ECE 656: Electronic Transport in Semiconductors Fall 2017

Resistors at T > 0 K

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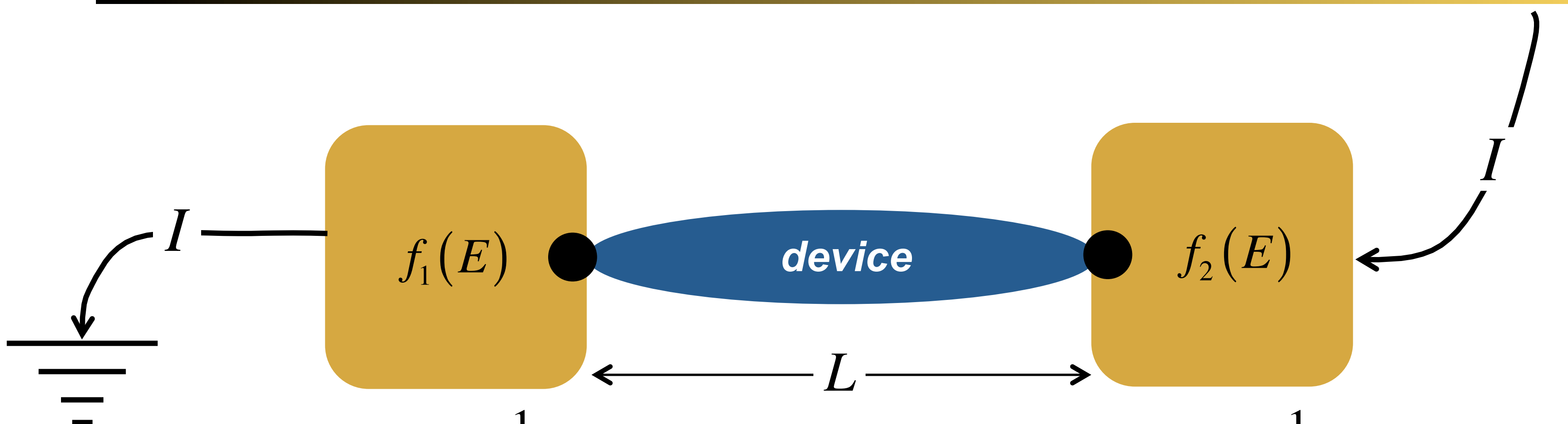
Landauer Approach



$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}} \qquad f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) (f_1 - f_2)\, dE$$

transmission, modes (channels), differences in Fermi functions

Near-equilibrium (constant temperature)

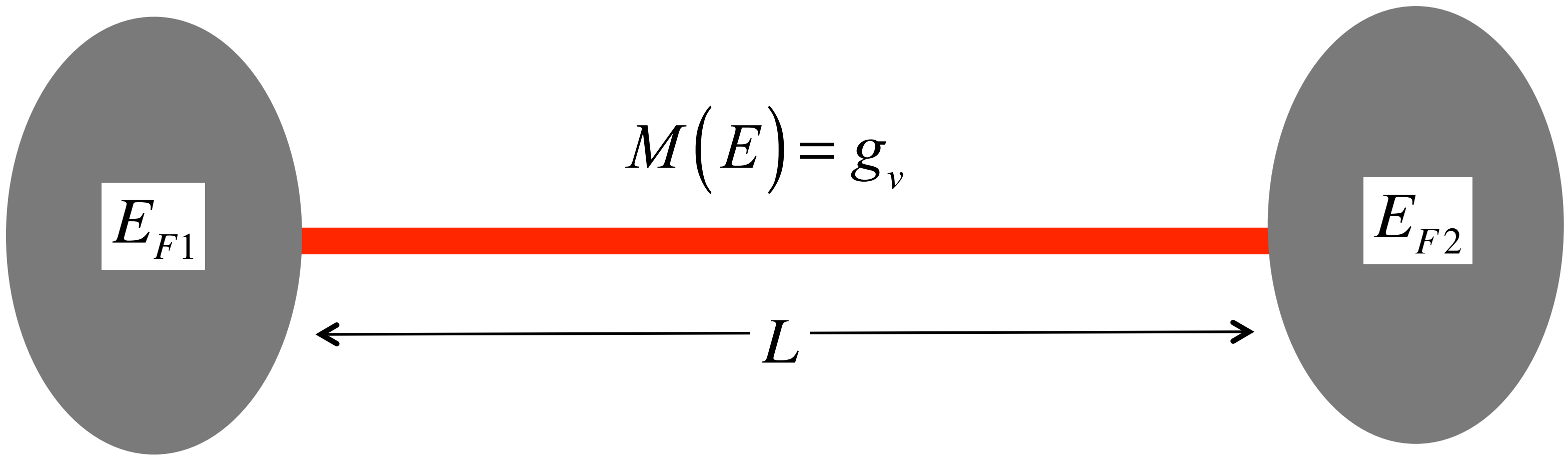
$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)(f_1 - f_2)dE$$

