ECE 656: Electronic Transport in Semiconductors Fall 2017

Electrical Characterization of Materials: I

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Coupled charge and heat current equations

electrical current:

$$\mathcal{E} = \rho J + S\frac{dT}{dx}$$

heat current (electronic):

$$J_Q = \pi J - \kappa_e \frac{dT}{dx}$$

heat current (lattice):

$$q_x = -\kappa_L \left(\frac{dT}{dx}\right)$$

How do we measure the four transport parameters?

(Note that we only need to measure 3 of them, because the Kelvin Relation relates the Seebeck and Peltier coefficients.)

Outline

1. **Introduction**
2. Resistivity / conductivity measurements
3. Hall effect measurements
4. The van der Pauw method
5. Seebeck coefficient
6. Summary

Measurement of conductivity / resistivity

1) Commonly-used to characterize electronic materials.

2) Results can be clouded by several effects – e.g. contacts, thermoelectric effects, etc.

3) Measurements in the absence of a magnetic field are often combined with those in the presence of a B-field.

This lecture is a brief introduction to the measurement and characterization of near-equilibrium transport.

Resistivity / conductivity measurements

$$J_{nx} = \sigma_n \frac{d\left(F_n/q\right)}{dx}$$

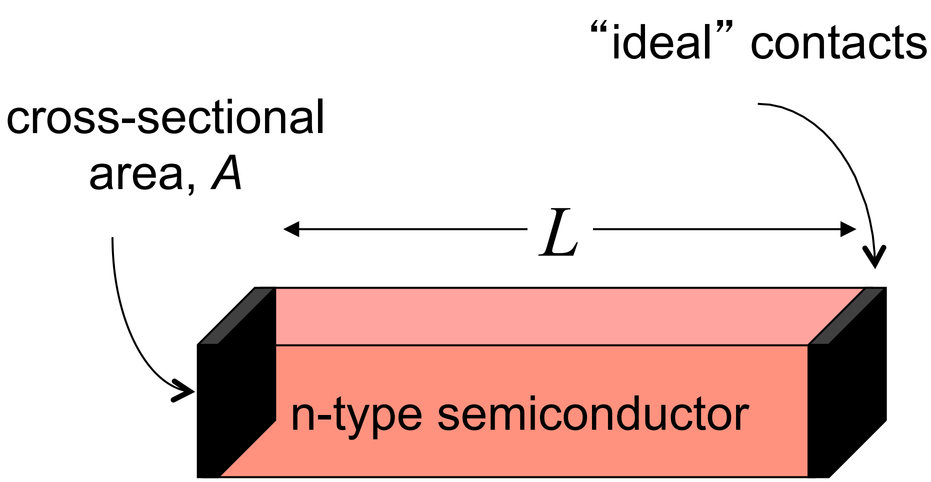
(diffusive transport assumed)

For uniform carrier concentrations:

$$J_{nx} = \sigma_n \mathcal{E}_x \quad \mathcal{E}_x = \rho_n J_{nx}$$

We generally measure **resisitivity** (or **conductivity**) because for diffusive samples, these parameters depend on material properties and not on the length of the resistor or its width or cross-sectional area.

Landauer conductance and conductivity



$$G = \sigma_n \frac{A}{L}$$

For ballistic or quasi-ballistic transport, replace the mfp with the "apparent" mfp:

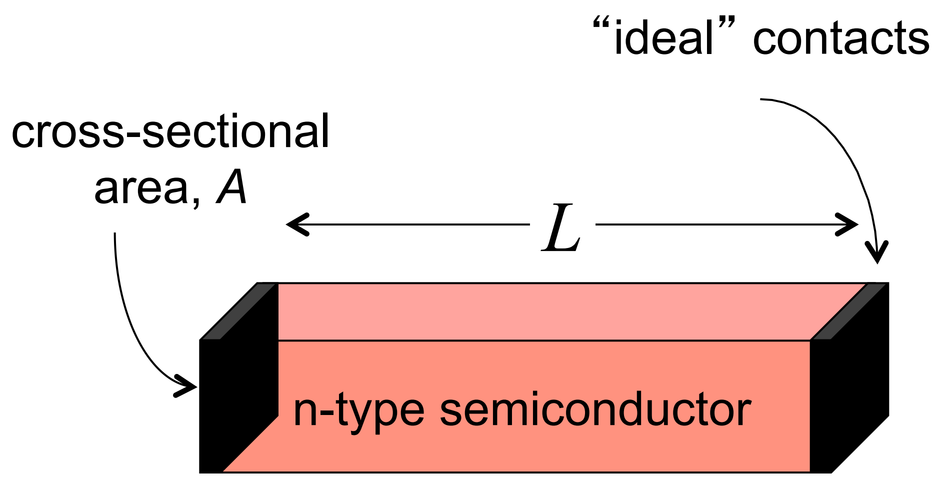
$$1/\lambda_{app}(E) = 1/\lambda(E) + 1/L$$

$$G = \frac{2q^2}{h}\int M(E)T(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \qquad T(E) = \frac{\lambda(E)}{L}$$

$$\sigma = \frac{G}{A/L} = \frac{2q^2}{h}\int M_{3D}(E)\lambda(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \qquad M_{3D}(E) = \frac{M(E)}{A}$$

(diffusive)

Conductivity and mobility



$$\sigma = \frac{G}{A/L} = \frac{2q^2}{h}\int M_{3D}(E)\lambda(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

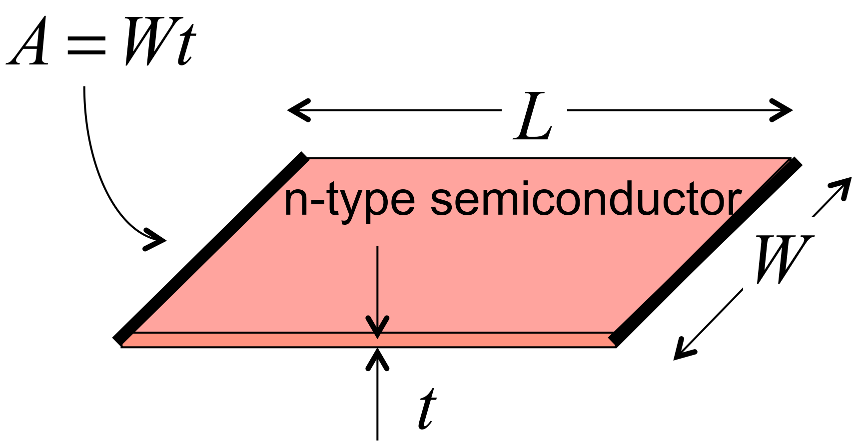
So we need techniques to measure two quantities:

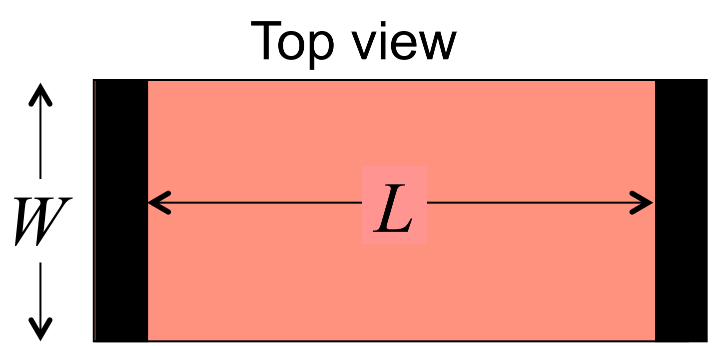
1) conductivity 2) carrier density

1) Conductivity depends on E_F.
2) E_F *depends on* carrier density.
3) So it is common to characterize the conductivity at a given carrier density.
4) Mobility is often the quantity that is quoted.

$$\sigma_n = nq\mu_n$$

2D: conductivity and sheet conductance





$$I = GV$$

2D electrons

$$G = \sigma_n \frac{A}{L}$$

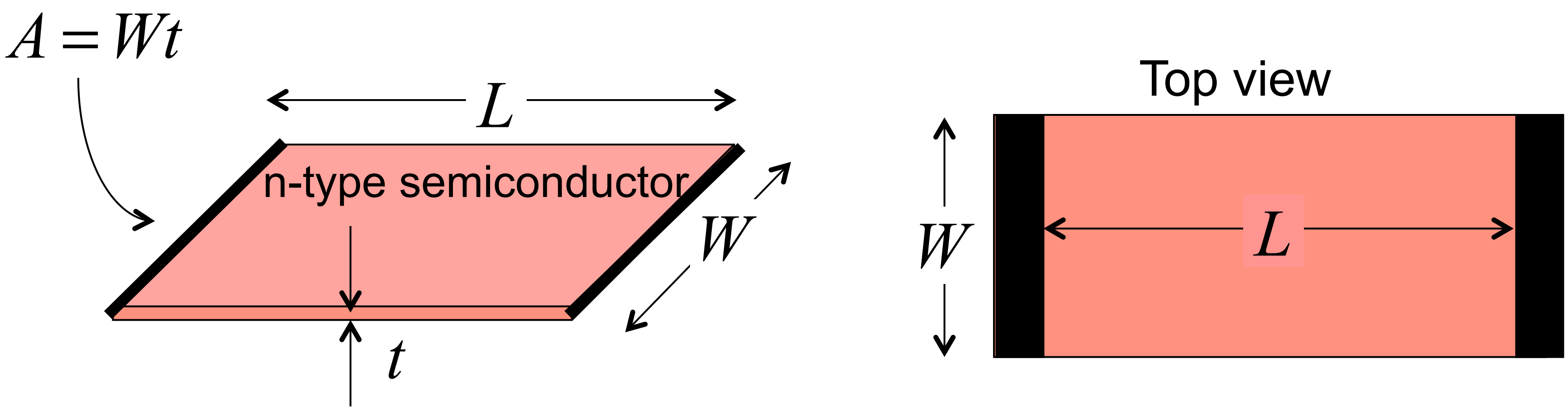
$$G = \sigma_n \left(\frac{Wt}{L} \right) = \sigma_n t \left(\frac{W}{L} \right)$$

$$G = \sigma_S \left(\frac{W}{L} \right)$$

$$\sigma_S = n_S \, q \, \mu_n \quad (1/\Omega)$$

"sheet conductance"

2D electrons vs. 3D electrons



3D electrons:

$$G = \sigma \frac{A}{L} = \sigma \frac{Wt}{L} \rightarrow \sigma_S = \frac{G}{W/L} = \frac{2q^2}{h} \int t M_{3D}(E) \lambda(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

2D electrons:

$$G = \sigma_S \frac{W}{L} \rightarrow \sigma_S = \frac{G}{W/L} = \frac{2q^2}{h} \int M_{2D}(E) \lambda(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

Mobility

1) Measure the conductivity: σ_S

2) Measure the sheet carrier density: n_S

3) Deduce the mobility from: $\sigma_S \equiv n_S q \mu_n$

4) Relate the mobility to material parameters:

$$\sigma_S = \frac{2q^2}{h} \int M_{2D}(E) \lambda(E) \left(-\frac{\partial f_0}{\partial E} \right) \equiv n_S q \mu_n$$

Recap

There are three near-equilibrium transport coefficients: conductivity, Seebeck (and Peltier) coefficient, and the electronic thermal conductivity. We can measure all three, but in this brief lecture, we will focus on the conductivity and briefly mention the Seebeck coefficient.

Conductivity depends on the location of the Fermi level, which can be set by controlling the carrier density.

So we need to discuss how to measure the conductivity (or resistivity) and the carrier density. Let's discuss the resistivity first.

