ECE 656: Electronic Transport in Semiconductors Fall 2017

The BTE with a B-field:

Simple Approach

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Introduction

Solving the BTE with a small B-field gives:

$$\vec{J}_n = \sigma_S \vec{\mathcal{E}} - \sigma_S \mu_H \left(\vec{\mathcal{E}} \times \vec{B} \right) \qquad \omega_c \tau_m << 1$$

$$\sigma_S = n_S q \mu_n \qquad \mu_H = \mu_n r_H$$ Hall mobility

$$\mu_n \equiv \frac{q \langle\langle \tau_m \rangle\rangle}{m^*} \qquad r_H \equiv \frac{\langle\langle \tau_m^2 \rangle\rangle}{\langle\langle \tau_m \rangle\rangle^2}$$ Hall factor

A question…

Solving the BTE with a B-field can get mathematically complicated. Is there a simpler way?

A simpler (approximate) approach

$$\vec{F}_e = -q\vec{\mathcal{E}} - q\vec{\upsilon}\times\vec{B} = \frac{d\vec{p}}{dt}$$

Assume that scattering makes *p* = 0 after a scattering time, *τ*

$$\frac{d\vec{p}}{dt} \approx \frac{\vec{p}}{\tau_m} = -q\vec{\mathcal{E}} - q\vec{\upsilon}\times\vec{B}$$

$$\vec{p} = \left(-q\vec{\mathcal{E}} - q\vec{\upsilon}\times\vec{B}\right)\tau_m$$

A simpler (approximate) approach

$$\vec{p} = \left(-q\vec{\mathcal{E}} - q\vec{\upsilon}\times\vec{B}\right)\tau_m = m^*\vec{\upsilon}$$

$$\vec{\upsilon} = -\frac{q\tau_m}{m^*}\vec{\mathcal{E}} - \frac{q\tau_m}{m^*}\vec{\upsilon}\times\vec{B}$$

(Can be solved exactly for the velocity. See prob. 4.18, Lundstrom.)

$$\vec{\upsilon} \approx -\frac{q\tau_m}{m^*}\vec{\mathcal{E}}$$

$$\vec{\upsilon} \approx -\frac{q\tau_m}{m^*}\vec{\mathcal{E}} + \frac{q^2\tau_m^2}{\left(m^*\right)^2}\vec{\mathcal{E}}\times\vec{B}$$

(Low B-fields)

B-field dependent current equation

$$\vec{\upsilon} \approx -\frac{q\tau_m}{m^*}\vec{\mathcal{E}} + \frac{q^2\tau_m^2}{\left(m^*\right)^2}\vec{\mathcal{E}} \times \vec{B}$$

$$\left\langle \vec{\upsilon} \right\rangle \approx -\frac{q\left\langle \tau_m \right\rangle}{m^*}\vec{\mathcal{E}} + \frac{q^2\left\langle \tau_m^2 \right\rangle}{\left(m^*\right)^2}\vec{\mathcal{E}} \times \vec{B}$$

Average velocity of all electrons with different scattering times.

$$\vec{J}_n = -nq\left\langle \vec{\upsilon} \right\rangle = nq\left(\frac{q\left\langle \tau_m \right\rangle}{m^*}\right)\vec{\mathcal{E}} - nq\frac{q^2\left\langle \tau_m^2 \right\rangle}{\left(m^*\right)^2}\vec{\mathcal{E}} \times \vec{B}$$

Current equation

$$\vec{J}_n = nq\left(\frac{q\langle\tau_m\rangle}{m^*}\right)\vec{\mathcal{E}} - nq\frac{q^2\langle\tau_m^2\rangle}{\left(m^*\right)^2}\vec{\mathcal{E}}\times\vec{B}$$

$$\vec{J}_n = nq\mu_n\vec{\mathcal{E}} - nq\frac{q\langle\tau_m\rangle}{m^*}\frac{q\langle\tau_m\rangle}{m^*}\left(\frac{\langle\tau_m^2\rangle}{\langle\tau_m\rangle^2}\right)\vec{\mathcal{E}}\times\vec{B}$$

$$\vec{J}_n = \sigma_n\vec{\mathcal{E}} - \sigma_n\mu_n\left(\frac{\langle\tau_m^2\rangle}{\langle\tau_m\rangle^2}\right)\vec{\mathcal{E}}\times\vec{B}$$

$$\vec{J}_n = \sigma_n\vec{\mathcal{E}} - \left(\sigma_n\mu_H\right)\vec{\mathcal{E}}\times\vec{B}$$

$$\mu_H = r_H\,\mu_n$$

$$r_H = \frac{\langle\tau_m^2\rangle}{\langle\tau_m\rangle^2}$$

Summary

The simple approach gives the same current equation as the BTE:

$$\vec{J}_n = nq\mu_n\vec{\mathcal{E}} - (\sigma_n\mu_H)\vec{\mathcal{E}} \times \vec{B}$$

But not a prescription for computing the averages:

$$\mu_H = r_H \mu_n \qquad r_H = \frac{\langle \tau_m^2 \rangle}{\langle \tau_m \rangle^2}$$

Comment

We have assumed a low B-field.