$$(f_1 - f_2) \rightarrow \left(-\frac{\partial f_0}{\partial E}\right)qV$$

$$I = GV \qquad G = \frac{2q^2}{h}\int \mathcal{T}(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

Linear (near-equilibrium) transport.

1D Resistor



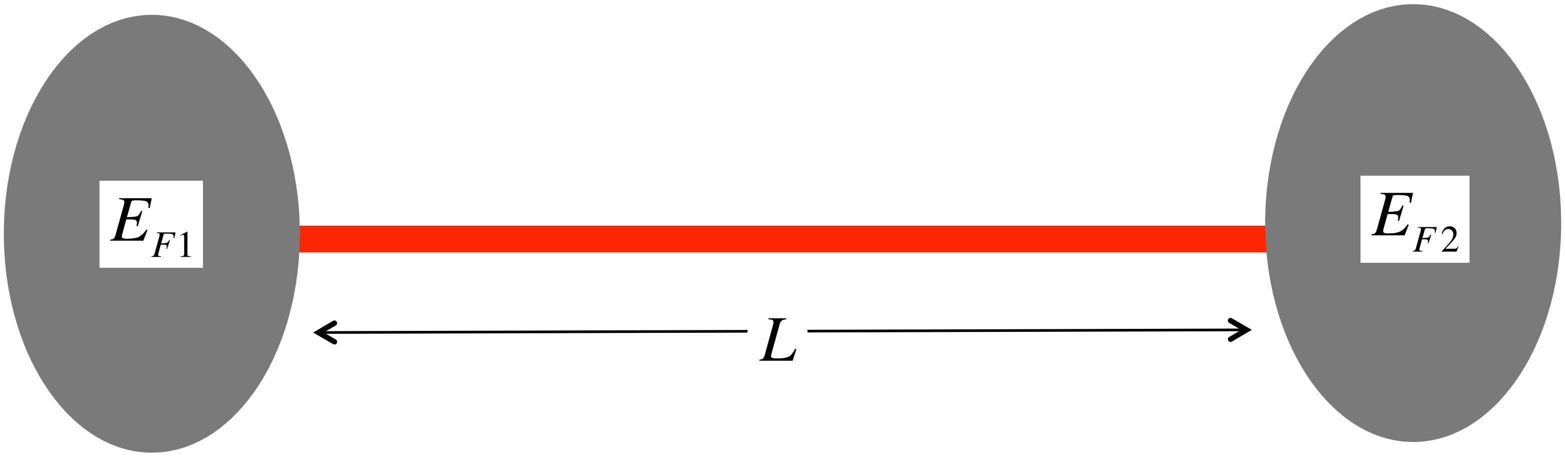
Independent of bandstructure (except for valley degeneracy).

Because it simplifies the math, we will use a 1D resistor to illustrate several, more general points.

Outline

1) Introduction / recap
2) **Ballistic resistor for T > 0 K**
3) Constant MFP
4) Energy-dependent MFP
5) Diffusive limit
6) Power law scattering
7) Other materials and dimensions
8) Questions

Ballistic transport at T > 0 K



$$\mathcal{T}(E) = 1 \qquad G_B = \frac{2q}{h}\int M_{1D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$G_B = \frac{2q^2}{h}\langle M_{1D}\rangle \qquad \langle M\rangle = \int M_{1D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

Average number of channels in the Fermi window.