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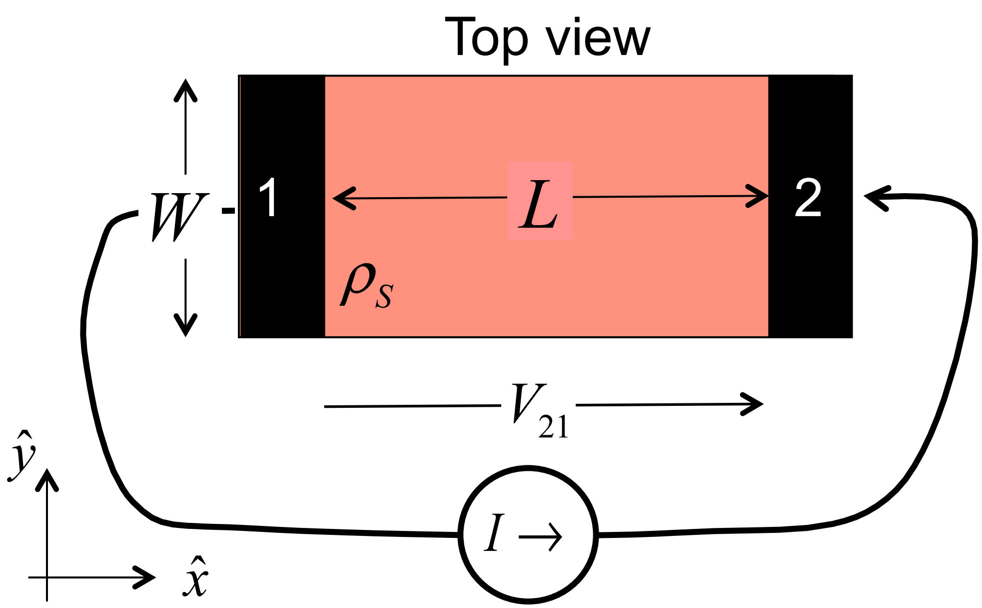
2-probe measurements

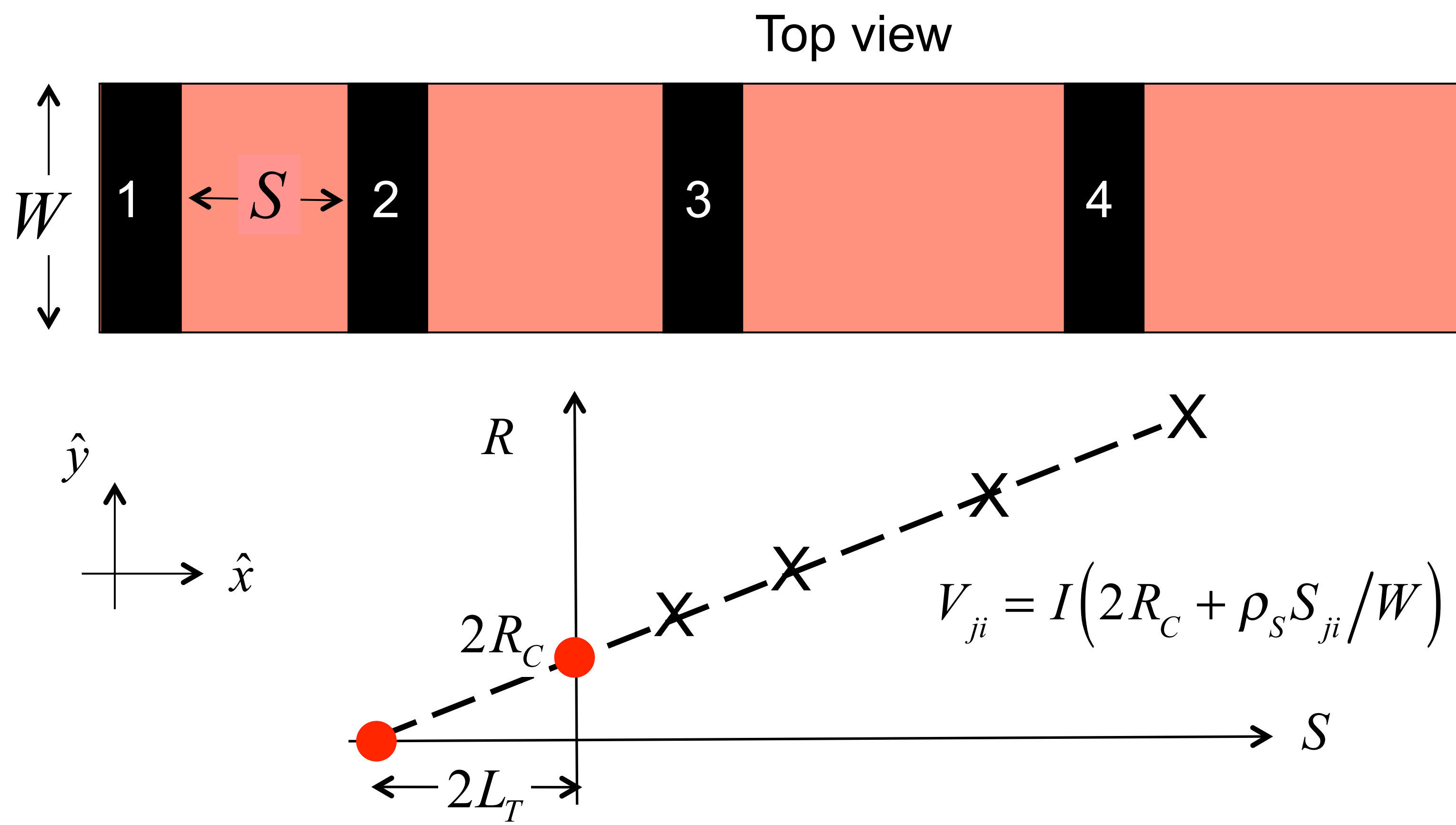


$$R_{CH} = \rho_S \frac{L}{W}$$

$$V_{21} = I\left(2R_C + R_{CH}\right)$$

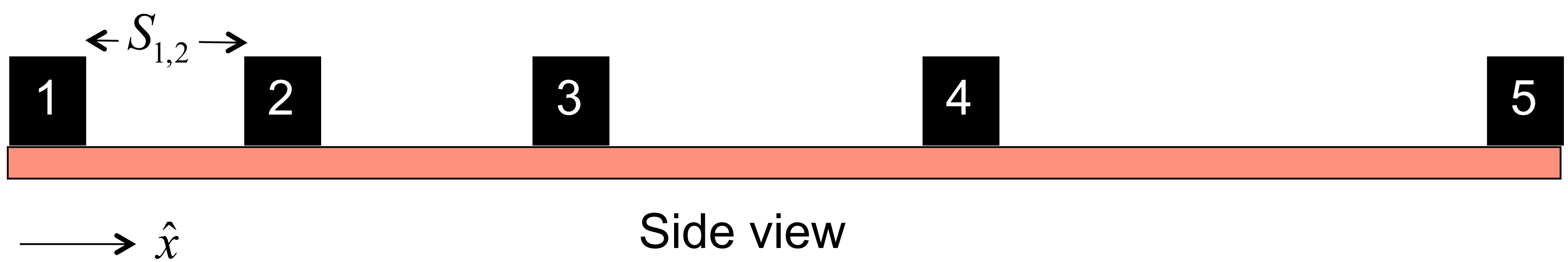
$$R_{CH} \neq \frac{V_{21}}{I}$$

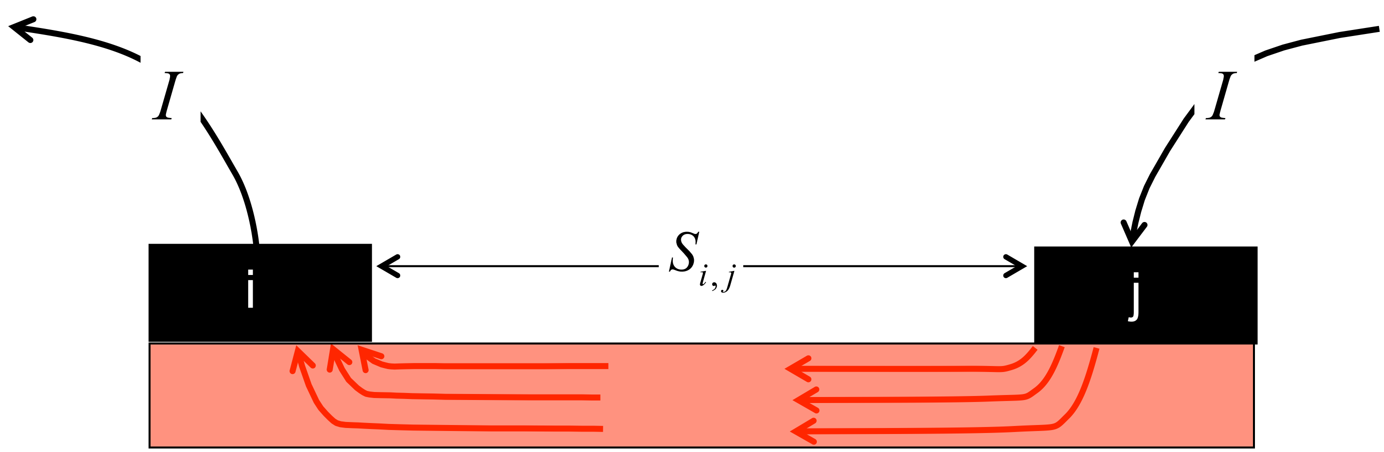
"Transmission line measurements"



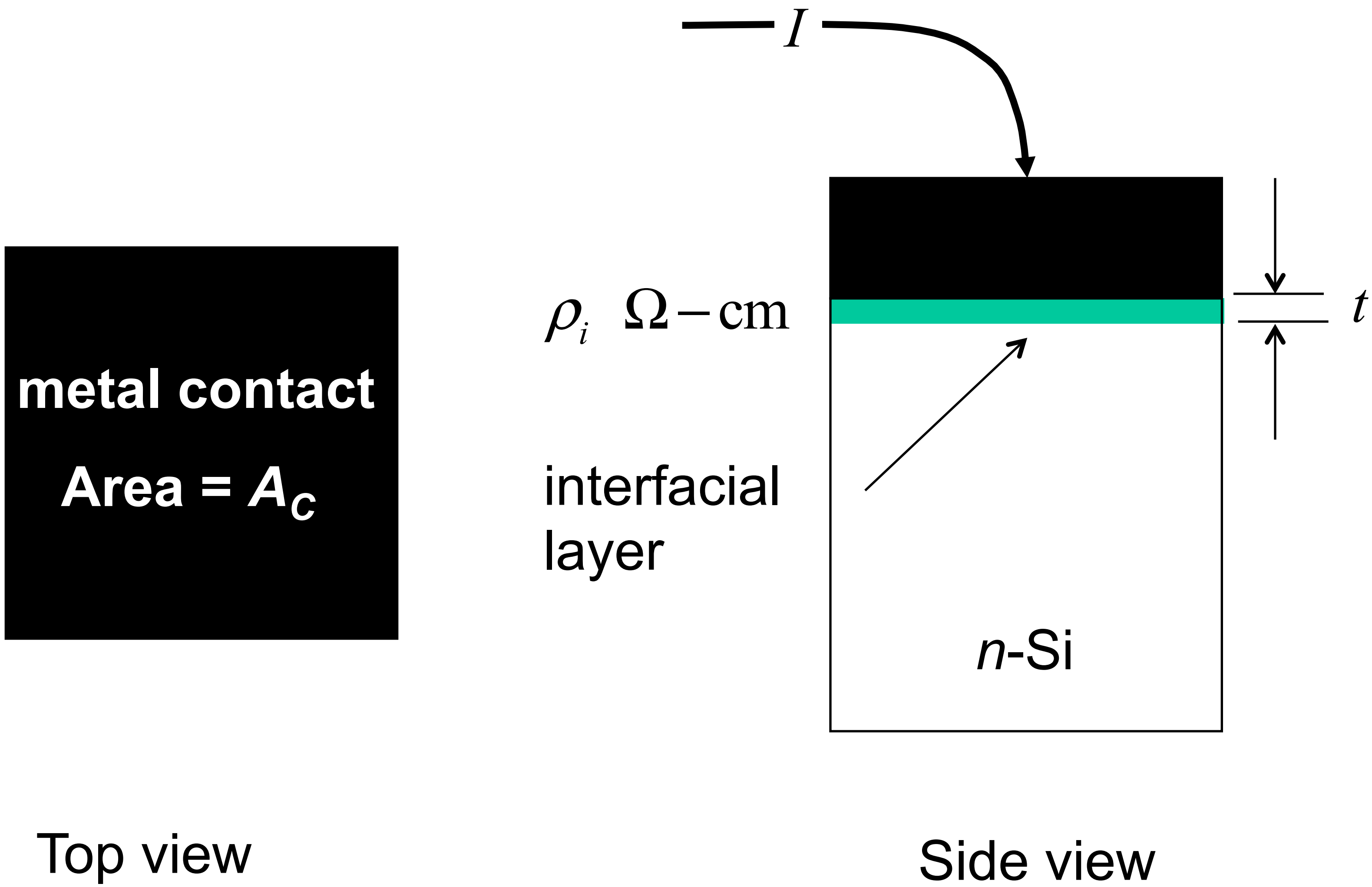
H.H. Berger, "Models for Contacts to Planar Devices," *Solid-State Electron*., **15**, 145-158, 1972.

