ECE 656: Electronic Transport in Semiconductors Fall 2017

# The Hall Effect

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# Introduction

$$\vec{J}_n = \sigma_S \vec{\mathcal{E}} - \left(\sigma_S \mu_n r_H\right) \vec{\mathcal{E}} \times \vec{B}$$

Hall effect measurements are an important application of the low B-field current equation we have derived.

They are widely used to characterize semiconductor materials.

# Basic equations

$$\vec{J}_n = nq\mu_n\vec{\mathcal{E}} - (\sigma_n\mu_H)\vec{\mathcal{E}}\times\vec{B}$$

$$\mu_H = r_H\,\mu_n$$ "Hall mobility"

$$r_H = \frac{\langle\langle \tau_m{}^2\rangle\rangle}{\langle\langle \tau_m\rangle\rangle^2}$$ "Hall factor"

Power Law Scattering:

$$r_H = \frac{\Gamma(2s+5/2)\Gamma(5/2)}{\Gamma(s+5/2)^2}$$

(3D electrons)

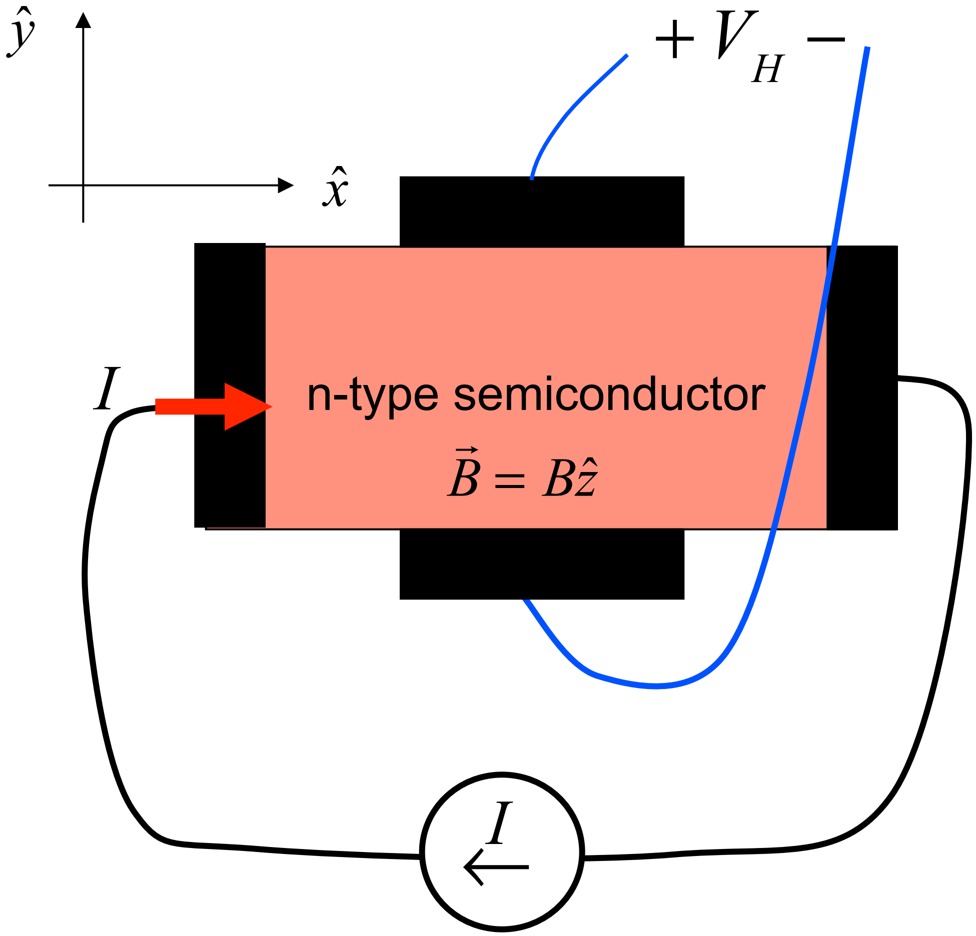
ADP scattering: *s* = -1/2
II scattering: *s* = 3/2

$$1.18 < r_H < 1.93$$

# Outline

1) Introduction
2) **Hall effect: Physics**
3) Hall Effect: Math
4) Hall bar procedure
5) Discussion

# Hall effect




current in *x*-direction:

$$I_x$$

B-field in *z*-direction:

