ECE 656: Electronic Transport in Semiconductors Fall 2017

Hot Carrier Transport in the bulk

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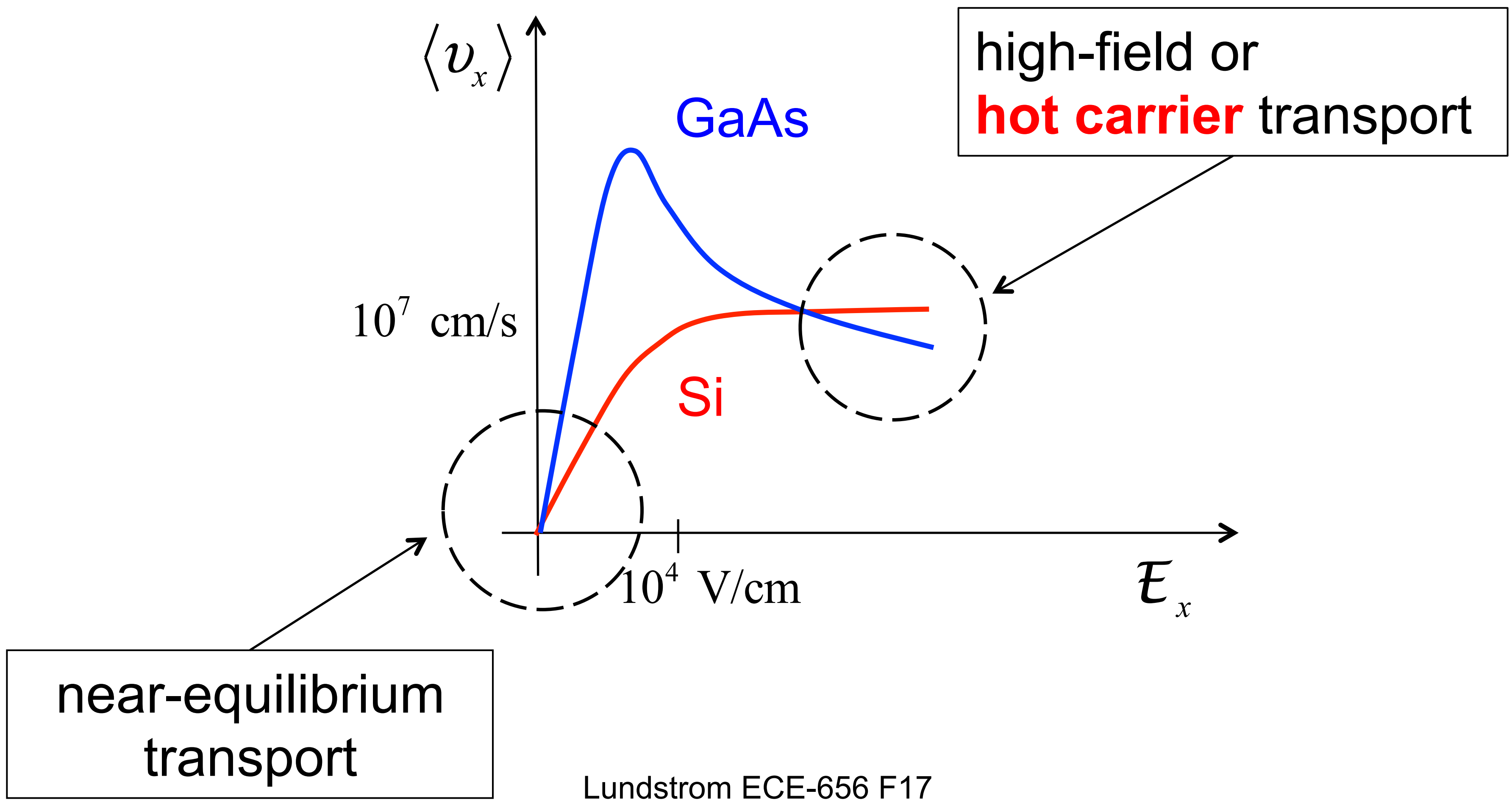
Velocity vs. field characteristics

(***bulk*** semiconductors assumed)



Outline

1) Brief Introduction
2) **Current Equation**
3) Qualitative features of high field transport
4**)** Saturated velocity
5) Survey of results
6) Electron temperature model
7) Summary

Current equation

$$J_{nx} = nq\mu_n \mathcal{E}_x + 2\mu_n \frac{dW_{xx}}{dx} \qquad W_{xx} = \frac{1}{\Omega}\sum_{\vec{k}} \frac{1}{2} m^* \upsilon_x^2 = n u_{xx}$$

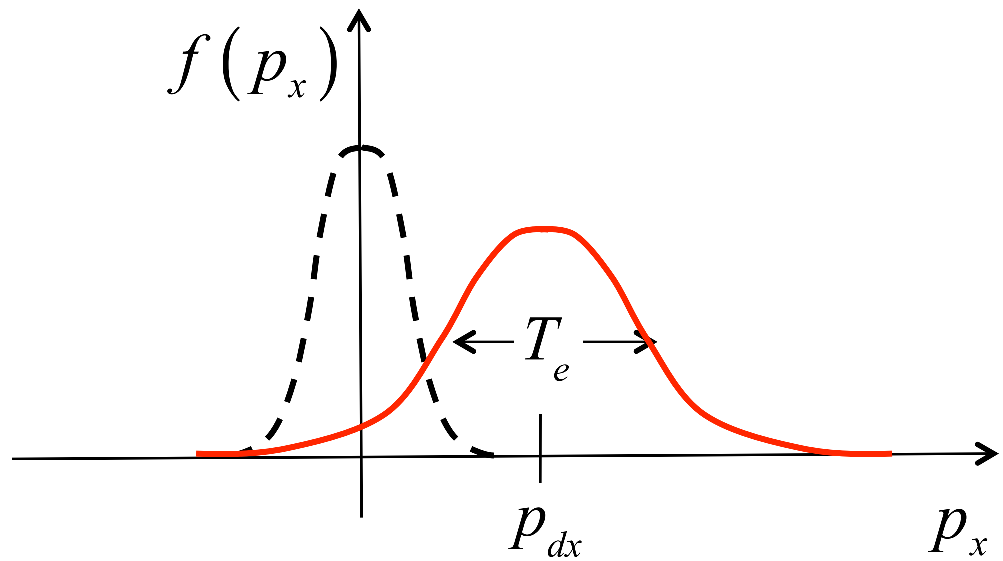
This is an "exact" steady-state current equation, but….

$$\mu_n\left[f\left(\vec{r},\vec{p},t\right)\right] \qquad u_{xx}\left[f\left(\vec{r},\vec{p},t\right)\right] = \left\langle \frac{1}{2} p_z \upsilon_z \right\rangle$$

Electron kinetic energy

$$W_{xx} = \frac{1}{\Omega}\sum_{\vec{k}} \frac{1}{2} m^* \upsilon_x^2 = n u_{xx}$$

$$u_{xx} = \frac{1}{2} m^* \upsilon_{dx}^2 + \frac{3}{2} k_B T_e$$



Aside: neglect of the drift energy

$$u = \frac{1}{2} m^* \upsilon_d^2 + \frac{3}{2} k_B T_e \approx \frac{3}{2} k_B T_e$$

$$\frac{m^* \upsilon_d^2 / 2}{3 k_B T_e / 2} = \frac{\langle \tau_m \rangle}{\langle \tau_E \rangle} << 1$$

The drift energy is small when the energy relaxation time is much larger than the momentum relaxation time (which is the typical case).

