ECE 656: Electronic Transport in Semiconductors Fall 2017

The
Energy Transport Equations

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The balance equation prescription

$$\frac{\partial n}{\partial t} = -\nabla \bullet \frac{\vec{J}_n}{(-q)}$$

$$\frac{\partial \vec{J}_n}{\partial t} = \frac{2q}{3m^*}\nabla W + \frac{nq^2}{m^*}\vec{\mathcal{E}} - \frac{\vec{J}_n}{\langle \tau_m \rangle}$$

← Terminate here and get the near-equilibrium DD or coupled current equations.

$$\frac{\partial W}{\partial t} = -\nabla \vec{F}_W - \vec{J}_n \vec{\mathcal{E}} - \frac{W - W_0}{\langle \tau_E \rangle}$$

$$\frac{\partial \vec{F}_W}{\partial t} = \ldots$$

← Terminate here and get the energy balance equations.

Balance equation prescription

$$\frac{1}{\Omega}\sum_{\vec{p}}\phi(\vec{p})\left\{\frac{\partial f}{\partial t}+\vec{\upsilon}\bullet\nabla_r f-q\vec{\mathcal{E}}\bullet\nabla_p f=\hat{C}f\right\}$$

$$\frac{\partial n_\phi}{\partial t}=-\nabla\bullet\vec{F}_\phi+G_\phi-R_\phi$$

$$n_\phi(\vec{r},t)=\frac{1}{\Omega}\sum_{\vec{p}}\phi(\vec{p})f(\vec{r},\vec{p},t)$$

$$G_\phi=-q\vec{\mathcal{E}}\bullet\left\{\frac{1}{\Omega}\sum_{\vec{p}}\vec{\nabla}_p\phi f\right\}$$

$$\vec{F}_\phi\equiv\frac{1}{\Omega}\sum_{\vec{p}}\phi(\vec{p})\vec{\upsilon}f(\vec{r},\vec{p},t)$$

$$R_\phi\equiv\frac{n_\phi-n_\phi^0}{\langle\tau_\phi\rangle}$$

First two moments

$$\phi(\vec{p}) = p^0 = 1$$

$$\frac{\partial n_\phi}{\partial t} = -\nabla \bullet \vec{F}_\phi + G_\phi - R_\phi \quad \rightarrow \quad \rightarrow \quad \frac{\partial n}{\partial t} = -\nabla \bullet \frac{\vec{J}_n}{(-q)}$$

$$\phi(\vec{p}) = p_x$$

$$\frac{\partial n_\phi}{\partial t} = -\frac{\partial F_{\phi x}}{\partial x} + G_\phi - R_\phi \quad \rightarrow \quad \rightarrow \quad \frac{\partial P_x}{\partial t} = -\frac{\partial (2W_{xx})}{\partial x} - nq\mathcal{E}_x - \frac{P_x}{\langle \tau_m \rangle}$$

$$J_{nx} = nq\mu_n \mathcal{E}_x + 2\mu_n \frac{\partial W_{xx}}{\partial x}$$

W_{xx}

$$J_{nx} = nq\mu_n \mathcal{E}_x + 2\mu_n \frac{\partial W_{xx}}{\partial x}$$

What is W_{xx}?

Answer: Write another balance equation to find out.

$$n_\phi(\vec{r},t) = \frac{1}{\Omega}\sum_{\vec{p}} \phi(\vec{p}) f\left(\vec{r},\vec{p},t\right) \qquad W_{xx} = \frac{1}{\Omega}\sum_{\vec{p}} \left(\frac{1}{2}p_x \upsilon_x\right) f\left(\vec{r},\vec{p},t\right)$$

