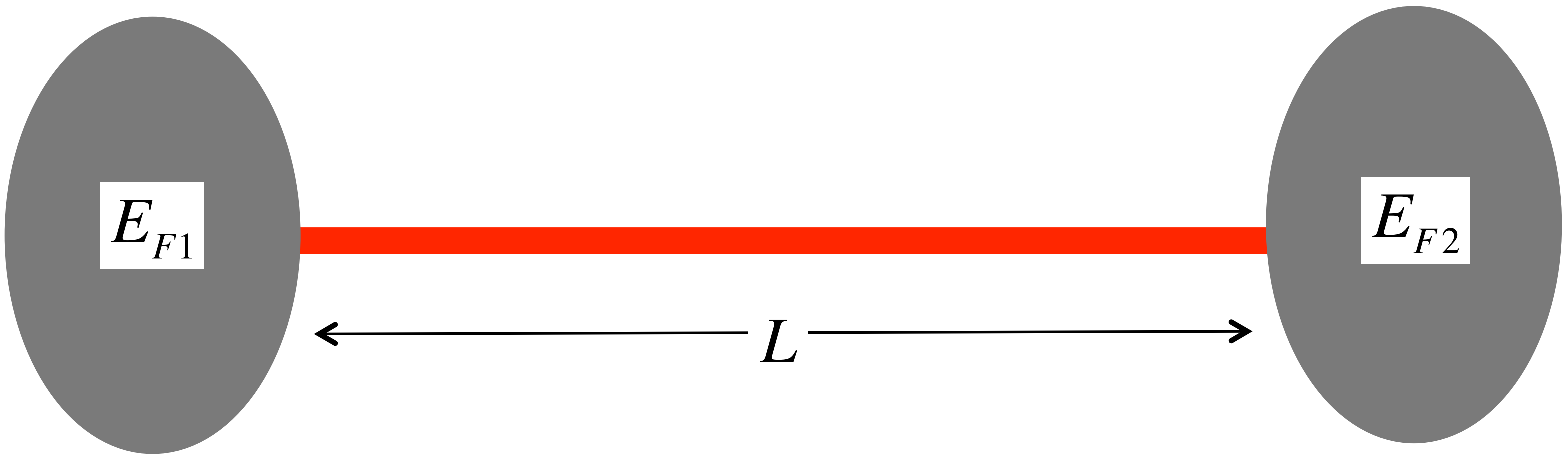
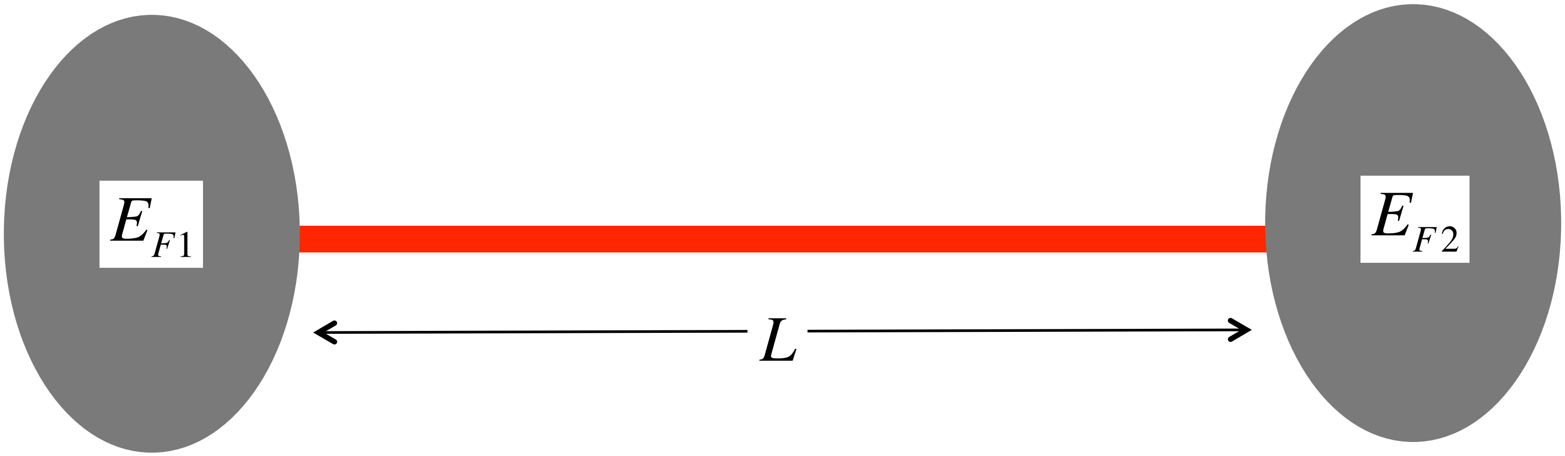
1D Resistor: at T = 0 K with scattering