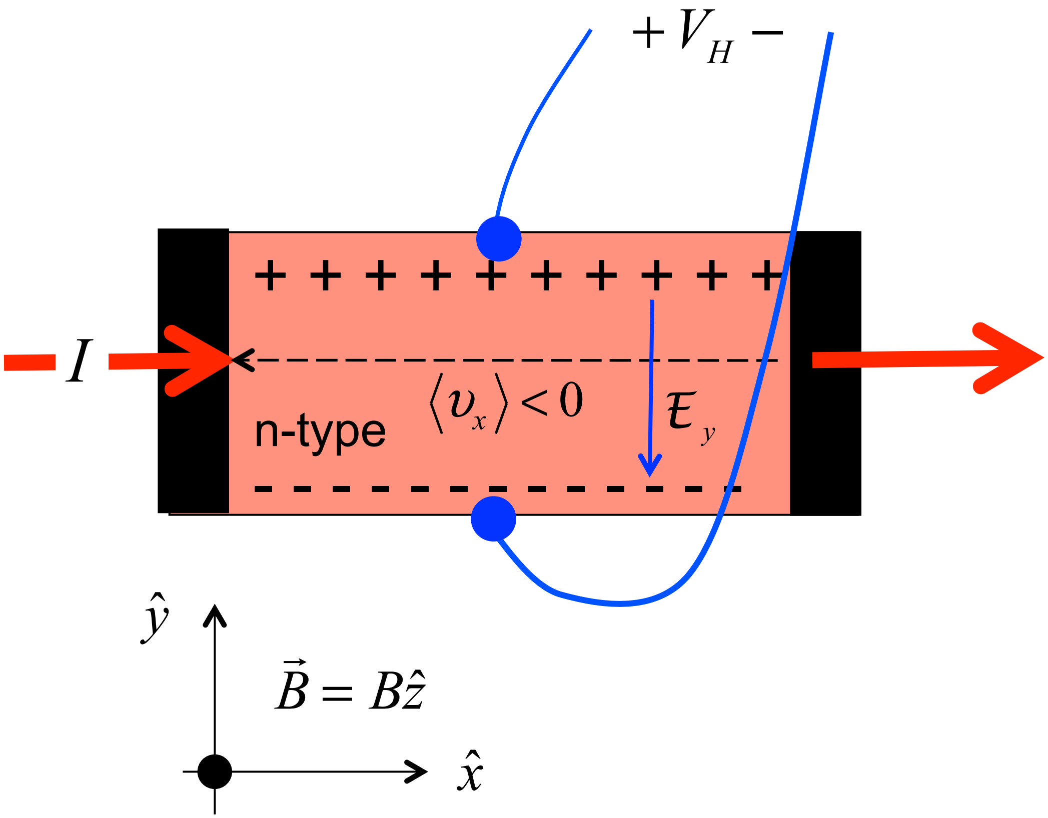
$$\vec{B} = B\hat{z}$$

Hall voltage measured in the *y*-direction:

$$V_H > 0 \quad (\text{n-type})$$

*The Hall effect was discovered by Edwin Hall in 1879 and is widely used to characterize electronic materials. It also finds use magnetic field sensors.*