Transmission line measurements (TLM)





Contact resistance (vertical flow)



Contact resistance (vertical flow)

$$R_C = \frac{\rho_i t}{A_C} = \frac{\rho_C}{A_C} \; \Omega$$

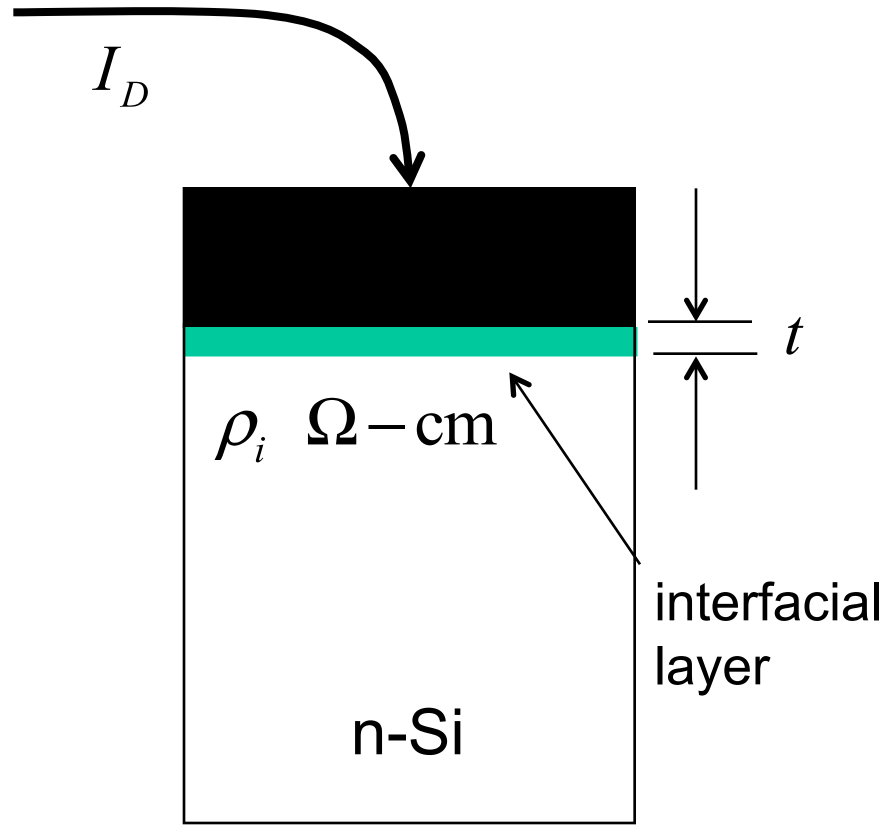
$$10^{-8} < \rho_C < 10^{-6} \; \Omega\text{-cm}^2$$

"interfacial contact resistivity"

$$A_C = 0.10 \; \mu m \times 1.0 \mu m$$

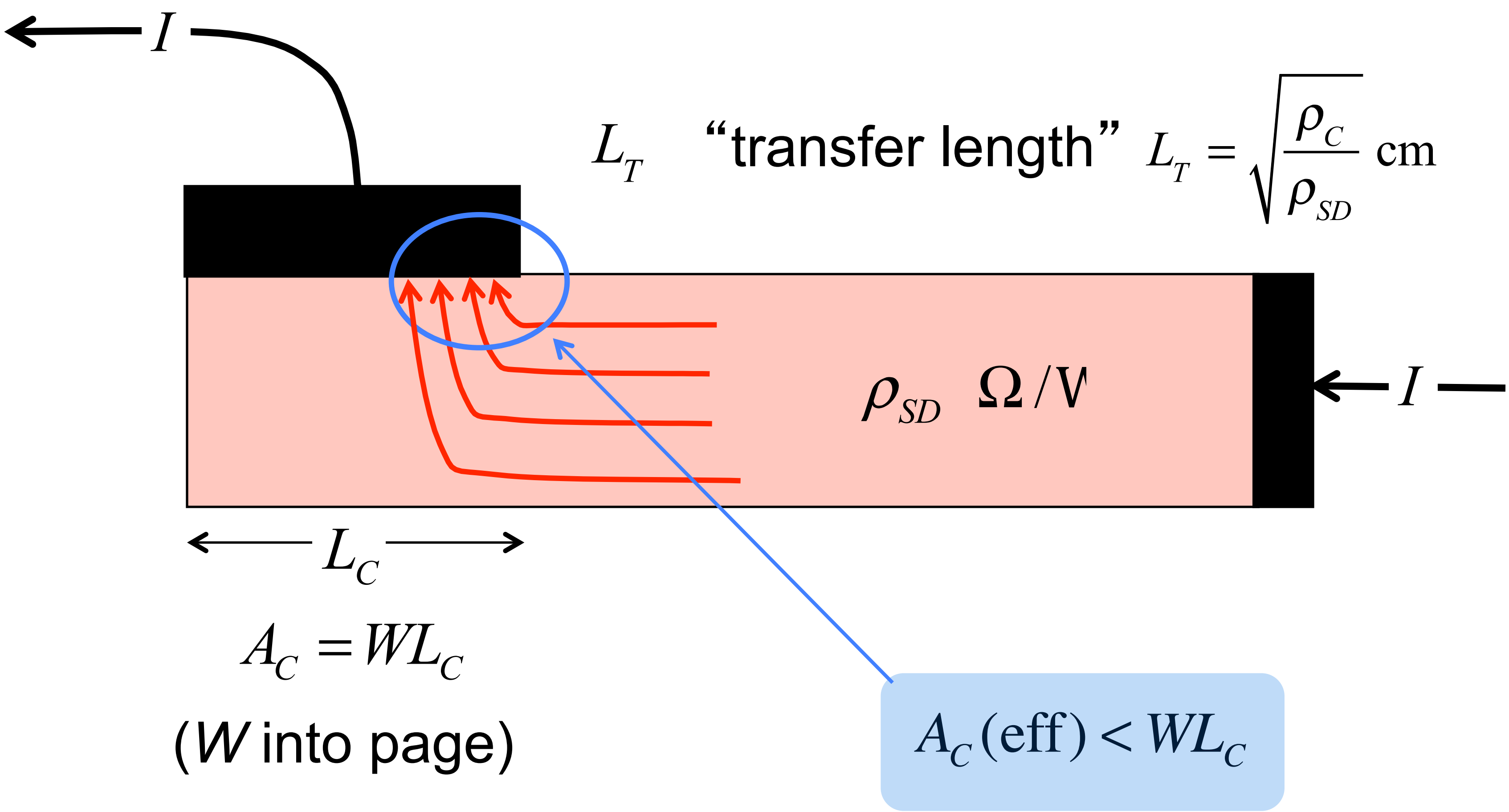
$$\rho_C = 10^{-7} \; \Omega\text{-cm}^2$$

$$R_C = 100 \;\; \Omega$$

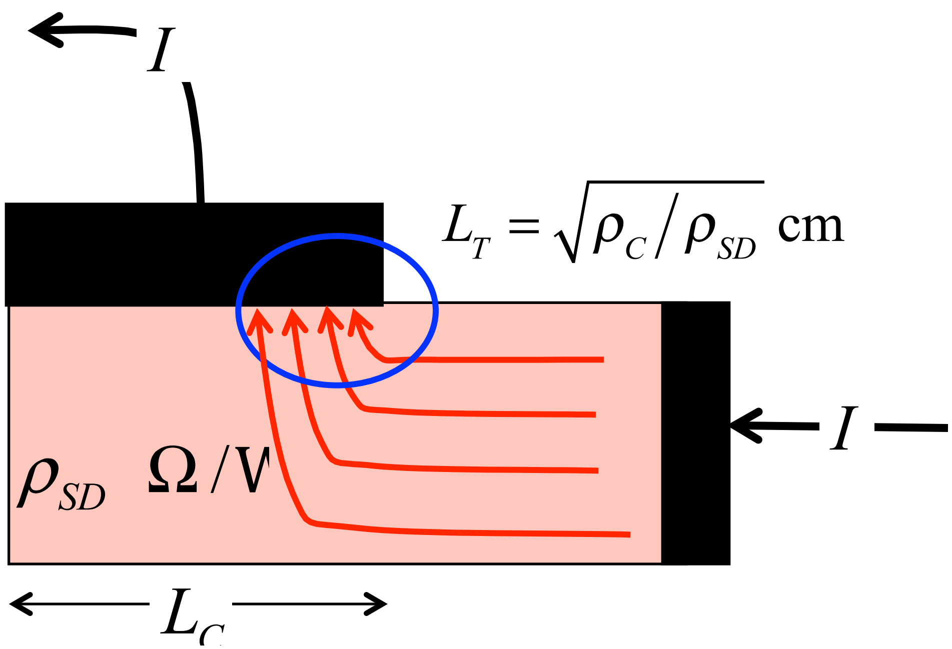


Side view

Contact resistance (vertical + lateral flow)



Contact resistance



$$L_T = \sqrt{\rho_C / \rho_{SD}} \text{ cm}$$

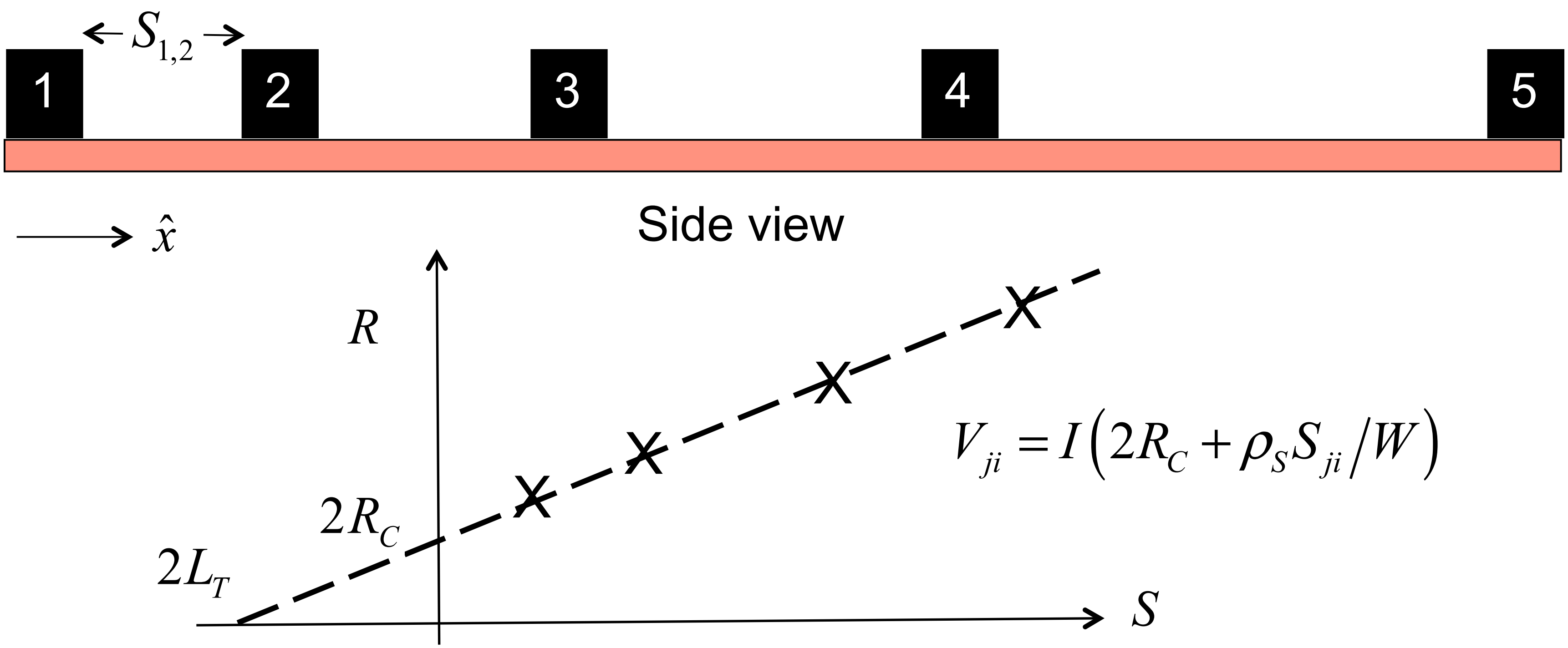
$$A_C = W L_C$$

$$R_C = \frac{\sqrt{\rho_C \rho_{SD}}}{W} \coth\left(L_C / L_T\right)$$

i) $L_C \ll L_T$: $\quad R_C = \dfrac{\rho_C}{L_C W}$

ii) $L_C \gg L_T$: $\quad R_C = \dfrac{\rho_C}{L_T W}$

Transfer length measurments (TLM)



