Electrical Characterization of Materials: II

Mark Lundstrom

Electrical and Computer Engineering
Purdue University
West Lafayette, IN USA

11/3/17

Coupled charge and heat current equations

electrical current:

$$\mathcal{E} = \rho J + S \frac{dT}{dx}$$

heat current (electronic):

$$J_Q = \pi J - \kappa_e \frac{dT}{dx}$$

heat current (lattice):

$$q_x = -\kappa_L \left(\frac{dT}{dx} \right)$$

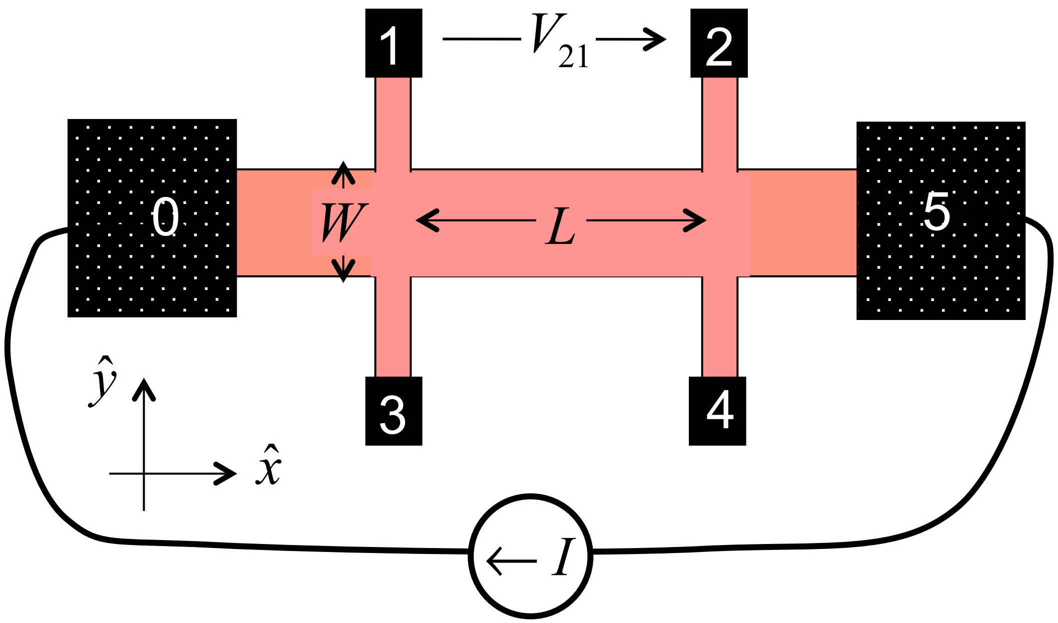
We have been discussing how to measure conductivity (and carrier density and mobility), and we briefly the Seebeck coefficient.

Outline

1. **Review**
2. Temperature-dependent measurements
3. Errors in Hall effect measurements
4. Graphene: a case study
5. Summary

Hall bar geometry

Top view



$$R_H = \frac{-V_H}{I_x B_z} = \frac{1}{(-q)(n_S/r_H)}$$

Hall coefficient

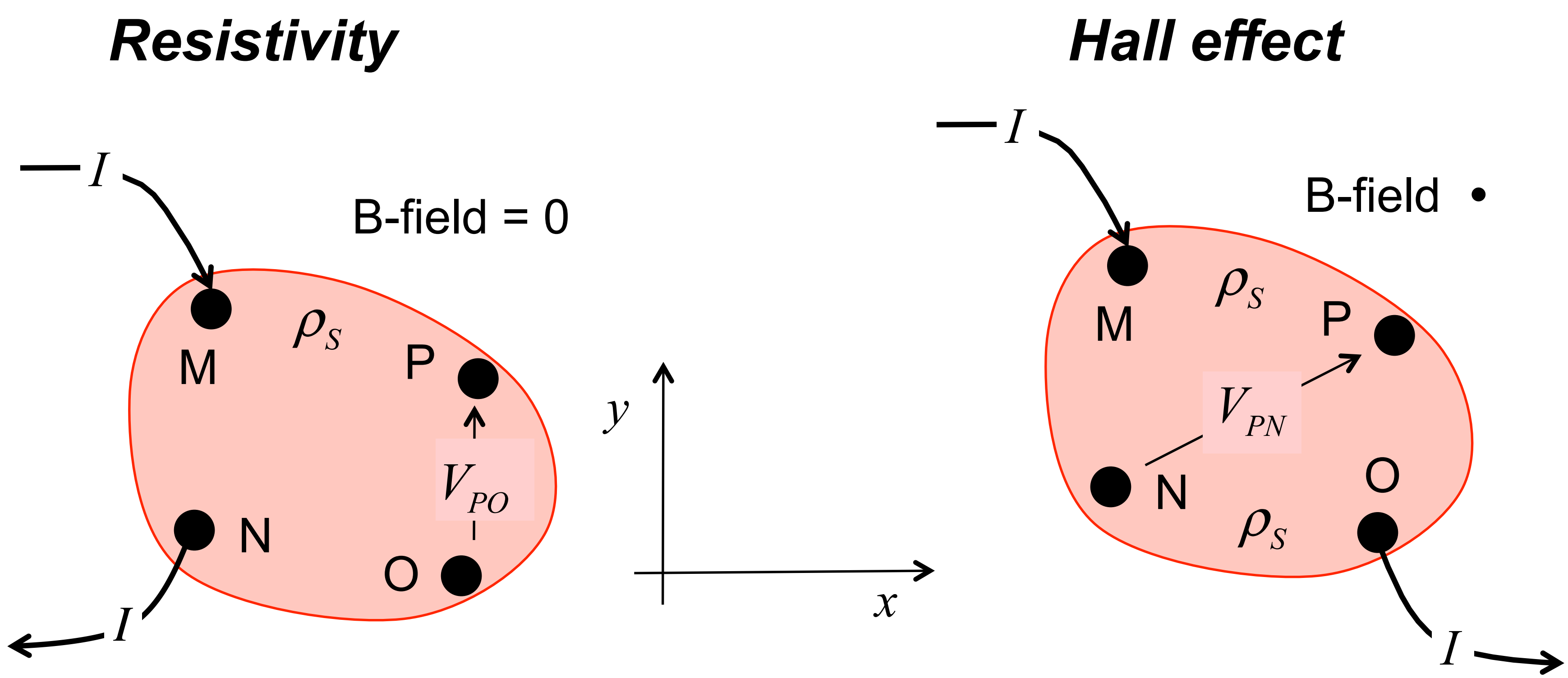
$$\frac{V_{21}}{I} = \rho_S \frac{L}{W}$$

$$\mu_H = r_H \mu_n = \frac{1}{n_H q \rho_S}$$

$$r_H \equiv \langle\langle \tau_m^2 \rangle\rangle / \langle\langle \tau_m \rangle\rangle^2$$

"Hall factor"

van der Pauw geometry

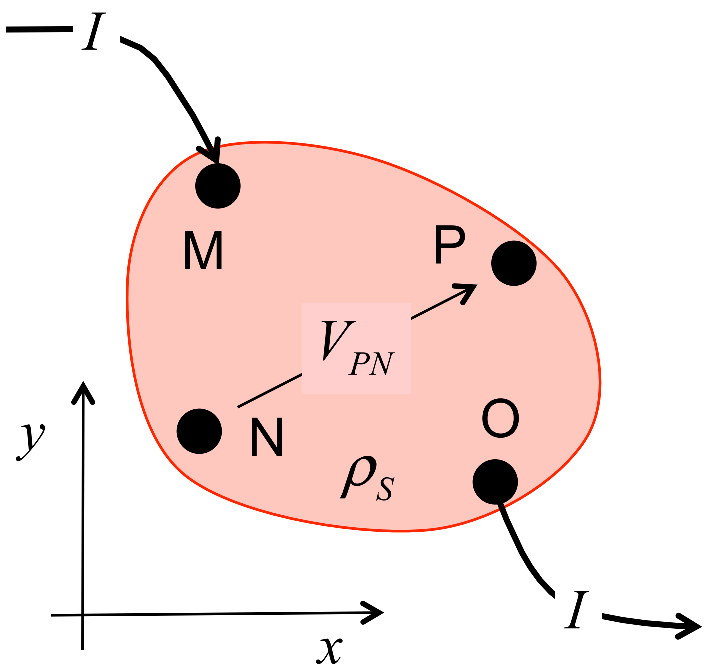


1) force a current in *M* and out *N*
2) measure V_{PO}
3) $R_{MN,\ OP} = V_{PO} / I$ related to ρ_S

1) force a current in *M* and out *O*
2) measure V_{PN}
3) $R_{MO,\ NP} = V_{PN} / I$ related to V_H

van der Pauw approach: Hall effect

Hall effect



$$\vec{J}_n = nq\mu_n\vec{\mathcal{E}} - \left(\sigma_n\mu_n r_H\right)\vec{E}\times\vec{B}$$

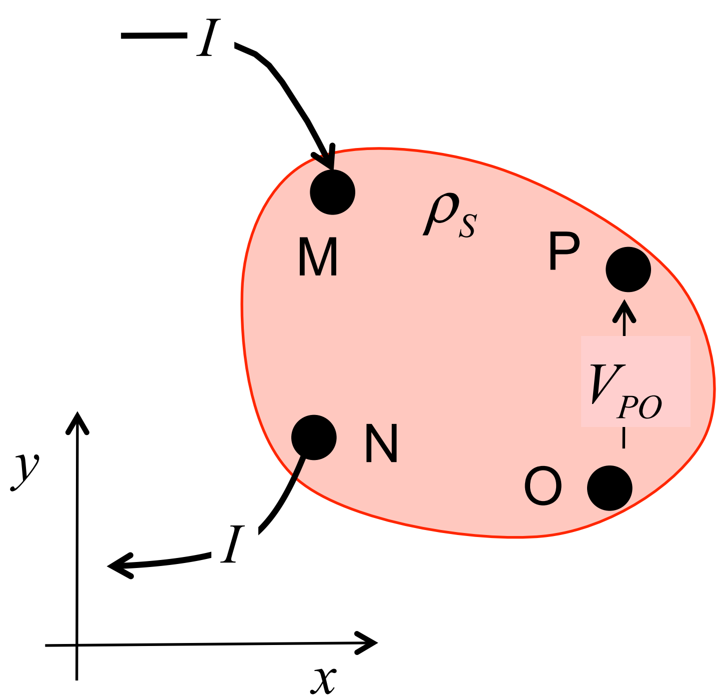
$$V_H \equiv \frac{1}{2}\left[V_{PN}\left(+B_z\right) - V_{PN}\left(-B_z\right)\right]$$

$$V_H = \rho_n\mu_H B_z I$$

So we can do Hall effect measurements on such samples.