# Hall effect:  physics




$$I_x = nq\langle \upsilon_x \rangle$$

$$\langle \upsilon_x \rangle < 0$$

$$\vec{F}_e = -q\vec{\upsilon} \times \vec{B}$$

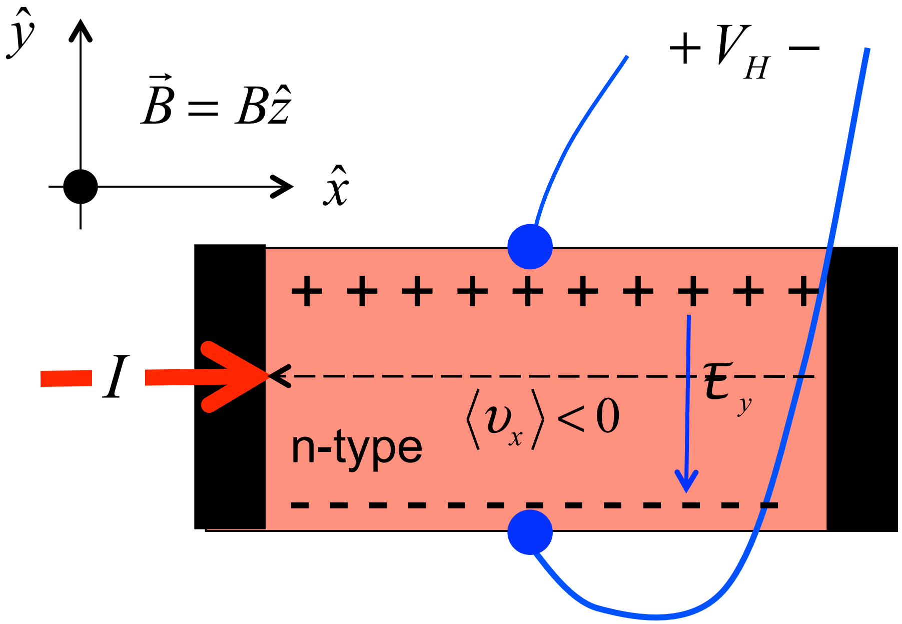
$$\langle F_{ey} \rangle < 0$$

$$\mathcal{E}_y < 0$$

$$V_H > 0 \quad (\text{n-type})$$

# Hall effect: 2D analysis (i)

Top view of a 2D film




$$\vec{J}_n = \sigma_n \vec{\mathcal{E}} - (\sigma_n \mu_n r_H) \vec{\mathcal{E}} \times \vec{B}$$

low B-field assumed

$$J_x = \sigma_S \mathcal{E}_x - (\sigma_S \mu_n r_H) \mathcal{E}_y B_z$$

$$J_y = \sigma_S \mathcal{E}_y + (\sigma_S \mu_n r_H) \mathcal{E}_x B_z$$

$$\mathcal{E}_x = \rho_S J_x + (\rho_S \mu_n r_H B_z) J_y$$

$$\mathcal{E}_y = \rho_S J_y - (\rho_S \mu_n r_H B_z) J_x$$

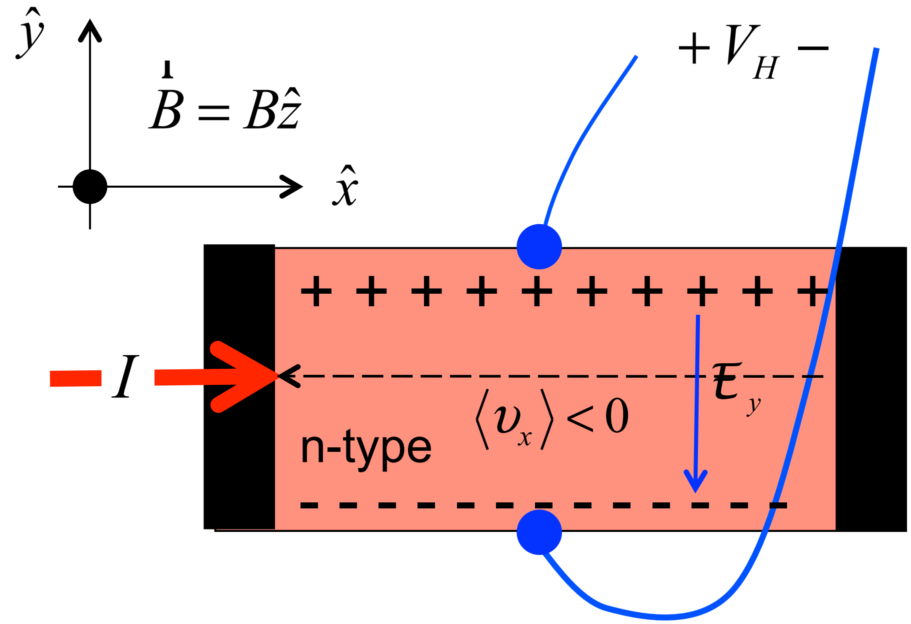
$$J_y = 0 \Rightarrow$$

$$\mathcal{E}_y = -(\rho_S \mu_n r_H B_z) J_x = -\frac{r_H B_z J_x}{q n_S}$$

7

# Hall effect: 2D analysis (ii)

Top view of a 2D film




$$\vec{J}_n = \sigma_n \vec{\mathcal{E}} - (\sigma_n \mu_n r_H) \vec{\mathcal{E}} \times \vec{B}$$

