ECE 656: Electronic Transport in Semiconductors Fall 2017

The BTE with a High B-field

Mark Lundstrom

Electrical and Computer Engineering
Purdue University
West Lafayette, IN USA

nanoHUB.org

10/11/17

PURDUE UNIVERSITY

Outline

1) Introduction
2) Landau levels
3) Shubnikov-deHaas oscillations
4) Quantum Hall Effect

High magnetic fields

$$\omega_c = \frac{qB}{m^*} \qquad \omega_c \tau_m >> 1 \qquad \mu_n B_z >> 1$$

Interesting things happen when the B-field is large.

D. F. Holcomb, “Quantum electrical transport in samples of limited dimensions,” *American Journal of Physics*, **67**, pp. 278-297, April 1999.

Reference

Longitudinal magneto-resistance

"Shubnikov-deHaas (SdH) oscillations"

quantized Hall voltage
zero longitudinal resistance, R_{xx}

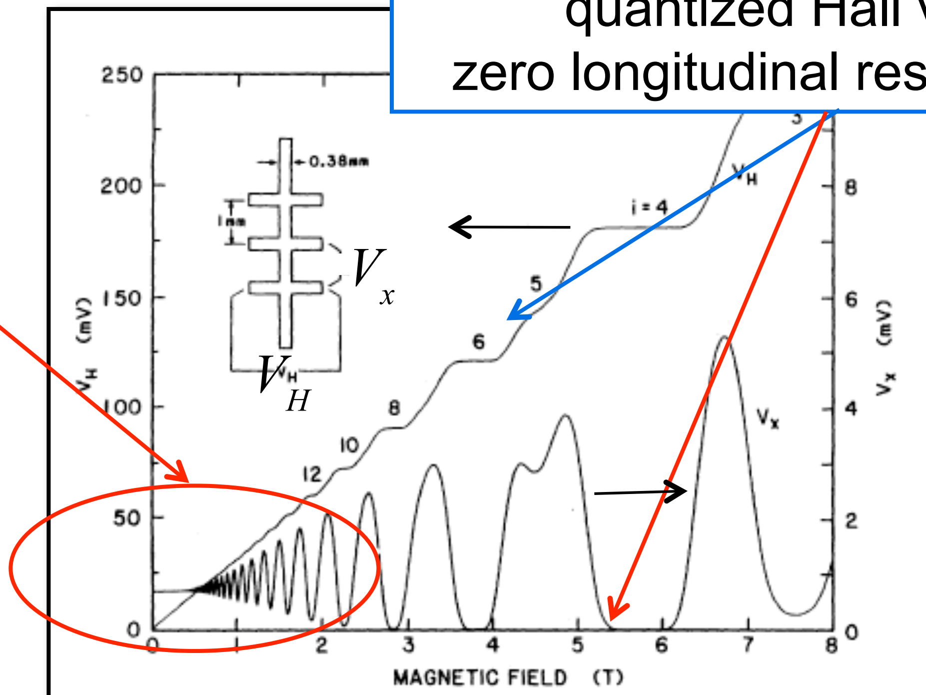


Fig. 1. Recording of V_H and V_x versus magnetic field for a GaAs device cooled to 1.2 K. The current is 25.5 μA.

M.E. Cage, R.F. Dziuba, and B.F. Field, "A Test of the Quantum Hall Effect as a Resistance Standard," *IEEE Trans. Instrumentation and Measurement*," Vol. IM-34, pp. 301-303, 1985

Some numbers

silicon

$$\mu_n = 1000 \quad \text{cm}^2/\text{V-s}$$

$$B_z = 2{,}000 \quad \text{Gauss}$$

$$B_z = 0.2 \quad \text{Tesla}$$

$$\mu_n B_z \approx 0.02 << 1$$

Hall effect measurements with typical laboratory-sized magnets are in the low B-field regime. Except – for very high mobility sample such as modulation doped films.)