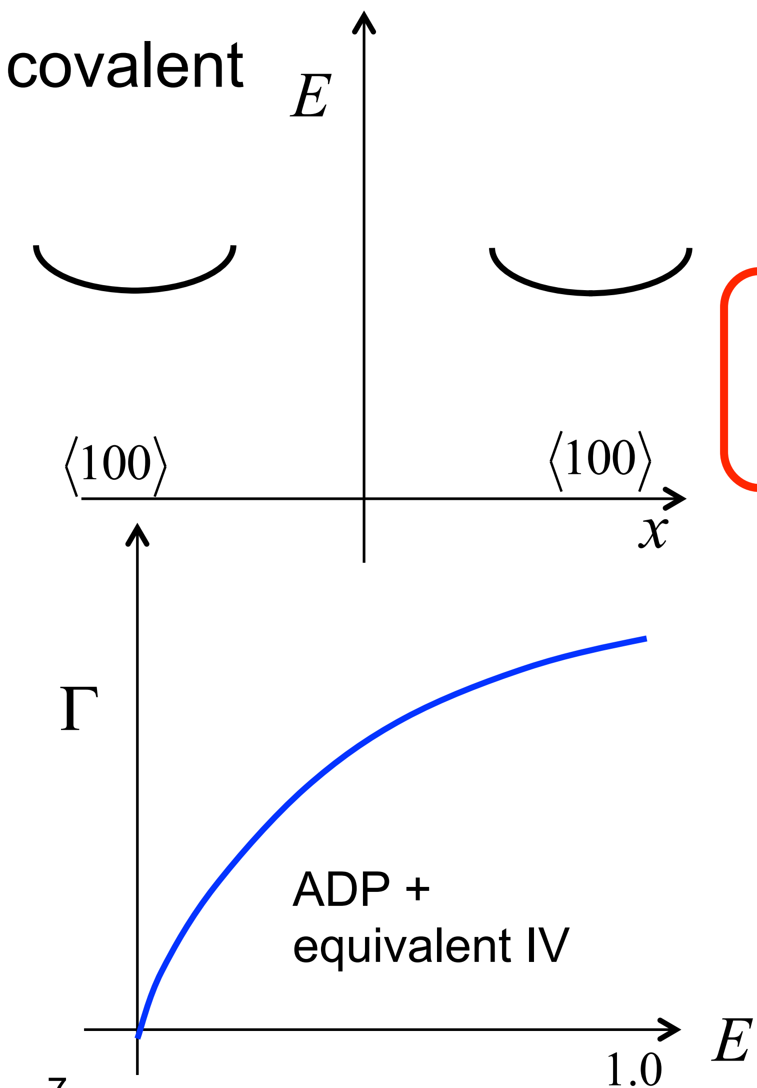
W_{xx} vs. W

$$W_{xx} \approx W/3 \qquad J_{nx} = nq\mu_n \mathcal{E}_x + \frac{2}{3}\mu_n \frac{\partial W}{\partial x} \qquad \mu_n(W)$$

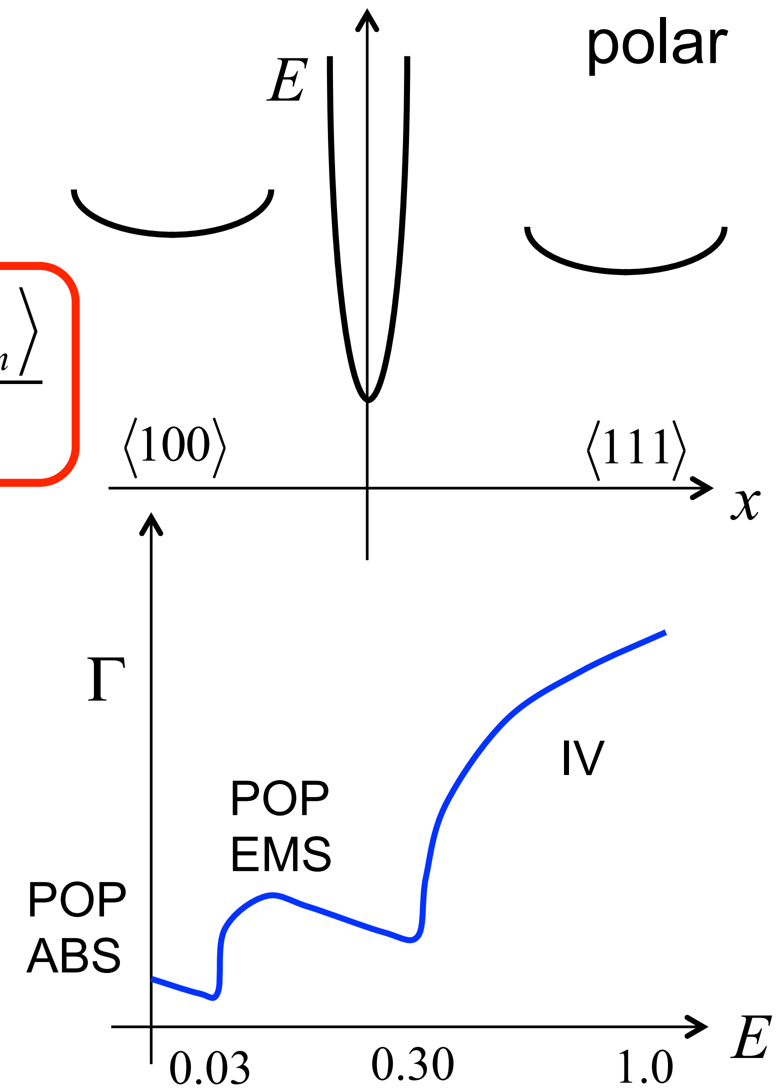
$$R_\phi \equiv -\frac{1}{\Omega}\sum_{\vec{p}} \phi(\vec{p})\hat{C} f \qquad R_{P_x} \equiv -\frac{1}{\Omega}\sum_{\vec{p}} p_x \hat{C} f \equiv \frac{P_x}{\langle \tau_m \rangle} \qquad \mu_n \equiv \frac{q\langle \tau_m \rangle}{m^*}$$