$$\mathcal{E}_y = \left( -\frac{r_H}{n_S q} \right) B_z J_x$$

$$\frac{\mathcal{E}_y}{J_x B_z} \equiv R_H = \frac{r_H}{(-q) n_S}$$

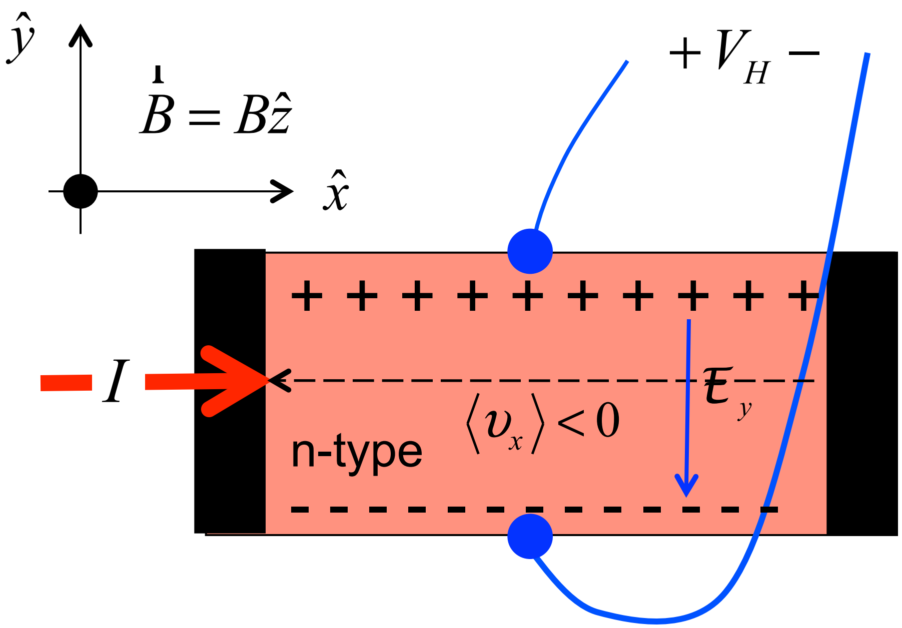
$R_H$ is the "Hall coefficient"

- $R_H < 0$ for n-type
- $R_H > 0$ for p-type

$r_H$ is the "Hall factor"

# Hall effect: 2D analysis (iii)

Top view of a 2D film




$$\vec{J}_n = nq\mu_n \vec{\mathcal{E}} - (\sigma_n \mu_n r_H)\vec{E} \times \vec{B}$$

$$R_H \equiv \frac{\mathcal{E}_y}{J_x B_z} = \frac{-V_H}{I_x B_z}$$

$$R_H = \frac{r_H}{(-q)n_S} \qquad r_H \equiv \frac{\langle\langle \tau_m^2 \rangle\rangle}{\langle\langle \tau_m \rangle\rangle^2}$$

"Hall factor"

$$R_H = \frac{1}{(-q)n_H} \qquad n_H \equiv \frac{n_S}{r_H}$$

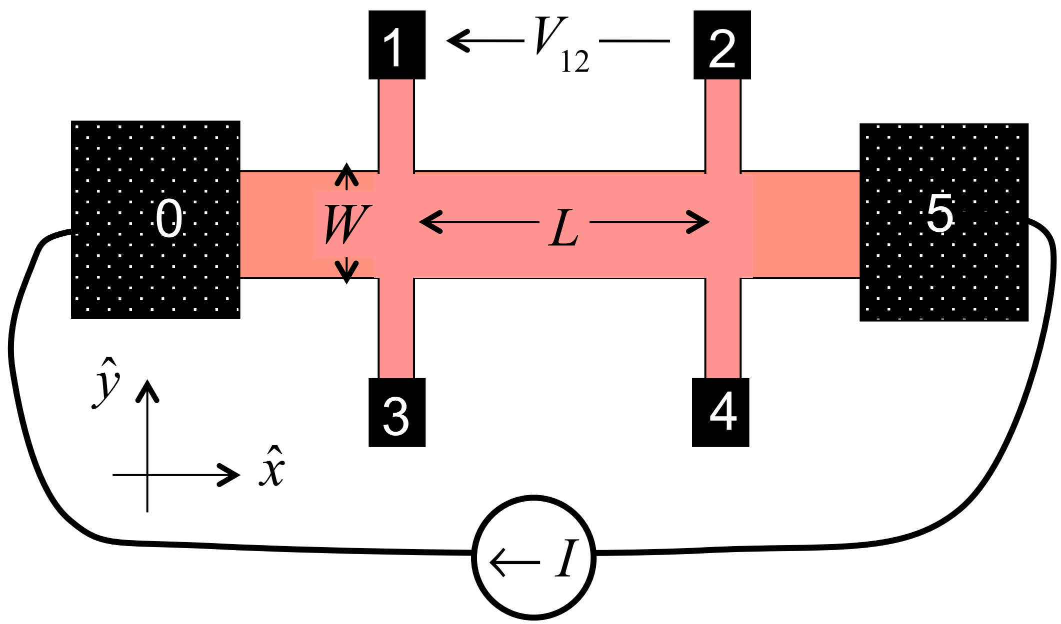
"Hall concentration"