Birck Nanotechnology Center: 1-8 T

National High Magnetic Field Lab (Florida State Univ.): 45 T

Large magnetic fields

InAlAs/InGaAs

T_L = 300K

$$\mu_n \approx 10{,}000 \quad \text{cm}^2/\text{V-s}$$

$$B_z = 2{,}000 \quad \text{Gauss}$$

$$B_z = 0.2 \quad \text{Tesla}$$

$$\mu_n B_z \approx 0.2 < 1$$

InAlAs/InGaAs

T_L = 77K

$$\mu_n \approx 100{,}000 \quad \text{cm}^2/\text{V-s}$$

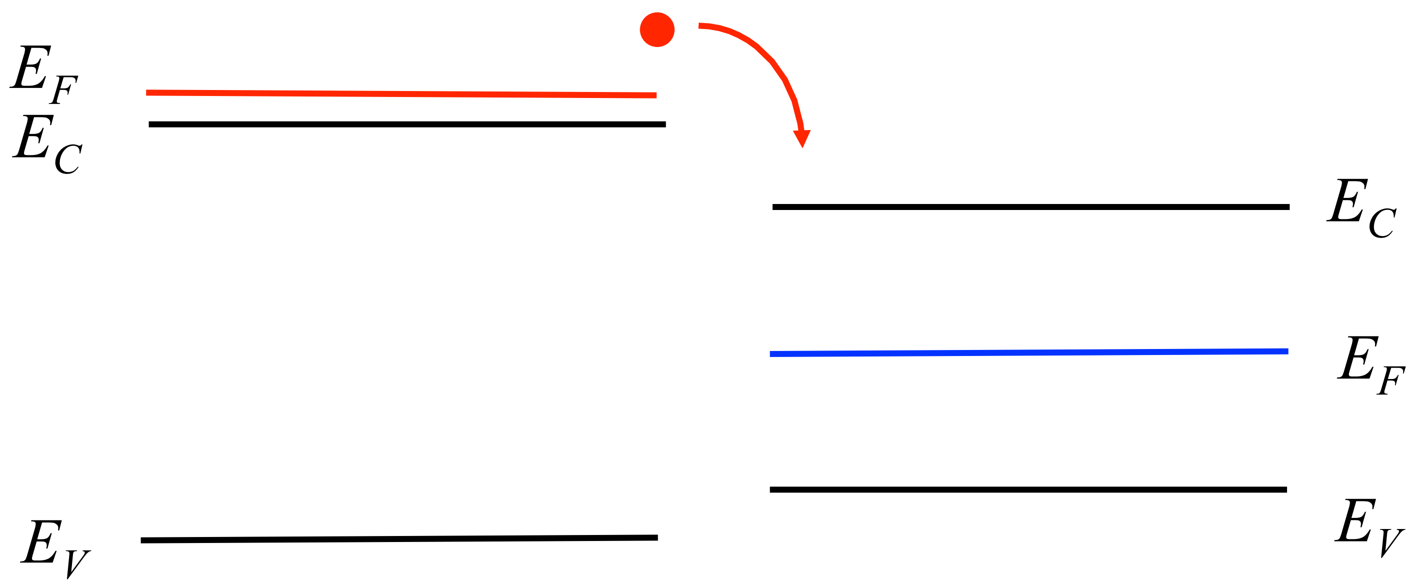
$$B_z = 2{,}000 \quad \text{Gauss}$$

$$B_z = 0.2 \quad \text{Tesla}$$

$$\mu_n B_z \approx 2 > 1$$

"Shubnikov-deHaas (SdH) oscillations" and quantum Hall effect

Modulation doping



wide band gap
doped n^+

small band gap
nominally undoped

R. Dingle, et al, *Appl. Phys. Lett.*, **33**, 665, 1978.

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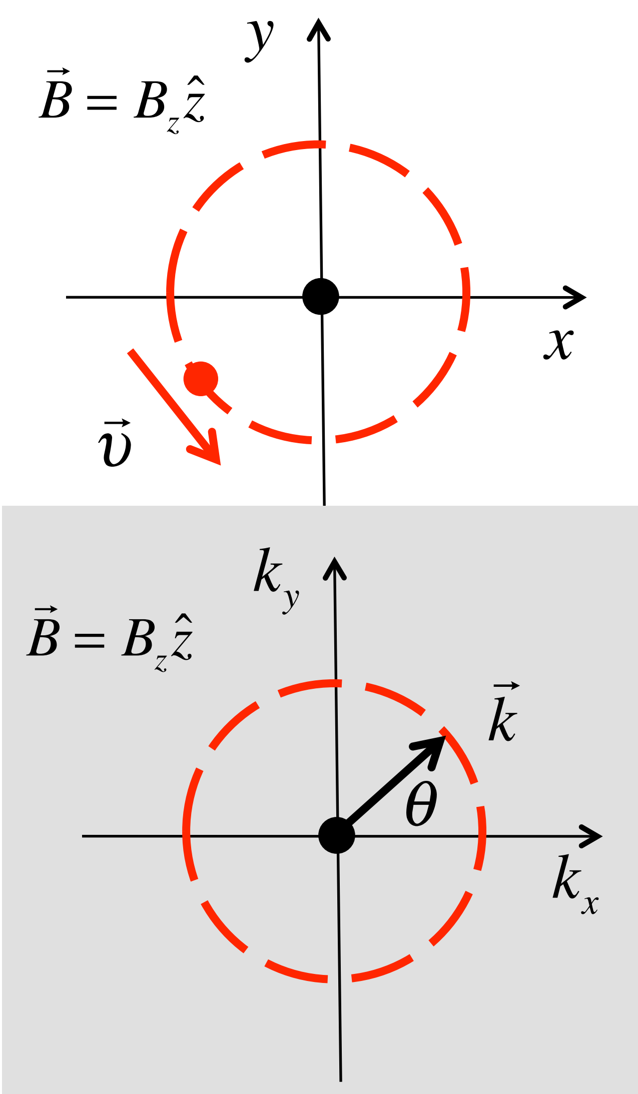
Schrödinger equation with a B-field

$$\left[\varepsilon_1 + \frac{\left(i\hbar\nabla + q\vec{A}\right)^2}{2m^*} + U\left(y\right)\right]\Psi\left(x,y\right) = E\Psi\left(x,y\right)$$

$$\vec{B} = \nabla \times \vec{A}$$

See S. Datta, *Electronic Transport in Mesoscopic Systems*, Cambridge, 1995, pp. 29-27.

Cyclotron frequency



$$\frac{d\left(\hbar\vec{k}\right)}{dt} = -q\vec{\upsilon}\times\vec{B}$$

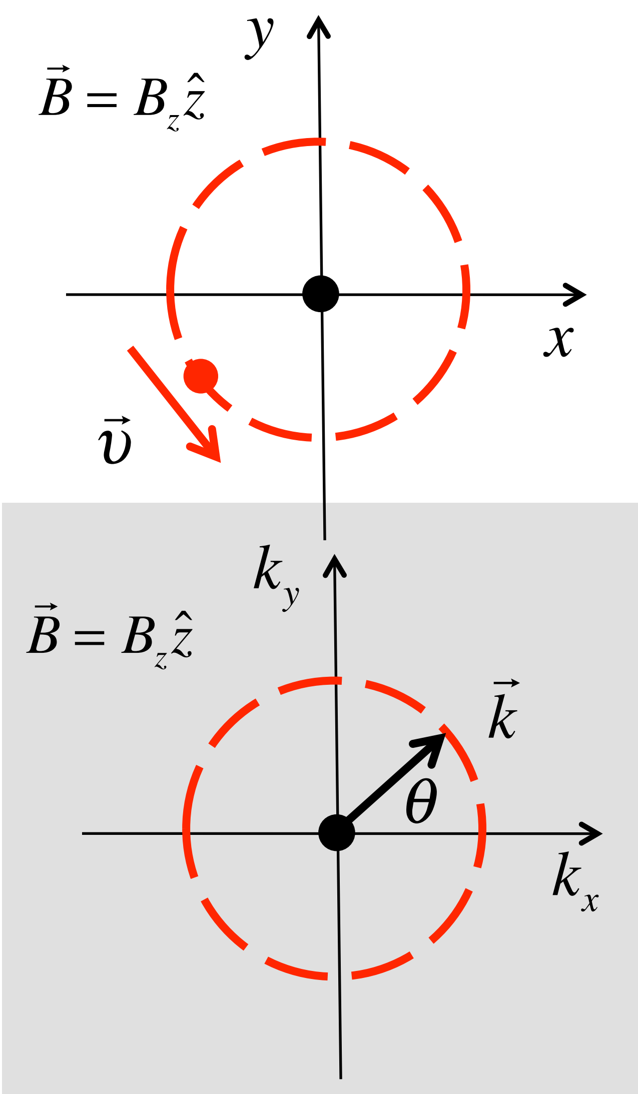
$$\hbar\frac{dk_x}{dt} = -q\upsilon_y B_z = \hbar k\frac{d\cos\theta}{dt} = -q\upsilon\sin\theta B_z$$