$$G = \frac{2q^2}{h}\int \mathcal{T}(E) M_{1D}(E)\left(-\frac{\partial f_0}{\partial E}\right) dE \rightarrow \frac{2q^2}{h}\mathcal{T}(E_F)M_{1D}(E_F) = \frac{1}{R}$$

$$\mathcal{T}(E_F) = \frac{\lambda(E_F)}{\lambda(E_F)+L} \qquad M_{1D}(E) = g_v$$

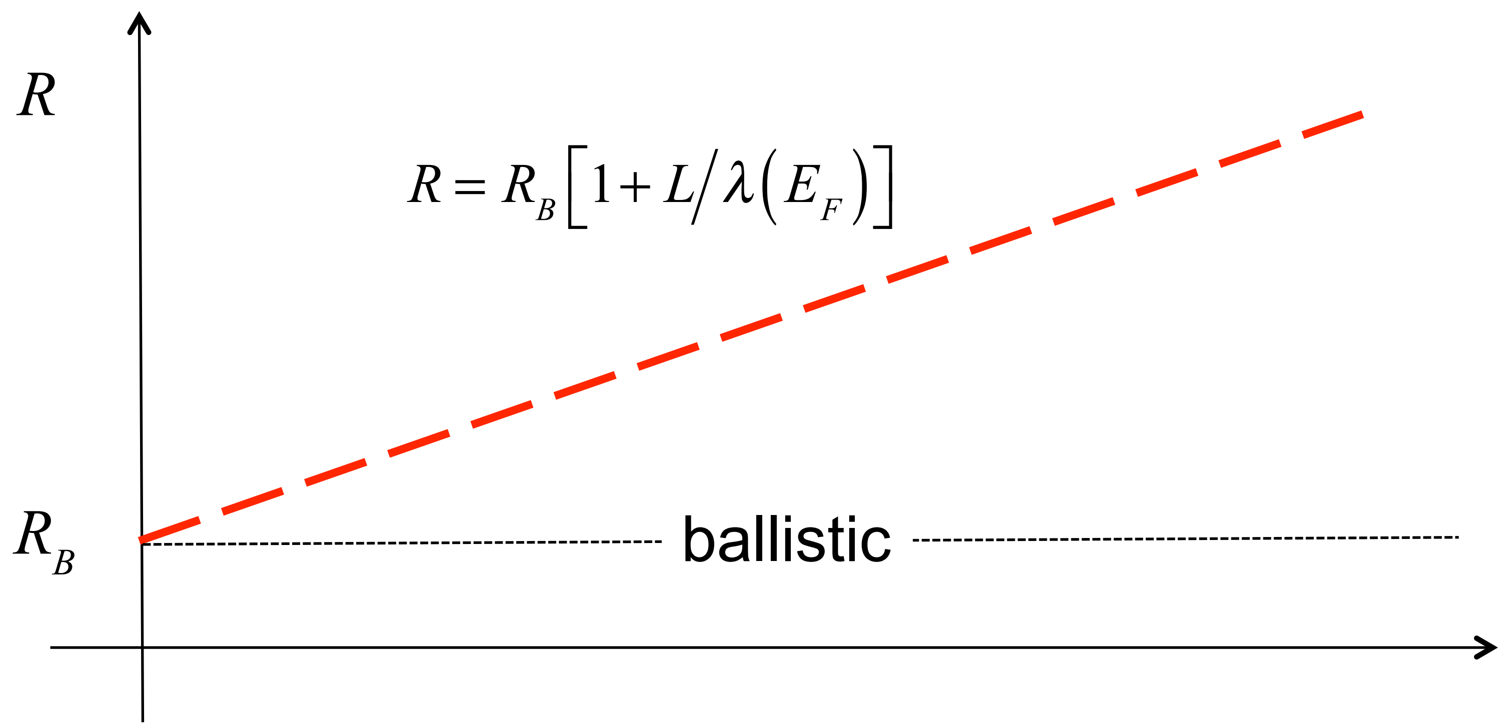
1D Resistor: with scattering (degenerate)