See Lundstrom, *Fundamentals of Carrier Transport*, 2nd Ed., Sec. 4.7.1.

van der Pauw approach: resistivity



$$R_{MN,OP} = \frac{\rho_S}{\pi} \ln\left(\frac{(a+b)(b+c)}{b(a+b+c)} \right)$$

$$R_{NO,PM} = \frac{\rho_S}{\pi} \ln\left(\frac{(a+b)(b+c)}{ac} \right)$$

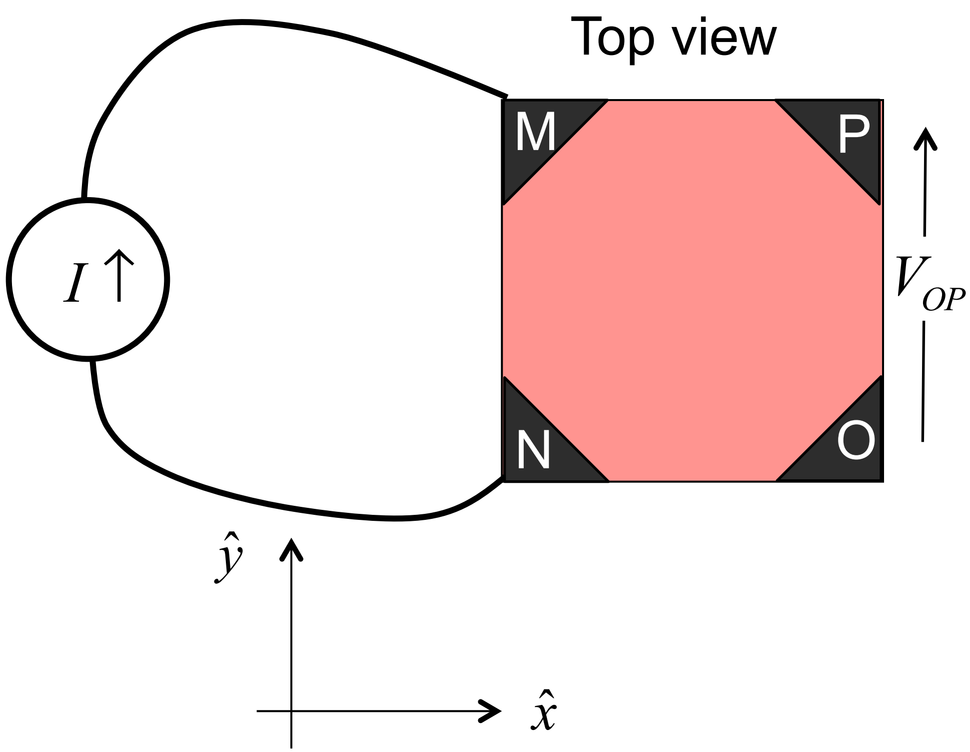
it can be shown that:

$$e^{-\frac{\pi}{\rho_S} R_{MN,OP}} + e^{-\frac{\pi}{\rho_S} R_{NO,PM}} = 1$$

So we can do resistivity measurements on such samples.

Given two measurements of resistance, this equation can be solved for the sheet resistance.

van der Pauw technique: regular sample



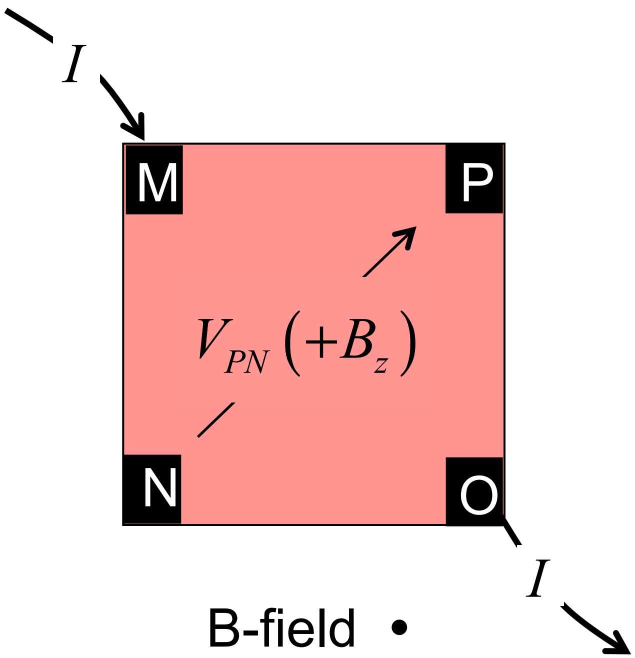
$$e^{-\frac{\pi}{\rho_S} R_{MN,OP}} + e^{-\frac{\pi}{\rho_S} R_{NO,PM}} = 1$$

$$R_{MN,OP} = R_{NO,PM} = \frac{V}{I}$$

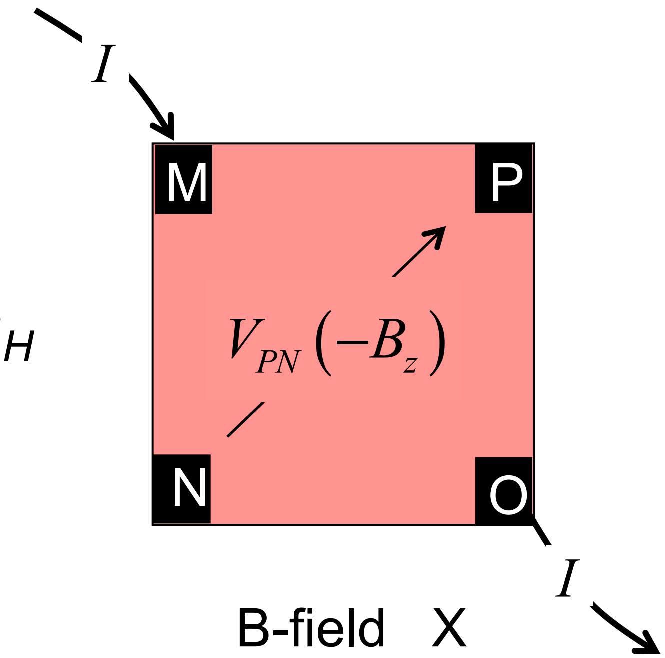
$$\rho_S = \frac{\pi}{\ln 2} \frac{V}{I}$$

Force *I* through two contacts, measure *V* between the other two contacts.

van der Pauw technique: summary



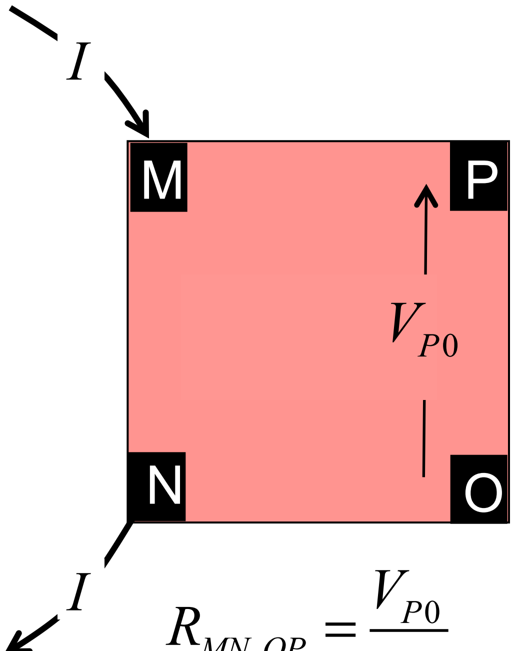
1) measure n_H



$$V_H = \frac{1}{2}\left[V_{PN}(+B_Z) - V_{PN}(-B_Z)\right] = \frac{r_H}{qn_S} B_z I = \frac{B_z I}{qn_H}$$

van der Pauw technique: summary

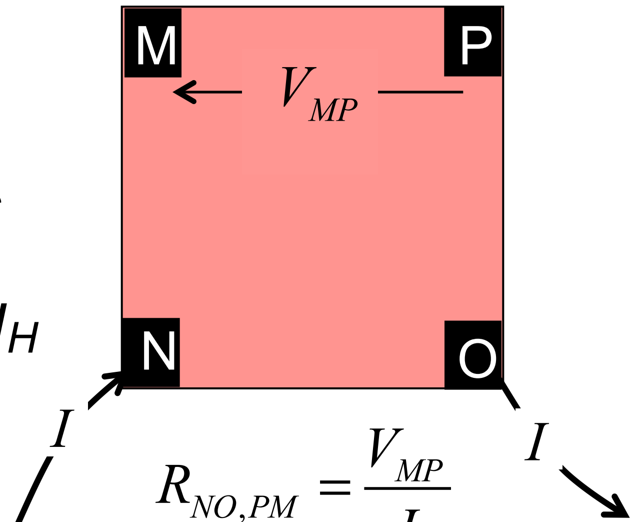
B = 0



$$R_{MN,OP} = \frac{V_{P0}}{I}$$

2) measure ρ_S

3) determine μ_H



$$R_{NO,PM} = \frac{V_{MP}}{I}$$

$$e^{-\frac{\pi}{\rho_S} R_{MN,OP}} + e^{-\frac{\pi}{\rho_S} R_{NO,PM}} = 1 \qquad \sigma_S = n_S q \mu_n = \frac{n_S}{r_H} q\, r_H \mu_n = n_H\, q\, \mu_H$$