$$\hbar\frac{dk_y}{dt} = +q\upsilon_x B_z = \hbar k\frac{d\sin\theta}{dt} = +q\upsilon\cos\theta B_z$$

$$\frac{d^2\cos\theta}{dt^2} = -\left(\frac{q\upsilon B_z}{\hbar k}\right)^2\cos\theta = -\omega_c^2\cos\theta$$

$$\cos\theta(t) = \cos\theta(0)e^{i\omega_c t}$$

Cyclotron frequency



$$\frac{d\left(\hbar\vec{k}\right)}{dt} = -q\,\vec{\upsilon}\times\vec{B}$$

harmonic oscillator:

$$\omega_c = \left(\frac{q\,\upsilon B_z}{\hbar k}\right)$$

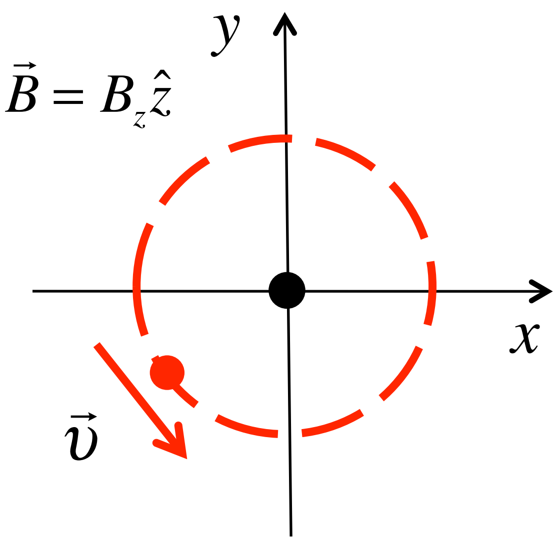
Quantum mechanically:

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$$

"Landau levels"

Cyclotron frequency

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c \qquad \omega_c = \left(\frac{q\upsilon B_z}{\hbar k}\right)$$



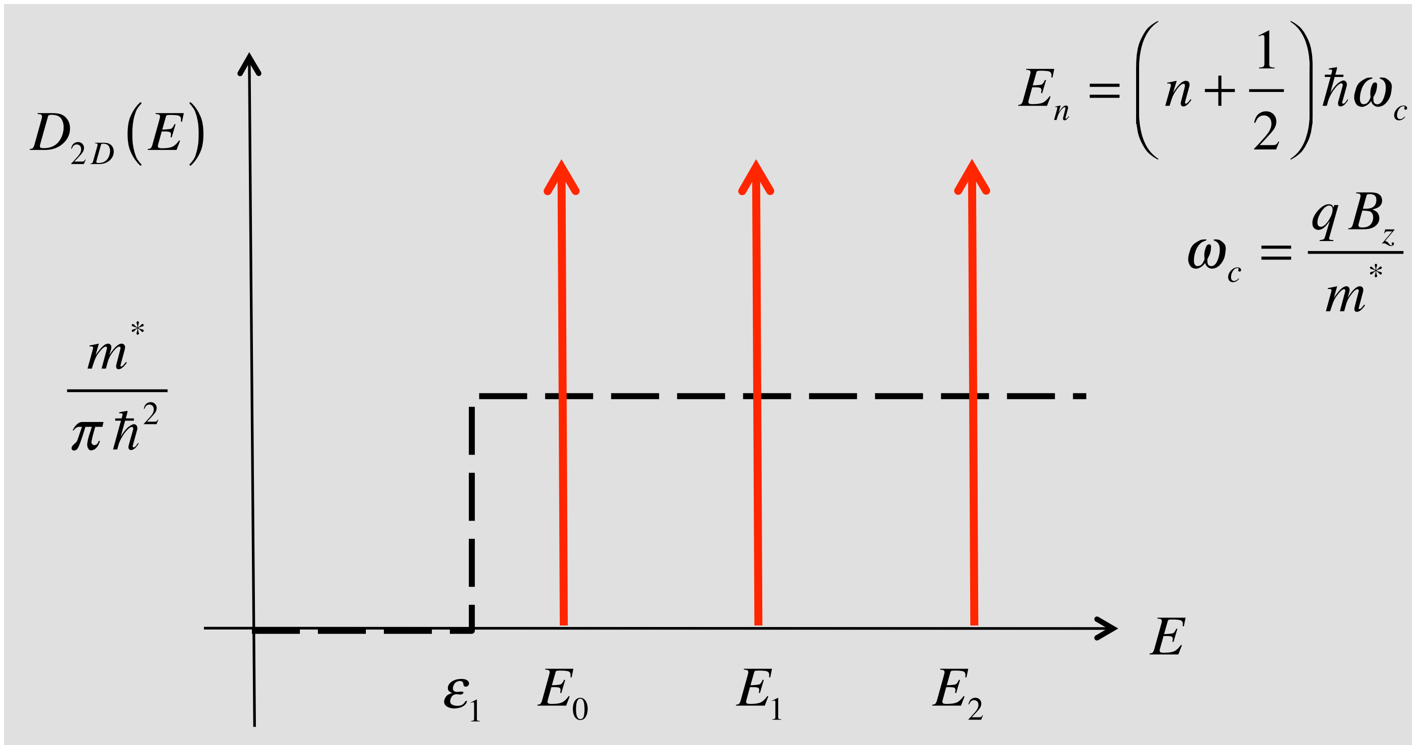
i) parabolic energy bands:

$$\upsilon = \hbar k / m^* \qquad \omega_c = \frac{qB_z}{m^*}$$

ii) graphene:

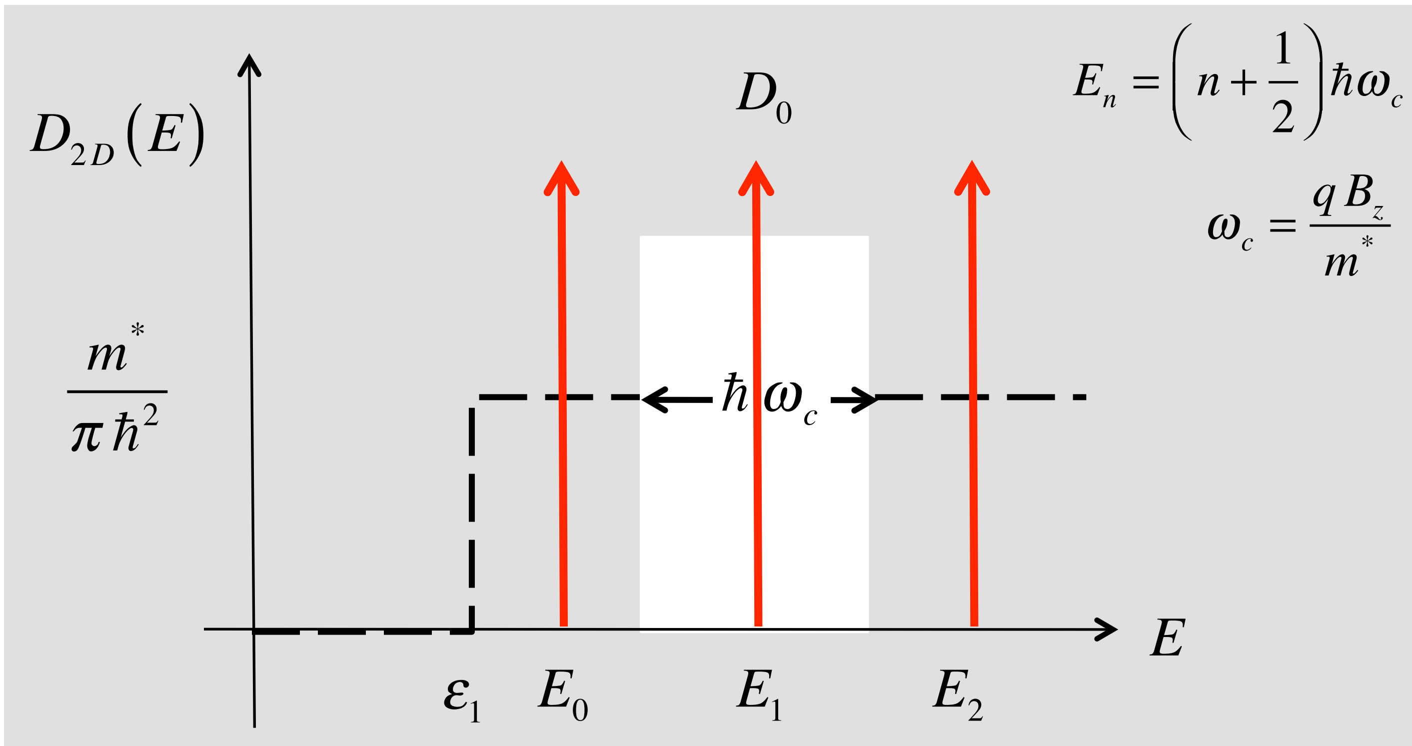
$$E = \hbar \upsilon_F k \qquad \omega_c = \frac{qB_z}{\left(E/\upsilon_F^2\right)}$$

Effect on DOS



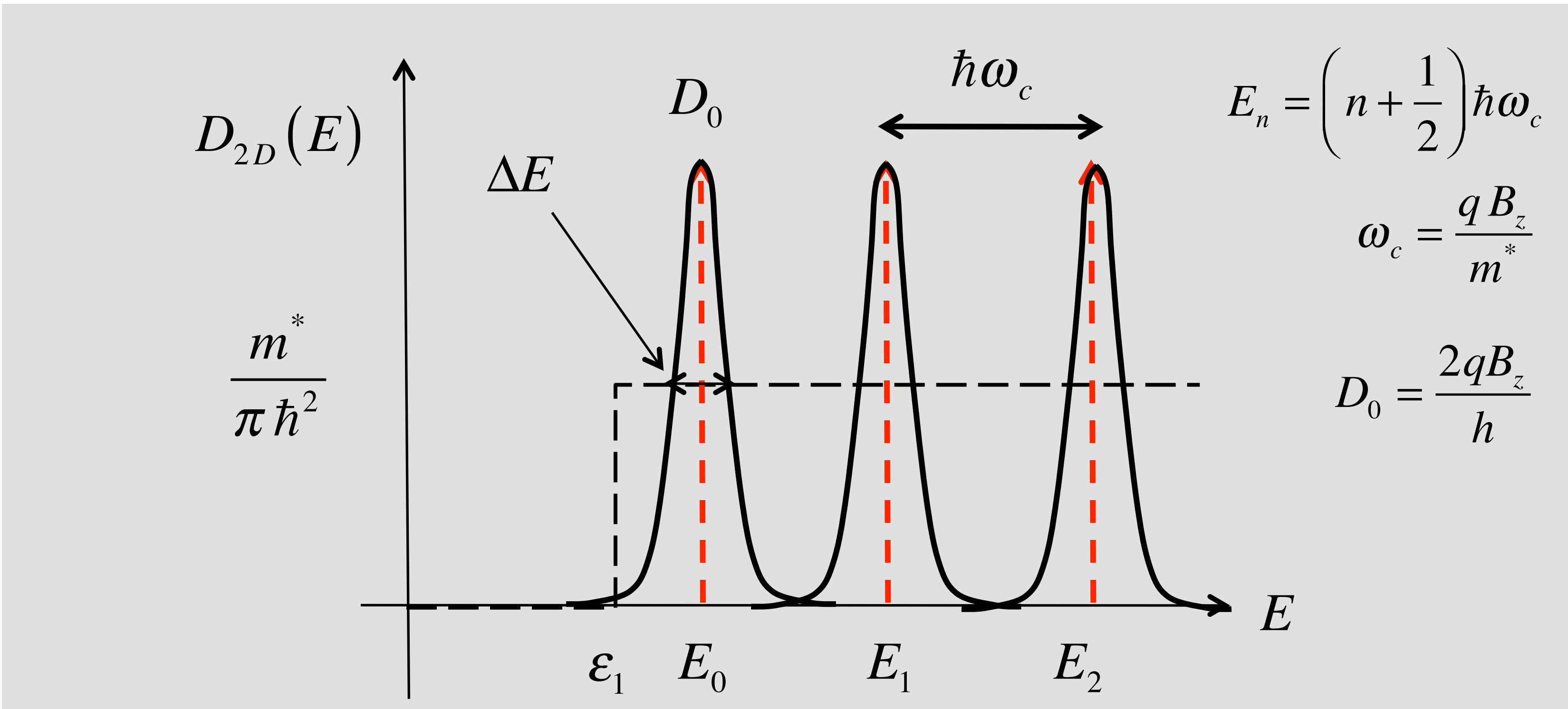
$$D_{2D}(E, B_z) = D_0 \sum_{n=0}^{\infty} \delta\left[E - \varepsilon_1 - \left(n+\frac{1}{2}\right)\hbar\omega_c\right]$$