# Hall bar geometry

B-field in *z*-direction:

$$\vec{B} = B\hat{z}$$

Top view




$$\vec{\mathcal{E}} = \rho_S \vec{J}_n + \left(\rho_S \mu_n r_H\right) \vec{J}_n \times \vec{B}$$

# Hall bar measurements

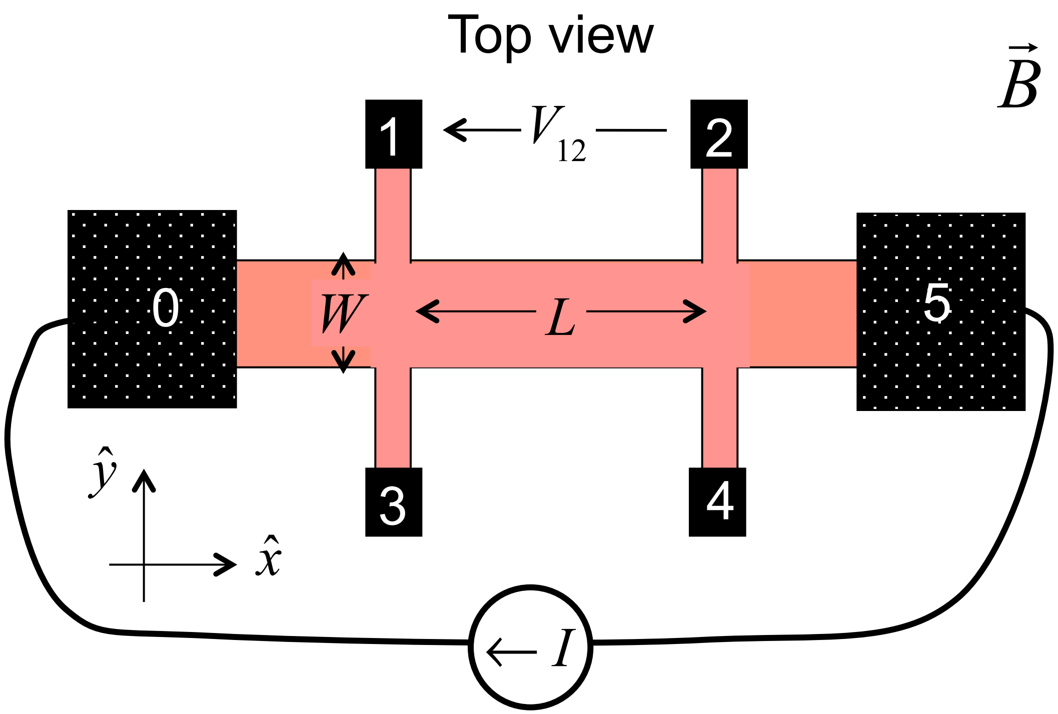
**Goal:** To measure the sheet carrier density and the mobility.

$$n_S \quad \text{cm}^{-2} \qquad\qquad \mu_n \quad \text{cm}^2/\text{V-s}$$

# Step 1: Longitudinal resistance, $R_{xx}$

B-field in *z*-direction:

$$\vec{B} = B\hat{z}$$




$$\vec{\mathcal{E}} = \rho_S \vec{J}_n + \left(\rho_S \mu_n r_H\right) \vec{J}_n \times \vec{B}$$