Current equation: bulk semiconductor

$$J_{nx} = nq\mu_n\mathcal{E}_x + 2\mu_n \frac{d\left(nu_{xx}\right)}{dx}$$

$$J_{nx} = nq\mu_n\mathcal{E}_x + qD_n \frac{dn}{dx}$$

$$\frac{D_n}{\mu_n} = \frac{2u_{xx}}{q}$$

near equilibrium: $u_{xx} \approx \frac{k_B T_L}{2}$

high-fields: $u_{xx} \approx \frac{k_B T_e}{2}$

$$T_e > T_L$$

(local) Field-dependent mobility

$$J_{nx} = nq\mu_n \mathcal{E}_x + qD_n \frac{dn}{dx}$$

In general: $\mu_n\left[f\left(\vec{r},\vec{p},t\right)\right] \quad D_n\left[f\left(\vec{r},\vec{p},t\right)\right]$

In a bulk semiconductor, *f* is determined by $\mathcal{E}$, so there is a one-to-one mapping between $\mathcal{E}$ and *f*.

$\mu_n\left(\mathcal{E}\right) \quad D_n\left(\mathcal{E}\right)$ Electric ***field dependent*** mobility and diffusion coefficient.

$$J_{nx} = nq\mu_n\left(\mathcal{E}\right)\mathcal{E}_x + qD_n\left(\mathcal{E}\right)\frac{dn}{dx}$$

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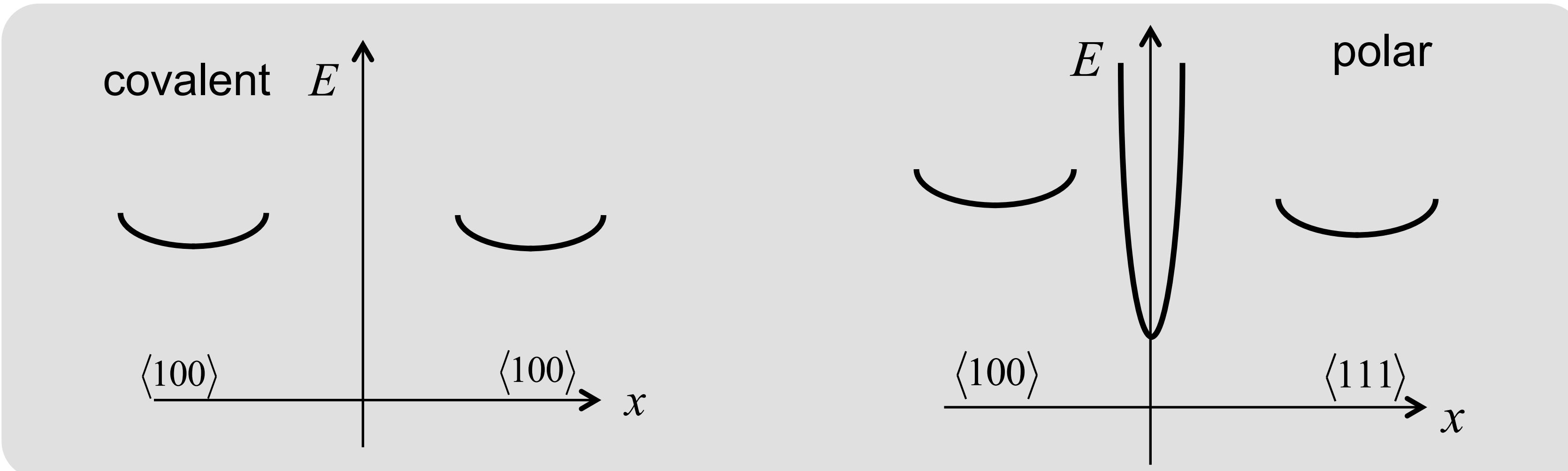
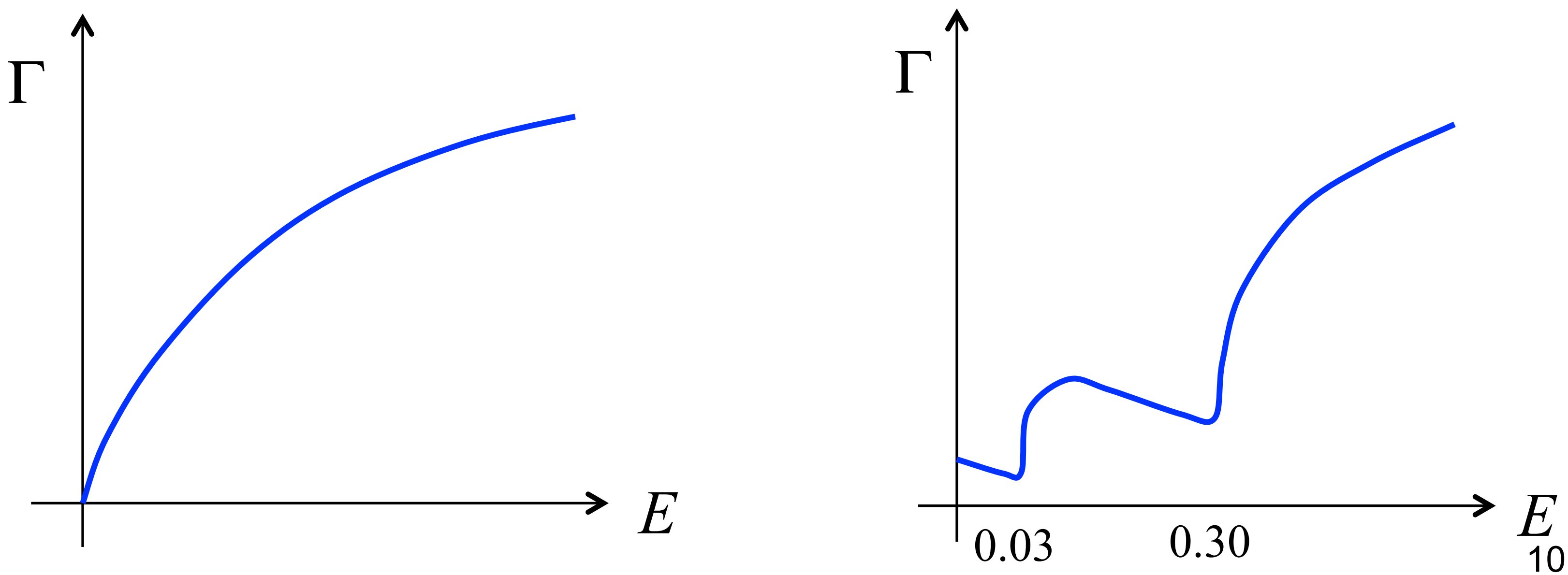
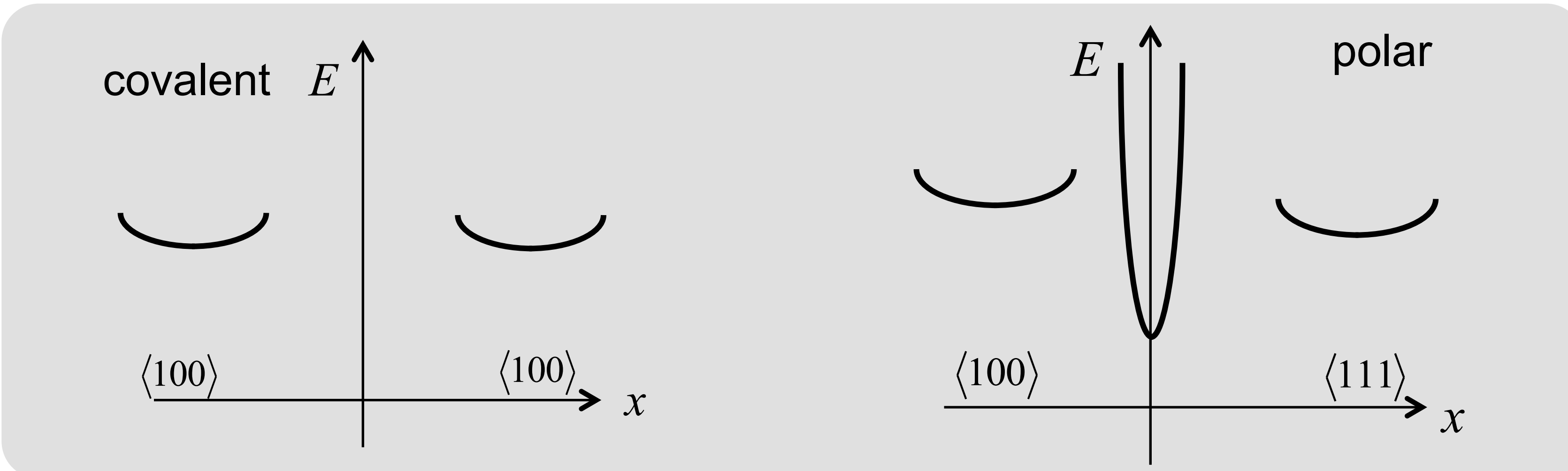
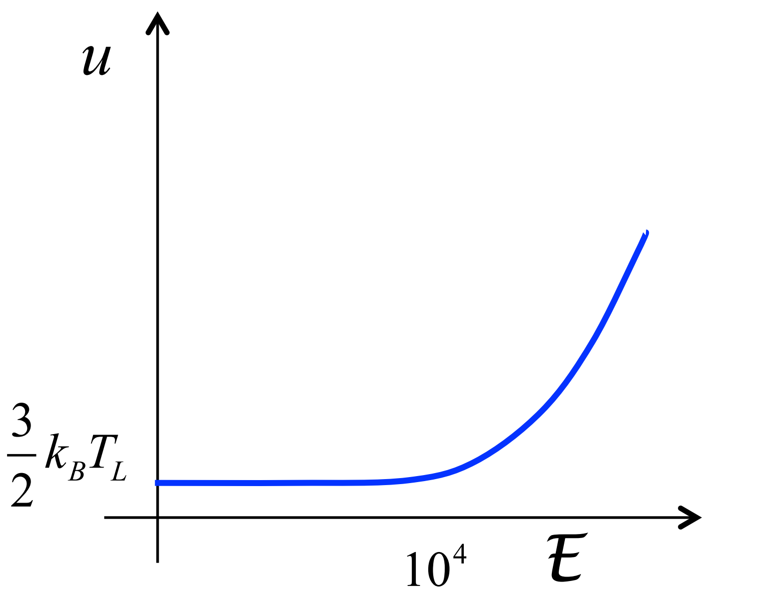
Covalent vs. polar semiconductors