$$R = R_B\left(1 + \frac{L}{\lambda(E_F)}\right) \qquad R_B = \frac{h}{2q^2}\frac{1}{M_{1D}(E_F)}$$

$$R = R_B + \rho_{1D}L \qquad \rho_{1D} = \frac{R_B}{\lambda(E_F)} = \frac{h}{2q^2 M_{1D}(E_F)}\frac{1}{\lambda(E_F)}$$

1D Resistor: with scattering (degenerate)



$$R_B = \frac{h}{2q^2}\frac{1}{M_{1D}(E_F)}$$

ballistic resistance

1D Resistor: diffusive limit

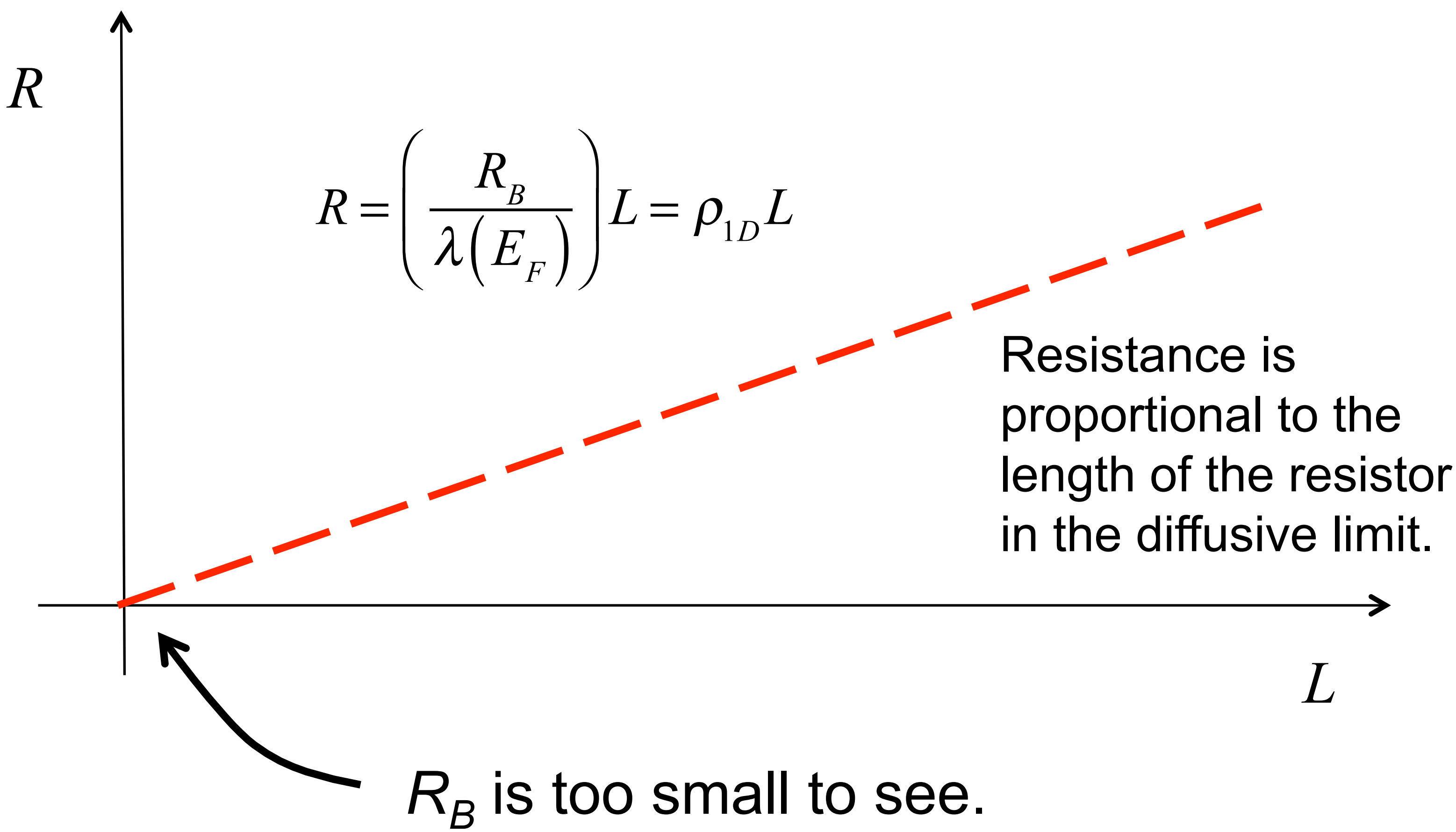
$$L >> \lambda\left(E_F\right)$$

$$R = R_B\left(1+\frac{L}{\lambda\left(E_F\right)}\right) \rightarrow R_B\frac{L}{\lambda\left(E_F\right)}$$

$$R = \rho_{1D}L \qquad \rho_{1D} = \frac{h}{2q^2 M_{1D}\left(E_F\right)}\frac{1}{\lambda\left(E_F\right)}$$