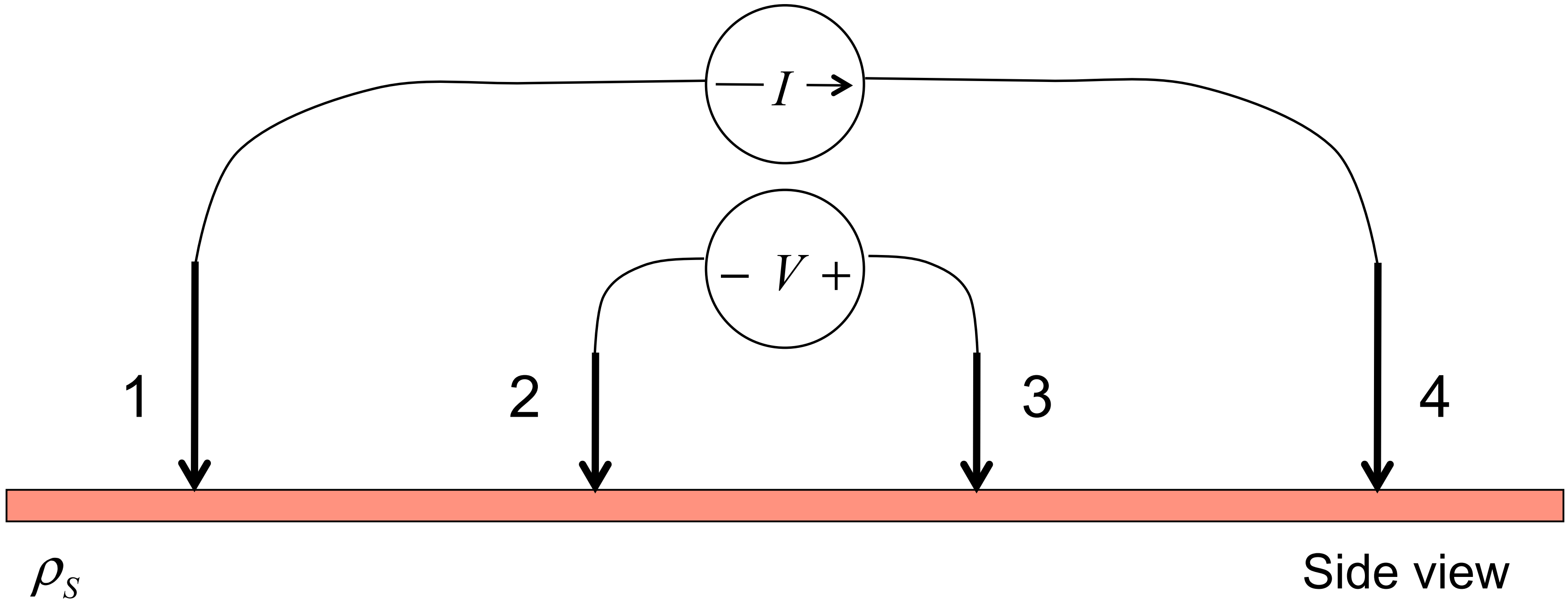
$$V_{ji} = I\left(2R_C + \rho_S S_{ji}/W\right)$$

1) Slope gives sheet resistance, intercept gives contact resistance

2) Determine specific contact resistivity and transfer length:

$$R_C = \frac{\sqrt{\rho_C \rho_{SD}}}{W} \coth\left(L_C / L_T\right) \qquad L_T = \sqrt{\rho_C/\rho_{SD}}\ \text{cm}$$

Four probe measurements

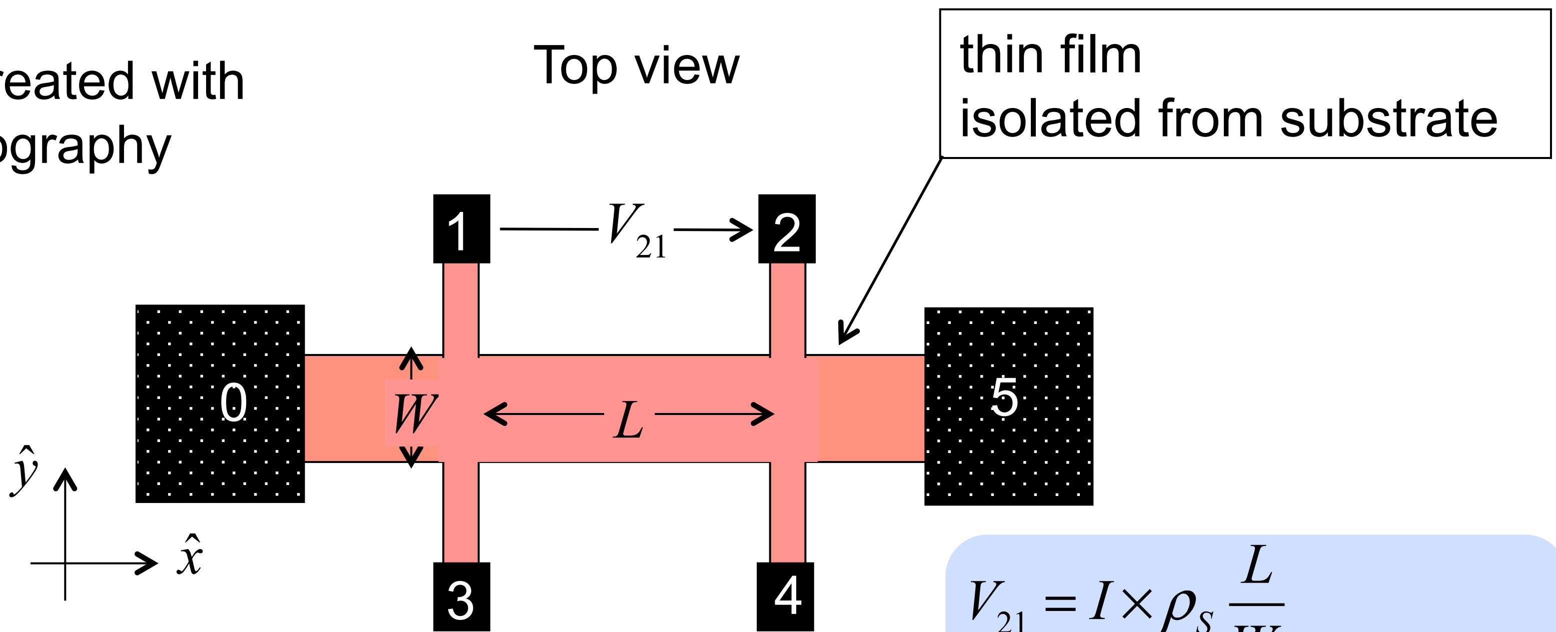


1) force a current through probes 1 and 4
2) with a high impedance voltmeter, measure the voltage between probes 2 and 3

$$R = \frac{V}{I} = f\left(\rho_S\right) \quad \text{(no series resistance)}$$

Hall bar geometry

pattern created with photolithography



$$V_{21} = I \times \rho_S \frac{L}{W}$$

(high impedance voltmeter)

no contact resistance

Contacts 0 and 5: "current probes"
Contacts 1 and 2 (3 and 4): "voltage probes"

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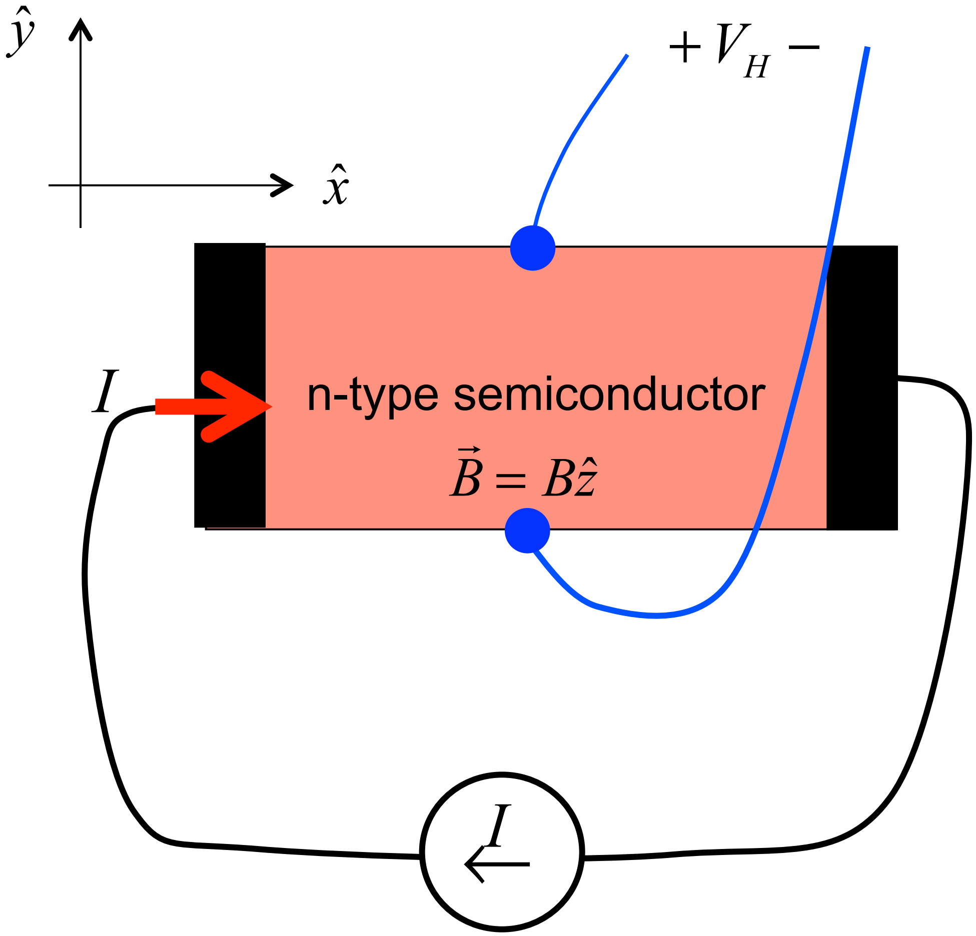
Hall effect



current in *x*-direction:

I_x

B-field in *z*-direction:

$\vec{B} = B\hat{z}$

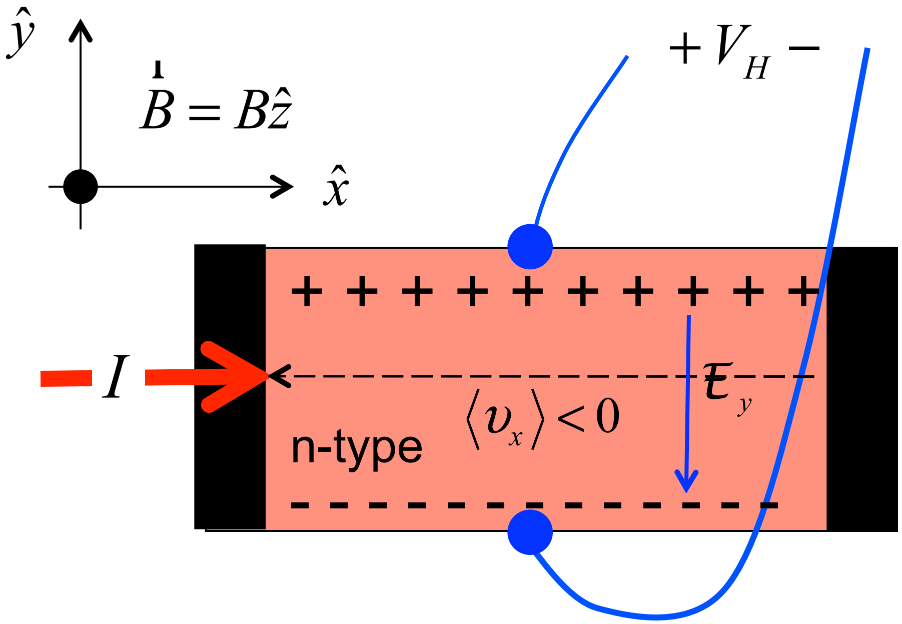
Hall voltage measured in the *y*-direction:

$V_H > 0 \quad \text{(n-type)}$

The Hall effect was discovered by Edwin Hall in 1879 and is widely used to characterize electronic materials. It also finds use magnetic field sensors.