Degeneracy of Landau levels



$$D_0 = \hbar\omega_c \times \frac{m^*}{\pi\hbar^2} = \frac{2qB_z}{h}$$

Broadening



$\Delta E \Delta t = \hbar$

$\Delta E \approx \hbar/\tau$

to observe Landau levels: $\hbar\omega_c >> \Delta E \rightarrow \omega_c \tau >> 1$

Example

If B = 1T, how many states are there in each LL?

$$D_0 = \frac{2qB_z}{h} = 4.8 \times 10^{10} \text{ cm}^{-2}$$

If n_S = 5 x 10^{11} cm^{-2}, then 10.4 LL's are occupied.

How high would the mobility need to be to observe these LL's?

$$\mu B > 1 \rightarrow \mu > 10{,}000 \text{ cm}^2/\text{V-s} \quad (B = 1\text{ T})$$

"modulation-doped semiconductors"

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4) Quantum Hall Effect

SdH oscillations

Longitudinal magneto-resistance

"Shubnikov-deHaas (SdH) oscillations"

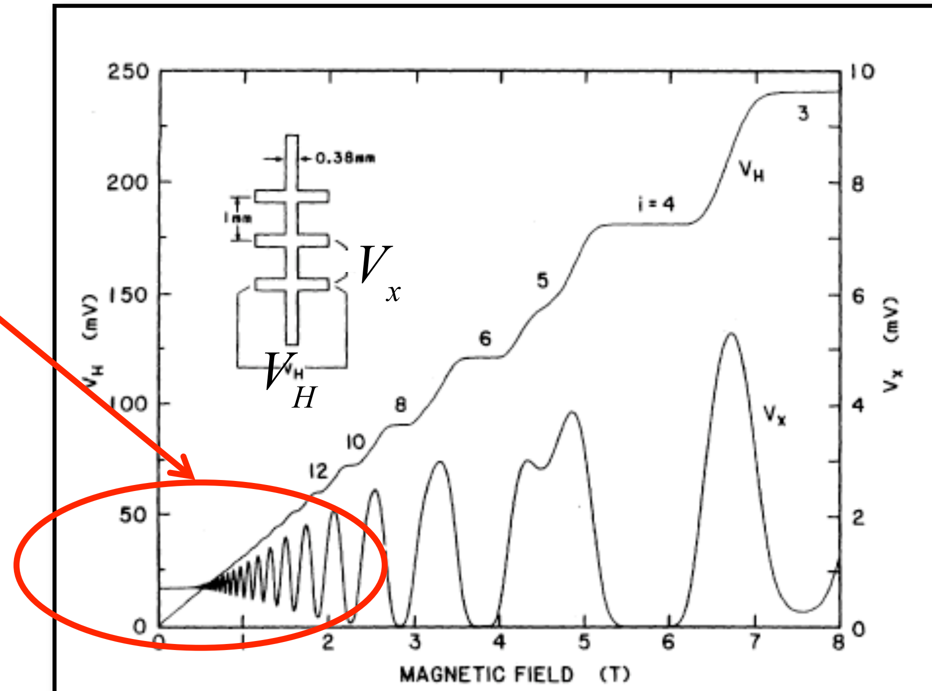
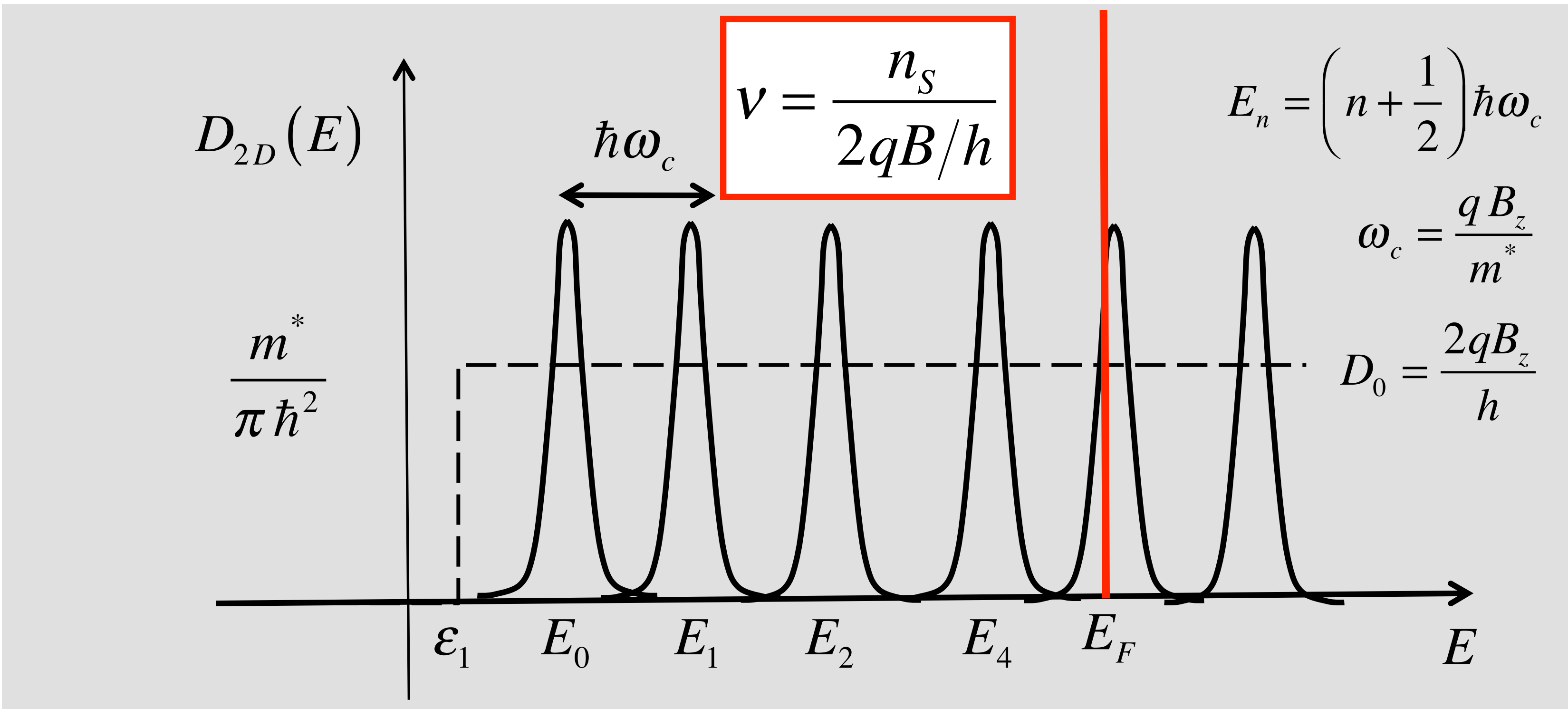


