Electrical Characterization of Materials: I

Mark Lundstrom

Electrical and Computer Engineering
Purdue University
West Lafayette, IN USA
Coupled charge and heat current equations

electrical current:
\[ \mathcal{E} = \rho J + S \frac{dT}{dx} \]

heat current (electronic):
\[ J_Q = \pi J - \kappa_e \frac{dT}{dx} \]

heat current (lattice):
\[ q_x = -\kappa_L \left( \frac{dT}{dx} \right) \]

How do we measure the four transport parameters?

(Note that we only need to measure 3 of them, because the Kelvin Relation relates the Seebeck and Peltier coefficients.)
Outline

1. **Introduction**
2. Resistivity / conductivity measurements
3. Hall effect measurements
4. The van der Pauw method
5. Seebeck coefficient
6. Summary
Measurement of conductivity / resistivity

1) Commonly-used to characterize electronic materials.

2) Results can be clouded by several effects – e.g. contacts, thermoelectric effects, etc.

3) Measurements in the absence of a magnetic field are often combined with those in the presence of a B-field.

This lecture is a brief introduction to the measurement and characterization of near-equilibrium transport.
Resistivity / conductivity measurements

\[ J_{nx} = \sigma_n \frac{d(F_n/q)}{dx} \]  

(diffusive transport assumed)

For uniform carrier concentrations:

\[ J_{nx} = \sigma_n \mathcal{E}_x \quad \mathcal{E}_x = \rho_n J_{nx} \]

We generally measure **resistivity** (or **conductivity**) because for diffusive samples, these parameters depend on material properties and not on the length of the resistor or its width or cross-sectional area.
Landauer conductance and conductivity

For ballistic or quasi-ballistic transport, replace the mfp with the “apparent” mfp:

\[
\frac{1}{\lambda_{app} (E)} = \frac{1}{\lambda (E)} + \frac{1}{L}
\]

\[
G = \sigma_n \frac{A}{L}
\]

\[
G = \frac{2q^2}{h} \int M(E) T(E) \left( - \frac{\partial f_0}{\partial E} \right) dE
\]

\[
\sigma = \frac{G}{A/L} = \frac{2q^2}{h} \int M_{3D}(E) \lambda (E) \left( - \frac{\partial f_0}{\partial E} \right) dE
\]

\[
T(E) = \frac{\lambda(E)}{L}
\]

\[
M_{3D}(E) = \frac{M(E)}{A}
\]
Conductivity and mobility

Conductivity depends on $E_F$. 

2) $E_F$ depends on carrier density.

3) So it is common to characterize the conductivity at a given carrier density.

4) Mobility is often the quantity that is quoted.

So we need techniques to measure two quantities:

1) conductivity  
2) carrier density

$$\sigma = \frac{G}{A/L} = \frac{2q^2}{h} \int M_{3D}(E) \lambda(E) \left( -\frac{\partial f_0}{\partial E} \right) dE$$

$$\sigma_n = nq \mu_n$$
2D: conductivity and sheet conductance

\[ I = G V \]

**2D electrons**

\[ G = \sigma_n \frac{A}{L} \]

\[ G = \sigma_n \left( \frac{Wt}{L} \right) = \sigma_n t \left( \frac{W}{L} \right) \]

\[ G = \sigma_s \left( \frac{W}{L} \right) \]

\[ \sigma_s = n_s q \mu_n \left( \frac{1}{\Omega} \right) \]

"sheet conductance"
2D electrons vs. 3D electrons

\[ A = Wt \]

3D electrons:

\[ G = \sigma \frac{A}{L} = \sigma \frac{Wt}{L} \rightarrow \sigma_s = \frac{G}{W/L} = \frac{2q^2}{h} \int tM_{3D}(E) \lambda(E) \left( -\frac{\partial f_0}{\partial E} \right) dE \]

2D electrons:

\[ G = \sigma_s \frac{W}{L} \rightarrow \sigma_s = \frac{G}{W/L} = \frac{2q^2}{h} \int M_{2D}(E) \lambda(E) \left( -\frac{\partial f_0}{\partial E} \right) dE \]
Mobility

1) Measure the conductivity: \( \sigma_S \)

2) Measure the sheet carrier density: \( n_S \)

3) Deduce the mobility from: \( \sigma_S \equiv n_S q \mu_n \)

4) Relate the mobility to material parameters:

\[
\sigma_S = \frac{2q^2}{h} \int M_{2D}(E) \lambda(E) \left( -\frac{\partial f_0}{\partial E} \right) \equiv n_S q \mu_n
\]
There are three near-equilibrium transport coefficients: conductivity, Seebeck (and Peltier) coefficient, and the electronic thermal conductivity. We can measure all three, but in this brief lecture, we will focus on the conductivity and briefly mention the Seebeck coefficient.