What does “low” mean?

What happens if the B-field is large?

B-field dependent DD equation: again

$$\vec{F}_e = -q\vec{\mathcal{E}} - q\vec{\upsilon} \times \vec{B} = \frac{d\vec{p}}{dt}$$

$$\vec{p} = \left(-q\vec{\mathcal{E}} - q\vec{\upsilon} \times \vec{B}\right)\tau_m = m^*\vec{\upsilon}$$

$$\vec{\upsilon} = -\frac{q\tau_m}{m^*}\vec{\mathcal{E}} - \frac{q\tau_m}{m^*}\vec{\upsilon} \times \vec{B}$$

Consider a 2D geometry. Electric field in the x-y plane with B-field normal to the plane.

2D solution with a z-directed B-field

$$\vec{\upsilon} = -\frac{q\tau_m}{m^*}\vec{\mathcal{E}} - \frac{q\tau_m}{m^*}\vec{\upsilon}\times\vec{B}$$

$$\upsilon_x = -\frac{q\tau_m}{m^*}\mathcal{E}_x - \frac{q\tau_m}{m^*}\upsilon_y B_z$$

$$\upsilon_y = -\frac{q\tau_m}{m^*}\mathcal{E}_y + \frac{q\tau_m}{m^*}\upsilon_x B_z$$

2D problem

z-directed B-field

Solution in 2D with z-directed B-field

$$\upsilon_x = -\frac{q\tau_m}{m^*}\mathcal{E}_x - \frac{q\tau_m}{m^*}\upsilon_y B_z \qquad \upsilon_y = -\frac{q\tau_m}{m^*}\mathcal{E}_y + \frac{q\tau_m}{m^*}\upsilon_x B_z$$

$$\upsilon_x = -\frac{q\tau_m}{m^*}\mathcal{E}_x + \left(\frac{q\tau_m}{m^*}\right)^2 \mathcal{E}_y B_z - \left(\frac{q\tau_m}{m^*}\right)^2 \upsilon_x B_z^2$$

$$\upsilon_x\left(1+\left(\frac{qB_z}{m^*}\right)^2 \tau_m^2\right) = -\frac{q\tau_m}{m^*}\mathcal{E}_x + \left(\frac{q\tau_m}{m^*}\right)^2 \mathcal{E}_y B_z$$