Fig. 1. Recording of V_H and V_x versus magnetic field for a GaAs device cooled to 1.2 K. The current is 25.5 μA.

M.E. Cage, R.F. Dziuba, and B.F. Field, "A Test of the Quantum Hall Effect as a Resistance Standard," *IEEE Trans. Instrumentation and Measurement*," Vol. IM-34, pp. 301-303, 1985

LL filling



For a given sheet carrier density, n_s, some (fractional) number of LL's are occupied.

SdH oscillations

As the *B*-field is varied, the longitudinal resistance will oscillate as the number of filled LL's changes. The period of the oscillation corresponds to the change in filling factor from one integer to the next.

$$\nu_1 - \nu_2 = 1$$

$$\frac{n_S}{2qB_1/h} - \frac{n_S}{2qB_2/h} = 1$$

$$n_S = \frac{2q}{h}\left(\frac{1}{1/B_1 - 1/B_2}\right)$$

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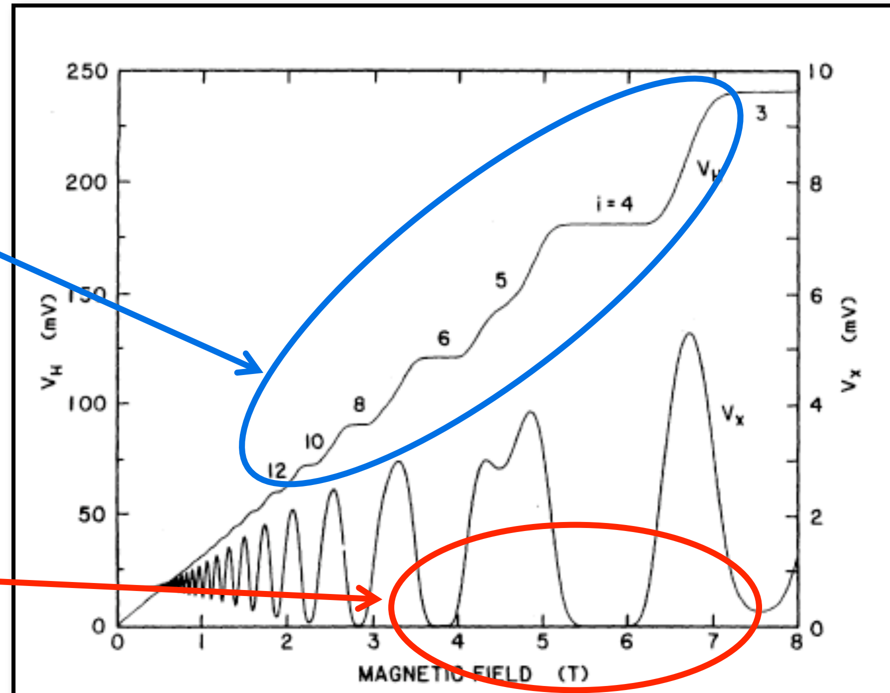
SdH oscillations