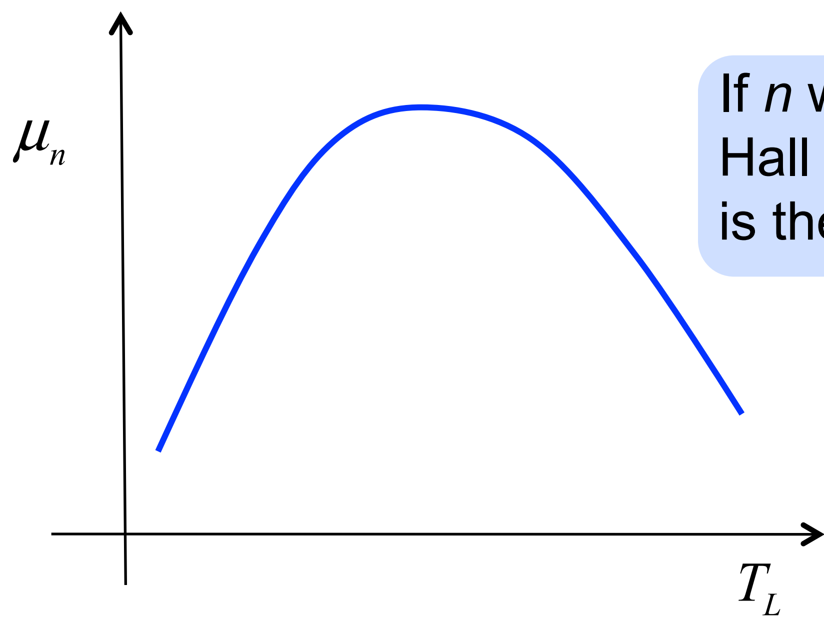
outline

1. Review
2. **Temperature-dependent measurements**
3. Errors in Hall effect measurements
4. Graphene: a case study
5. Summary

Temperature-dependent measurements

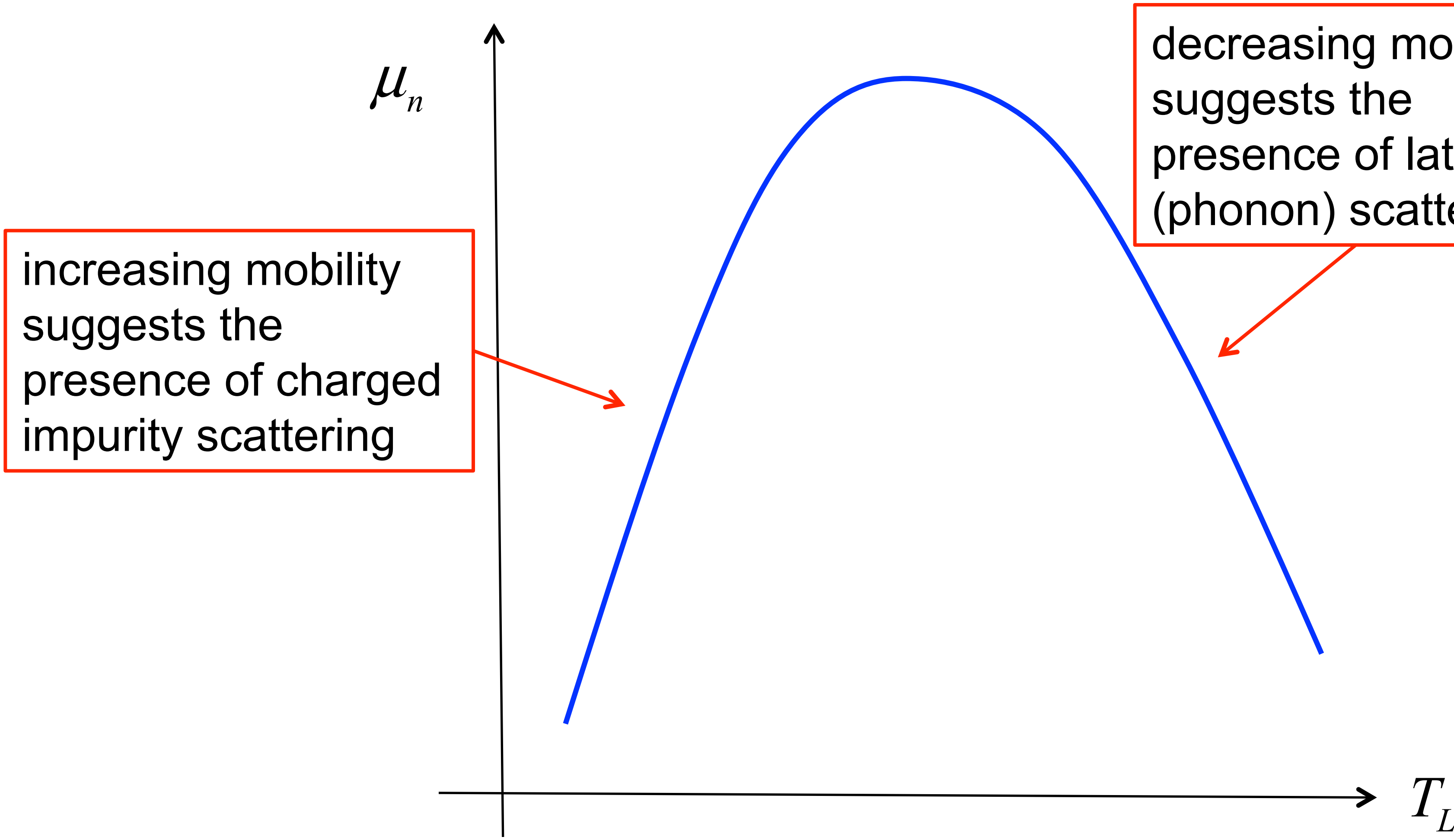
It is common practice to measure the temperature-dependent conductivity.

Assuming that the carrier density is known (or can be measured), a mobility is then extracted from: $\sigma = n q \mu_n$



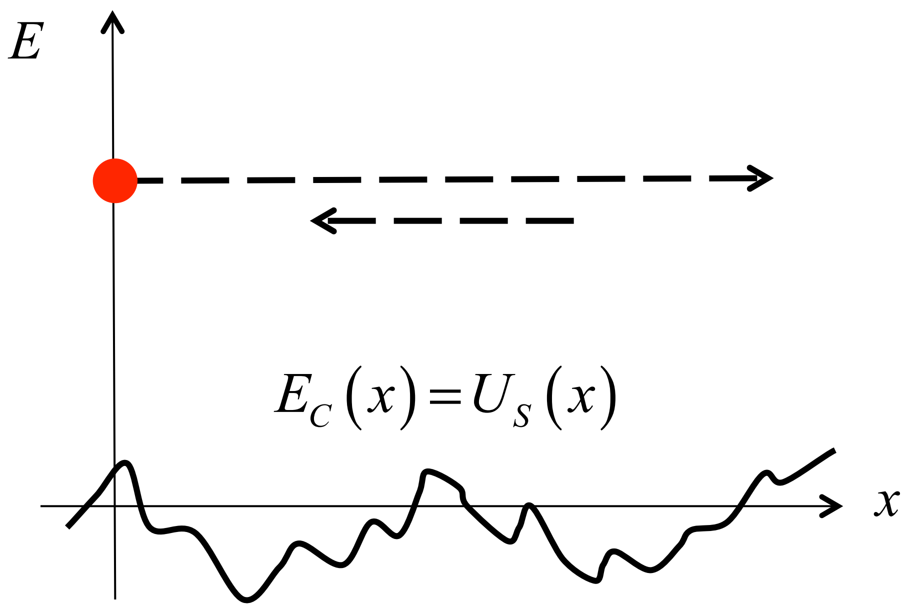
If *n* was measured by Hall effect, then mobility is the Hall mobility.

Interpretation



Charged impurity scattering

$$\tau(E)\uparrow \text{ as } E\uparrow$$



Random charges introduce random fluctuations in E_C, which act a scattering centers.

High energy electrons don't "see" these fluctuations and are not scattered as strongly.

Average carrier energy ~ $k_B T_L$.

Mobility should increase with increasing temperature.

LAttice (phonon) scattering

$$\frac{1}{\tau(E)} \propto n_{ph}$$

Carrier scattering rate is proportional to the number of phonons.

$$n_{ph} = \frac{1}{e^{\hbar\omega/k_B T_L} - 1}$$

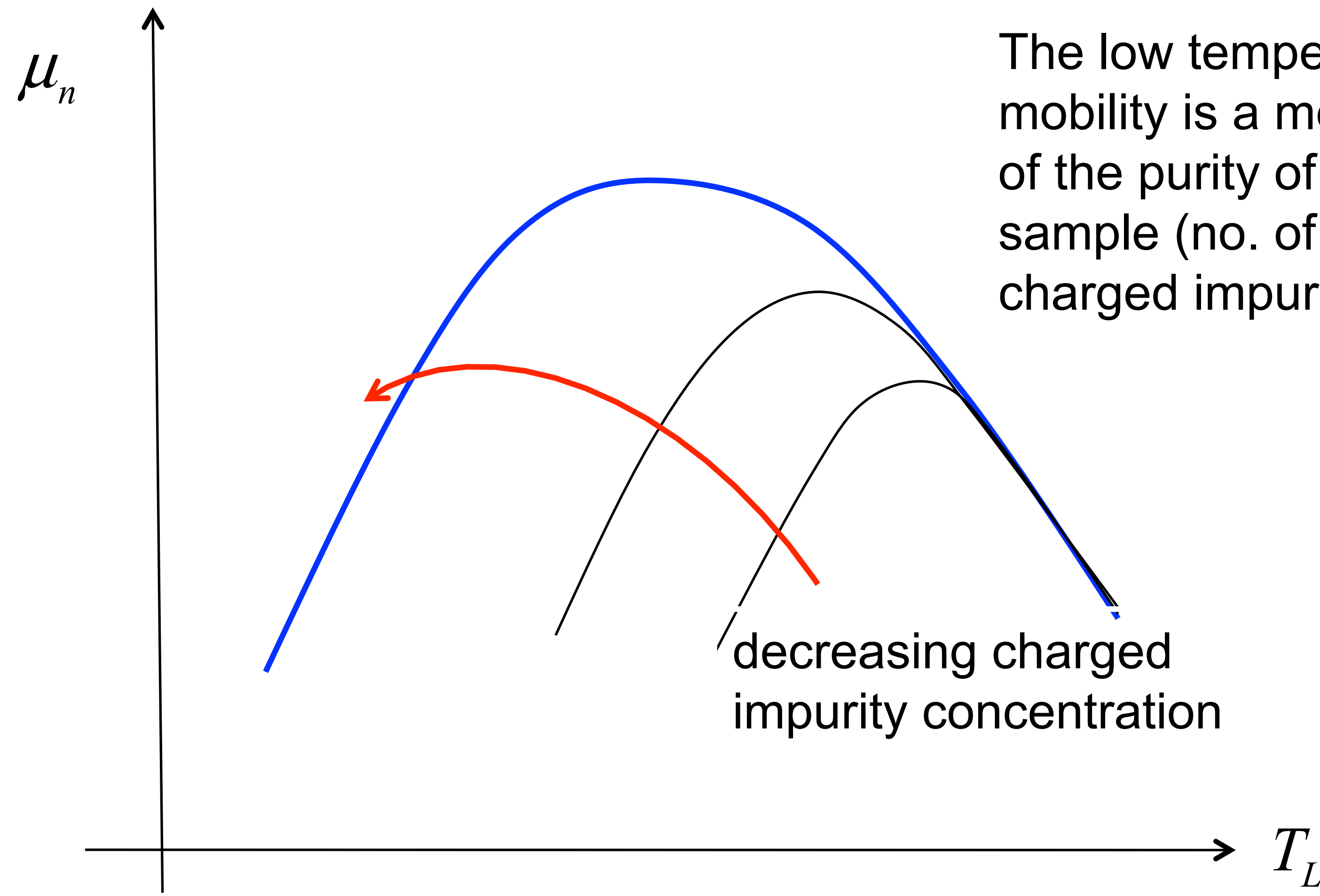
Phonon occupation number given by the Bose-Einstein distribution.