In principle, mobility depends on the unknown distribution function, *f*. To first order, mobility is a function of the average kinetic energy.

Scattering rate vs. energy



$$\mu_n = \frac{q\langle \tau_m \rangle}{m^*}$$



Third moment

$$\phi(\vec{p}) = E(p)$$

$$\frac{\partial n_\phi}{\partial t} = -\frac{\partial F_{\phi x}}{\partial x} + G_\phi - R_\phi \quad \rightarrow \quad \rightarrow \quad \frac{\partial W}{\partial t} = -\frac{\partial F_{Wx}}{\partial x} - J_{nx}\mathcal{E}_x - \frac{W - W_0}{\langle \tau_E \rangle}$$

$$n_\phi(\vec{r},t) = \frac{1}{\Omega}\sum_{\vec{p}} E(\vec{p}) f(\vec{r},\vec{p},t) = W(\vec{r},t) \qquad G_\phi = -q\mathcal{E}_x \left\{ \frac{1}{\Omega}\sum_{\vec{p}} \frac{\partial\left(E(\vec{p})\right)}{\partial p_x} f \right\} = J_{nx}\mathcal{E}_x$$

$$F_{\phi x} \equiv \frac{1}{\Omega}\sum_{\vec{p}} \left(E(\vec{p})\right)\upsilon_x f(\vec{r},\vec{p},t) \equiv F_{Wx} \qquad R_\phi \equiv \frac{n_\phi - n_\phi^0}{\langle \tau_\phi \rangle} = \frac{W - W_0}{\langle \tau_E \rangle}$$

The balance equations

$$\phi(\vec{p}) = p^0 = 1$$

$$\frac{\partial n}{\partial t} = -\nabla \bullet \frac{J_{nx}}{(-q)}$$

$$\phi(\vec{p}) = p_x$$

$$J_{nx} = nq\mu_n \mathcal{E}_x + \frac{2}{3}\mu_n \frac{\partial W}{\partial x}$$

$$\phi(\vec{p}) = E(p)$$

$$\frac{\partial W}{\partial t} = -\frac{\partial F_{Wx}}{\partial x} - J_{nx}\mathcal{E}_x - \frac{W - W_0}{\langle \tau_E \rangle}$$

Energy flux balance equations

$$\phi(\vec{p}) = E(p)\upsilon_x \qquad n_\phi(\vec{r},t) = \frac{1}{\Omega}\sum_{\vec{p}}\left(E(\vec{p})\right) f(\vec{r},\vec{p},t) = F_W(\vec{r},t)$$