Conductivity depends on the location of the Fermi level, which can be set by controlling the carrier density.

So we need to discuss how to measure the conductivity (or resistivity) and the carrier density. Let’s discuss the resistivity first.
Outline

1. Introduction
2. Resistivity / conductivity measurements
3. Hall effect measurements
4. The van der Pauw method
5. Seebeck coefficient
6. Summary
2-probe measurements

\[ R_{CH} = \rho_s \frac{L}{W} \]

\[ V_{21} = I \left( 2R_C + R_{CH} \right) \]

\[ R_{CH} \neq \frac{V_{21}}{I} \]
“Transmission line measurements”

Transmission line measurements (TLM)

\[ S_{1,2} \]

\[ \hat{x} \]

Side view

\[ S_{i,j} \]

Lundstrom ECE-656 F17
Contact resistance (vertical flow)

metal contact
Area = $A_c$

Top view

Side view

$n$-Si

$\rho_i \ \Omega \cdot \text{cm}$

interfacial layer

$I$

$\frac{t}{\text{cm}}$
Contact resistance (vertical flow)

\[ R_C = \frac{\rho_i t}{A_C} = \frac{\rho_C}{A_C} \, \Omega \]

\[ 10^{-8} < \rho_C < 10^{-6} \, \Omega \text{-cm}^2 \]

“interfacial contact resistivity”

\[ A_C = 0.10 \, \mu m \times 1.0 \, \mu m \]
\[ \rho_C = 10^{-7} \, \Omega \text{-cm}^2 \]
\[ R_C = 100 \, \Omega \]
Contact resistance (vertical + lateral flow)

$L_T$ “transfer length” \[ L_T = \sqrt{\frac{\rho_C}{\rho_{SD}}} \text{ cm} \]

\[ A_C = W L_C \]

($W$ into page)

$A_C(\text{eff}) < W L_C$
Contact resistance

\[ L_T = \sqrt{\frac{\rho_C}{\rho_{SD}}} \text{ cm} \]

\[ R_C = \frac{\sqrt{\rho_C \rho_{SD}}}{W} \coth \left( \frac{L_C}{L_T} \right) \]

i) \( L_C \ll L_T \) : \( R_C = \frac{\rho_C}{L_C W} \)

ii) \( L_C \gg L_T \) : \( R_C = \frac{\rho_C}{L_T W} \)
Transfer length measurements (TLM)

1) Slope gives sheet resistance, intercept gives contact resistance

2) Determine specific contact resistivity and transfer length:

\[ R_C = \frac{\sqrt{\rho_C \rho_{SD}}}{W} \coth \left( \frac{L_C}{L_T} \right) \]

\[ L_T = \sqrt{\frac{\rho_C}{\rho_{SD}}} \text{ cm} \]
Four probe measurements

1) force a current through probes 1 and 4
2) with a high impedance voltmeter, measure the voltage between probes 2 and 3

\[ R = \frac{V}{I} = f(\rho_s) \quad \text{(no series resistance)} \]
Hall bar geometry

pattern created with photolithography

Top view

thin film isolated from substrate

Contacts 0 and 5: “current probes”
Contacts 1 and 2 (3 and 4): “voltage probes”

\[ V_{21} = I \times \rho_s \frac{L}{W} \]

(high impedance voltmeter)

no contact resistance
Outline

1. Introduction
2. Resistivity / conductivity measurements
3. Hall effect measurements
4. The van der Pauw method
5. Seebeck coefficient
6. Summary
The Hall effect was discovered by Edwin Hall in 1879 and is widely used to characterize electronic materials. It also finds use in magnetic field sensors.

The diagram illustrates the setup for the Hall effect experiment. A current $I$ flows in the $x$-direction, with a magnetic field $\vec{B} = B\hat{z}$ applied in the $z$-direction. As a result, a Hall voltage $V_H$ is generated in the $y$-direction.

Mathematically, the Hall voltage is given by the formula:

$$V_H = \frac{1}{B} \frac{\rho}{I}$$

where $\rho$ is the resistivity of the material and $I$ is the current.