Hall effect: analysis

Top view of a 2D film



$$\vec{J}_n = nq\mu_n \vec{\mathcal{E}} - (\sigma_n \mu_n r_H)\vec{E} \times \vec{B}$$

$$R_H \equiv \frac{\mathcal{E}_y}{J_x B_z} = \frac{-V_H}{I_x B_z}$$

$$R_H = \frac{r_H}{(-q)n_S} \qquad r_H \equiv \frac{\langle\langle \tau_m^2 \rangle\rangle}{\langle\langle \tau_m \rangle\rangle^2}$$

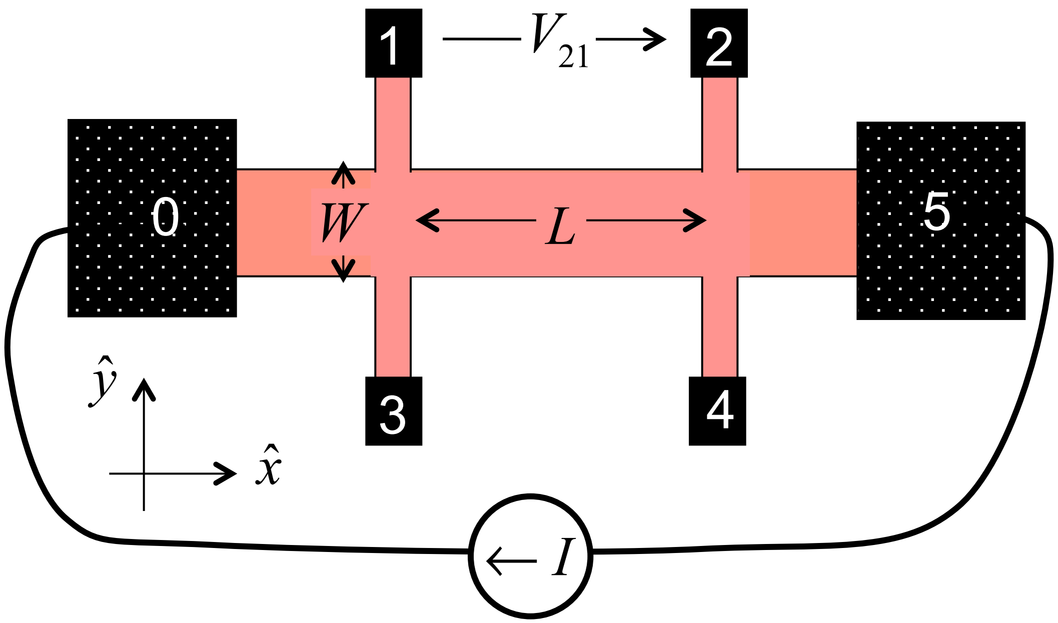
"Hall factor"

$$n_H \equiv \frac{n_S}{r_H}$$

"Hall concentration"

Example

Top view



What are the:

1) resistivity?
2) sheet carrier density?
3) mobility?

$$I = I_x = 1\ \mu\text{A}$$

$$B_z = 2{,}000 \text{ Gauss}$$

$$\left(1 \text{ Tesla} = 10^4 \text{ Gauss}\right)$$

$$L = 100\ \mu\text{m}$$

$$W = 50\ \mu\text{m}$$

B = 0:

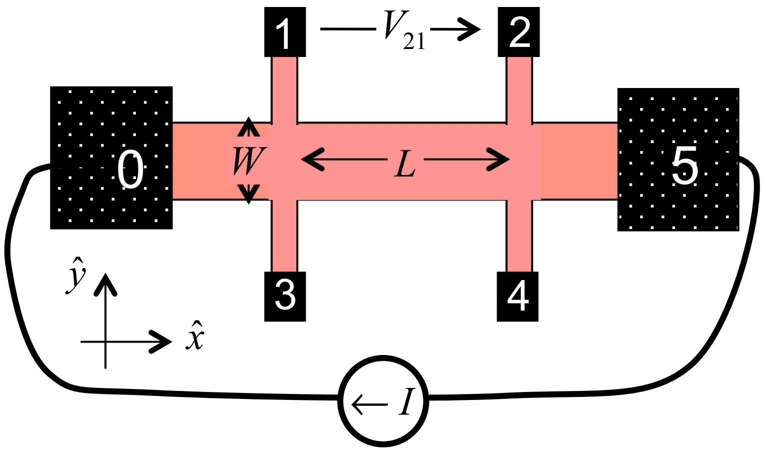
$$V_{21} = 0.4 \text{ mV}$$

B ≠ 0:

$$V_{24} = 13\ \mu\text{V}$$

Example: resistivity

Top view



$$I = I_x = 1\ \mu\text{A}$$

resistivity:

$$R_{xx} = \frac{V_{21}}{I} = 400\ \Omega$$

$$R_{xx} = \rho_S \frac{L}{W} \rightarrow \rho_S = 200\ \Omega/\square$$

$$B_z = 2{,}000 \text{ Gauss}$$

$$\left(1 \text{ Tesla} = 10^4 \text{ Gauss}\right)$$

$$L = 100\ \mu\text{m}$$

$$W = 50\ \mu\text{m}$$

B = 0:

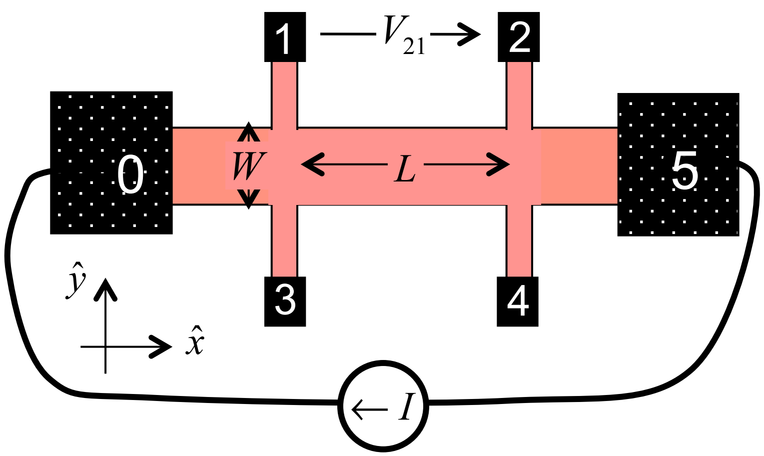
$$V_{21} = 0.4 \text{ mV}$$

B = 0.2T:

$$V_{24} = 13\ \mu$$

Example: sheet carrier density

Top view



$$I = I_x = 1\ \mu\text{A}$$

$$B_z = 2{,}000\ \text{Gauss}$$

$$\left(1\ \text{Tesla} = 10^4\ \text{Gauss}\right)$$

$$L = 100\ \mu\text{m}$$

$$W = 50\ \mu\text{m}$$

B = 0:

$$V_{21} = 0.4\ \text{mV}$$

B = 0.2T:

$$V_{24} = 13\ \mu\text{V}$$

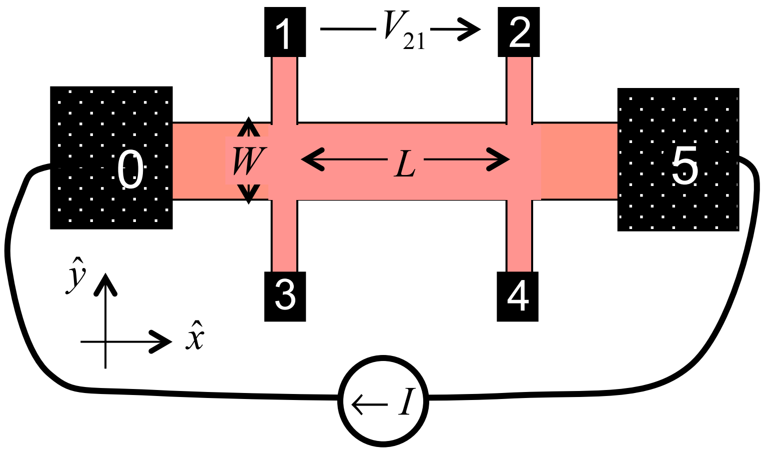
sheet carrier density:

$$n_H \equiv \frac{n_S}{r_H} = \frac{I_x B_z}{q V_H} = \frac{I_x B_z}{q V_{24}}$$

$$n_H = 9.6 \times 10^{12}\ \text{cm}^{-2}$$

Example: mobility

Top view



$$I = I_x = 1\ \mu\text{A}$$

mobility:

$$\sigma_S = \frac{1}{\rho_S} = n_S q \mu_n = \left(\frac{n_S}{r_H}\right) q \left(r_H \mu_n\right)$$

$$\mu_H \equiv r_H \mu_n = 3125\ \text{cm}^2/\text{V-s}$$

$$B_z = 2{,}000\ \text{Gauss}$$

$$\left(1\ \text{Tesla} = 10^4\ \text{Gauss}\right)$$

$$L = 100\ \mu\text{m}$$

$$W = 50\ \mu\text{m}$$

B = 0:

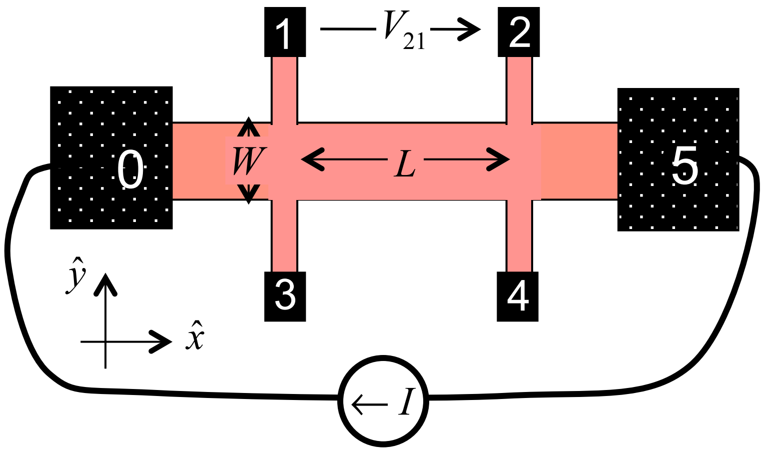
$$V_{21} = 0.4\ \text{mV}$$

B = 0.2T:

$$V_{24} = 13\ \mu\text{V}$$

Re-cap

Top view



$$I = I_x = 1\ \mu\text{A}$$

1) Hall coefficient:

$$R_H \equiv \frac{-V_H}{I_x B_z} = \frac{r_H}{(-q)n_S}$$

2) Hall concentration:

$$n_H \equiv n_S / r_H$$

3) Hall mobility:

$$\mu_H \equiv r_H \mu_n$$

4) Hall factor:

$$r_H \equiv \left\langle\left\langle \tau_m^2 \right\rangle\right\rangle \Big/ \left\langle\left\langle \tau_m \right\rangle\right\rangle$$

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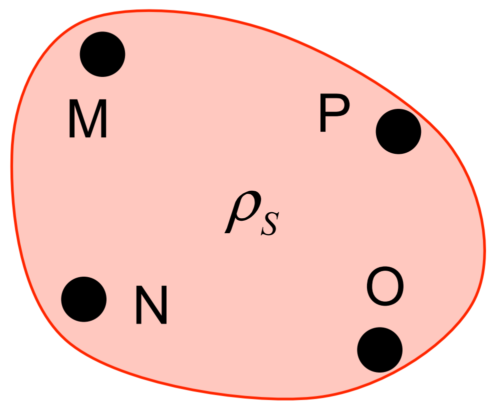
van der Pauw sample

2D film
arbitrarily shaped
homogeneous, isotropic
(no holes)