$$n_{ph} \uparrow \quad \text{as} \quad T_L \uparrow$$

Number of phonons increases as temperature increase. Scattering time decreases.

Mobility should decrease with increasing temperature.

Mobility vs. temperature

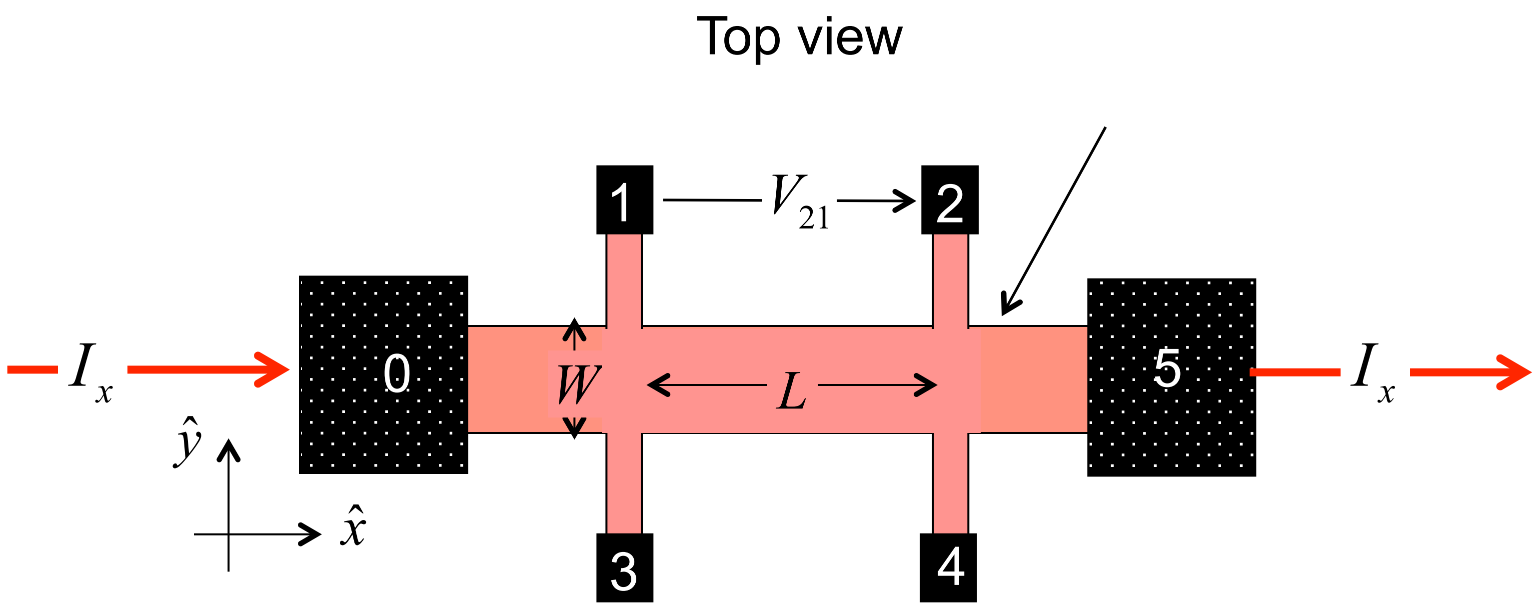


The low temperature mobility is a measure of the purity of the sample (no. of charged impurities).

Outline

1. Review
2. Temperature-dependent measurements
3. **Errors in Hall effect measurements**
4. Graphene: a case study
5. Summary

Hall effect measurements (errors)



We have assumed isothermal conditions to compute the Hall voltage, but we expect Peltier cooling at contact 0 and Peltier heating at contact 1. If the sample is not isothermal, how does the Hall voltage change?

Magnetoconductivity tensor

$$\begin{pmatrix} J_{nx} \\ J_{ny} \end{pmatrix} = \begin{pmatrix} \sigma_S & -\sigma_S \mu_H B_z \\ +\sigma_S \mu_H B_z & \sigma_S \end{pmatrix} \begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix}$$

$$J_{ni} = \sum_j \sigma_{ij}\left(B_z\right)\mathcal{E}_j \qquad \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} \sigma_S & -\sigma_S \mu_H B_z \\ +\sigma_S \mu_H B_z & \sigma_S \end{pmatrix}$$

$J_{ni} = \sigma_{ij}\left(B_z\right)\mathcal{E}_j$ (summation convention)

$$J_i = \sigma_S \mathcal{E}_i - \sigma_S \mu_H \varepsilon_{ijk} \mathcal{E}_j B_k$$

$$\begin{aligned} \varepsilon_{ijk} &= +1\left(i,j,k \ \text{ cyclic}\right) \\ &= -1\left(i,j,k \ \text{ anti-cyclic}\right) \\ &= 0\left(\text{otherwise}\right) \end{aligned}$$

Recall...

$$\mathcal{E}_i = \rho_{ij}\left(\vec{B}\right) J_j + S_{ij}\left(\vec{B}\right) \partial_j T_L$$

$$J_i^Q = \pi_{ij}\left(\vec{B}\right) J_j - \kappa_{ij}^e\left(\vec{B}\right) \partial_j T_L$$

$$\rho_{ij}\left(\vec{B}\right) = \rho_0 + \rho_0 \mu_H \varepsilon_{ijk} B_k + \ldots$$

$$S_{ij}\left(\vec{B}\right) = S_0 + S_1 \varepsilon_{ijk} B_k + \ldots$$

$$\pi_{ij}\left(\vec{B}\right) = \pi_0 + \pi_1 \varepsilon_{ijk} B_k + \ldots$$

$$\kappa_{ij}^e\left(\vec{B}\right) = \kappa_0^e + \kappa_1 \varepsilon_{ijk} B_k + \ldots$$

For parabolic energy bands

Nernst effect

Assume that there is a temperature gradient in the *x*-direction. How is the electric field (Hall voltage) affected?)

$$\mathcal{E}_i = \rho_{ij}\left(\vec{B}\right) J_j + S_{ij}\left(\vec{B}\right)\partial_j T_L$$

$$\rho_{ij}\left(\vec{B}\right) = \rho_0 + \rho_0 \mu_H \varepsilon_{ijk} B_k + \ldots$$

$$S_{ij}\left(\vec{B}\right) = S_0 + S_1 \varepsilon_{ijk} B_k + \ldots$$

$$\mathcal{E}_y = \rho_0 J_y + \rho_0 \mu_H \varepsilon_{yjz} B_z J_j + S_0 \partial_y T_L + S_1 \varepsilon_{yjz} B_z \partial_j T_L$$

$$\mathcal{E}_y = +\rho_0 \mu_H \varepsilon_{yxz} B_z J_x + S_1 \varepsilon_{yxz} B_z \partial_x T_L$$

$$\mathcal{E}_y = -\rho_0 \mu_H B_z J_x - S_1 B_z \partial_x T_L$$

Nernst voltage
Reverse direction of B_z and J_x and average results to eliminate.

Other effects

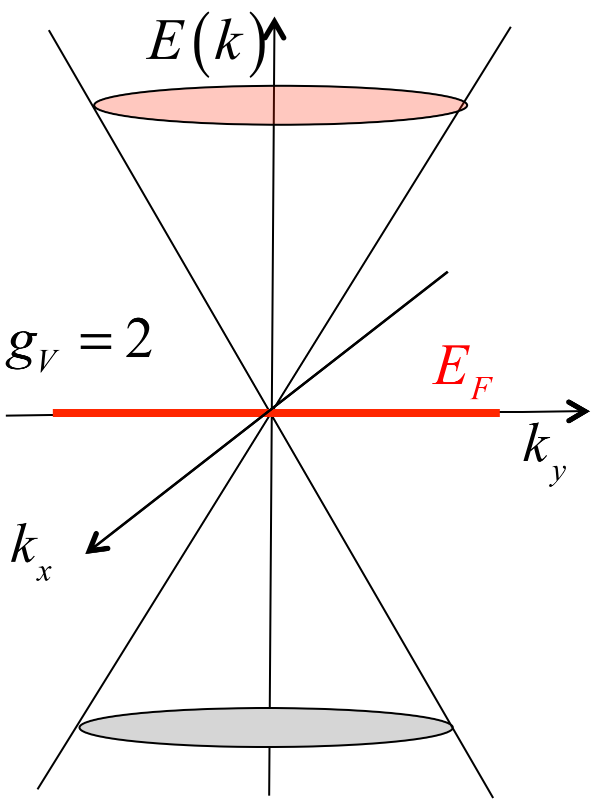
Other "thermomagnetic effects" such as the Ettingshaussen and Righi-Leduc effects also occur and affect the measured Hall voltage. See Lundstrom, Chapter 4, Sec. 4.6.2 for a discussion.