For an n-type semiconductor, the Hall voltage $V_H > 0$.
Hall effect: analysis

Top view of a 2D film

\[ \hat{B} = B\hat{z} \]

\[ \langle \nu_x \rangle < 0 \]

\[ \bar{J}_n = nq\mu_n \bar{E} - (\sigma_n \mu_n r_H) \bar{E} \times \bar{B} \]

Hall concentration

```
\[ R_H = \frac{\mathcal{E}_y}{J_x B_z} = - \frac{V_H}{I_x B_z} \]

\[ R_H = \frac{r_H}{(-q)n_S} \]

\[ r_H \equiv \frac{\langle \tau_m^2 \rangle}{\langle \tau_m \rangle^2} \]

“Hall factor”

\[ n_H \equiv \frac{n_S}{r_H} \]

“Hall concentration”
What are the:
1) resistivity?
2) sheet carrier density?
3) mobility?

\( I = I_x = 1 \mu \text{A} \)

\( B_z = 2,000 \text{ Gauss} \)

\( (1 \text{ Tesla} = 10^4 \text{ Gauss}) \)

\( L = 100 \mu \text{m} \)

\( W = 50 \mu \text{m} \)

\( B = 0: \)
\( V_{21} = 0.4 \text{ mV} \)

\( B \neq 0: \)
\( V_{24} = 13 \mu \text{V} \)
Example: resistivity

\[ B_z = 2,000 \text{ Gauss} \]
\( (1 \text{ Tesla} = 10^4 \text{ Gauss}) \)
\( L = 100 \, \mu\text{m} \)
\( W = 50 \, \mu\text{m} \)

\( B = 0: \)
\( V_{21} = 0.4 \, \text{mV} \)

\( B = 0.2 \, \text{T}: \)
\( V_{24} = 13 \, \mu \)

**resistivity:**

\[
R_{xx} = \frac{V_{21}}{I} = 400 \, \Omega
\]

\[
R_{xx} = \rho_s \frac{L}{W} \Rightarrow \rho_s = 200 \, \Omega/\square
\]
Example: sheet carrier density

Top view

\[ I = I_x = 1 \mu A \]

\[ B_z = 2,000 \text{ Gauss} \]

\[ (1 \text{ Tesla} = 10^4 \text{ Gauss}) \]

\[ L = 100 \text{ \mu m} \]

\[ W = 50 \text{ \mu m} \]

\[ B = 0: \quad V_{21} = 0.4 \text{ mV} \]

\[ B = 0.2T: \quad V_{24} = 13 \text{ \mu V} \]

sheet carrier density:

\[ n_H \equiv \frac{n_S}{r_H} = \frac{I_x B_z}{qV_H} = \frac{I_x B_z}{qV_{24}} \]

\[ n_H = 9.6 \times 10^{12} \text{ cm}^{-2} \]
Example: mobility

\[ B_z = 2,000 \text{ Gauss} \]
\[ (1 \text{ Tesla} = 10^4 \text{ Gauss}) \]
\[ L = 100 \mu\text{m} \]
\[ W = 50 \mu\text{m} \]
\[ B = 0: \]
\[ V_{21} = 0.4 \text{ mV} \]
\[ B = 0.2\text{T}: \]
\[ V_{24} = 13 \mu\text{V} \]

Mobility:

\[ \sigma_S = \frac{1}{\rho_S} = n_S q \mu_n = \left( \frac{n_S}{r_H} \right) q (r_H \mu_n) \]
\[ \mu_H \equiv r_H \mu_n = 3125 \text{ cm}^2/\text{V-s} \]
Re-cap

1) Hall coefficient:
\[ R_H \equiv \frac{-V_H}{I_x B_z} = \frac{r_H}{(-q)n_S} \]

2) Hall concentration:
\[ n_H \equiv \frac{n_S}{r_H} \]

3) Hall mobility:
\[ \mu_H \equiv r_H \mu_n \]

4) Hall factor:
\[ r_H \equiv \frac{\langle \tau_m^2 \rangle}{\langle \tau_m \rangle} \]
Outline

1. Introduction
2. Resistivity / conductivity measurements
3. Hall effect measurements
4. The van der Pauw method
5. Seebeck coefficient
6. Summary
van der Pauw sample

2D film arbitrarily shaped homogeneous, isotropic (no holes)

Top view

Four small contacts along the perimeter
van der Pauw approach

**Resistivity**

1) force a current in $M$ and out $N$
2) measure $V_{PO}$
3) $R_{MN, OP} = V_{PO} / I$ related to $\rho_S$

**Hall effect**

1) force a current in $M$ and out $O$
2) measure $V_{PN}$
3) $R_{MO, NP} = V_{PN} / I$ related to $V_H$
van der Pauw approach: Hall effect

**Hall effect**

\[ J_x = \sigma_n \mathcal{E}_x - \left( \sigma_n \mu_n r_H \right) E_y B_z \]
\[ J_y = \sigma_n \mathcal{E}_y + \left( \sigma_n \mu_n r_H \right) E_x B_z \]
\[ \mathcal{E}_x = \rho_n J_x + \left( \rho_n \mu_H B_z \right) J_y \]