Top view

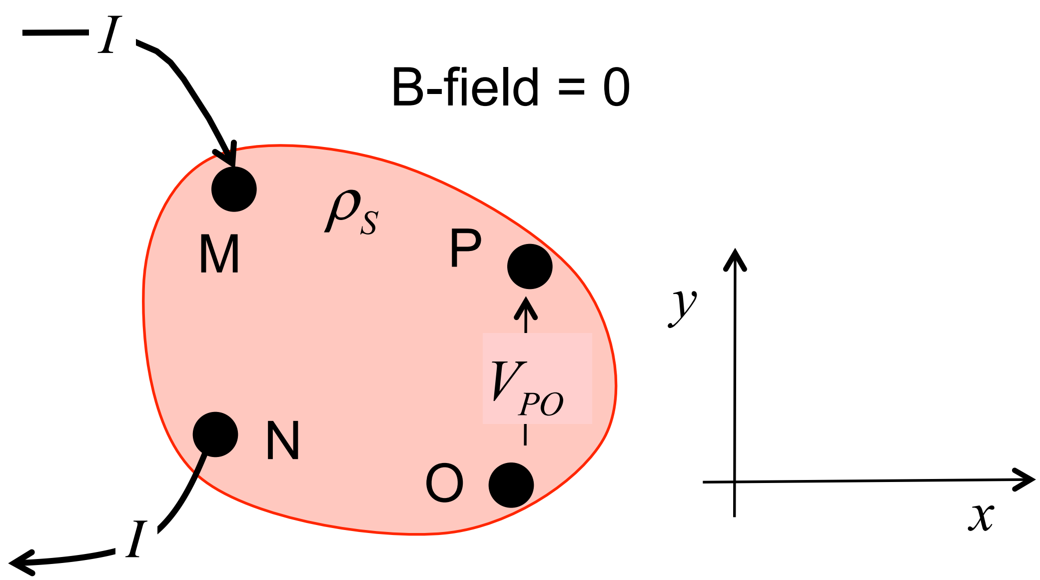


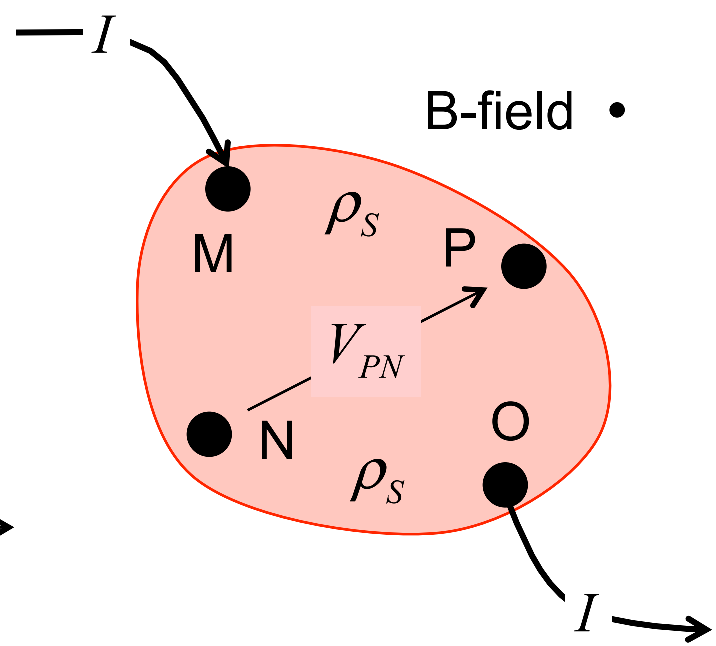
Four small contacts
along the perimeter

van der Pauw approach

Resistivity

Hall effect



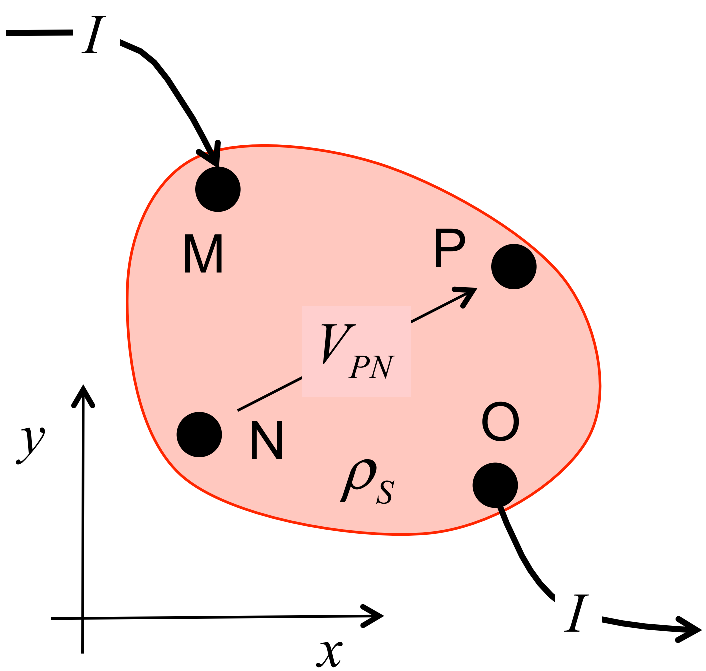


1) force a current in *M* and out *N*
2) measure V_{PO}
3) $R_{MN,\,OP} = V_{PO} / I$ related to ρ_S

1) force a current in *M* and out *O*
2) measure V_{PN}
3) $R_{MO,\,NP} = V_{PN} / I$ related to V_H

van der Pauw approach: Hall effect

Hall effect



$$\vec{J}_n = \sigma_n \vec{\mathcal{E}} - \left(\sigma_n \mu_n r_H\right) \vec{E} \times \vec{B}$$

$$J_x = \sigma_n \mathcal{E}_x - \left(\sigma_n \mu_n r_H\right) E_y B_z$$

$$J_y = \sigma_n \mathcal{E}_y + \left(\sigma_n \mu_n r_H\right) E_x B_z$$

$$\mathcal{E}_x = \rho_n J_x + \left(\rho_n \mu_H B_z\right) J_y$$

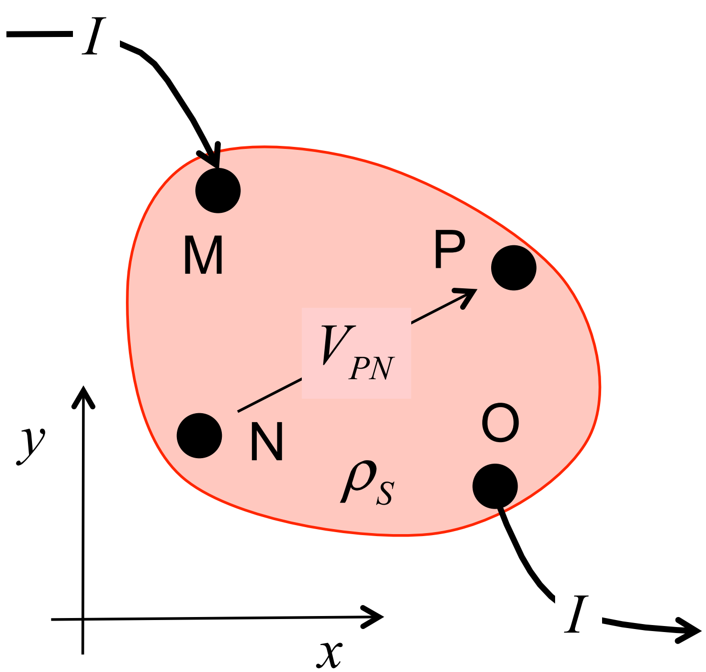
$$\mathcal{E}_y = -\left(\rho_n \mu_H B_z\right) J_x + \rho_n J_y$$

$$V_{PN}\left(B_z\right) = -\int_N^P \vec{\mathcal{E}} \bullet d\vec{l} = -\int_N^P \mathcal{E}_x\, dx + \mathcal{E}_y\, dy$$

$$V_H \equiv \frac{1}{2}\left[V_{PN}\left(+B_z\right) - V_{PN}\left(-B_z\right)\right]$$

van der Pauw approach: Hall effect

Hall effect



$$\vec{J}_n = nq\mu_n\vec{\mathcal{E}} - (\sigma_n\mu_n r_H)\vec{E}\times\vec{B}$$

$$V_H = \rho_n\mu_H B_z\left[\int_{y_N}^{y_P} J_x\,dy - \int_{x_N}^{x_P} J_y\,dx\right]$$

$$I = \int_N^P \vec{J}\bullet\hat{n}\,dl$$

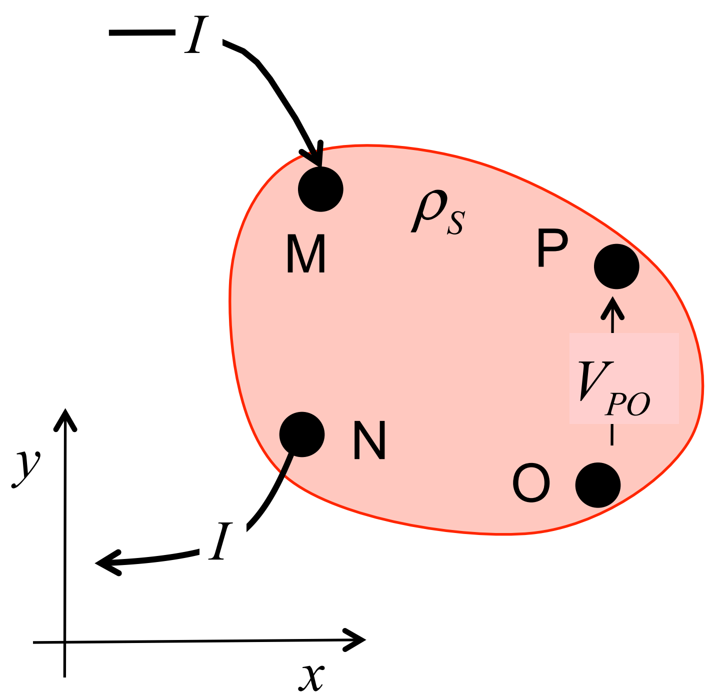
$$V_H = \rho_n\mu_H B_z I$$

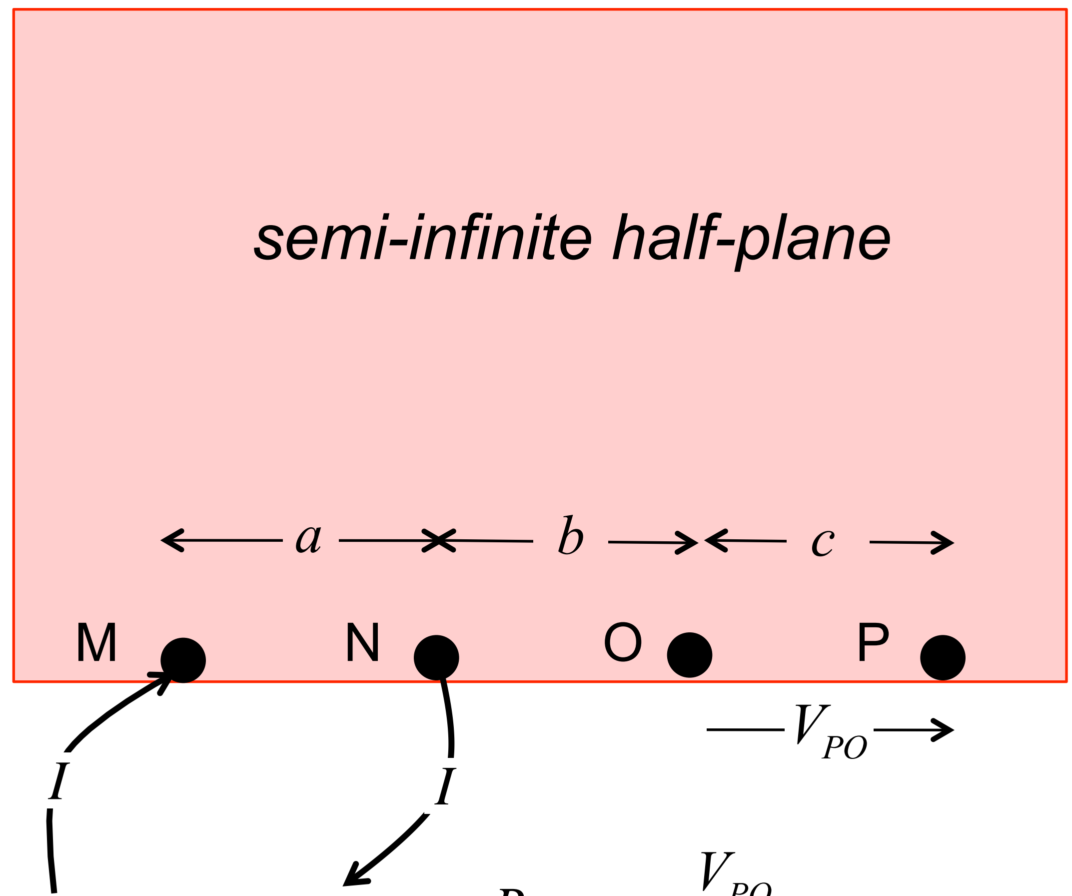
So we can do Hall effect measurements on such samples.