outline

1. Review
2. Temperature-dependent measurements
3. Errors in Hall effect measurements
4. **Graphene: a case study**
5. Summary

Taken from: 2011 NCN Summer School, “Near-equilibrum Transport, Lecture 10: Case study-Near-equilibrium Transport in Graphene,” http://nanohub.org/resources/11873

Graphene



$$E(k) = \pm\hbar\upsilon_F k = \pm\hbar\upsilon_F\sqrt{k_x^2 + k_y^2}$$

$$\upsilon(k) = \upsilon_F \approx 1\times 10^8 \text{ cm/s}$$

$$D(E) = 2\left|E\right|/\pi\hbar^2\upsilon_F^2$$

$$n_S(E_F) = \int_0^\infty D(E) f_0(E)\, dE \approx \frac{E_F^2}{\pi\hbar^2\upsilon_F^2}$$

$$M(E) = W\frac{2\left|E\right|}{\pi\hbar\upsilon_F}$$

$$\sigma_S(0\text{K}) = \frac{2q^2}{h}\lambda_{app}\left(\frac{2E_F}{\pi\text{h}\upsilon_F}\right)$$

$$\frac{1}{\lambda_{app}} = \frac{1}{\lambda(E_F)} + \frac{1}{L}$$

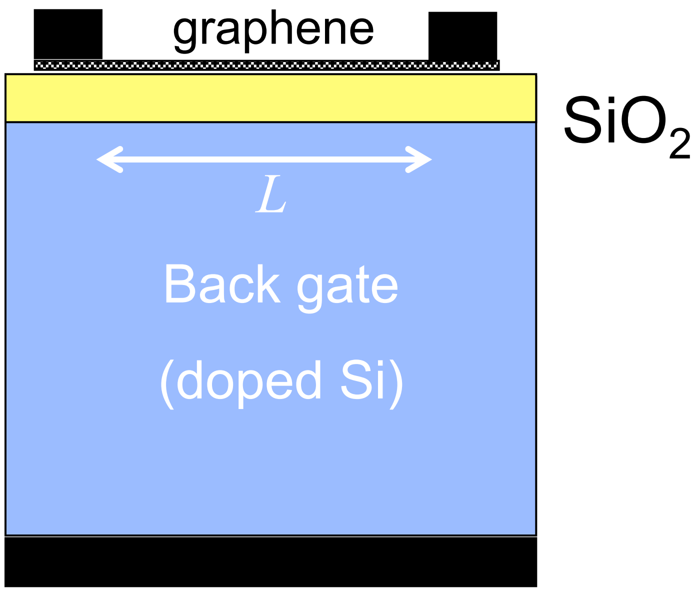
Gate-modulated conductance in graphene

1) The location of the Fermi level (or equivalently the carrier density) is experimentally controlled by a "gate."

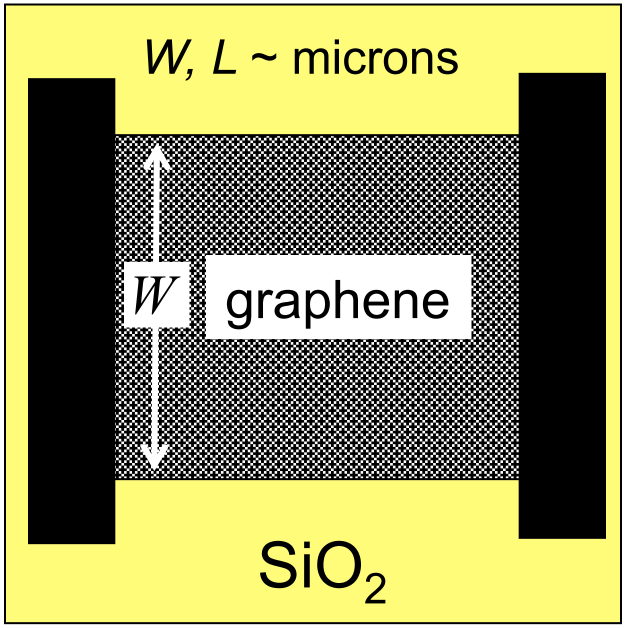
2) In a typical experiments, a layer of graphene is placed on a layer of SiO_2, which is on a doped silicon substrate. By changing the potential of the Si substrate (the "back gate"), the potential in the graphene can be modulated to vary E_F and, therefore, n_S.

Experimental structure (2-probe)

(4-probe techniques are used to eliminate series resistance and for Hall effect measurements.)



Side

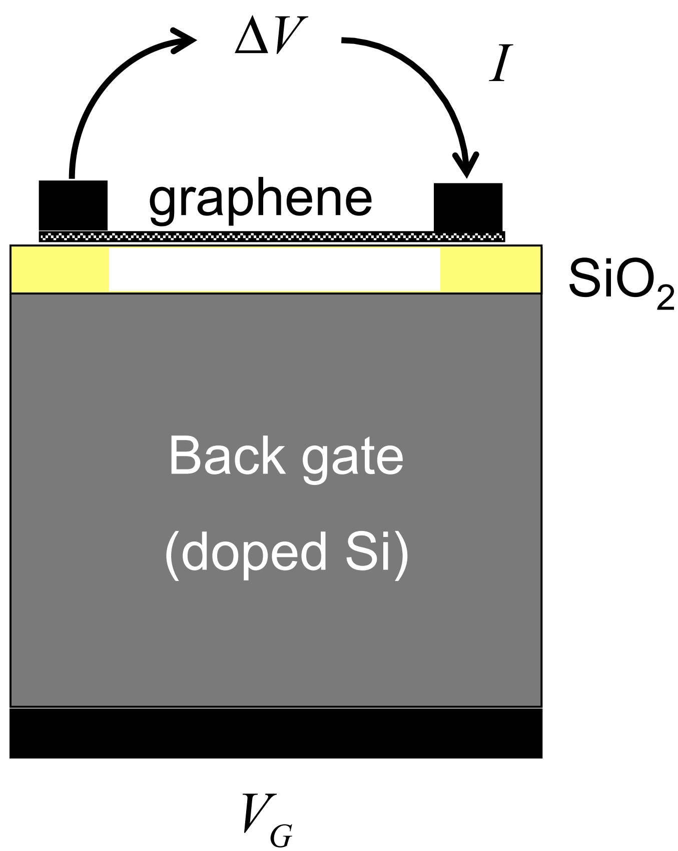


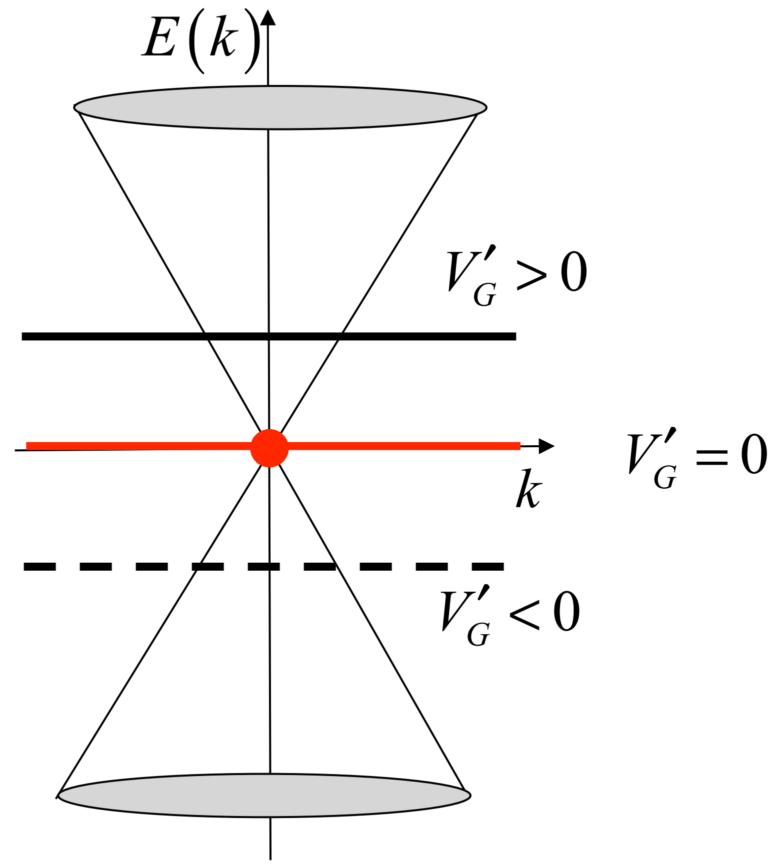
Top view

Typically, Cr/Au or Ti/Au are used for the metal contacts.

The thickness of SiO_2 is typically 300nm or 90nm, which makes it possible to see a single layer of graphene.

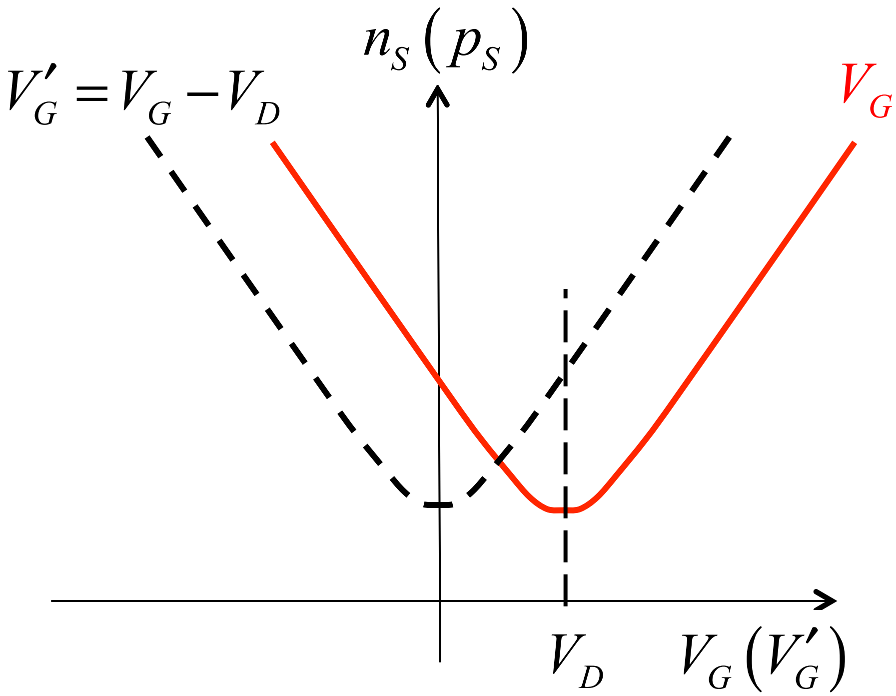
Using a gate voltage to change the Dirac point (or E_F)