quantized Hall voltage

Zero longitudinal resistance



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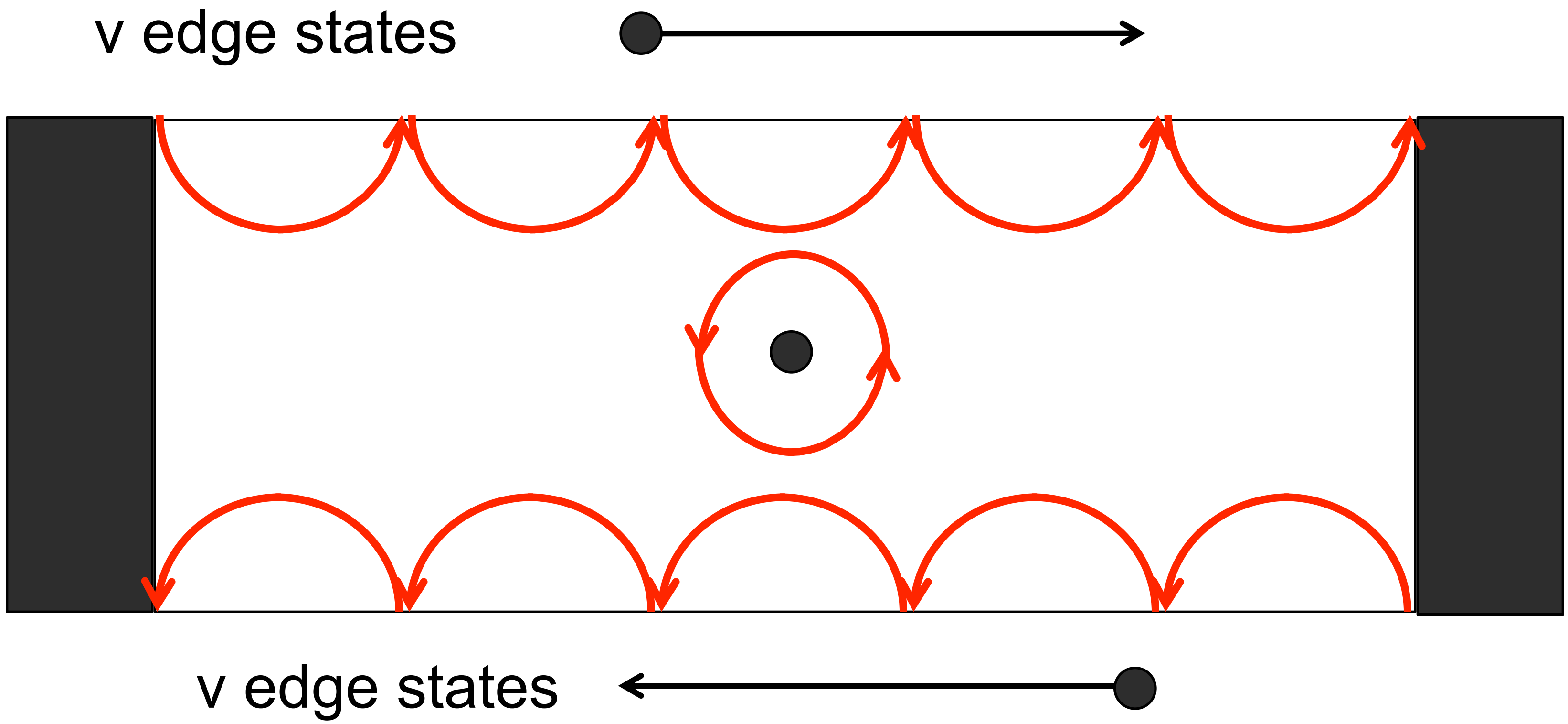
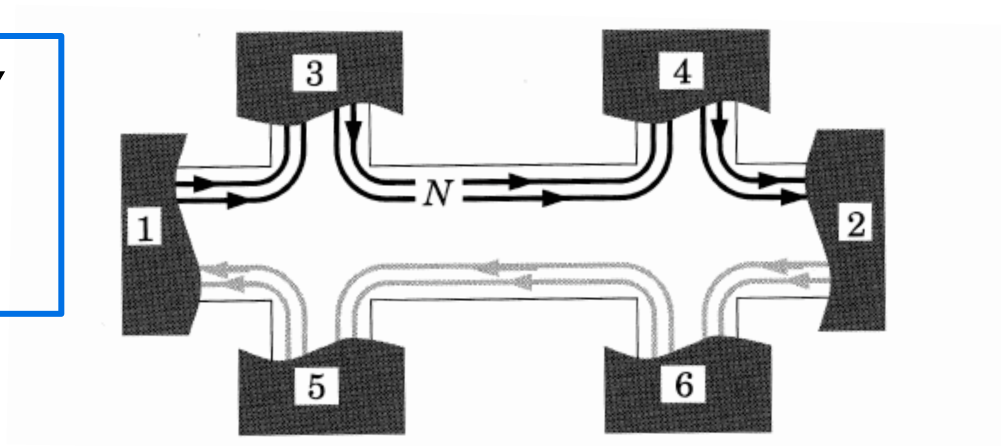
“Edge states”



Assume that the Fermi level lies between Landau levels in the bulk, then only the edge states are occupied (edge states play the role of modes).

QHE: quantized Hall Voltage

T = 1 because $+v$ and $-v$ channels are spatially isolated.



(from J.H. Davies, *The Physics of Low-Dimensional Semiconductors*, Cambridge, 1998, Fig. 6.18, p. 240)