For the missing steps, see Lundstrom, *Fundamentals of Carrier Transport*, 2nd Ed., Sec. 4.7.1.

van der Pauw approach: resistivity

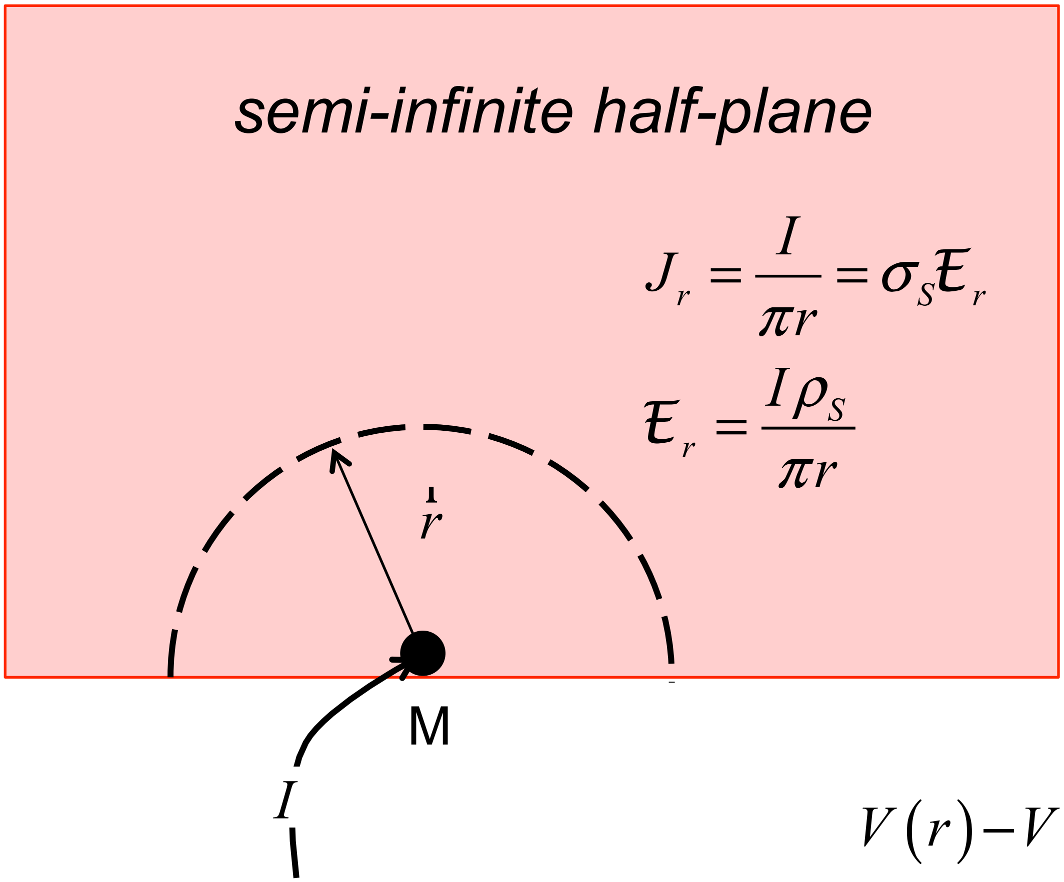
Resistivity





$$R_{MN,OP} = \frac{V_{PO}}{I}$$

van der Pauw approach: resistivity

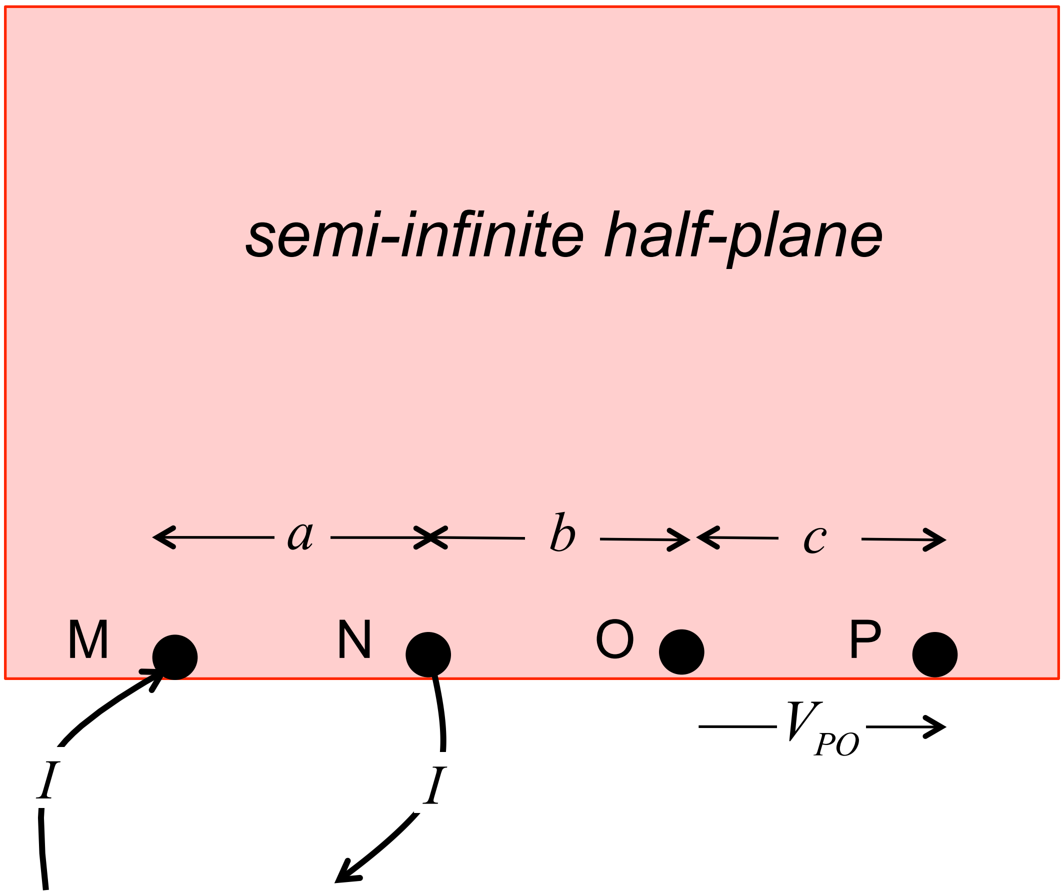


$$J_r = \frac{I}{\pi r} = \sigma_S \mathcal{E}_r$$

$$\mathcal{E}_r = \frac{I\rho_S}{\pi r}$$

$$V(r) - V(r_0) = -\frac{I\rho_S}{\pi} \ln\left(\frac{r}{r_0}\right)$$

van der Pauw approach: resistivity



$$V(r) - V(r_0) = -\frac{I\rho_S}{\pi}\ln\left(\frac{r}{r_0}\right)$$

$$V(P) = -\frac{I\rho_S}{\pi}\ln\left(\frac{a+b+c}{r_0}\right)$$

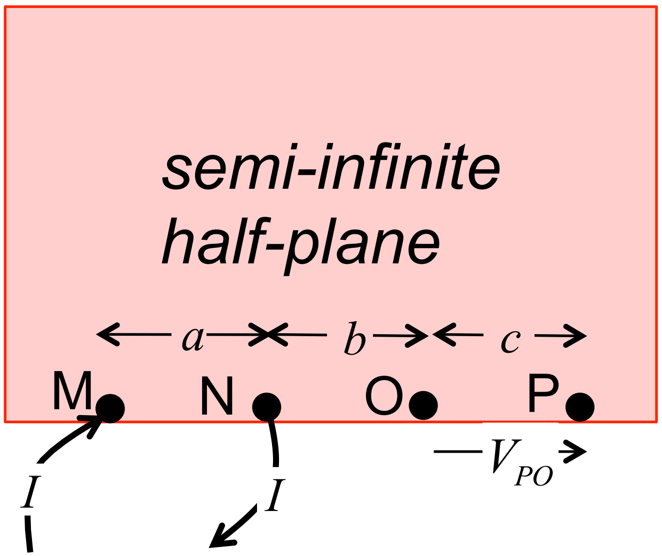
$$V(O) = -\frac{I\rho_S}{\pi}\ln\left(\frac{a+b}{r_0}\right)$$

$$V_{PO} = -\frac{I\rho_S}{\pi}\ln\left(\frac{a+b+c}{a+b}\right)$$

but there is also a contribution from contact *N*

$$V'_{PO} = +\frac{I\rho_S}{\pi}\ln\left(\frac{b+c}{b}\right)$$

van der Pauw approach: resistivity



$$R_{MN,OP} = \frac{V_{PO} + V'_{PO}}{I} = \frac{\rho_S}{\pi} \ln\left(\frac{(a+b)(b+c)}{b(a+b+c)} \right)$$

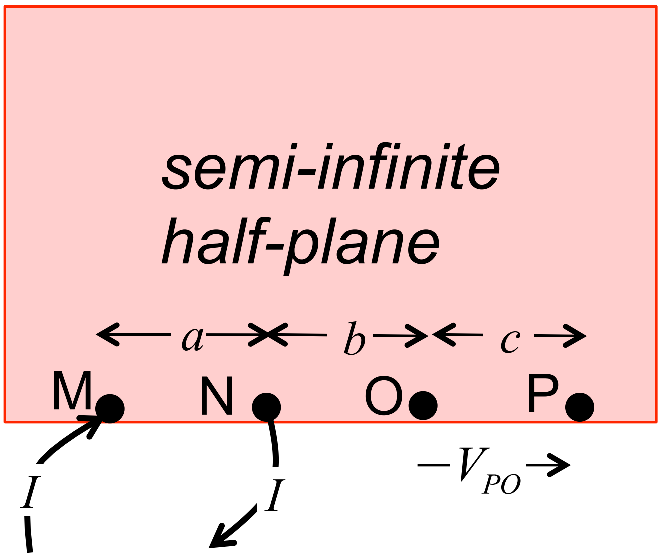
$$R_{NO,PM} = \frac{\rho_S}{\pi} \ln\left(\frac{(a+b)(b+c)}{ac} \right)$$

it can be shown that:

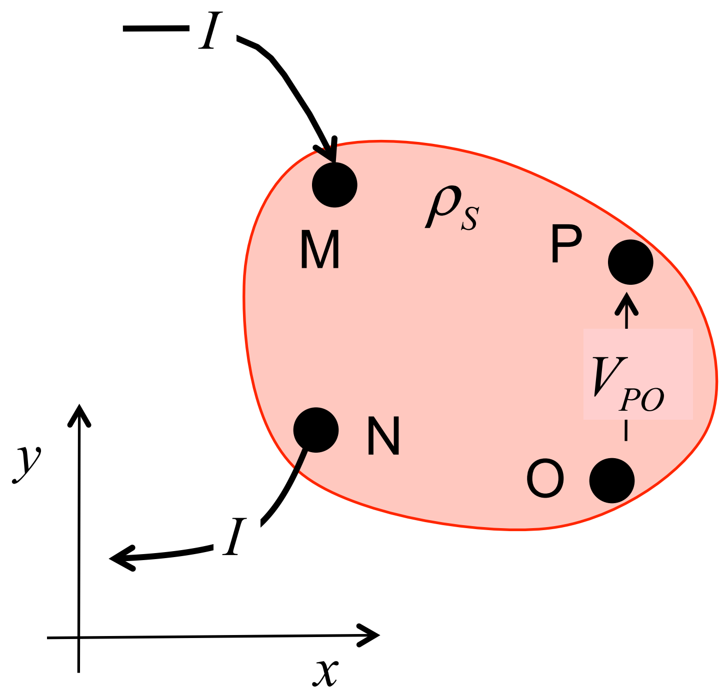
$$e^{-\frac{\pi}{\rho_S} R_{MN,OP}} + e^{-\frac{\pi}{\rho_S} R_{NO,PM}} = 1$$

Given two measurements of resistance, this equation can be solved for the sheet resistance.