\[ \mathcal{E}_y = - \left( \rho_n \mu_H B_z \right) J_x + \rho_n J_y \]
\[ V_{PN}(B_z) = - \int_{N}^{P} \mathcal{E} \cdot d\vec{l} = - \int_{N}^{P} \mathcal{E}_x \, dx + \mathcal{E}_y \, dy \]
\[ V_H \equiv \frac{1}{2} \left[ V_{PN}(+B_z) - V_{PN}(-B_z) \right] \]
van der Pauw approach: Hall effect

So we can do Hall effect measurements on such samples.

For the missing steps, see Lundstrom, Fundamentals of Carrier Transport, 2nd Ed., Sec. 4.7.1.
van der Pauw approach: resistivity

Resistivity

\[ R_{MN,OP} = \frac{V_{PO}}{I} \]

semi-infinite half-plane

Lundstrom ECE-656 F17
van der Pauw approach: resistivity

semi-infinite half-plane

\[ J_r = \frac{I}{\pi r} = \sigma_s \mathcal{E}_r \]

\[ \mathcal{E}_r = \frac{I \rho_s}{\pi r} \]

\[ V(r) - V(r_0) = -\frac{I \rho_s}{\pi} \ln \left( \frac{r}{r_0} \right) \]
van der Pauw approach: resistivity

\[ V(r) - V(r_0) = -\frac{I\rho_S}{\pi} \ln\left(\frac{r}{r_0}\right) \]

\[ V(P) = -\frac{I\rho_S}{\pi} \ln\left(\frac{a+b+c}{r_0}\right) \]

\[ V(O) = -\frac{I\rho_S}{\pi} \ln\left(\frac{a+b}{r_0}\right) \]

\[ V'_{PO} = +\frac{I\rho_S}{\pi} \ln\left(\frac{b+c}{b}\right) \]

but there is also a contribution from contact \( N \)
van der Pauw approach: resistivity

It can be shown that:

\[ e^{\frac{-\pi \rho_s R_{MN,OP}}{R_{MN,OP}}} + e^{\frac{-\pi \rho_s R_{NO,PM}}{R_{NO,PM}}} = 1 \]

Given two measurements of resistance, this equation can be solved for the sheet resistance.
van der Pauw approach: resistivity

The same equation applies for an arbitrarily shaped sample!
van der Pauw technique: regular sample

Force $I$ through two contacts, measure $V$ between the other two contacts.
van der Pauw technique: summary

1) measure $n_H$

$$V_H = \frac{1}{2} [V_{PN} (+B_z) - V_{PN} (-B_z)] = \frac{r_H}{qn_S} B_z I = \frac{B_z I}{qn_H}$$
van der Pauw technique: summary

\[ B = 0 \]

2) measure \( \rho_S \)

3) determine \( \mu_H \)

\[ R_{MN,OP} = \frac{V_{P0}}{I} \]

\[ R_{NO,PM} = \frac{V_{MP}}{I} \]

\[
\begin{align*}
\rho_S &= \frac{n_S q \mu_n}{r_H} \\
\sigma_S &= \frac{n_S q r_H \mu_n}{r_H} = n_H q \mu_H
\end{align*}
\]
outline

1. Introduction
2. Resistivity / conductivity measurements
3. Hall effect measurements
4. The van der Pauw method
5. Seebeck coefficient
6. Summary
Measuring the Seebeck coefficient

\[ \Delta V_s = -S_s \Delta T \]

\[ \Delta V_l = -S_l \Delta T \]

\[ V_{meas} = \Delta V_s - \Delta V_l \]

\[ V_{meas} = -(S_s - S_l) \Delta T \]
1. Introduction
2. Resistivity / conductivity measurements
3. Hall effect measurements
4. The van der Pauw method
5. Seebeck coefficient
6. Summary
summary

1) Hall bar or van der Pauw geometries allow measurement of both resistivity and Hall concentration from which the Hall mobility can be deduced.

2) Temperature-dependent measurements (to be discussed in the next lecture) provide information about the dominant scattering mechanisms.

3) Care must be taken to exclude thermoelectric effects (also to be discussed in the next lecture).
for more about low-field measurements


Questions?

1. Introduction
2. Resistivity / conductivity measurements
3. Hall effect measurements
4. The van der Pauw method
5. Seebeck coefficient
6. Summary