Ballistic transport at T > 0 K

$$G_B = \frac{2q^2}{h} M_{1D}\left(E_F\right) \rightarrow \frac{2q^2}{h}\left\langle M_{1D}\right\rangle$$

T = 0 K $\qquad\qquad$ T > 0 K

$$\left\langle M_{1D}\right\rangle = \frac{\int M_{1D}\left(E\right)\left(-\partial f_0/\partial E\right)dE}{\int\left(-\partial f_0/\partial E\right)dE}$$

At T = 0 K, the ballistic conductance is proportional to the number of channels at the Fermi energy.

For T > 0 K, the ballistic conductance is proportional the average number of channels in the Fermi window.

Exercise

Assume a parabolic band, ballistic nanowire.

$$M(E) = M_{1\mathrm{D}}(E) = g_V$$

Evaluate the ballistic conductance:

$$G_B = \frac{2q^2}{h}\langle M_{1D}\rangle \qquad \langle M_{1D}\rangle = \frac{\int M_{1D}(E)\left(-\partial f_0/\partial E\right)dE}{\int\left(-\partial f_0/\partial E\right)dE}$$

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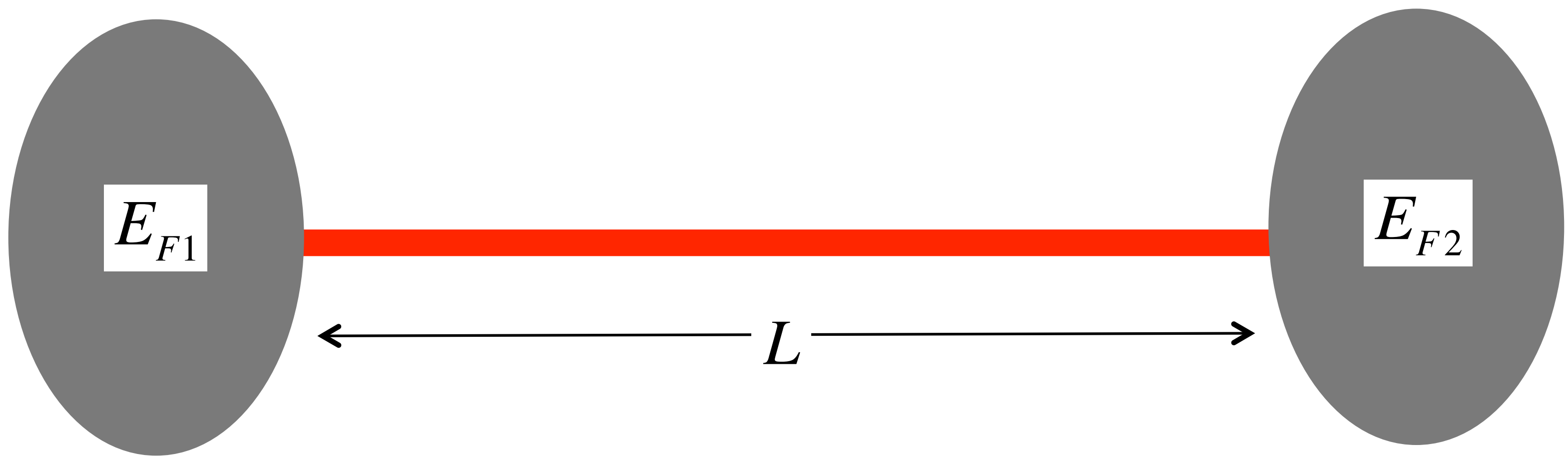
Energy-independent scattering



$$G = \frac{2q^2}{h}\int \mathcal{T}(E) M(E)\left(-\frac{\partial f_0}{\partial E}\right) dE \qquad \mathcal{T}(E) = \frac{\lambda_0}{\lambda_0 + L}$$

$$G = \frac{2q^2}{h}\left(\frac{\lambda_0}{\lambda_0 + L}\right)\int M_{1D}(E)\left(-\frac{\partial f_0}{\partial E}\right) dE = \frac{2q^2}{h}\left(\frac{\lambda_0}{\lambda_0 + L}\right)\langle M_{1D}\rangle$$