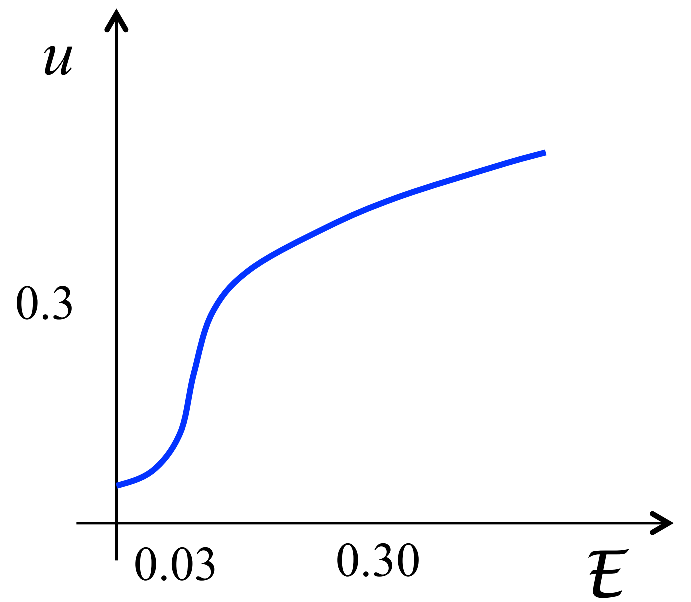




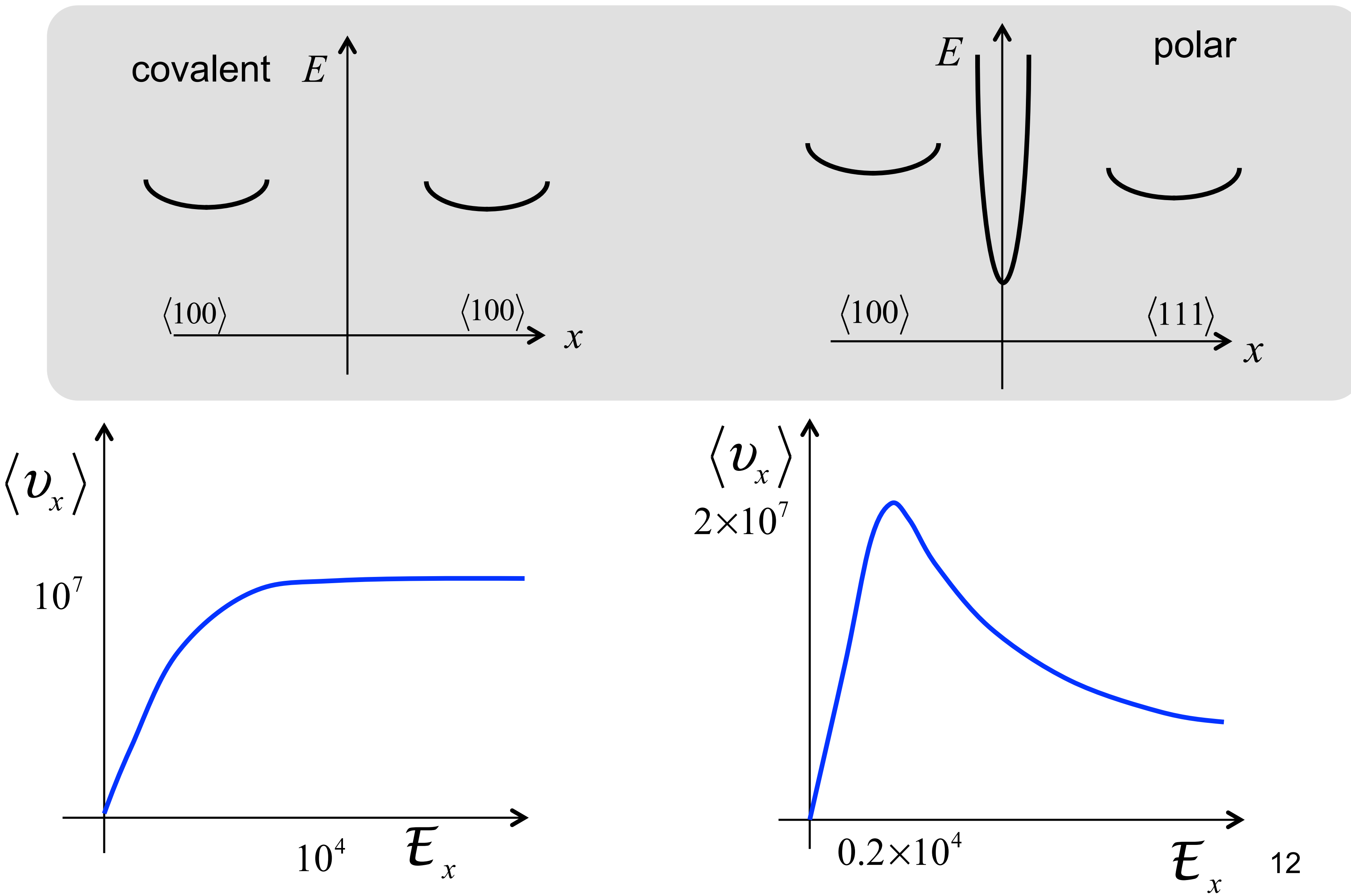
Average energy vs. electric field



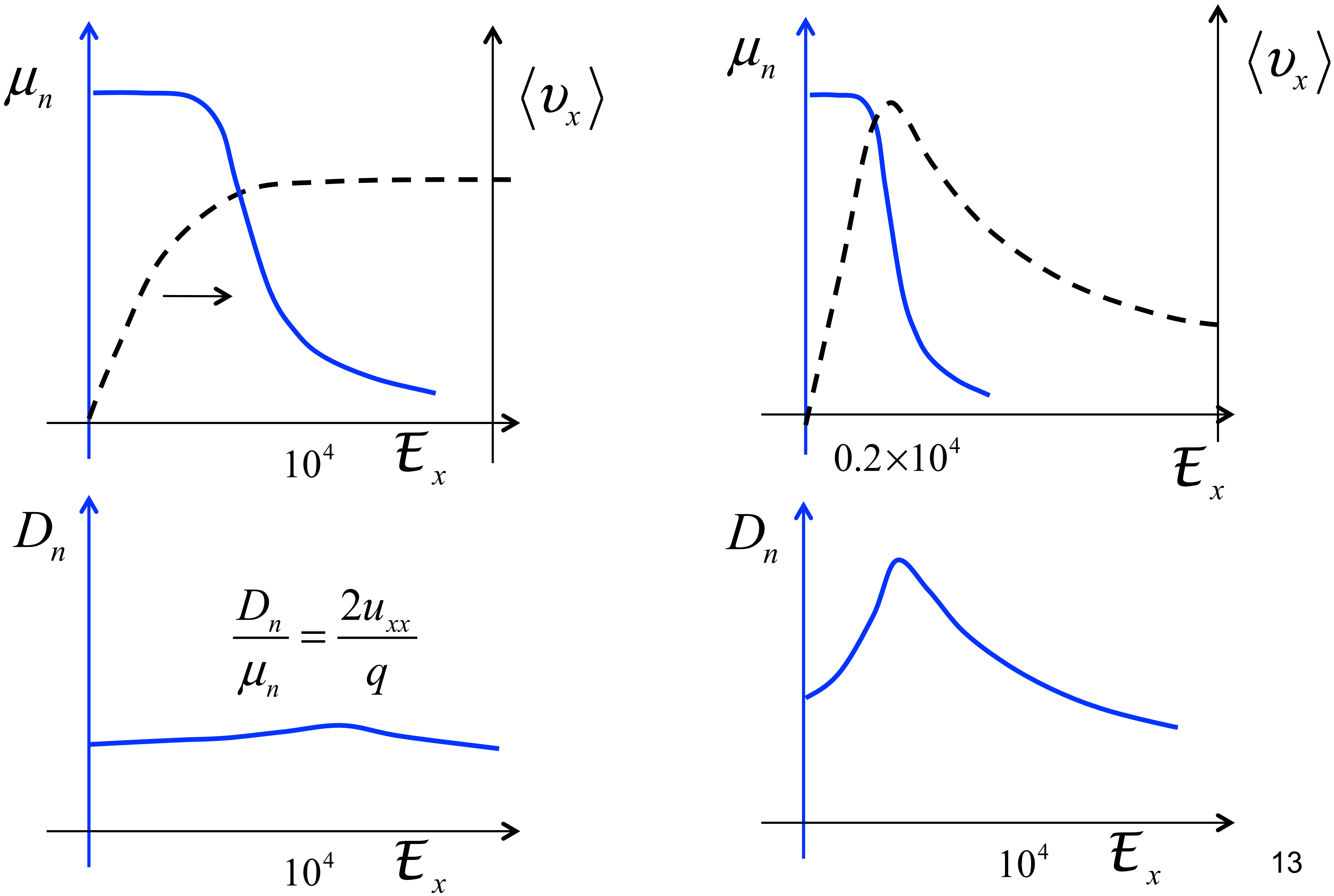




Average velocity vs. electric field



Mobility and diffusion coefficient



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Saturation velocity

$$\upsilon_{SAT} \approx \sqrt{\frac{\hbar\omega_0}{m^*}}$$

Si: $\hbar\omega_0 = 0.063 \text{ eV}$ $\upsilon_{SAT} \approx 1.0 \times 10^7 \text{ cm/s}$