$$V_G' = V_G - V_D$$

Gate voltage - carrier density relation



If the oxide is not too thin (so that the quantum capacitance of the graphene is not important), then

$$qn_S = C_{ins} V_G'$$

$$C_{ins} = \frac{\varepsilon_{ins}}{t_{ins}}$$

Sheet conductance vs. V_G

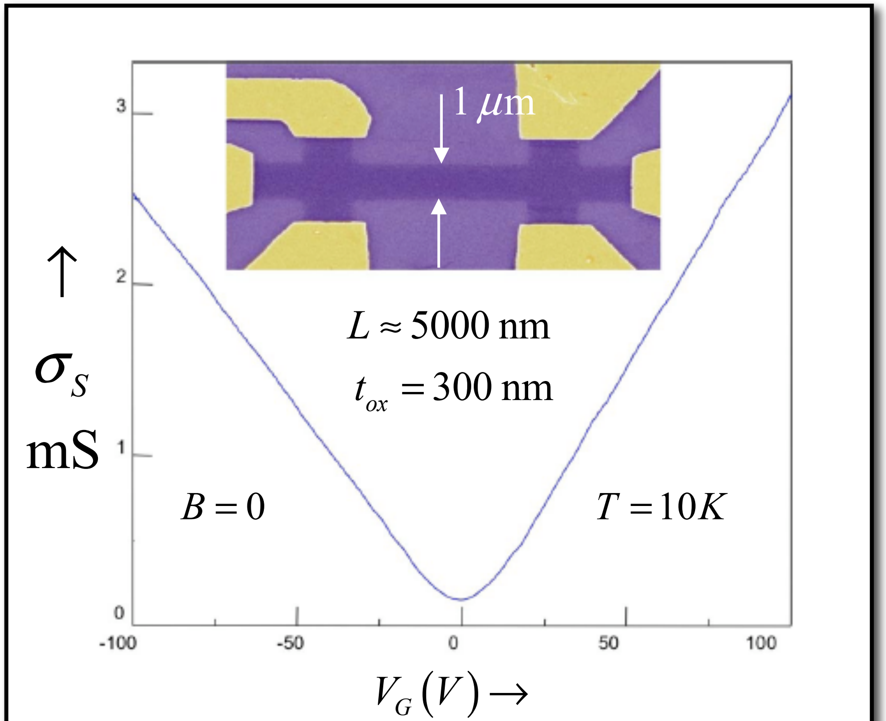


Fig. 30 in A. H. Castro, et al.,"The electronic properties of graphene," Rev. of Mod. Phys., **81**, 109, 2009.

$$G = \sigma_S W/L$$

$$\sigma_S(E_F) \approx \frac{2q^2}{h}\lambda_{app}(E_F)\left(\frac{2E_F}{\pi\hbar\upsilon_F}\right)$$

$$n_S = C_{ox}V_G \approx \frac{1}{\pi}\left(\frac{E_F}{\hbar\upsilon_F}\right)^2$$

$$\lambda_{app}(E_F) = \frac{\sigma_S/(2q^2/h)}{2\sqrt{n_S/\pi}}$$

$$(T_L = 0\text{ K})$$

Mean-free-path (V_G = 100V)

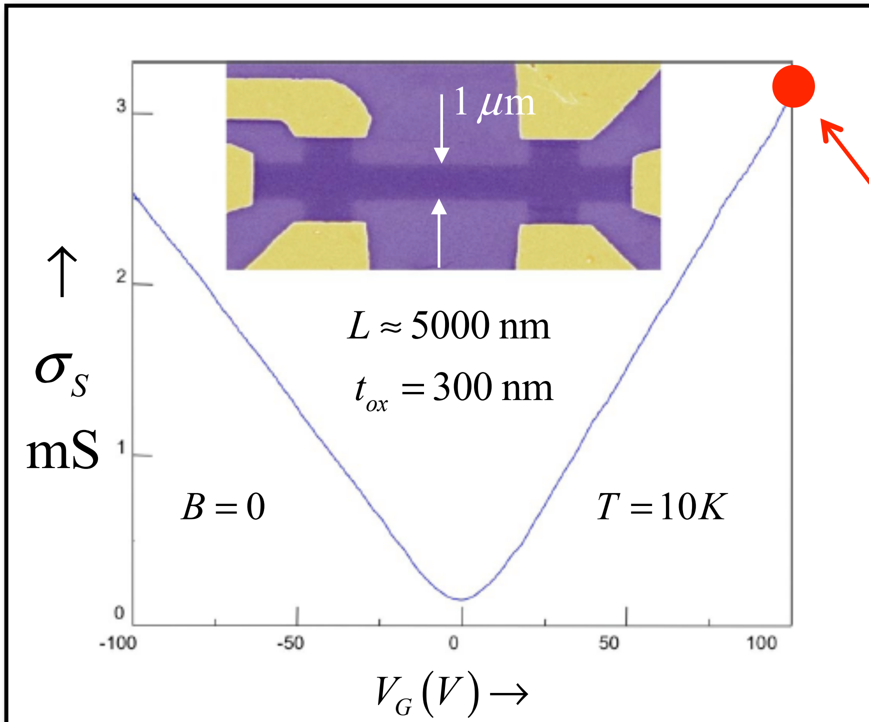


Fig. 30 in A. H. Castro, et al.,"The electronic properties of graphene," Rev. of Mod. Phys., **81**, 109, 2009.

$$\sigma_S \approx 3.0 \text{ mS}$$

$$n_S \approx 7.1\times10^{12} \text{ cm}^{-2}$$

$$E_F \approx 0.3 \text{ eV}$$

$$\lambda_{app}(0.3 \text{ eV}) \approx 130 \text{ nm}$$

$$\lambda(0.3 \text{ eV}) << L$$

Mean-free-path (V_G = 50V)

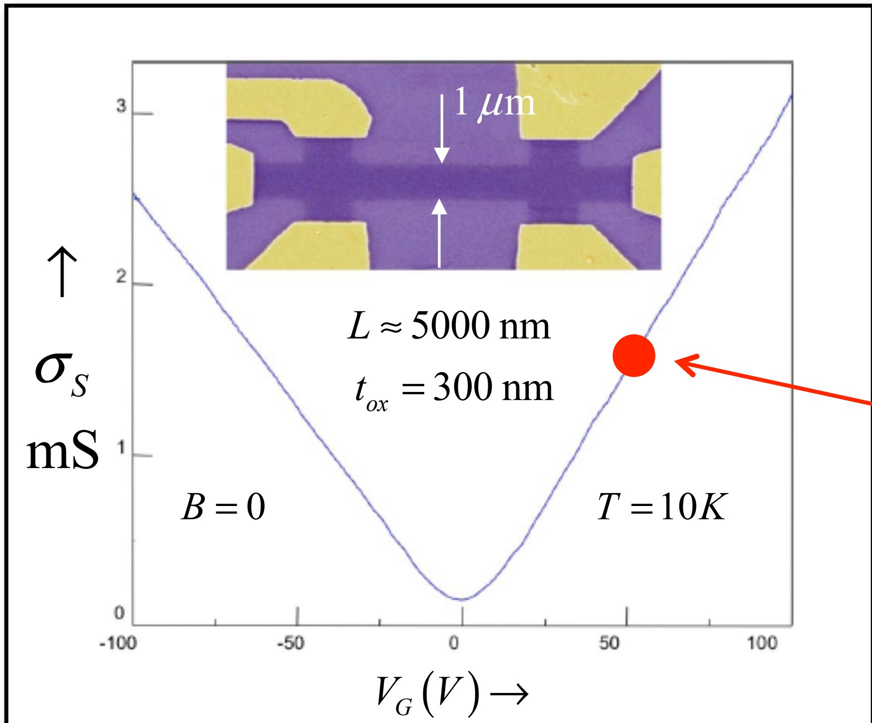


Fig. 30 in A. H. Castro, et al.,"The electronic properties of graphene," Rev. of Mod. Phys., **81**, 109, 2009.

$\sigma_S \approx 1.5$ mS

$n_S \approx 3.6\times10^{12}$ cm^{-2}

$E_F \approx 0.2$ eV

$\lambda_{app}(0.2 \text{ eV}) \approx 90$ nm

$$\frac{\lambda(0.2 \text{ eV})}{\lambda(0.3 \text{ eV})} \approx 0.69$$

$$\frac{0.2 \text{ eV}}{0.3 \text{ eV}} \approx 0.67$$

$$\lambda(E_F) \propto E_F$$

Mobility

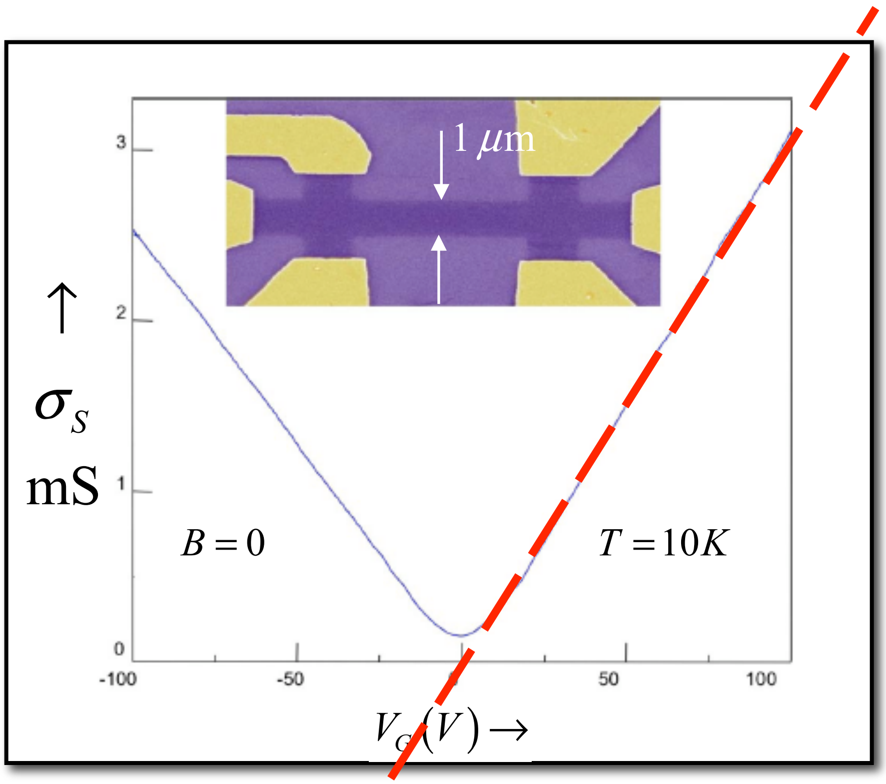


Fig. 30 in A. H. Castro, et al.,"The electronic properties of graphene," Rev. of Mod. Phys., **81**, 109, 2009.