Energy-independent scattering

$$G = \frac{2q^2}{h}\left(\frac{\lambda_0}{\lambda_0 + L}\right)\langle M(E)\rangle \qquad \langle M_{1D}\rangle = \frac{\int M_{1D}(E)\left(-\partial f_0/\partial E\right)dE}{\int\left(-\partial f_0/\partial E\right)dE}$$

$$R = \frac{1}{G} = \frac{h}{2q^2}\frac{1}{\langle M(E)\rangle}\left(1 + \frac{L}{\lambda_0}\right) = R_B\left(1 + \frac{L}{\lambda_0}\right)$$

A plot of *R* vs. *L* is linear and can be used to determine the ballistic resistance and the MFP.

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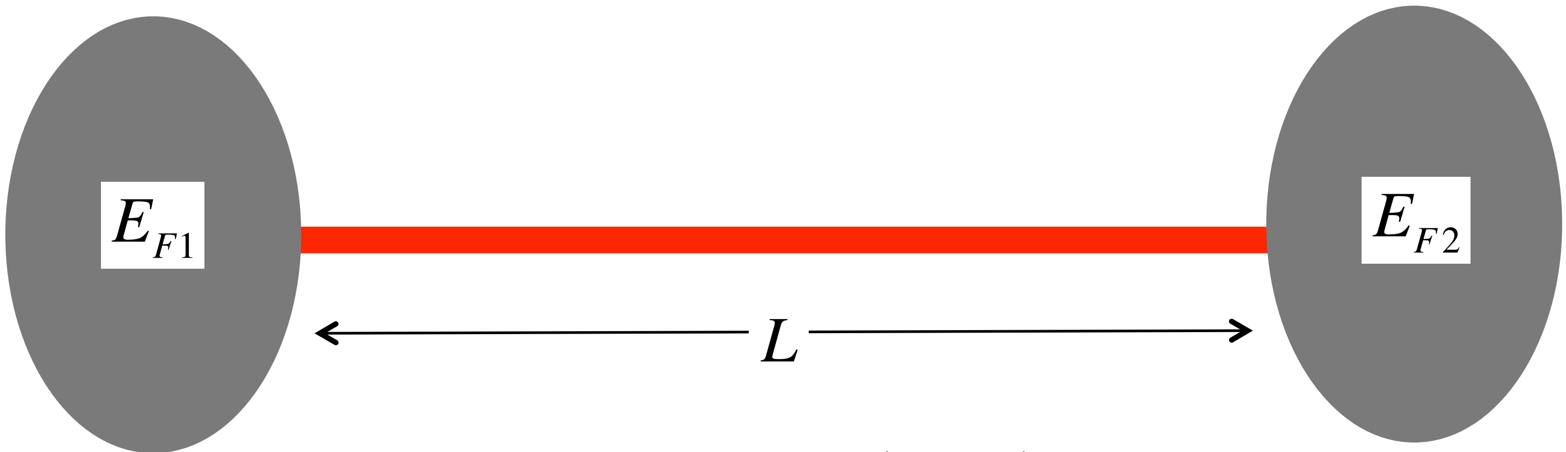
Energy-dependent MFP



$$G = \frac{2q^2}{h} \int \mathcal{T}(E) M_{1D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$\mathcal{T}(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

Energy dependent MFP

$$G = \frac{2q^2}{h} \int \mathcal{T}(E) M_{1D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$G = \frac{2q^2}{h} \left\{ \frac{\int \mathcal{T}(E) M_{1D}(E) \left(-\partial f_0 / \partial E\right) dE}{\int M_{1D}(E) \left(-\partial f_0 / \partial E\right) dE} \right\} \int M_{1D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$G = \frac{2q^2}{h} \left\langle \left\langle \mathcal{T}(E) \right\rangle \right\rangle \left\langle M_{1D}(E) \right\rangle$$