Ge: $\hbar\omega_0 = 0.037 \text{ eV}$ $\upsilon_{SAT} \approx 0.6 \times 10^7 \text{ cm/s}$

SiC: $\hbar\omega_0 = 0.12 \text{ eV}$ $\upsilon_{SAT} \approx 1.5 \times 10^7 \text{ cm/s}$

Can we calculate υ_{SAT}?

$$\mu_n = \frac{q\langle\tau_m\rangle}{m^*}$$ (momentum balance)

$$J_{nx}\mathcal{E}_x = \frac{n(u-u_0)}{\langle\tau_E\rangle}$$ (energy balance) $$nq\mu_n\mathcal{E}_x^{\,2} = \frac{n(u-u_0)}{\langle\tau_E\rangle}$$

$$u = u_0 + \langle\tau_E\rangle q\mu_n\mathcal{E}_x^{\,2} = u_0 + \frac{\langle\tau_E\rangle\langle\tau_m\rangle}{m^*}q^2\mathcal{E}_x^{\,2} \approx \frac{\langle\tau_E\rangle\langle\tau_m\rangle}{m^*}q^2\mathcal{E}_x^{\,2}$$

$\langle\tau_m\rangle \approx \langle\tau\rangle$ (ave. time between collisions)

$$\langle\tau_E\rangle \approx \frac{u}{\hbar\omega_0}\langle\tau\rangle = \frac{u}{\hbar\omega_0}\langle\tau_m\rangle$$

Calculation of υ_{SAT}

$$\mu_n = \frac{q\langle\tau_m\rangle}{m^*} \qquad u \approx \frac{\langle\tau_E\rangle\langle\tau_m\rangle}{m^*} q^2\mathcal{E}_x^{\,2} \qquad \langle\tau_E\rangle \approx \frac{u}{\hbar\omega_0}\langle\tau\rangle = \frac{u}{\hbar\omega_0}\langle\tau_m\rangle$$

$$u \approx \frac{\langle\tau_m\rangle^2 u}{\hbar\omega_0 m^*} q^2\mathcal{E}_x^{\,2}$$

$$\langle\tau_m\rangle \approx \frac{\sqrt{\hbar\omega_0 m^*}}{q\mathcal{E}_x} \rightarrow \mu_n = \sqrt{\frac{\hbar\omega_0}{m^*}}\frac{1}{\mathcal{E}_x}$$

$$\langle\upsilon_x\rangle = \mu_n\mathcal{E}_x \rightarrow \upsilon_{SAT} = \sqrt{\frac{\hbar\omega_0}{m^*}}$$

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<111> Silicon: low-field

$$\mathcal{E}_z = -100 \text{ V/cm}$$

$$\langle \upsilon_z \rangle = 8.1 \times 10^4 \text{cm/s}$$

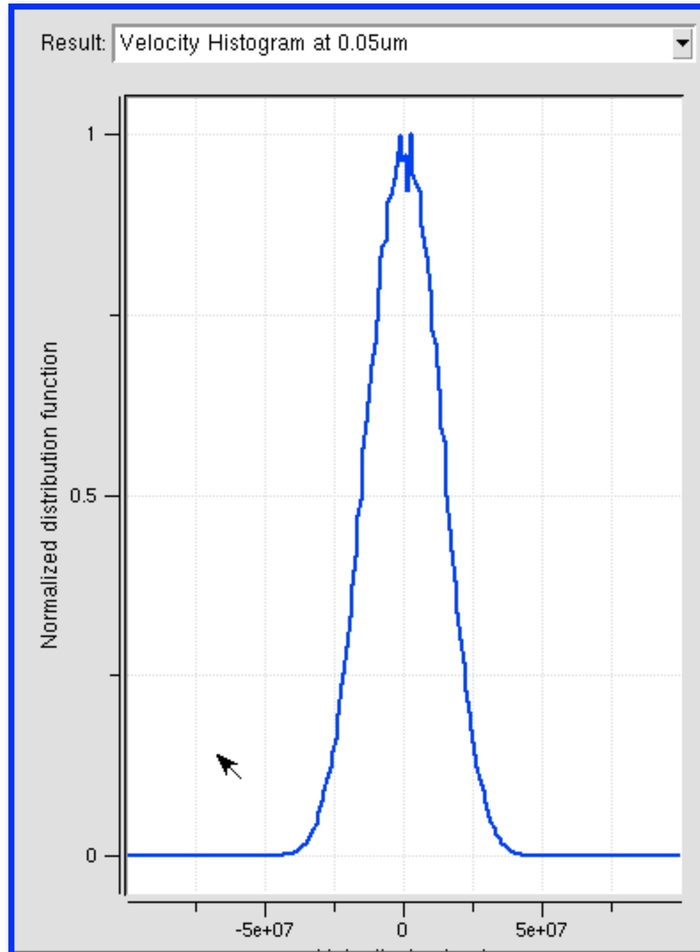
$$\mu_n(\mathcal{E}_z) = 810 \text{ cm}^2/\text{V-s}$$

$$u = 0.04 \text{ eV} \quad \left(1.5 k_B T_L / q = 0.039 \text{ eV}\right)$$

$$u_{zz} = 0.0135 \text{ eV} \quad \left(u_{zz} / u = 0.34\right)$$

$$u_{drift} \sim 10^{-7} \text{ eV} \quad \left(u_{drift} / u \sim 10^{-5}\right)$$

$$n(x, y, z)/n = 0.33 / 0.335 / 0.335$$



(simulations performed with DEMONs on www.nanoHUB.org)