HW prob. 5.7 in FCT

$$F_{Wx} = -W\mu_E \mathcal{E}_x - \frac{d\left(D_E W\right)}{dx}$$

$$\mu_E = \frac{5}{3}\frac{q}{m^*\left\langle 1/\tau_{FW}\right\rangle}$$

$$D_E = \frac{k_B T_e}{q}\mu_E$$

$$J_{nx} = nq\mu_n \mathcal{E}_x + \frac{2}{3}\mu_n \frac{\partial W}{\partial x}$$

$$\frac{J_{nx}}{(-q)} = n\mu_n \mathcal{E}_x + \frac{2}{3}\mu_n \frac{\partial W}{\partial x}$$

$$D_n = +\frac{k_B T_e}{q}\mu_n$$

The energy balance equations

$$\frac{\partial n}{\partial t} = -\frac{d}{dx}\frac{J_{nx}}{(-q)}$$

$$J_{nx} = nq\mu_n \mathcal{E}_x + \frac{2}{3}\mu_n \frac{\partial W}{\partial x}$$

$$\frac{\partial W}{\partial t} = -\frac{\partial F_{Wx}}{\partial x} - J_{nx}\mathcal{E}_x - \frac{W - W_0}{\langle \tau_E \rangle}$$

$$F_{Wx} = -W\mu_E \mathcal{E}_x - \frac{d(D_E W)}{dx}$$

$n(x,t)$

$W(x,t)$

$\mu_n(W)$

$\mu_E(W)$

$\langle \tau_E \rangle (W)$

About temperature

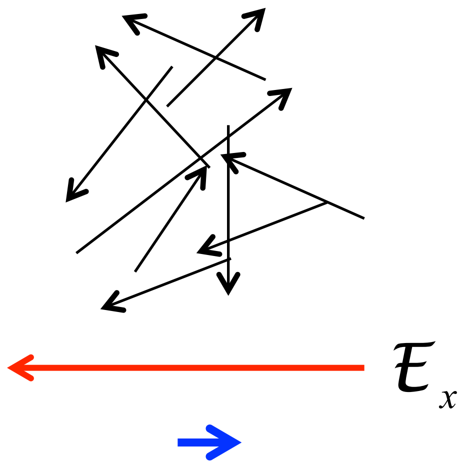
We are tempted to write: $W(x,t) = \frac{3}{2} n(x,t) k_B T_e(x,t)$

Hot carrier transport when: $T_e > T_L$

Can we be more precise?

Electron temperature

random thermal
motion of electrons



$$\langle \upsilon_x \rangle = \upsilon_{dx}$$

$$W = \frac{1}{\Omega}\sum_{\vec{p}} E(\vec{p})f = \frac{1}{\Omega}\sum_{\vec{p}} \frac{1}{2} m^* \upsilon^2 f$$

$$W = n\frac{1}{2}m^*\langle \upsilon^2 \rangle$$

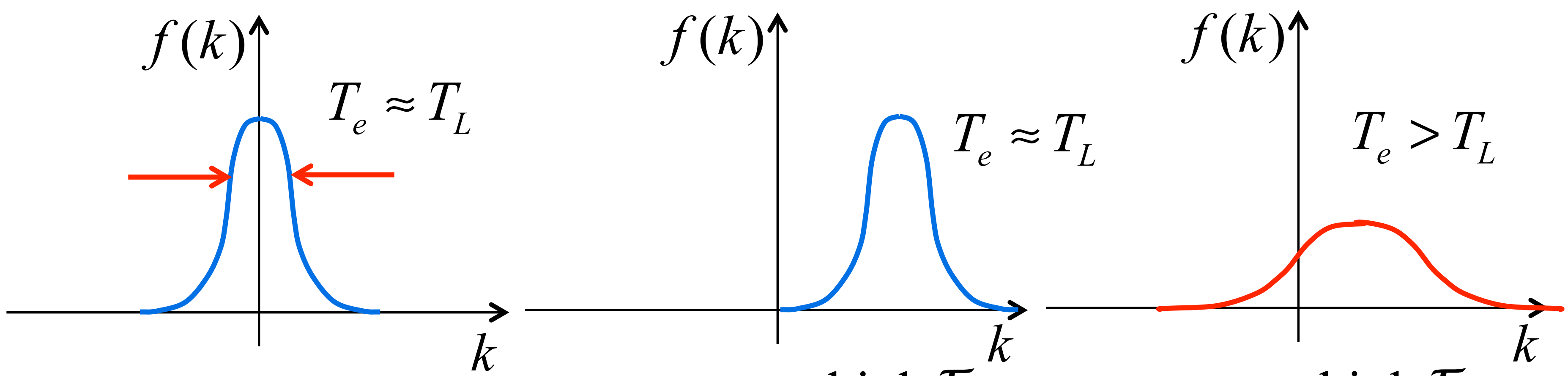
$$\vec{\upsilon} = \vec{\upsilon}_d + \vec{c} \qquad \langle \vec{c} \rangle \equiv 0$$

$$\upsilon^2 = \upsilon_d^2 + 2\vec{c}\bullet\vec{\upsilon}_d + c^2$$

$$W = n\left(\frac{1}{2}m^*\upsilon_d^2 + \frac{1}{2}m^*\langle c^2 \rangle\right)$$