van der Pauw approach: resistivity

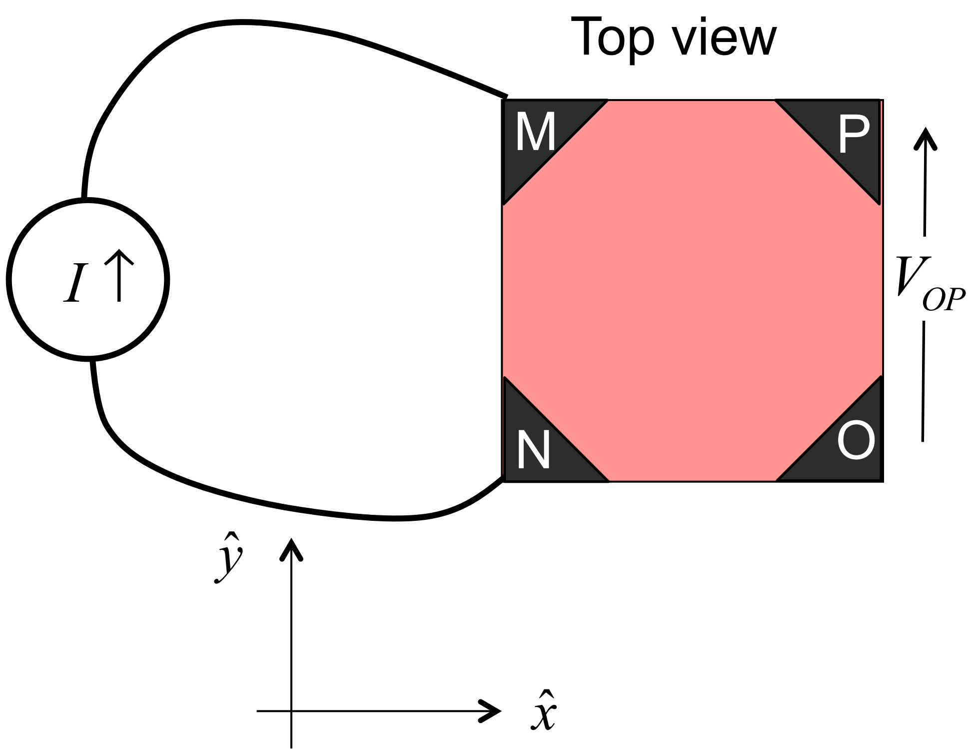


$$e^{-\frac{\pi}{\rho_S}R_{MN,OP}} + e^{-\frac{\pi}{\rho_S}R_{NO,PM}} = 1$$



The same equation applies for an arbitrarily shaped sample!

van der Pauw technique: regular sample



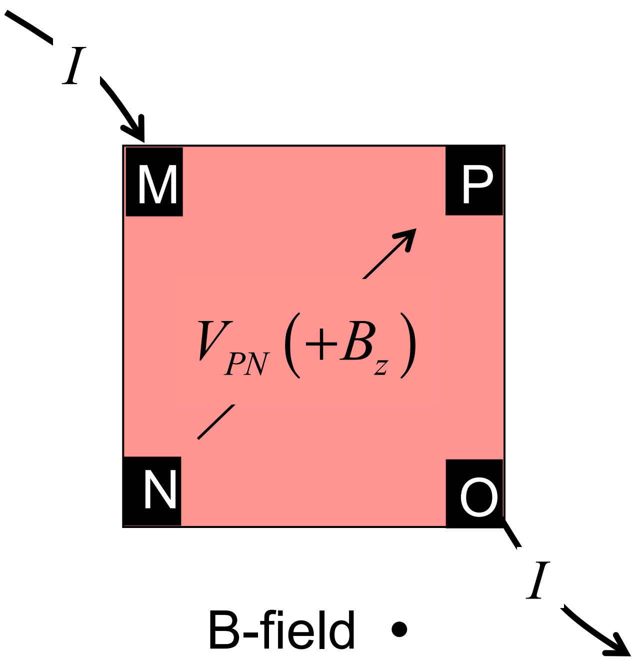
$$e^{-\frac{\pi}{\rho_S} R_{MN,OP}} + e^{-\frac{\pi}{\rho_S} R_{NO,PM}} = 1$$

$$R_{MN,OP} = R_{NO,PM} = \frac{V}{I}$$

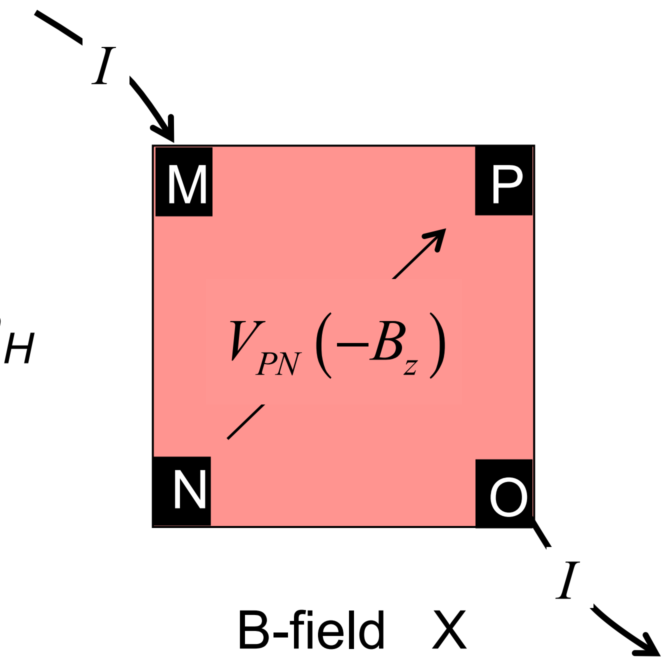
$$\rho_S = \frac{\pi}{\ln 2} \frac{V}{I}$$

Force *I* through two contacts, measure *V* between the other two contacts.

van der Pauw technique: summary



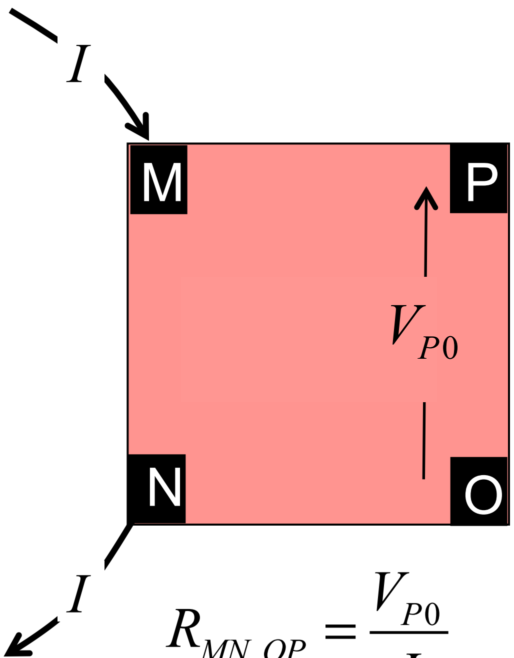
1) measure n_H



$$V_H = \frac{1}{2}\left[V_{PN}(+B_Z) - V_{PN}(-B_Z)\right] = \frac{r_H}{qn_S} B_z I = \frac{B_z I}{qn_H}$$

van der Pauw technique: summary

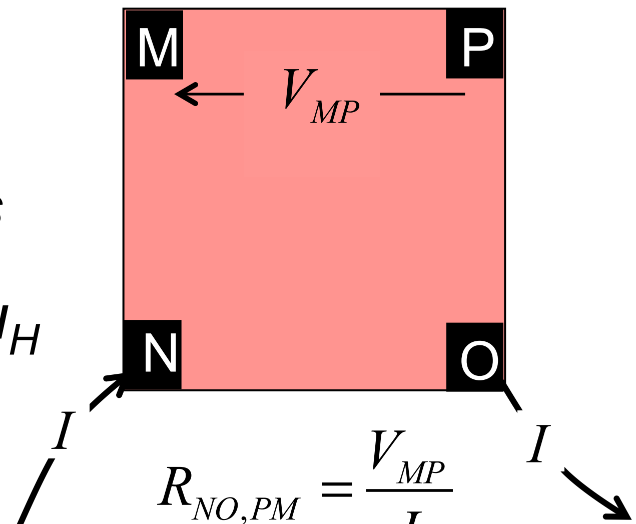
B = 0



$$R_{MN,OP} = \frac{V_{P0}}{I}$$

2) measure ρ_S

3) determine μ_H



$$R_{NO,PM} = \frac{V_{MP}}{I}$$

$$e^{-\frac{\pi}{\rho_S} R_{MN,OP}} + e^{-\frac{\pi}{\rho_S} R_{NO,PM}} = 1$$

$$\sigma_S = n_S q \mu_n = \frac{n_S}{r_H} q\, r_H \mu_n = n_H\, q\, \mu_H$$

outline

1. Introduction
2. Resistivity / conductivity measurements
3. Hall effect measurements
4. The van der Pauw method
5. **Seebeck coefficient**
6. Summary

Measuring the Seebeck coefficient

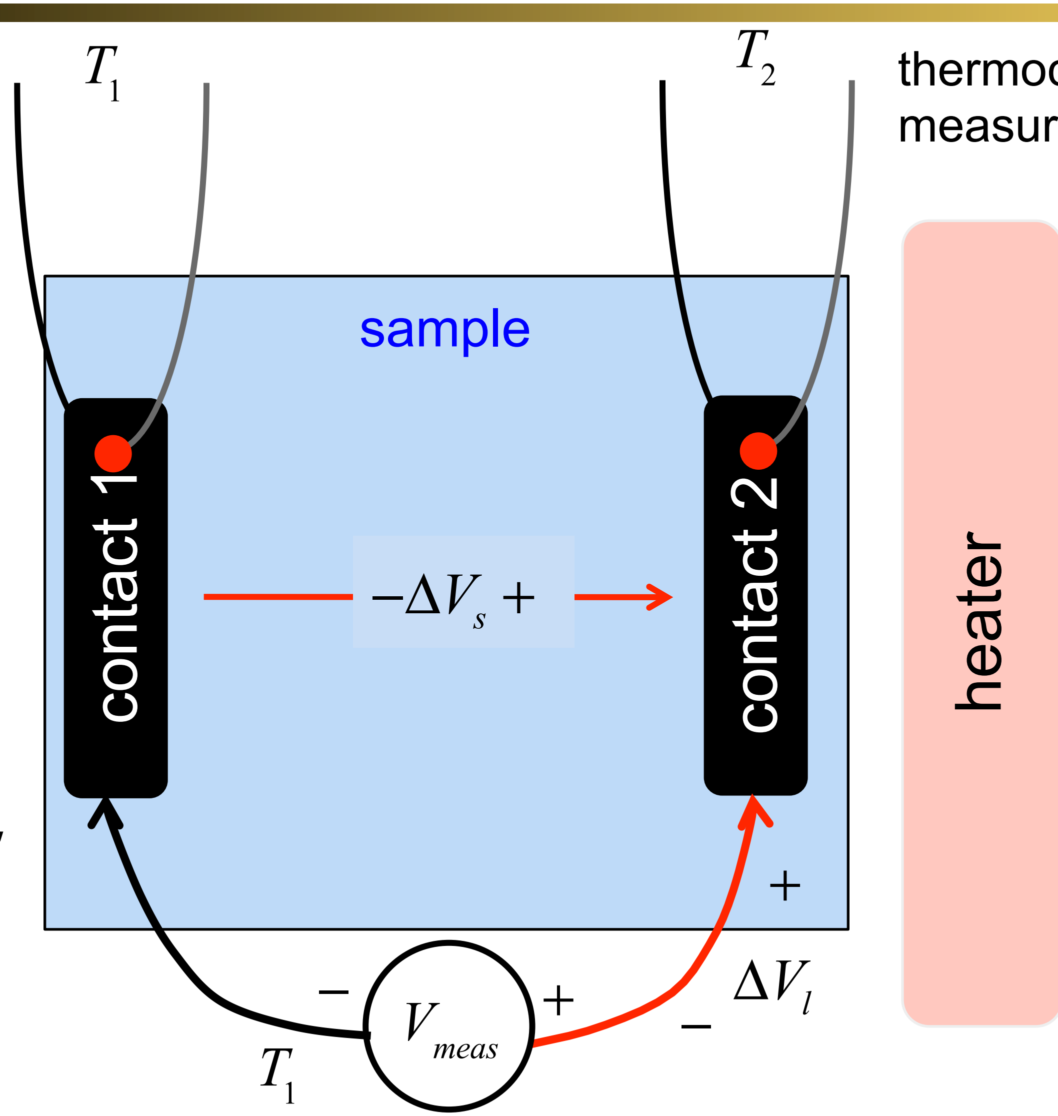
thermocouples to measure temperature

$$\Delta V_s = -S_s \Delta T$$

$$\Delta V_l = -S_l \Delta T$$

$$V_{meas} = \Delta V_s - \Delta V_l$$

$$V_{meas} = -\left(S_s - S_l\right)\Delta T$$



outline

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summary

1) Hall bar or van der Pauw geometries allow measurement of both resistivity and Hall concentration from which the Hall mobility can be deduced.

2) Temperature-dependent measurements (to be discussed in the next lecture) provide information about the dominant scattering mechanisms.

3) Care must be taken to exclude thermoelectric effects (also to be discussed in the next lecture).

for more about low-field measurments

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Questions?

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