<111> Silicon: high-field

$$\mathcal{E}_z = -10^5 \text{ V/cm}$$

$$\langle \upsilon_z \rangle = 1.04 \times 10^7 \text{ cm/s}$$

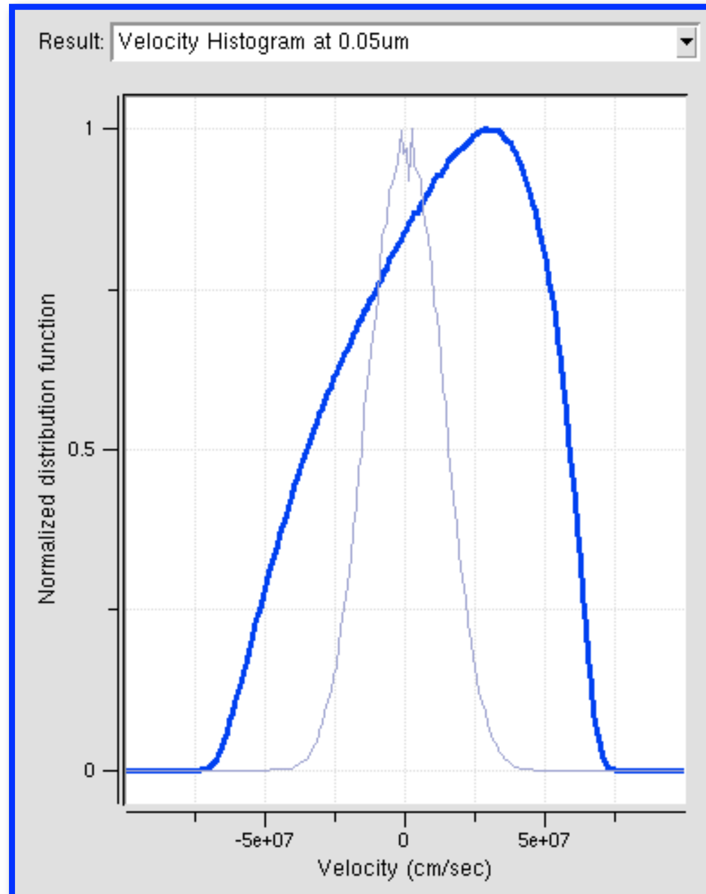
$$\mu_n(\mathcal{E}_z) = 104 \text{ cm}^2/\text{V-s}$$

$$u = 0.364 \text{ eV} \quad \left(1.5 k_B T_L / q = 0.039 \text{ eV}\right)$$

$$u_{zz} = 0.145 \text{ eV} \quad \left(u_{zz} / u = 0.40\right)$$

$$u_{drift} = 0.008 \text{ eV} \quad \left(u_{drift} / u = 0.02\right)$$

$$n(x, y, z)/n = 0.336 / 0.331 / 0.333$$



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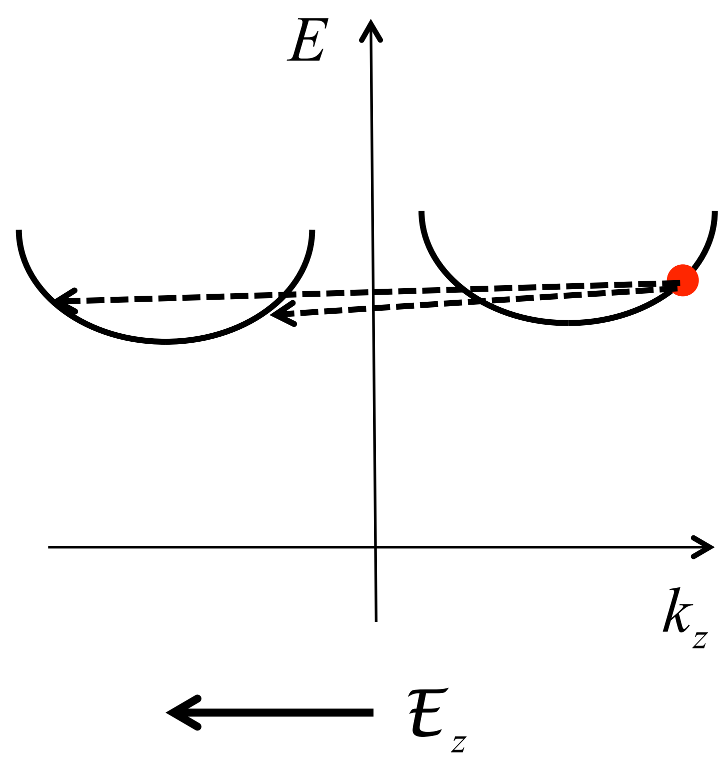
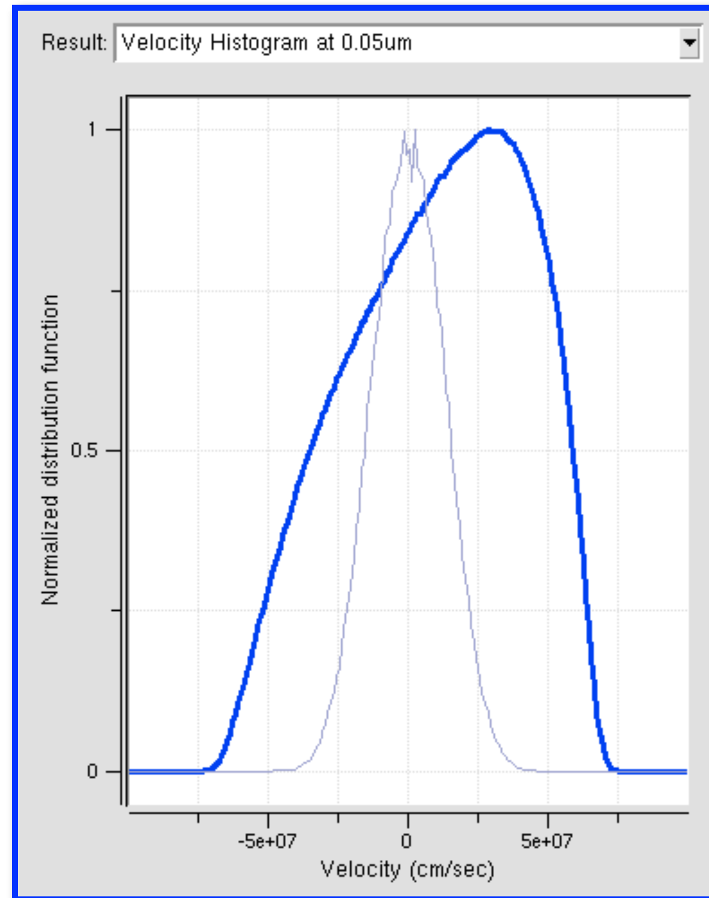
<111> Silicon: high-field





(simulations performed with DEMONs on www.nanoHUB.org)