Energy dependent MFP

$$G = \frac{2q^2}{h} \langle\langle \mathcal{T}(E) \rangle\rangle \langle M(E) \rangle = 1/R$$

$$G = \frac{2q^2}{h} \mathcal{T}(E_F) M(E_F) \quad T = 0\text{ K}$$

A Plot of *R* vs. *L* is not strictly linear for *T* > 0 K.

$$\langle\langle \mathcal{T}(E) \rangle\rangle = \left\{ \frac{\int \left\{ \frac{\lambda(E)}{\lambda(E) + L} \right\} M_{1D}(E)\left(-\partial f_0/\partial E\right) dE}{\int M_{1D}(E)\left(-\partial f_0/\partial E\right) dE} \right\}$$

“Differential conductance”

Because, in general, the integrals need to be done numerically, it is convenient to express the conductance in terms of a “differential conductance”.

$$G = \frac{2q^2}{h}\int \mathcal{T}(E) M_{1D}(E)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$G = \int G'(E)\,dE \qquad G' \equiv \frac{2q^2}{h}\mathcal{T}(E) M_{1D}(E)\left(-\frac{\partial f_0}{\partial E}\right)$$

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Diffusive limit

$$L >> \lambda(E)$$

$$\mathcal{T}(E) = \frac{\lambda(E)}{\lambda(E) + L} \rightarrow \frac{\lambda(E)}{L}$$

$$G = \frac{2q^2}{h} \int \mathcal{T}(E) M_{1D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \rightarrow \frac{2q^2}{h} \int \frac{\lambda(E)}{L} M_{1D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

In the diffusive limit, it is convenient to define a conductivity.

Conductivity and differential conductivity

$$G = \frac{2q^2}{h}\int \frac{\lambda(E)}{L} M_{1D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \;= \sigma_{1D}\frac{1}{L}$$ (diffusive)

Conductivity:

$$\sigma_{1D} = \frac{2q^2}{h}\int \lambda(E) M_{1D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

Differential conductivity:

$$\sigma_{1D} = \int \sigma'_{1D}(E)\,dE \qquad \sigma'_{1D} \equiv \frac{2q^2}{h}\lambda(E) M_{1D}(E)\left(-\frac{\partial f_0}{\partial E}\right)$$

General expression for the conductivity

$$\sigma_{1D} = \frac{2q^2}{h} \int \lambda(E) M_{1D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$\sigma_{1D} = \frac{2q^2}{h} \langle\langle \lambda(E) \rangle\rangle \langle M_{1D}(E) \rangle$$

$$\langle\langle \lambda(E) \rangle\rangle \equiv \frac{\int \lambda(E) M_{1D}(E) \left(-\partial f_0 / \partial E\right) dE}{\int M_{1D}(E) \left(-\partial f_0 / \partial E\right) dE} \qquad \langle M_{1D}(E) \rangle = \int M_{1D}(E) \left(-\partial f_0 / \partial E\right) dE$$

Four ways to write the conductivity

$$\sigma_{1D} = \frac{2q^2}{h}\lambda\left(E_F\right)M_{1D}\left(E_F\right)$$

$$\sigma_{1D} = q^2\upsilon_F^2\tau_m\left(E_F\right)D_{1D}\left(E_F\right)$$

$$\sigma_{1D} = q^2 D_n\left(E_F\right)D_{1D}\left(E_E\right)$$

$$\sigma_{1D} = n_L q\mu_n$$

T = 0K

$$\sigma_{1D} = \frac{2q^2}{h}\int\lambda\left(E\right)M_{1D}\left(E\right)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$\sigma_{1D} = q^2\int\upsilon^2\left(E\right)\tau_m\left(E\right)D_{1D}\left(E\right)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$\sigma_{1D} = q^2\int D_n\left(E\right)D_{1D}\left(E\right)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$\sigma_{1D} = n_L q\mu_n$$

Finite temperature

Discussion

Just as we did for T = 0 K, we can define a mobility and derive an expression for it from the conductivity.