Electron temperature

$$W = n\left(\frac{1}{2}m^*\upsilon_d^2 + \frac{1}{2}m^*\left\langle c^2\right\rangle\right) \qquad \frac{3}{2}k_B T_e \equiv \frac{1}{2}m^*\left\langle c^2\right\rangle$$



low $\mathcal{E}_x$

high $\mathcal{E}_x$
low T_e

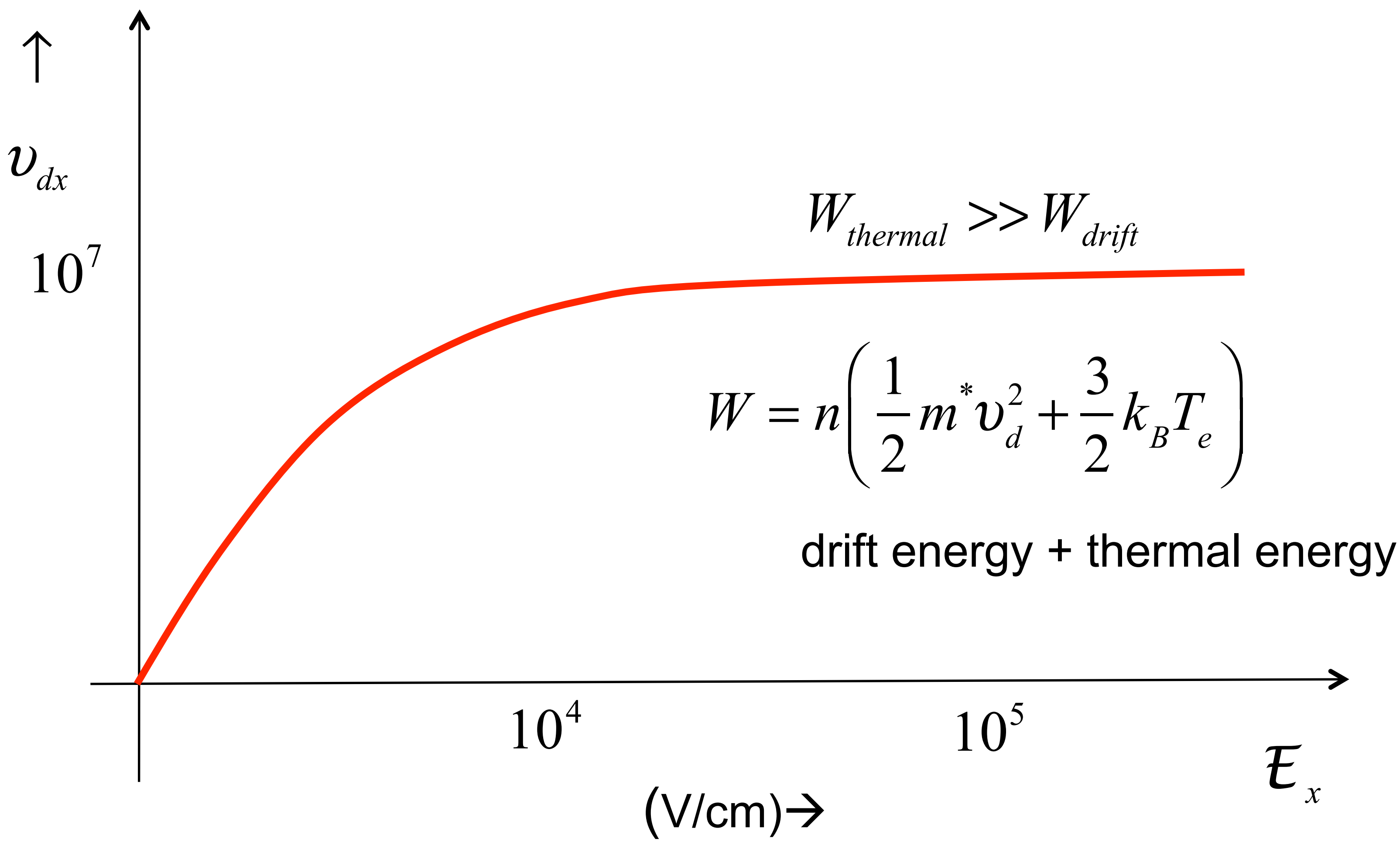
high $\mathcal{E}_x$
high T_e

$$f(k) \sim e^{-\hbar^2 k^2 / 2m^* k_B T_L}$$

$$f(k) \sim e^{-\hbar^2 (k-k_d)^2 / 2m^* k_B T_L}$$

$$f(k) \sim e^{-\hbar^2 (k-k_d)^2 / 2m^* k_B T_e}$$

Electron temperature



Energy flux

$$F_{Wx} = \frac{1}{\Omega}\sum_{\vec{p}} E(\vec{p})\upsilon_x f$$

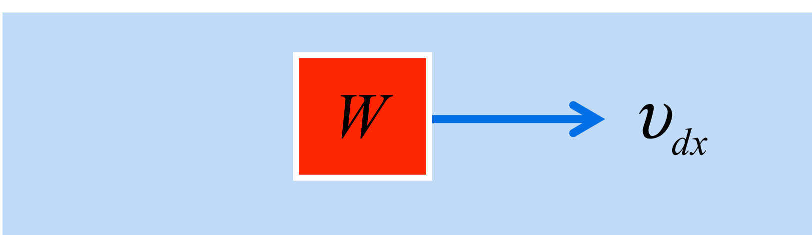
$$\upsilon_x = \upsilon_{dx} + c_x$$

$$F_{Wx} = \frac{1}{\Omega}\sum_{\vec{p}} \left(\frac{1}{2} m^* \upsilon^2\right)\upsilon_x f$$

$$F_{Wx} = \frac{1}{2} m^* n \left\langle \upsilon^2 \upsilon_x \right\rangle$$

$$F_{Wx} = W\upsilon_{dx} + nk_B T_e \upsilon_{dx} + Q_x$$

$$Q_x \equiv n\frac{1}{2} m^* \left\langle c^2 c_x \right\rangle$$



$$P\Omega = Nk_B T_e$$

The temperature balance equations

$$\frac{\partial n}{\partial t} = -\frac{d}{dx}\frac{J_{nx}}{(-q)}$$