<100> Silicon: high-field

$$\mathcal{E}_z = -10^5 \text{ V/cm}$$

$$\langle \upsilon_z \rangle = 0.98 \times 10^7 \text{cm/s}$$

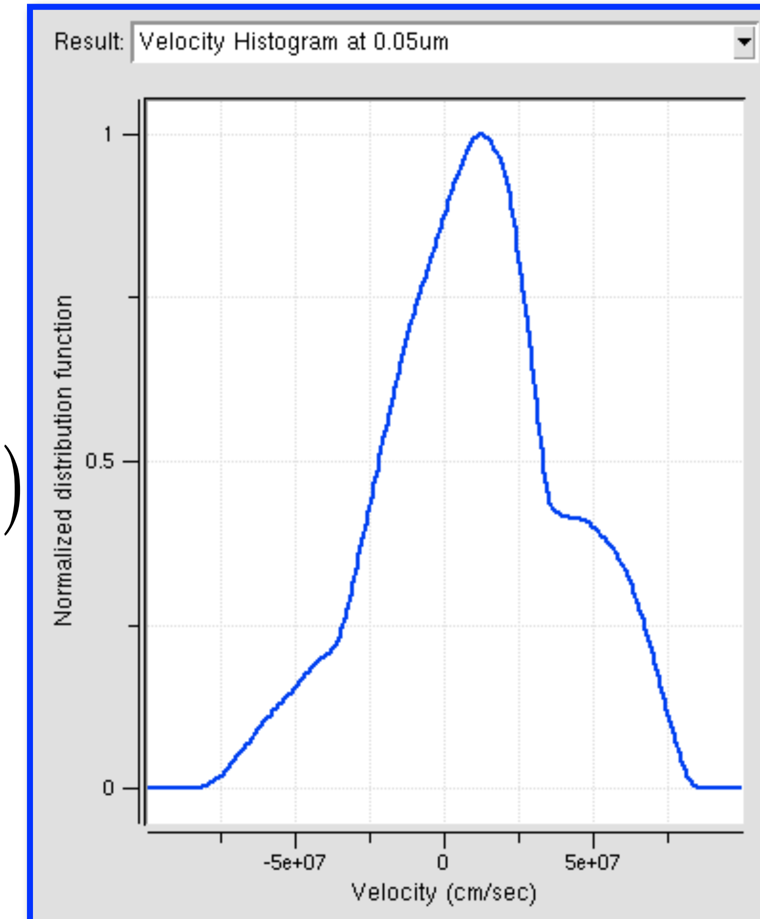
$$\mu_n(\mathcal{E}_z) = 98 \text{ cm}^2/\text{V-s}$$

$$u = 0.346 \text{ eV} \quad \left(1.5 k_B T_L / q = 0.039 \text{ eV}\right)$$

$$u_{zz} = 0.138 \text{ eV} \quad \left(u_{zz} / u = 0.40\right)$$

$$u_{drift} = 0.007 \text{ eV} \quad \left(u_{drift} / u = 0.02\right)$$

$$n(x,y,z)/n = 0.306 / 0.309 / 0.385$$



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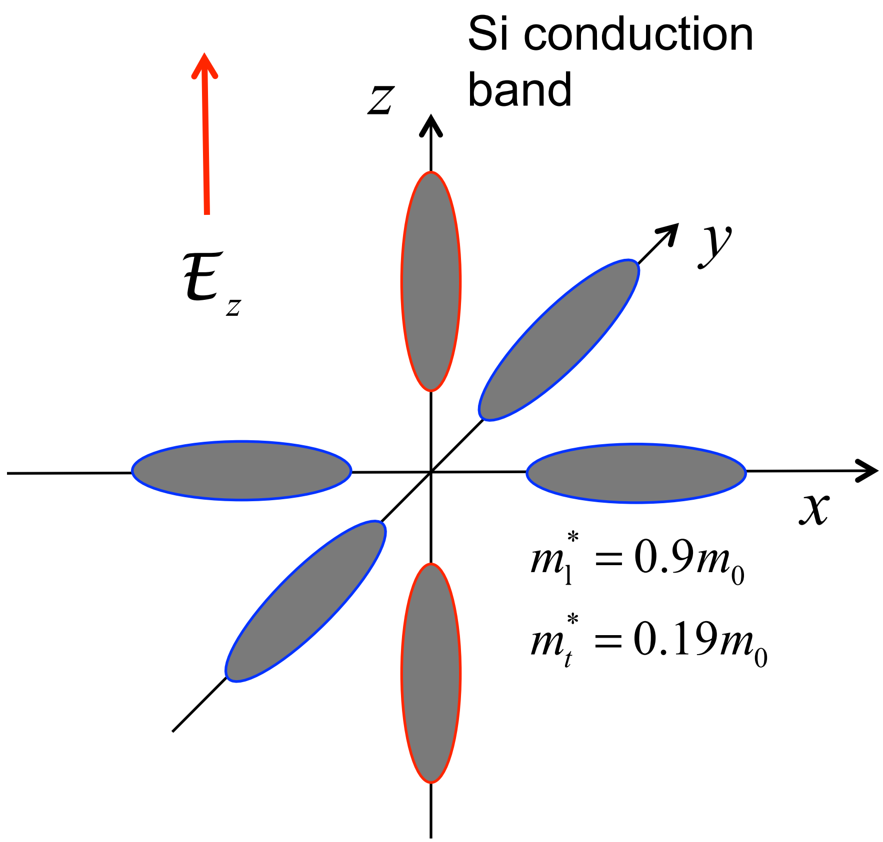
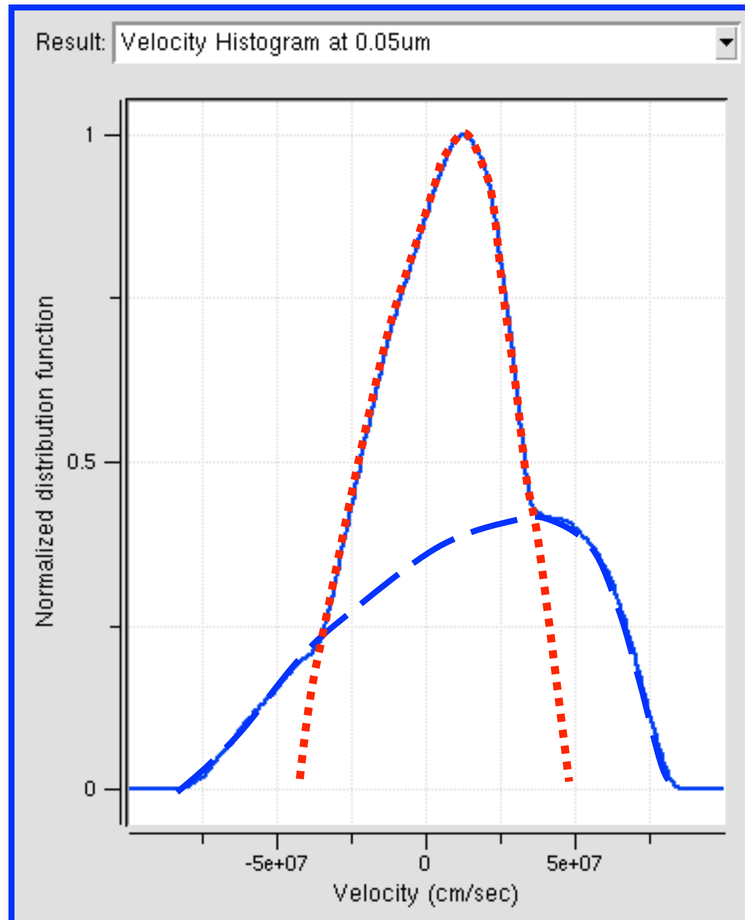
<100> Silicon: high-field





(simulations performed with DEMONs on www.nanoHUB.org)

Suggested exercises

1) Estimate the momentum and energy relaxation times from the high-field simulations (assume that the effective mass is the conductivity effective mass).