Conventional approach: $\rho_{1D} = \dfrac{1}{n_L q \mu_n}$

1D Resistor: diffusive limit



Outline

1) Fully degenerate, ballistic, 1D conductance
2) Effect of scattering
3) **Apparent conductivity and apparent MFP**
4) Mobility and apparent mobility
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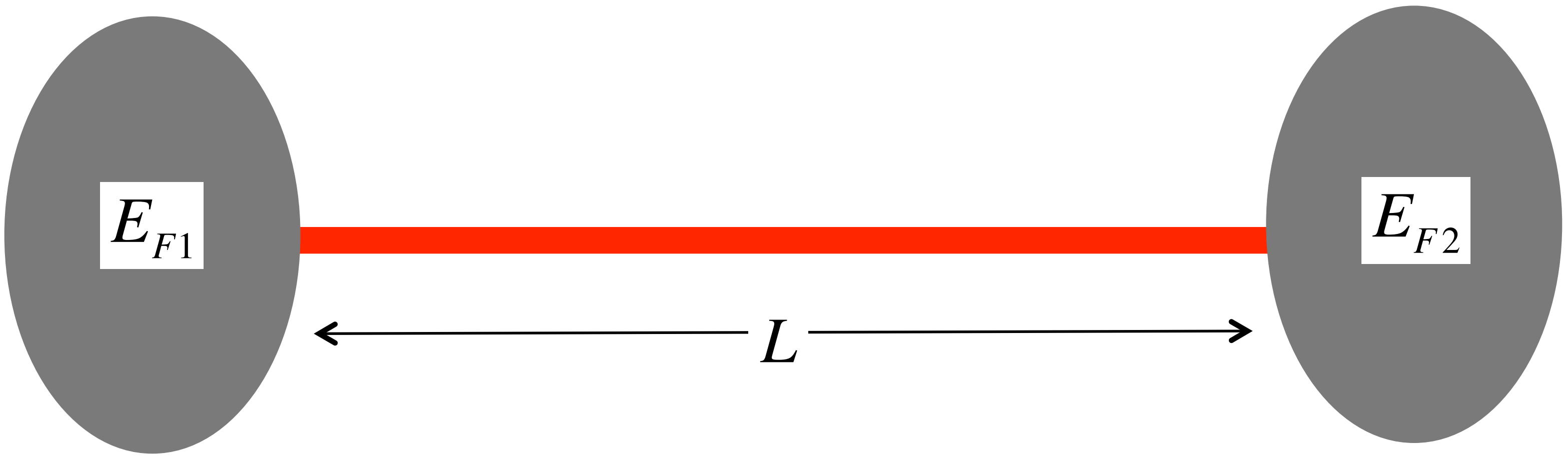
1D Resistor: apparent mfp



$$R = R_B\left(1+\frac{L}{\lambda(E_F)}\right) = R_B\left(\frac{1}{L}+\frac{1}{\lambda(E_F)}\right)L = \left(\frac{R_B}{\lambda_{app}(E_F)}\right)L$$

$$\frac{1}{\lambda_{app}(L)} \equiv \left(\frac{1}{L}+\frac{1}{\lambda(E_F)}\right)$$