Since, $\sigma_S \sim n_S$, we can write:

$$\sigma_S \equiv n_S q \mu_n$$

and deduce a mobility:

$$\mu_n \approx 12{,}500\ \mathrm{cm}^2/\mathrm{V\text{-}sec}$$

Mobility is constant, but mean-free-path depends on the Fermi energy (or n_S).

Electron-hole puddles

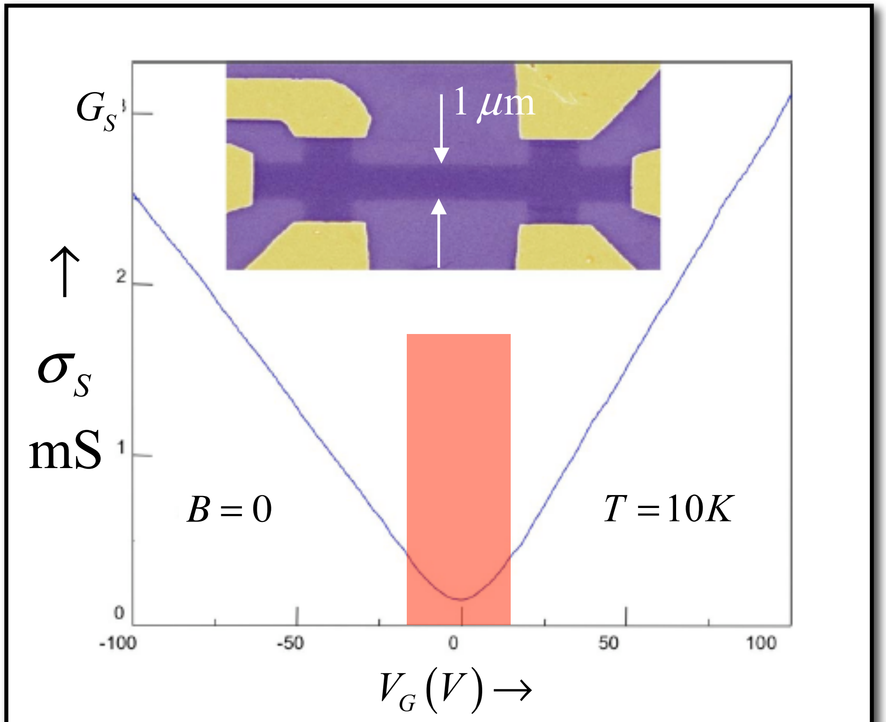
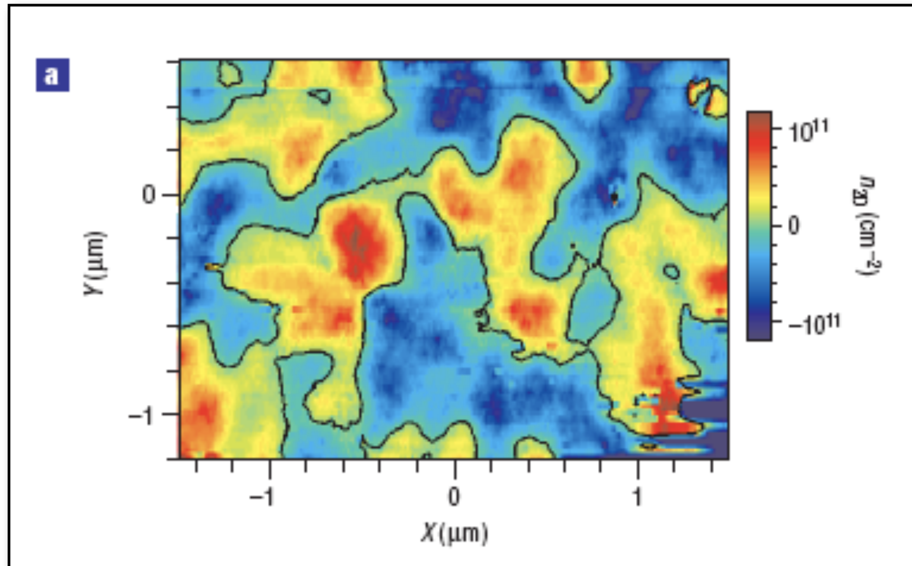
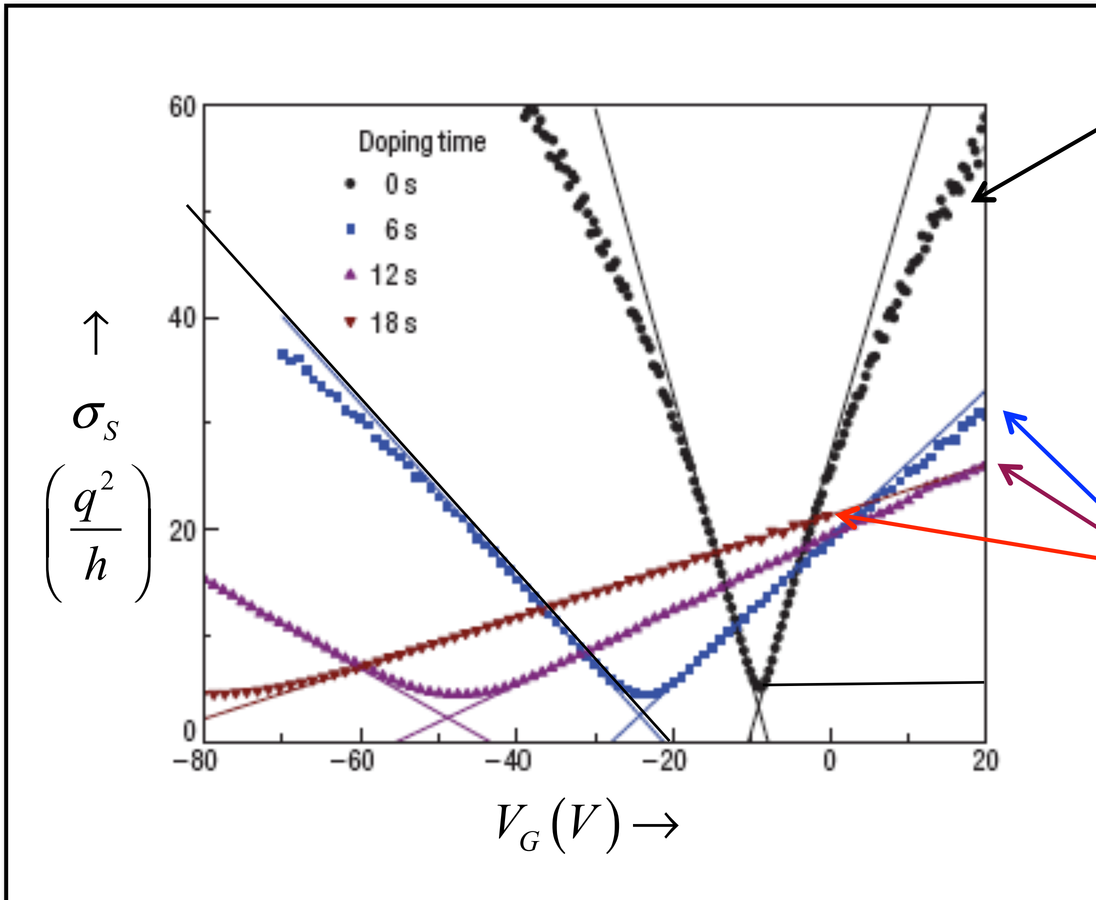


Fig. 30 in A. H. Castro, et al.,"The electronic properties of graphene," Rev. of Mod. Phys., **81**, 109, 2009.



J. Martin, et al, "Observation of electron–hole puddles in graphene using a scanning single-electron transistor," Nature Phys., **4**, 144, 2008