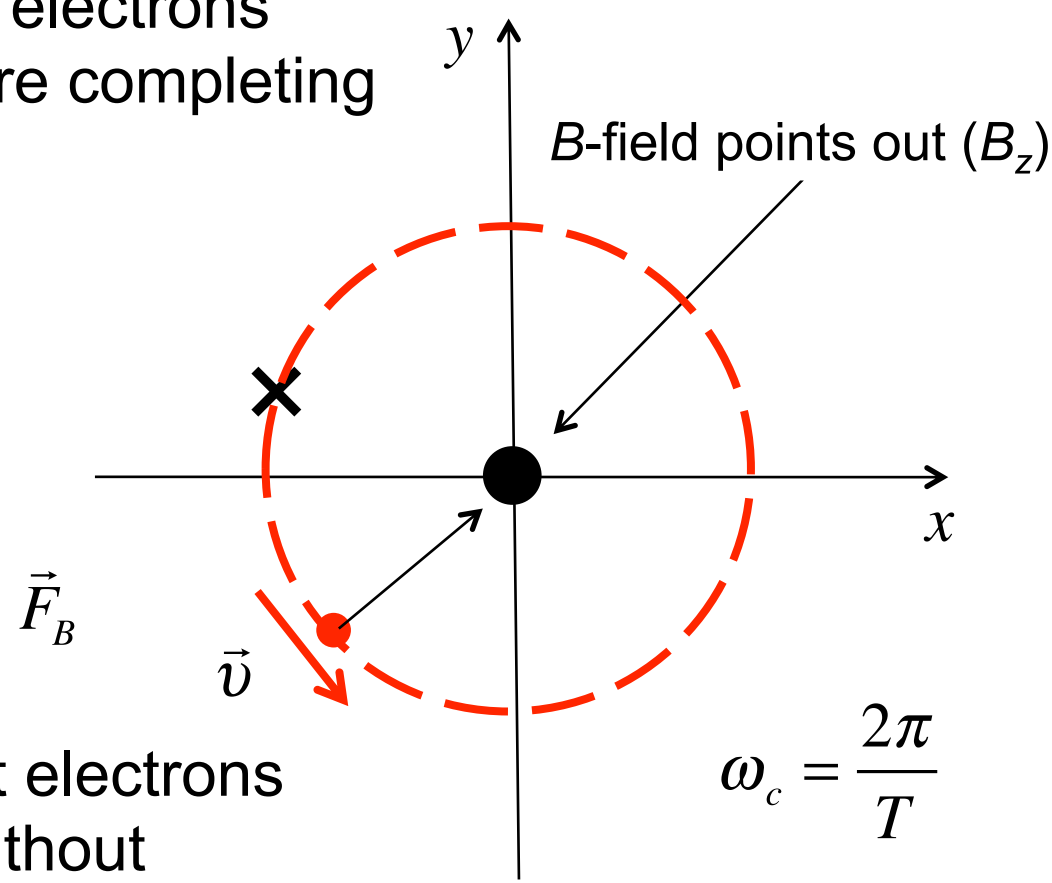
$$\upsilon_x = \frac{-\mu_n \mathcal{E}_x + \mu_n^2 \mathcal{E}_y B_z}{1+\left(\omega_c \tau_m\right)^2}$$

$$\omega_c = \frac{qB_z}{m^*}$$ "cyclotron frequency"

Cyclotron frequency

"Low B-field" means that electrons scatter many times before completing an orbit.

$$\omega_c \tau_m << 1 \rightarrow T >> \tau_m$$



$$\omega_c = \frac{2\pi}{T}$$

"High B-field" means that electrons can complete an orbit without scattering. $\omega_c \tau_m >> 1 \rightarrow T << \tau_m$

Cyclotron frequency

$$\omega_c \tau_m = \frac{qB_z}{m^*}\tau_m = \mu_n B_z$$

Some numbers

assume "silicon"

$$n_0 = 10^{16} \quad \text{cm}^{-3}$$

$$J_n = 10^2 \quad \text{A/cm}^2$$

$$\mu_n = 1000 \quad \text{cm}^2/\text{V-s}$$

$$r_H = 1$$

$$B_z = 2{,}000 \quad \text{Gauss}$$

$$\left(10^4 \text{ Gauss} = 1\text{T}\right)$$

$$W_y = 1\,\mu\text{m}$$

$$\mu_H B_z \approx 0.02 << 1$$

For moderate mobilities and typical B-fields, we are usually in the low B-field regime.

Solution

$$\upsilon_x = \frac{-\mu_n \mathcal{E}_x + \mu_n^2 \mathcal{E}_y B_z}{1+(\omega_c \tau_m)^2} \qquad \upsilon_y = \frac{-\mu_n \mathcal{E}_y - \mu_n^2 \mathcal{E}_y B_z}{1+(\omega_c \tau_m)^2}$$

Assume T = 0 K, so there is no need to average the scattering time.

$$J_x = -nq\upsilon_x = \frac{\sigma_n}{1+(\mu_n B_z)^2}\left(\mathcal{E}_x - \mu_n B_z \mathcal{E}_y\right)$$

$$J_y = -nq\upsilon_y = \frac{\sigma_n}{1+(\mu_n B_z)^2}\left(\mathcal{E}_x + \mu_n B_z \mathcal{E}_y\right)$$

$$\mu_n B_z = \frac{q\tau_m}{m^*} B_z = \omega_c \tau$$

Magneto-conductivity tensor

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \frac{\sigma_n}{1+(\mu_n B_z)^2} \begin{bmatrix} 1 & -\mu_n B_z \\ \mu_n B_z & 1 \end{bmatrix} \begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix}$$

$$(\omega_c \tau_m = \mu_n B_z)$$

$$J_i = \sigma_{ij}(B_z)\mathcal{E}_j$$

Comments

$$\sigma_{ij}\left(B_z\right) = \frac{nq\mu_n}{1+\mu_n B_z}\begin{bmatrix} 1 & -\mu_n B_z \\ \mu_n B_z & 1 \end{bmatrix}$$

1) A magnetic field affects **both** the diagonal and off-diagonal components of the magneto-conductivity tensor.