Looks like the diffusive result, but with a length-dependent MFP.

1D Resistor: apparent conductivity

$$R = \left(\frac{R_B}{\lambda_{app}(L)} \right) L = \frac{1}{\sigma_{1D}^{app}(L)} L$$

$$R_B = \frac{h}{2q^2} \frac{1}{M_{1D}(E_F)}$$

$$\sigma_{1D}^{app} = \frac{2q^2}{h} M_{1D}(E_F) \lambda_{app}(E_F)$$

1) The physical conductivity is a material parameter – independent of the length of the resistor.

2) The apparent conductivity is a length dependent quantity. It is a way to write the resistance in a way that looks like the traditional diffusive result.

3) This equation is valid all the way from the ballistic to diffusive limit.

1D Resistor: apparent MFP

$$\frac{1}{\lambda_{app}(L)} = \left(\frac{1}{L} + \frac{1}{\lambda(E_F)} \right)$$

1) The physical MFP is a material parameter − generally independent of the length of the resistor.

2) The apparent MFP is a length dependent quantity. It is used to write the resistance in a way that looks like the traditional diffusive result.

3) The apparent MFP is the shorter of the actual MFP and the length of the resistor.

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Mobility

$$G = \sigma_{1D} / L$$ (diffusive limit)

Conventional expression:

$$\sigma_{1D} = n_L q \mu_n$$

Landauer expression:

$$\sigma_{1D} = \frac{2q^2}{h} M_{1D}(E_F)\lambda(E_F)$$

Equate the two expressions:

$$\mu_n = \frac{2q}{h} \frac{M_{1D}(E_F)\lambda(E_F)}{n_L}$$

Exercise:

For a parabolic band semiconductor, show that:

$$\mu_n = \frac{2q}{h} \frac{M_{1D}\left(E_F\right)\lambda\left(E_F\right)}{n_L} = \frac{q\tau_m\left(E_F\right)}{m^*}$$

Mobility from conductivity

For materials with non-parabolic band structures, one should NOT try to deduce the mobility from:

$$\mu_n = q\tau_m\left(E_F\right)/m^*$$

Instead, compute conductivity first: $\sigma_{1D} = \dfrac{2q^2}{h} M_{1D}\left(E_F\right)\lambda\left(E_F\right)$

Then deduce the mobility from: $\mu_n = \dfrac{\sigma_{1D}}{n_L q}$

Apparent mobility

$$G = \sigma_{1D}^{app}(L)/L \qquad \sigma_{1D}^{app} = \frac{2q^2}{h} M_{1D}(E_F)\lambda_{app}(E_F) \qquad = n_L q \mu_{app}$$

