The
Drift-Diffusion Equation

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11/9/17

The balance equations

$$\frac{\partial n}{\partial t} = -\nabla \bullet \frac{\vec{J}_n}{(-q)}$$

$$\frac{\partial \vec{J}_n}{\partial t} = \frac{2q}{3m^*}\nabla W + \frac{nq^2}{m^*}\vec{\mathcal{E}} - \frac{\vec{J}_n}{\langle \tau_m \rangle}$$

2 eqns.
3 unkns

$$\frac{\partial W}{\partial t} = -\nabla \vec{F}_W - \vec{J}_n \vec{\mathcal{E}} - \frac{W - W_0}{\langle \tau_E \rangle}$$

3 eqns.
4 unkns

$$\frac{\partial \vec{F}_W}{\partial t} = \ldots$$

4 eqns.
5 unkns

The balance equation prescription

$$\frac{\partial n}{\partial t} = -\nabla \bullet \frac{\vec{J}_n}{(-q)}$$

$$\frac{\partial \vec{J}_n}{\partial t} = \frac{2q}{3m^*}\nabla W + \frac{nq^2}{m^*}\vec{\mathcal{E}} - \frac{\vec{J}_n}{\langle \tau_m \rangle}$$

Terminate here
(find a way to estimate *W* in terms of know quantities).

$$\frac{\partial W}{\partial t} = -\nabla \vec{F}_W - \vec{J}_n \vec{\mathcal{E}} - \frac{W - W_0}{\langle \tau_E \rangle}$$

$$\frac{\partial \vec{F}_W}{\partial t} = \ldots$$

Momentum balance equation

$$\phi(\vec{p}) = p_x \qquad \frac{\partial P_x}{\partial t} = -\frac{\partial(2W_{xx})}{\partial x} - nq\mathcal{E}_x - \frac{P_x}{\langle \tau_m \rangle}$$

$$W_{xx} = \frac{1}{\Omega}\sum_{\vec{p}}\left(\frac{1}{2}p_x \upsilon_x\right) f(\vec{r},\vec{p},t)$$

Assume parabolic energy bands.

$$\vec{P} = nm^* \langle \vec{\upsilon} \rangle \qquad \vec{J}_n = n(-q)\langle \vec{\upsilon} \rangle \qquad (-q)\frac{\vec{P}}{m^*} = \vec{J}_n$$

Current equation (ii)

$$\frac{\partial J_{nx}}{\partial t} = \frac{q}{m^*}\frac{\partial\left(2W_{xx}\right)}{\partial x} + \frac{nq^2}{m^*}\mathcal{E}_x - \frac{J_{nx}}{\langle \tau_m \rangle}$$

$$W \equiv \frac{1}{\Omega}\sum_{\vec{p}} E\left(\vec{p}\right) f = \frac{1}{\Omega}\sum_{\vec{p}} \frac{1}{2} m^* \upsilon^2 f$$

$$\upsilon^2 = \upsilon_x^2 + \upsilon_y^2 + \upsilon_z^2 \approx 3\upsilon_x^2$$

(nearly isotropic)

$$W = 3W_{xx}$$

$$\frac{\partial J_{nx}}{\partial t} = \frac{2}{3}\frac{q}{m^*}\frac{\partial W}{\partial x} + \frac{nq^2}{m^*}\mathcal{E}_x - \frac{J_{nx}}{\langle \tau_m \rangle}$$

Current equation (iii)

$$\frac{\partial J_{nx}}{\partial t} = \frac{2}{3}\frac{q}{m^*}\frac{\partial W}{\partial x} + \frac{nq^2}{m^*}\mathcal{E}_x - \frac{J_{nx}}{\langle \tau_m \rangle}$$

$$J_{nx} + \langle \tau_m \rangle \frac{\partial J_{nx}}{\partial t} = \frac{2}{3}\frac{q\langle \tau_m \rangle}{m^*}\frac{\partial W}{\partial x} + nq\frac{q\langle \tau_m \rangle}{m^*}\mathcal{E}_x$$

$$J_{nx} + \langle \tau_m \rangle \frac{\partial J_{nx}}{\partial t} = \frac{2}{3}\mu_n \frac{\partial W}{\partial x} + nq\mu_n\mathcal{E}_x$$

$$\mu_n \equiv \frac{q\langle \tau_m \rangle}{m^*}$$

Assume time variation slow on the scale of a momentum relaxation time.

$$J_{nx} = nq\mu_n\mathcal{E}_x + \frac{2}{3}\mu_n\frac{\partial W}{\partial x}$$

Current equation (iv)

$$J_{nx} = nq\mu_n \mathcal{E}_x + \frac{2}{3}\mu_n \frac{\partial W}{\partial x} \qquad \mu_n \equiv \frac{q\langle \tau_m \rangle}{m^*}$$

Assumptions:

- Parabolic energy bands (not essential)
- Nearly isotropic distribution of velocities
- Slow time variations

Current equation (iv)

$$J_{nx} = nq\mu_n \mathcal{E}_x + \frac{2}{3}\mu_n \frac{\partial W}{\partial x} \qquad \mu_n \equiv \frac{q\langle \tau_m \rangle}{m^*}$$

Now assume: $W = \frac{3}{2} n k_B T_e$ (We will be more precise about temperature later.)

$$J_{nx} = nq\mu_n \mathcal{E}_x + \frac{2}{3}\mu_n \frac{\partial \left(\frac{3}{2} n k_B T_e \right)}{\partial x}$$

Now assume a constant temperature (no thermoelectric effects)

Current equation (iv)

$$J_{nx} = nq\mu_n \mathcal{E}_x + qD_n \frac{\partial n}{\partial x} \qquad \mu_n \equiv \frac{q\langle \tau_m \rangle}{m^*} \qquad D_n \equiv \frac{k_B T_e}{q}\mu_n$$

$T_e > T_L$ Hot electron transport

$T_e = T_L = T$ Near equilibrium

The hierarchy is terminated

The balance equation prescription

$$\frac{\partial n}{\partial t} = -\nabla \bullet \frac{\vec{J}_n}{(-q)}$$

$$\frac{\partial \vec{J}_n}{\partial t} = \frac{2q}{3m^*}\nabla W + \frac{nq^2}{m^*}\vec{\mathcal{E}} - \frac{\vec{J}_n}{\langle \tau_m \rangle}$$

Terminate here
(find a way to estimate *W* in terms of know quantities).

$$J_{nx} = nq\mu_n \mathcal{E}_x + qD_n \frac{\partial n}{\partial x}$$

$$W = \frac{3}{2} n k_B T$$

Two equations in two unknowns − electron concentration and current density.

Far from equilibrium

What if we are not close to equilibrium?

$$\frac{\partial n}{\partial t} = -\nabla \bullet \frac{\vec{J}_n}{(-q)}$$

$$J_{nx} = nq\mu_n \mathcal{E}_x + qD_n \frac{\partial n}{\partial x}$$

$$D_n \equiv \frac{k_B T_e}{q}\mu_n \qquad T_e > T_L$$

Now we must solve the energy balance equation to find T_e.

First moment of the BTE

$$\phi(\vec{p}) = p_x$$

$$\frac{\partial P_x}{\partial t} = -\frac{\partial(2W_{xx})}{\partial x} - nq\mathcal{E}_x - \frac{P_x}{\langle \tau_m \rangle} \quad \rightarrow \quad \rightarrow \quad J_{nx} = nq\mu_n \mathcal{E}_x + qD_n \frac{\partial n}{\partial x}$$

The first moment of the BTE leads to the DD equation….

Given the right assumptions. Can we name them?

Questions