ECE 656: Electronic Transport in Semiconductors Fall 2017

Landauer Approach and the BTE

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Landauer Approach vs. BTE (bulk)

$$J_{nx} = \sigma_n \mathcal{E}_x$$

Landauer

$$\sigma_n = \frac{2q^2}{h} \int_{E_C}^{\infty} \lambda(E) M_{1,2,3D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

BTE

$$\sigma_n = \frac{1}{L^D} \sum_{\vec{k}} q^2 \upsilon_x^2 \tau_m \left(-\frac{\partial f_0}{\partial E} \right)$$

By replacing the MFP in the Landauer approach with the apparent MFP, the Landauer Approach works from the ballistic to diffusive limit. The question is: **Are these two answers the same in the diffusive limit?**

Approach

Begin with the diffusive conductivity from the BTE:

$$\sigma_n = \frac{1}{L^D} \sum_{\vec{k}} q^2 \upsilon_x^2 \tau_m \left(-\frac{\partial f_0}{\partial E} \right) \quad D = 1,2,3...$$

And show that it is identical to the diffusive conductivity from the Landauer Approach:

$$\sigma_n = \frac{2q^2}{h} \int_{E_C}^{\infty} \lambda(E) M_{1,2,3D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

Step 1: Convert the sum to an integral

$$\sigma_n = \frac{1}{L}\sum_{\vec{k}} q^2 \upsilon_x^2 \tau_m \left(-\frac{\partial f_0}{\partial E}\right) \quad D = 1$$

$$\sigma_n = \frac{1}{L}\frac{L}{2\pi}\times 2\times 2\int_0^\infty q^2 \upsilon_x^2 \tau_m \left(-\frac{\partial f_0}{\partial E}\right) dk$$

The first 2 is for spin and the second 2 is to include both the +k and −k branches.

$$\sigma_n = \frac{2q^2}{\pi}\int_0^\infty \upsilon_x^2 \tau_m \left(-\frac{\partial f_0}{\partial E}\right) dk$$

Step 2: Assume parabolic energy bands

$$\sigma_n = \frac{1}{L}\sum_{\vec{k}} q^2 \upsilon_x^2 \tau_m \left(-\frac{\partial f_0}{\partial E}\right) \rightarrow \qquad \sigma_n = \frac{2q^2}{\pi}\int_0^{\infty} \upsilon_x^2 \tau_m \left(-\frac{\partial f_0}{\partial E}\right) dk$$

Now, assume parabolic bands:

$$E - E_C = \frac{\hbar^2 k^2}{2m^*} \qquad dE = \frac{\hbar^2}{m^*} k dk \qquad k = \frac{\sqrt{2m^*\left(E - E_C\right)}}{\hbar}$$

$$dk = \frac{m^*}{\hbar}\frac{dE}{\sqrt{2m^*\left(E - E_C\right)}}$$

Next, convert from an integral over *k* to an integral over *E*.

Step 3: Convert to integral over energy

$$\sigma_n = \frac{2q^2}{\pi}\left\{\int_0^\infty \upsilon_x^2 \tau_m \frac{m^*}{\hbar}\frac{dE}{\sqrt{2m^*\left(E-E_C\right)}}\left(-\frac{\partial f_0}{\partial E}\right)dE\right\}$$

Recall: $$D_{1D}\left(E\right)dE = \frac{2}{\pi\hbar}\sqrt{\frac{m^*}{2\left(E-E_C\right)}}dE$$

$$\sigma_n = 2q^2\left\{\int_0^\infty \frac{\upsilon_x^2\tau_m}{2}D_{1D}\left(E\right)\left(-\frac{\partial f_0}{\partial E}\right)dE\right\}$$

Step 4: Identify terms in the integrand

$$\sigma_n = 2q^2 \left\{ \int_0^\infty \frac{\upsilon_x^2 \tau_m}{2} D_{1D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \right\}$$

Re-write this as:

$$\sigma_n = \frac{4q^2}{h} \left\{ \int_0^\infty \upsilon_x \tau_m \left\{ \frac{h}{4} \upsilon_x(E) D_{1D}(E) \right\} \left(-\frac{\partial f_0}{\partial E} \right) dE \right\}$$

Step 5: Identify modes

$$\sigma_n = \frac{4q^2}{h}\left\{\int_0^{\infty} \upsilon_x \tau_m \left\{\frac{h}{4}\upsilon_x(E) D_{1D}(E)\right\}\left(-\frac{\partial f_0}{\partial E}\right) dE\right\}$$

Recall:

$$M(E) \equiv L^{D-1}\frac{h}{4}\left\langle \upsilon_x^{+}(E)\right\rangle D_{1,2,3D}(E)$$

$$M_{1D}(E) \equiv \frac{h}{4}\upsilon_x(E) D_{1D}(E)$$

$$\sigma_n = \frac{4q^2}{h}\left\{\int_0^{\infty} \upsilon_x \tau_m M_{1D}(E)\left(-\frac{\partial f_0}{\partial E}\right) dE\right\}$$

Step 6: Identify MFP for backscattering

$$\sigma_n = \frac{4q^2}{h}\left\{\int_0^\infty \upsilon_x \tau_m M_{1D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE\right\}$$

$$\sigma_n = \frac{2q^2}{h}\left\{\int_0^\infty 2\upsilon_x \tau_m M_{1D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE\right\}$$

Recall: $\lambda(E) \equiv 2\upsilon_x(E)\tau_m(E)$

$$\sigma_n = \frac{2q^2}{h}\left\{\int_0^\infty \lambda(E) M_{1D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE\right\}$$

Step 7: Final result

$$\sigma_n = \frac{2q^2}{h}\left\{\int_0^\infty \lambda(E) M_{1D}(E)\left(-\frac{\partial f_0}{\partial E}\right) dE\right\}$$

Summary

We have shown that the BTE solution for a 1D nanowire:

$$\sigma_n = q^2 \frac{1}{L} \sum_{\vec{k}} \upsilon_x^2 \tau_m \left(-\frac{\partial f_0}{\partial E} \right)$$

Can be written as:

$$\sigma_n = \frac{2q^2}{h} \left\{ \int_0^\infty \lambda(E) M_{1D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \right\}$$

Where:

$$\lambda(E) = 2\upsilon_x(E)\tau_m(E)$$

$$M_{1D}(E) \equiv \frac{h}{4} \upsilon_x(E) D_{1D}(E)$$

$$D_{1D}(E)dE = \frac{2}{\pi\hbar} \sqrt{\frac{m^*}{2(E - E_C)}} dE$$

Conclusion

For a bulk material (much longer than the MFP), the Landauer approach gives the same result as the BTE, but the Landauer Approach also works for short, ballistic or quasi ballistic conductors.

The BTE can also be used for ballistic and quasi-ballistic transport, but not in such a simple, physically insightful way.