$$\mu_n = \frac{2q}{h}\frac{M_{1D}(E_F)\lambda(E_F)}{n_L}$$

$$\mu_n^{app} = \frac{2q}{h}\frac{M_{1D}(E_F)\lambda_{app}(E_F)}{n_L}$$

i) $L >> \lambda$ $\qquad \mu_{app} \to \mu_n$

ii) $L << \lambda$ $\qquad \mu_{app} \to \mu_B$

$$\mu_B = \frac{2q}{h}\frac{M_{1D}(E_F)L}{n_L}$$

Ballistic mobility

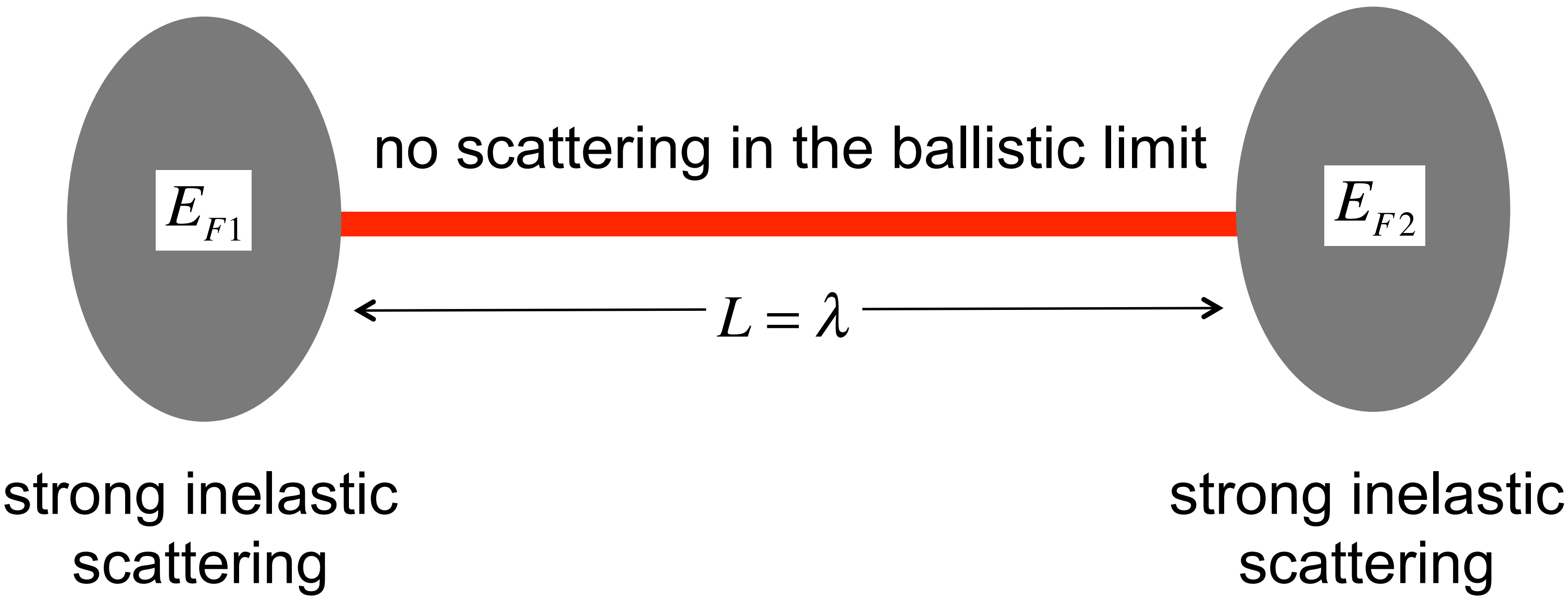
$$\mu_n^{app} = \frac{2q}{h}\frac{M_{1D}\left(E_F\right)\lambda_{app}\left(E_F\right)}{n_L}$$

$$\frac{1}{\lambda_{app}(L)} = \left(\frac{1}{L} + \frac{1}{\lambda\left(E_F\right)}\right)$$

$L << \lambda$ $\lambda_{app}(L) \rightarrow L$ $\mu_n^{app} \rightarrow \mu_B = \frac{2q}{h}\frac{M_{1D}\left(E_F\right)L}{n_L}$

In the ballistic limit, the MFP is replaced by the length of the resistor. How do we interpret this result?

Ballistic mobility



All scattering occurs in the contacts. No scattering occurs in the device. An electron that travels from contact 1 to contact 2 travels a distance of *L* between scattering events, so in the ballistic limit, the MFP is *L*.

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Alternative expression for diffusive conductivity

$$\sigma_{1D} = \frac{2q^2}{h}\lambda\left(E_F\right)M_{1D}\left(E_F\right)$$

Landauer

$$\sigma_{1D} = n_L q \mu_n$$

Traditional

There are several other ways to write the conductivity...