$$J_{nx} = nq\mu_n \mathcal{E}_x + \frac{2}{3}\mu_n \frac{\partial W}{\partial x}$$

$$W = n\left(\frac{1}{2}m^* \upsilon_d^2 + \frac{3}{2}k_B T_e\right)$$

$$\frac{\partial W}{\partial t} = -\frac{\partial F_{Wx}}{\partial x} - J_{nx}\mathcal{E}_x - \frac{W - W_0}{\langle \tau_E \rangle}$$

$$Q_x \approx -\kappa_e \frac{dT_e}{dx}$$

$$F_{Wx} = W\upsilon_{dx} + nk_B T_e \upsilon_{dx} + Q_x$$

$$n(x,t)$$

$$T_e(x,t)$$

The energy balance equations

$$\frac{\partial n}{\partial t} = -\frac{d}{dx}\frac{J_{nx}}{(-q)}$$

$$J_{nx} = nq\mu_n \mathcal{E}_x + \frac{2}{3}\mu_n \frac{\partial W}{\partial x}$$

$$\frac{\partial W}{\partial t} = -\frac{\partial F_{Wx}}{\partial x} - J_{nx}\mathcal{E}_x - \frac{W - W_0}{\langle \tau_E \rangle}$$

$$F_{Wx} = -W\mu_E \mathcal{E}_x - \frac{d(D_E W)}{dx}$$

$n(x,t)$

$W(x,t)$

Questions

Energy transport equations allow us to treat hot carrier in bulk semiconductors and non-local transport in devices.

But there are many simplifying assumptions that are hard to justify in general.

They provide a useful, qualitative understanding, but more rigorous technique is necessary.