$V_1 = 0$ and $V_2 = +V$

$V_3 = V_4 = V_1 = 0$

$$I = \frac{2q^2}{h}\nu V$$

$V_6 = V_5 = V_2 = +V$

$$V_H = V_3 - V_5 = \frac{1}{\nu}\frac{h}{2q^2}I$$

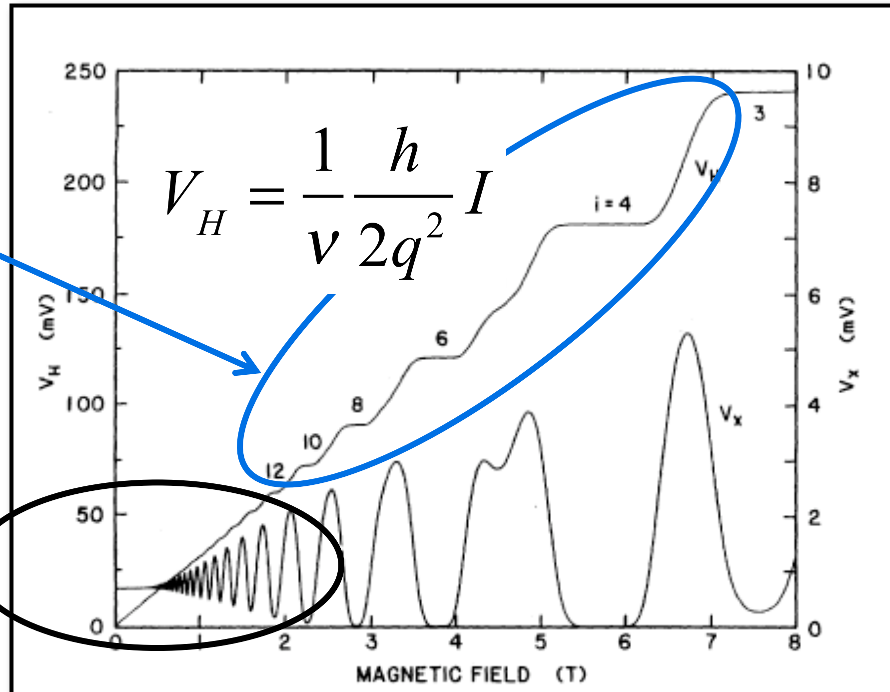
"quantized Hall resistance"

SdH oscillations

quantized Hall voltage

$$V_H = \frac{1}{\nu}\frac{h}{2q^2} I$$

"Shubnikov-deHaas (sdH) oscillations"



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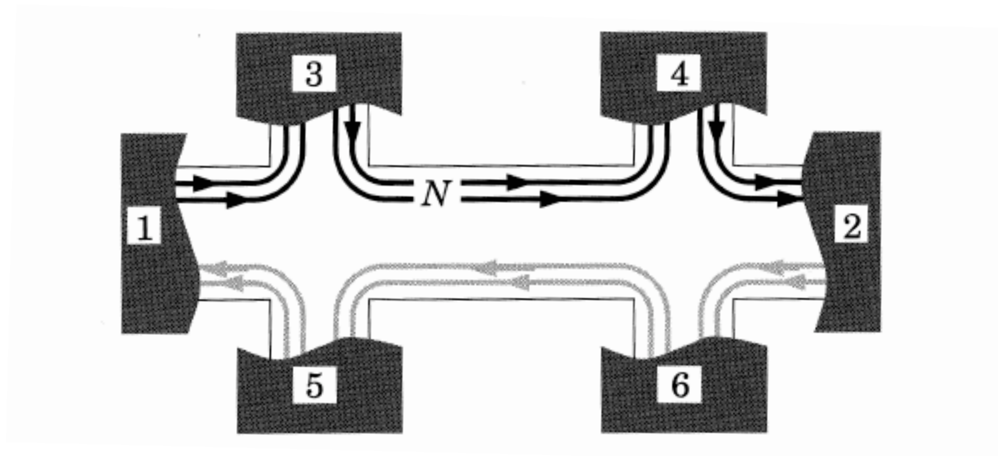
QHE: zero longitudinal magnetoresistance



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$V_1 = 0$ and $V_2 = +V$

$V_3 = V_4 = V_1 = 0$

$$I = \frac{2q^2}{h} \nu V$$

$V_6 = V_5 = V_2 = +V$

$$V_x = V_4 - V_3 = 0$$

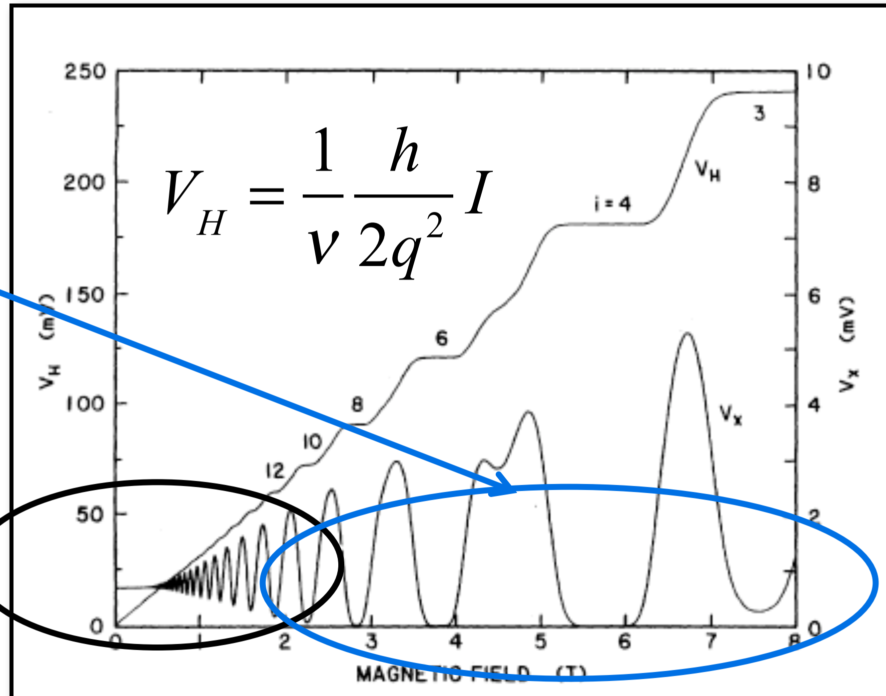
zero longitudinal resistance

SdH oscillations

vanishing longitudinal resistance

"Shubnikov-deHaas (sdH) oscillations"

$$V_H = \frac{1}{\nu}\frac{h}{2q^2} I$$



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For a more complete discussion

See Chapter 4 in *Electronic Transport in Mesoscopic Systems*, Supriyo Datta, Cambridge, 1995.

J.H. Davies, *The Physics of Low-Dimensional Semiconductors*, Cambridge, 1998.

D. F. Holcomb, "Quantum electrical transport in samples of limited dimensions," *American J. Physics*, **64**, pp. 278-297, 1999.

For a more complete discussion

Chapter 17 in the following book provides a good, general discussion of transport in higg magnetic fields.

J. Singh, *The Physics of Semiconductors and Their Heterostructures*, McGraw Hill, New York, 1993.