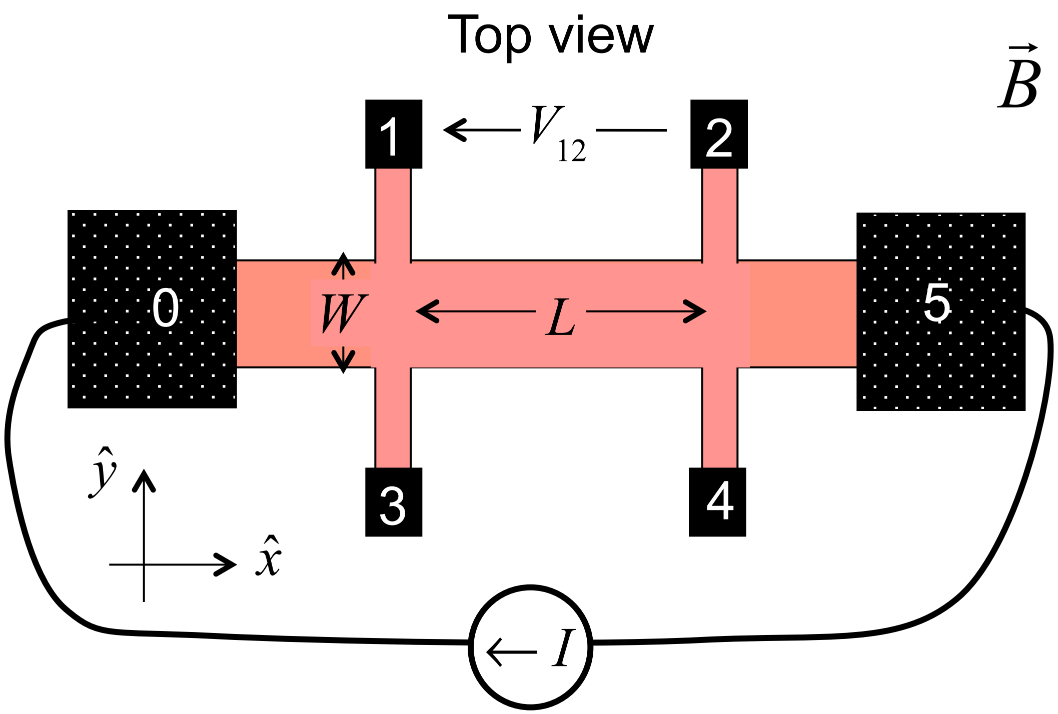
$$\mathcal{E}_x = \rho_S J_x$$

$$R_{xx} = \frac{V_{1,2}}{I} = \frac{\mathcal{E}_x L}{J_n W} = \rho_S \frac{L}{W}$$

# Step 2: Transverse resistance: $R_{xy}$

B-field in *z*-direction:

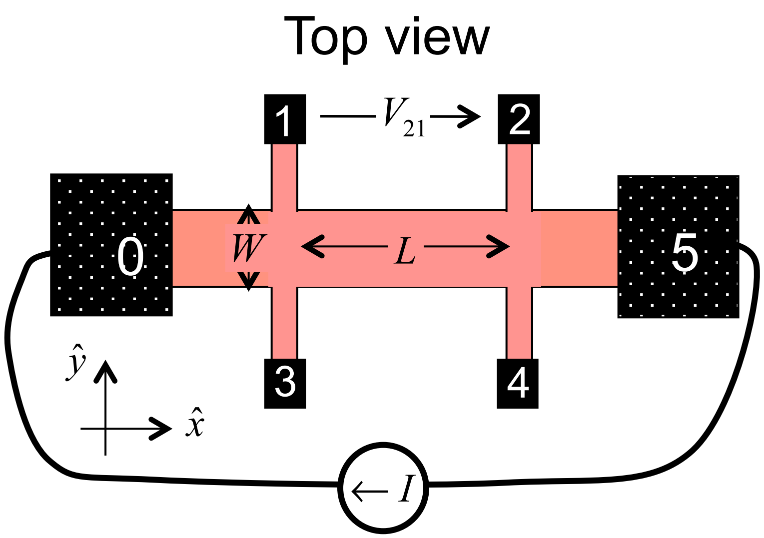
$$\vec{B} = B\hat{z}$$




$$\frac{\mathcal{E}_y}{J_x B_z} = -\left(\rho_s \mu_n r_H\right) = \frac{r_H}{(-q) n_S}$$

$$R_{xy} = \frac{V_{1,3}}{I} - \frac{-\mathcal{E}_y W}{W J_x B_z} = \frac{r_H}{q n_S} = \frac{1}{q n_H}$$