Four ways to write the conductivity

$$\sigma_{1D} = \frac{2q^2}{h}\lambda\left(E_F\right)M_{1D}\left(E_F\right)$$

$$\sigma_{1D} = q^2\upsilon_F^2\tau_m\left(E_F\right)D_{1D}\left(E_E\right)$$

$$\sigma_{1D} = q^2 D_n\left(E_F\right)D_{1D}\left(E_F\right)$$

$$\sigma_{1D} = n_L q\mu_n$$

frequently derived from the BTE

$$D_n\left(E\right) \equiv \frac{\upsilon_F\lambda\left(E_F\right)}{2}$$

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Other materials and dimensions

$$G = \frac{2q}{h} \int \mathcal{T}(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$G = \frac{2q}{h} \int \left\{ \frac{\lambda(E)}{\lambda(E) + L} \right\} M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

To treat other materials and dimensions, use the appropriate *M*(*E)* and MFP(*E*).

Other materials and dimensions

$$M(E) = M_{1D}(E) = \frac{h}{4}\langle \upsilon_x^+ \rangle D_{1D}(E)$$

DOS per unit energy and length

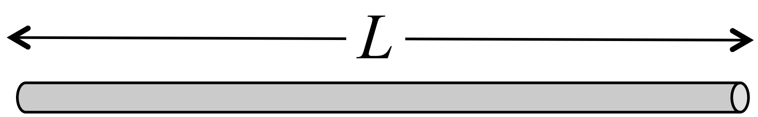
$$M(E) = M_{2D}(E)W = W\frac{h}{4}\langle \upsilon_x^+ \rangle D_{2D}(E)$$

DOS per unit energy and area

$$M(E) = M_{3D}(E)A = A\frac{h}{4}\langle \upsilon_x^+ \rangle D_{3D}(E)$$

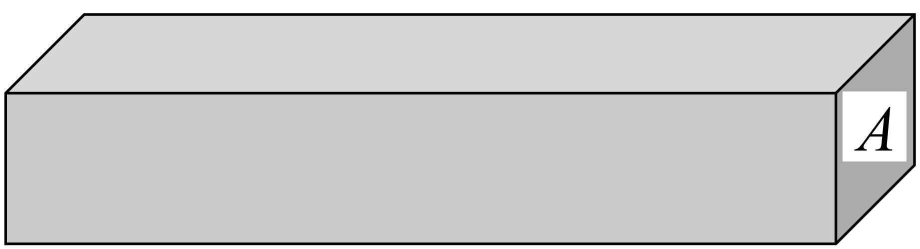
DOS per unit energy and volume

1D, 2D, and 2D Resistors



$$R_{1D} = \rho_{1D} L \qquad \rho_{1D} = \frac{1}{n_L q \mu_n}$$

$$R_{2D} = \rho_{2D} \frac{L}{W} \qquad \rho_{2D} = \frac{1}{n_S q \mu_n}$$



$$R_{3D} = \rho_{3D} \frac{L}{A} \qquad \rho_{3D} = \frac{1}{n q \mu_n}$$

Conductivity in 1D, 2D, and 3D

$$G_{1D} = \sigma_{1D} \frac{1}{L} \qquad \sigma_{1D} = \frac{2q^2}{h} \lambda(E_F) M(E_F) \qquad \sigma_{1D} = \frac{2q^2}{h} \lambda(E_F) M_{1D}(E_F)$$

$$G_{2D} = \sigma_{1D} \frac{W}{L} \qquad \sigma_{2D} = \frac{2q^2}{h} \lambda(E_F) M(E_F) / W \qquad \sigma_{2D} = \frac{2q^2}{h} \lambda(E_F) M_{2D}(E_F)$$

$$G_{3D} = \sigma_{1D} \frac{A}{L} \qquad \sigma_{3D} = \frac{2q^2}{h} \lambda(E_F) M(E_F) / A \qquad \sigma_{3D} = \frac{2q^2}{h} \lambda(E_F) M_{3D}(E_F)$$

All of the results we discussed in 1D, can be done in 2D and 3D for any material (e.g. mobility, apparent mobility, apparent MFP, apparent conductivity, etc.

Questions?

Ballistic resistors

Diffusive resistors

Ballistic to diffusive

Conductivity and apparent conductivity

Mobility, apparent mobility, and ballistic mobility