Effect of potassium doping

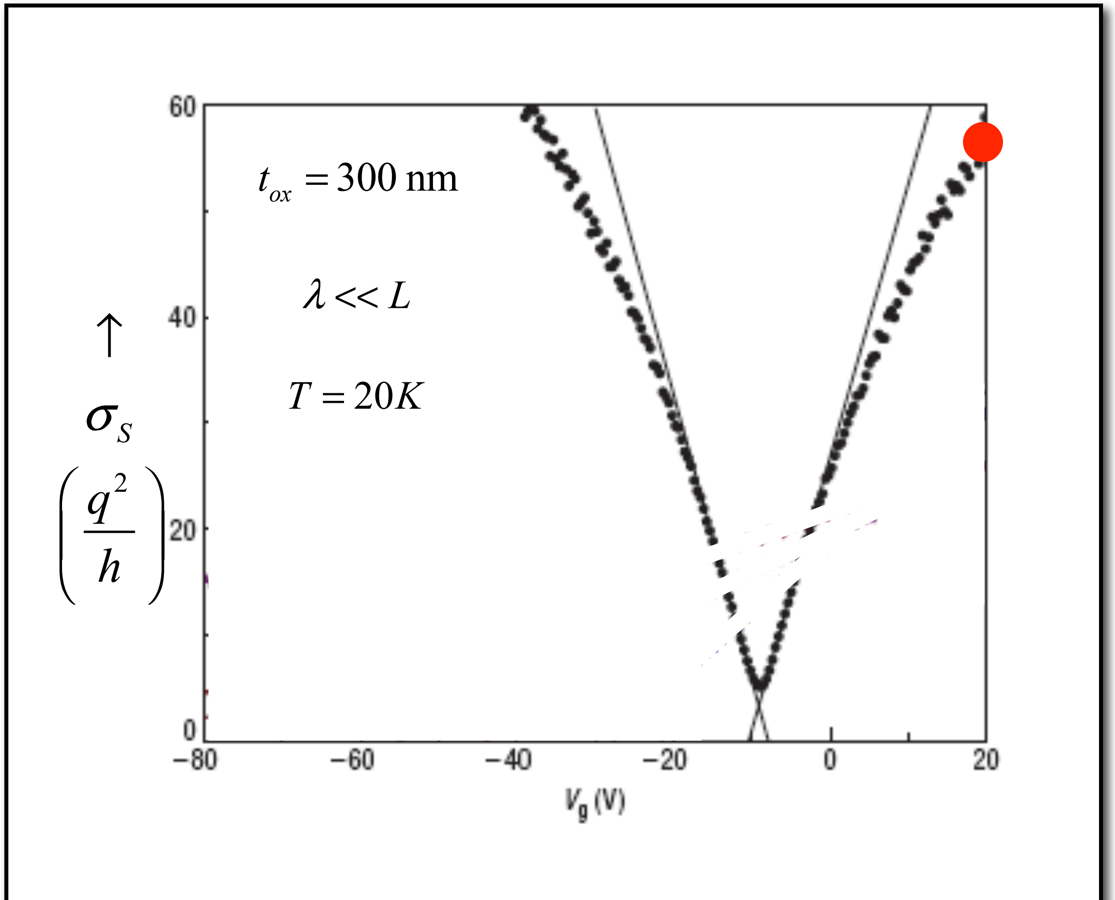


For nominally undoped samples, σ_S vs. n_S is non-linear.

As doping increases, σ_S vs. n_S becomes more linear, mobility decreases, and the NP shifts to the left.

J.-H. Chen, C. Jang, S. Adam, M. S. Fuhrer, E. D. Williams,and M. Ishigami, "Charged-impurity scattering in graphene," *Nature Phys.*, **4**, 377-381, 2008.

Nominally undoped sample: is it ballistic?



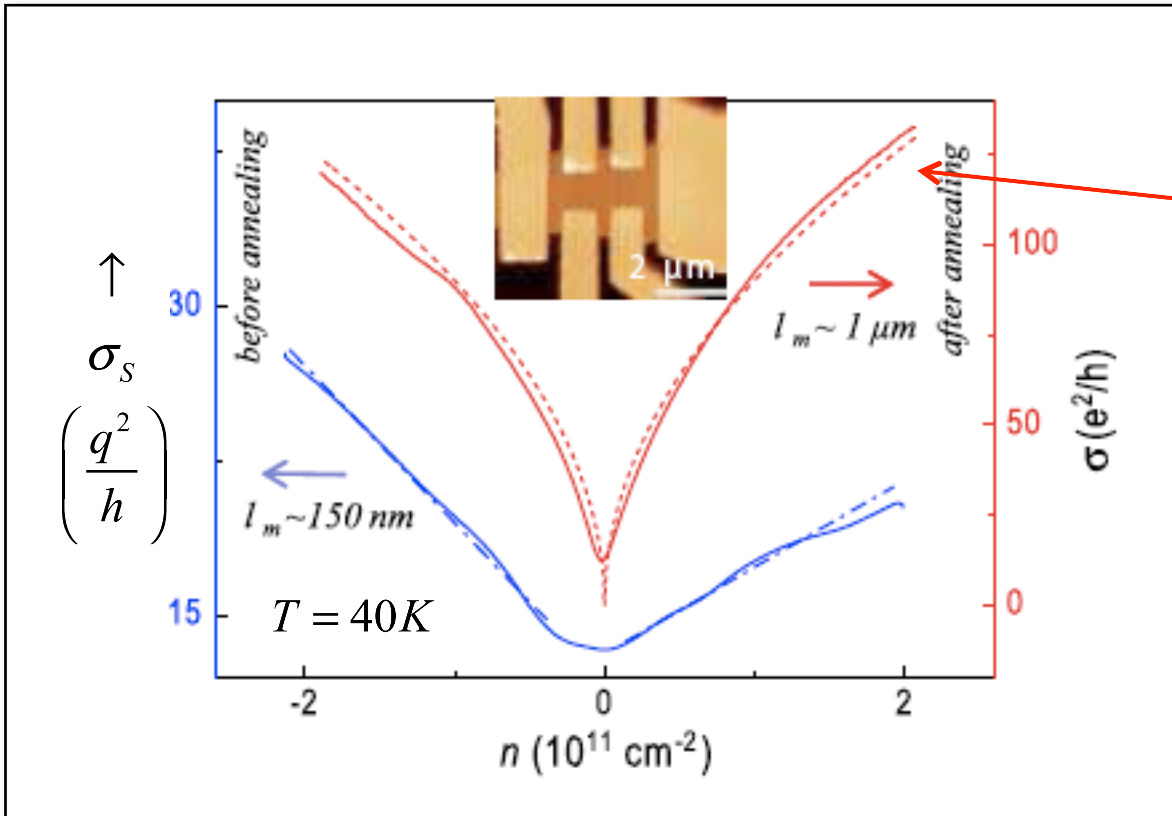
$$\frac{1}{\lambda_{app}} = \frac{1}{\lambda} + \frac{1}{L}$$

$$\lambda_{app} = \frac{\sigma_S / \left(2q^2/h\right)}{2\sqrt{n_S/\pi}} \approx 164 \text{ nm}$$

$$\lambda << L$$

J.-H. Chen, C. Jang, S. Adam, M. S. Fuhrer, E. D. Williams,and M. Ishigami, "Charged-impurity scattering in graphene," *Nature Phys*., **4**, 377-381, 2008.

Unannealed vs. annealed suspended graphene



$$\sigma_S \propto \sqrt{n_S}$$

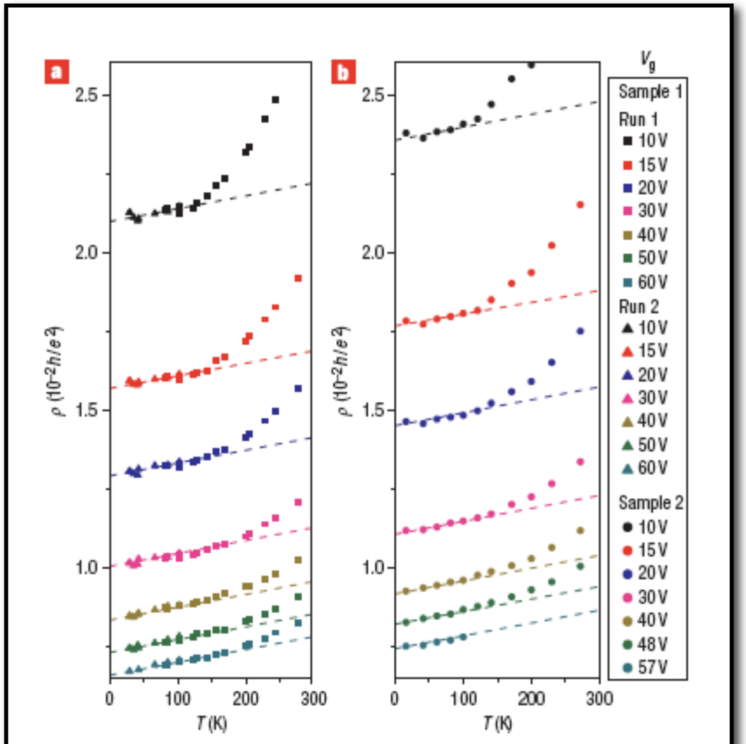
$$\lambda_{app} \approx 1300\ \text{nm}$$

expected from ballistic theory

K. I. Bolotin, K. J. Sikes, J. Hone, H. L. Stormer, and P. Kim, "Temperature dependent transport in suspended graphene," 2008

Temperature dependence

Away from the conductance minimum, the conductance decreases as T_L increases (or resistivity increases as temperature increases).



$$T_L < 100K: \quad R_S \propto T_L$$

(acoustic phonon scattering - intrinsic)

$$T_L > 100K: \quad R_S \propto e^{\hbar\omega_0 / k_B T_L}$$

(optical phonons in graphene or surface phonons at SiO_2 substrate)

J.-H. Chen, J. Chuan, X. Shudong, M. Ishigami, and M.S. Fuhrer, "Intrinsic and extrinsic performance limits of graphene devices on SiO_2," Nature Nanotechnology, **3**, pp. 206-209, 2008.

Phonons and temperature dependence

$$R_S = \frac{1}{G_S} \propto \frac{1}{\lambda} \propto \frac{1}{\tau} \propto n_0$$

$$n_0 = \frac{1}{e^{\hbar\omega(\beta)/k_B T_L} - 1}$$

acoustic phonons:

$$\hbar\omega < k_B T_L$$

$$N_\beta \approx \frac{k_B T_L}{\hbar\omega}$$

$$R_S \propto T_L$$

optical phonons:

$$\hbar\omega_0 \approx k_B T_L$$

$$n_0 = \frac{1}{e^{\hbar\omega_0/k_B T_L} - 1}$$

$$R_S \propto \frac{1}{e^{\hbar\omega_0/k_B T_L} - 1}$$

outline

1. Review
2. Temperature-dependent measurements
3. Errors in Hall effect measurements
4. Graphene: a case study
5. **Summary**

Summary

1) Hall bar or van der Pauw geometries allow measurement of both resistivity and Hall concentration from which the Hall mobility can be deduced.

2) Temperature-dependent measurements provide information about the dominant scattering mechanisms.

3) Care must be taken to exclude thermoelectric effects.

References

Kasper A. Borup, Johannes de Boor, Heng Wang,cFivos Drymiotis, Franck Gascoin, Xun Shi, Lidong Chen, Mikhail I. Fedorov, Eckhard Muller,Bo B. Iversen, and G. Jeffrey Snyder, "Measuring thermoelectric transport properties of Materials," *Energy & Environmental Science*, **8**, 423–435, 2015.

Questions

1. Review
2. Temperature-dependent measurements
3. Errors in Hall effect measurements
4. Graphene: a case study
5. Summary