We can also define an apparent MFP, apparent conductivity, apparent mobility, and ballistic mobility, so that we can use conventional expressions from the ballistic to diffusive regimes.

Kubo-Greenwood formula

$$\sigma_{1D} = \frac{2q^2}{h} \left\langle \left\langle \lambda(E) \right\rangle \right\rangle \left\langle M_{1D}(E) \right\rangle$$

diffusive

$$G = n_L q \mu_n \frac{1}{L}$$

$$\mu_n \equiv \frac{1}{n_L} \frac{2q}{h} \left\langle \left\langle \lambda(E) \right\rangle \right\rangle \left\langle M(E) \right\rangle$$

Just as we saw in the previous lecture, to derive the mobility, begin with the conductivity.

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Power law scattering

$$G = \frac{2q^2}{h}\int \mathcal{T}(E) M_{1D}(E)\left(-\frac{\partial f_0}{\partial E}\right) dE \qquad \mathcal{T}(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

For some common scattering mechanisms:

$$\lambda(E) = \lambda_0 \left(\frac{E - E_C}{k_B T}\right)^r$$

In the diffusive limit, this allows us to evaluate the integrals analytically.

1D Resistor: finite temperature

$$\sigma_{1D} = \frac{2q^2}{h}\langle\langle\lambda(E)\rangle\rangle\langle M_{1D}(E)\rangle$$

$$\langle M_{1D}(E)\rangle = \int_{E_C}^{\infty} M_{1D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$\langle M_{1D}(E)\rangle = \int_{E_C}^{\infty} g_V\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$\langle M\rangle = g_V \mathcal{F}_{-1}(\eta_F)$$

1D Resistor: finite temperature

$$\sigma_{1D} = \frac{2q^2}{h} \langle\langle \lambda(E) \rangle\rangle \langle M_{1D}(E) \rangle$$

$$\langle\langle \lambda(E) \rangle\rangle = \lambda_0 \frac{\int_{E_C}^{\infty} \left((E - E_C)/k_B T \right)^r g_V \left(-\frac{\partial f_0}{\partial E} \right) dE}{\int_{E_C}^{\infty} g_V \left(-\frac{\partial f_0}{\partial E} \right) dE}$$

$$\langle\langle \lambda(E) \rangle\rangle = \lambda_0 \, \Gamma(r+1) \frac{\mathcal{F}_{r-1}(\eta_F)}{\mathcal{F}_{-1}(\eta_F)}$$

1D Resistor: finite temperature

$$\sigma_{1D} = \frac{2q^2}{h} \left\langle \left\langle \lambda(E) \right\rangle \right\rangle \left\langle M_{1D}(E) \right\rangle$$

$$\sigma_{1D} = \frac{2q^2}{h} \left\{ \lambda_0 \, \Gamma(r+1) \frac{\mathcal{F}_{r-1}(\eta_F)}{\mathcal{F}_{-1}(\eta_F)} \right\} \left[g_V \mathcal{F}_{-1}(\eta_F) \right]$$

Exercise: Work out the conductivity for 2D and 3D materials with parabolic energy bands and for 2D graphene.

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Other materials and dimensions

$$G = \frac{2q^2}{h} \int \mathcal{T}(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$G = \frac{2q^2}{h} \int \left\{ \frac{\lambda(E)}{\lambda(E) + L} \right\} M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

To treat other materials and dimensions, use the appropriate *M*(*E)* and MFP(*E*).

Other materials and dimensions

Expressions for the conductance for 1D, 2D, and 3D parabolic bands and for 2D graphene can be found in the Appendix of:

Mark Lundstrom and Changwook Jeong, *Near-equilibrium Transport: Fundamentals and Applications*, World Scientific, Singapore, 2013.

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Questions?

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$$G = \frac{2q^2}{h}\langle\langle \mathcal{T}(E)\rangle\rangle\langle M(E)\rangle = 1/R \qquad \sigma = \frac{2q^2}{h}\langle\langle \lambda(E)\rangle\rangle\langle M_{1,2,3D}(E)\rangle$$