2) Small magnetic field means: $\mu_n B_z << 1$

$$\omega_c \tau << 1$$

Magneto-resistivity tensor

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \frac{\sigma_n}{1+\left(\mu_n B_z\right)^2} \begin{bmatrix} 1 & -\mu_n B_z \\ \mu_n B_z & 1 \end{bmatrix} \begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix}$$

$$J_i = \sigma_{ij}\left(B_z\right)\mathcal{E}_j$$

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \begin{bmatrix} \sigma_L & -\sigma_T \\ \sigma_T & \sigma_L \end{bmatrix} \begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix}$$

$$\begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix} = \begin{bmatrix} \rho_L & \rho_T \\ -\rho_T & \rho_L \end{bmatrix} \begin{pmatrix} J_x \\ J_y \end{pmatrix}$$

$$\rho_L = \frac{\sigma_L}{\sigma_L^2 + \sigma_T^2} = \frac{1}{\sigma_n}$$

$$\rho_T = \frac{\sigma_T}{\sigma_L^2 + \sigma_T^2} = \frac{\mu_n B_z}{\sigma_n}$$

Comparison

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \frac{\sigma_n}{1+\left(\mu_n B_z\right)^2} \begin{bmatrix} 1 & -\mu_n B_z \\ \mu_n B_z & 1 \end{bmatrix} \begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix}$$

$$\begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix} = \frac{1}{\sigma_n} \begin{bmatrix} 1 & \mu_n B_z \\ -\mu_n B_z & 1 \end{bmatrix} \begin{pmatrix} J_x \\ J_y \end{pmatrix}$$

1) Longitudinal magneto-resistance is independent of *B*
2) Hall voltage is proportional to *B*

Strong B-fields

For strong B-fields: $\mu_n B_z >> 1$ $\omega_c \tau >> 1$

Interesting quantum effects can occur.

Hall and integer quantum Hall effect

Hall effect

$V_H \sim B$
V_x independent of B

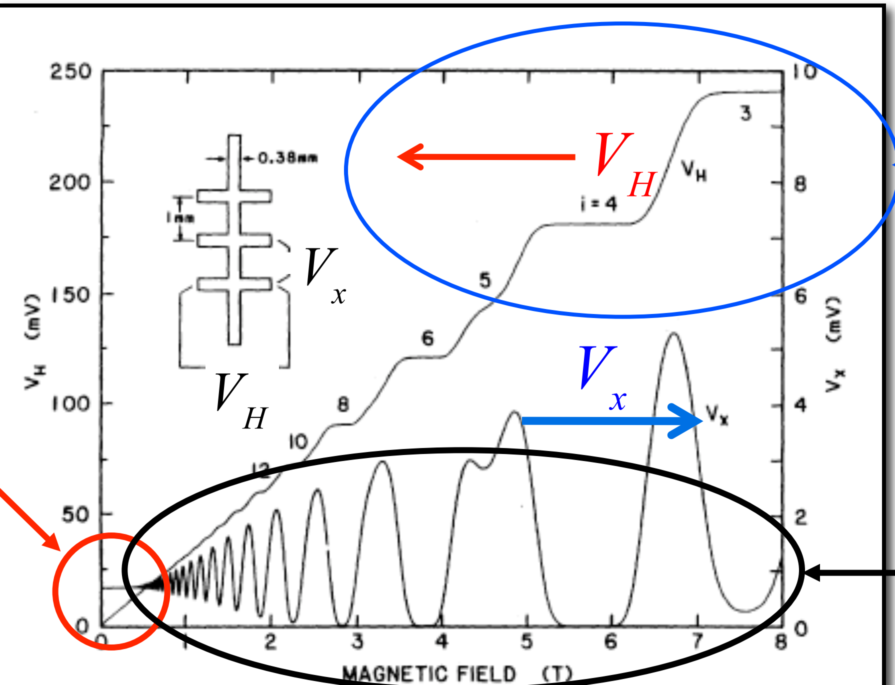


Fig. 1. Recording of V_H and V_x versus magnetic field for a GaAs device cooled to 1.2 K. The current is 25.5 μA.

Quantum Hall effect

Longitudinal magneto-resistance

M.E. Cage, R.F. Dziuba, and B.F. Field, “A Test of the Quantum Hall Effect as a Resistance Standard,” *IEEE Trans. Instrumentation and Measurement*,” Vol. IM-34, pp. 301-303, 1985