# Procedure




$$I = I_x = 1\,\mu\text{A}$$

"Hall bar" geometry

1) Measure $R_{xx}$:

$$R_{xx} = \frac{V_{1,2}}{I} = \rho_S \frac{L}{W}$$

2) Measure $R_{xy}$:

$$R_{xy} = \frac{V_{1,3}}{I} = \frac{r_H}{q n_S} = \frac{1}{q n_H}$$

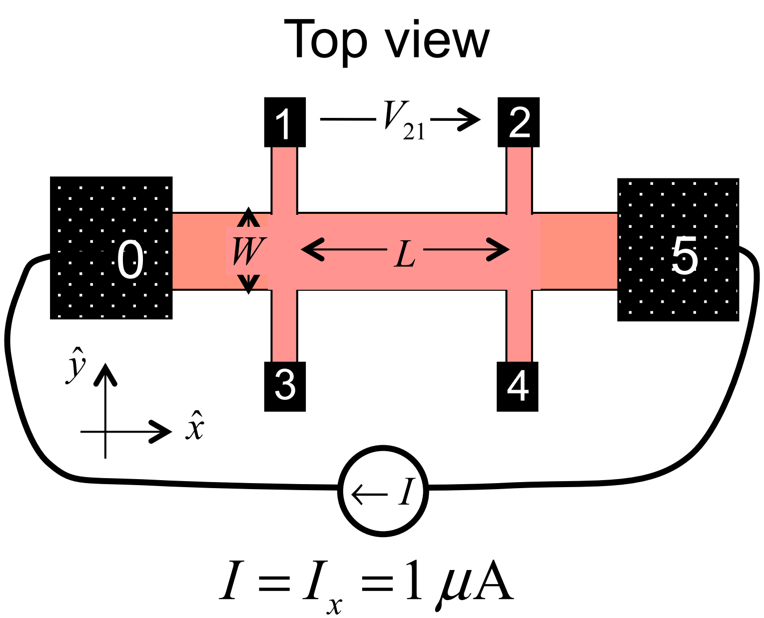
3) Hall concentration:

$$n_H \equiv n_S / r_H$$

4) Hall mobility:

$$\mu_H \equiv r_H \mu_n$$

# What can go wrong?




A lot…..

See *Fundamentals of Carrier Transport*, Chapter 4

# Hall bar measurements

**Goal:** To measure the sheet carrier density and the mobility.

$$n_S \quad \text{cm}^{-2} \qquad \qquad \mu_n \quad \text{cm}^2/\text{V-s}$$

**We actually measure:**

$$n_H = n_S / r_H \quad \text{cm}^{-2} \qquad \qquad \mu_n = r_H \mu_n \quad \text{cm}^2/\text{V-s}$$

"Hall concentration" "Hall mobility"

# References

For an excellent discussions about doing Hall effect measurements, see:

David C. Look, *Electrical Characterization of GaAs Materials and Devices*, John Wiley and Sons, New York, 1989.

Dieter K. Schroder, *Semiconductor Material and Device Characterization. 3rd Ed.*, John Wiley and Sons, New York, 2006. (Secs. 2.5 and 8.3).

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# Small magnetic fields

$$\omega_c \tau_m << 1 \qquad \mu_n B_z << 1$$

# Some numbers

**silicon**

$$\mu_n = 1000 \quad \text{cm}^2/\text{V-s}$$

$$B_z = 2{,}000 \quad \text{Gauss}$$

$$B_z = 0.2 \quad \text{Tesla}$$

$$\mu_n B_z \approx 0.02 << 1$$

Hall effect measurements with typical laboratory-sized magnets are in the low B-field regime. Except – for very high mobility sample such as modulation doped films.)

Birck Nanotechnology Center: 1-8 T

National High Magnetic Field Lab (Florida State Univ.): 45 T

# Questions

$$R_{xx} = \frac{V_{1,2}}{I} = \rho_S \frac{L}{W}$$

$$R_{xy} = \frac{V_{1,3}}{I} = \frac{1}{qn_H}$$

$$n_H \equiv n_S / r_H$$

$$\mu_H \equiv r_H \mu_n$$