2) Repeat the simulations for GaAs and compare the results to Si.

<100> Silicon: high-field

See Lundstrom, FCT, Sec. 7.5.1 for a discussion of how “valley repopulation” affect the velocity vs. electric field characteristics of <100> Si.

Summary

1) High-field transport leads to **local** field-dependent mobilities and diffusion coefficients (when the field varies slowly in space and time).

2) Balance equations provide a useful way to interpret detailed (e.g. Monte Carlo) simulations.

3) The electron temperature approach provides a qualitative (and sometimes quantitative) way to view high-field (hot carrier) transport.

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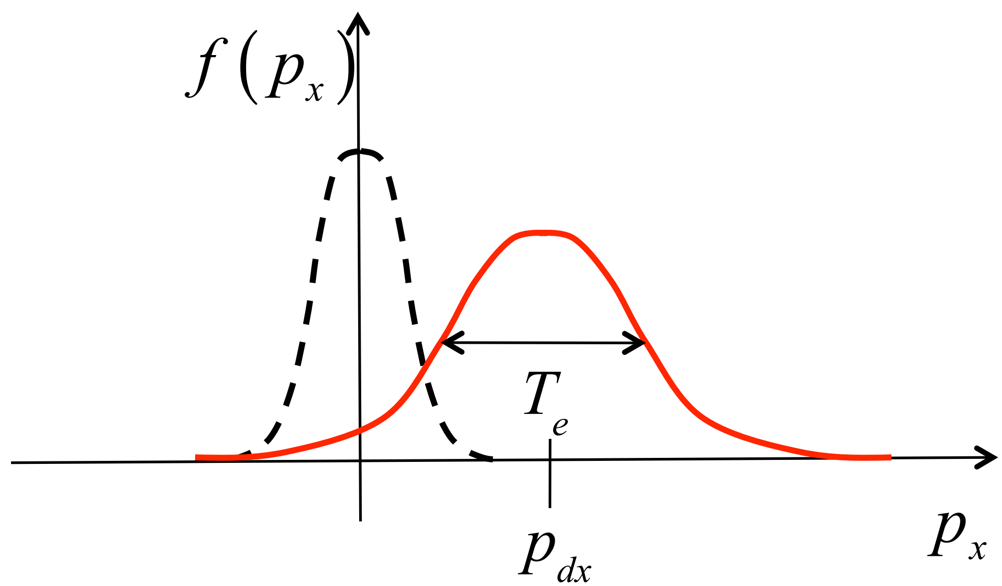
Electron temperature approach

1) Goal: to compute $\langle \upsilon_x \rangle = \upsilon_{dx}(\mathcal{E})$

2) Assume: $f(\vec{p}) = e^{-\left|\vec{p}-m^*\vec{\upsilon}_d\right|^2 / 2m^* k_B T_e}$



Electron temperature approach

$$f(\vec{p}) = e^{-\left|\vec{p}-m^*\vec{\upsilon}_d\right|^2 / 2m^* k_B T_e}$$

2 unknowns: υ_{dx}, T_e …need 2 equations

1) Momentum balance:

$$J_{nx} = nq\mu_n \mathcal{E}_x \rightarrow \upsilon_{dx} = -\mu_n \mathcal{E}_x$$

2) Energy balance:

$$J_{nx}\mathcal{E}_x = nq\mu_n \mathcal{E}_x^{\,2} = \frac{n(u-u_0)}{\langle \tau_E \rangle}$$

Electron temperature approach

$$J_{nx}\mathcal{E}_x = nq\mu_n\mathcal{E}_x^{\,2} = \frac{n(u-u_0)}{\langle\tau_E\rangle}$$

$$u = u_0 + q\langle\tau_E\rangle\mu_n\mathcal{E}_x^{\,2} = u_0 + \frac{q^2\langle\tau_E\rangle\langle\tau_m\rangle}{m^*}\mathcal{E}_x^{\,2}$$

$$u_0 \approx \frac{3}{2}k_B T_L$$

$$u \approx \frac{3}{2}k_B T_e$$ (neglects drift energy)

$$\frac{T_e}{T_L} = 1 + \frac{2q^2\langle\tau_E\rangle\langle\tau_m\rangle}{3k_B T_L m^*}\mathcal{E}_x^{\,2}$$

$$f(\vec{p}) = e^{-\left|\vec{p}-m^*\vec{\upsilon}_d\right|^2 / 2m^* k_B T_e}$$

$$\upsilon_{dx} = -\mu_n(T_e)\mathcal{E}_x$$

$$\frac{T_e}{T_L} = 1 + \frac{2q^2\langle\tau_E\rangle\langle\tau_m\rangle}{3k_B T_L m^*}\mathcal{E}_x^{\,2}$$

Aside: neglect of the drift energy

$$u = \frac{1}{2} m^* \upsilon_d^2 + \frac{3}{2} k_B T_e \approx \frac{3}{2} k_B T_e$$

The drift energy is small when the energy relaxation time is much larger than the momentum relaxation time (which is the typical case).

$$\frac{m^* \upsilon_d^2 / 2}{3 k_B T_e / 2} << 1 \, ?$$

$$\upsilon_d^2 = \mu_n^2 \mathcal{E}_x^2 = q^2 \langle \tau_m \rangle^2 \mathcal{E}_x^2 / m^{*2}$$

$$\frac{T_e}{T_L} = 1 + \frac{2q^2 \langle \tau_E \rangle \langle \tau_m \rangle}{3 k_B T_L m^*} \mathcal{E}_x^2 \rightarrow \frac{3}{2} k_B T_e \approx \frac{q^2 \langle \tau_E \rangle \langle \tau_m \rangle}{m^*} \mathcal{E}_x^2$$

$$\frac{m^* \upsilon_d^2 / 2}{3 k_B T_e / 2} = \frac{\langle \tau_m \rangle}{\langle \tau_E \rangle} << 1$$

Electron temperature model

$$f(\vec{p}) = e^{-\left|\vec{p}-m^*\vec{\upsilon}_d\right|^2 / 2m^* k_B T_e}$$

$$\upsilon_{dx} = -\mu_n(T_e)\mathcal{E}_x$$

$$\frac{T_e}{T_L} = 1 + \frac{2\mu_n\langle\tau_E\rangle}{3k_B T_L}\mathcal{E}_x^{\,2}$$

need to specify:

$$\mu_n(T_e) \text{ or } \langle\tau_m\rangle(T_e) \text{ and } \langle\tau_E\rangle(T_e)$$

"it can be shown"

$$\mu_n(T_e) = \mu_0\sqrt{T_L/T_e} \qquad \text{(ADP)}$$

$$\mu_n(T_e) = \mu_0(T_e/T_L)^{3/2} \qquad \text{(II)}$$

for ODP IV scattering in Si:

$$1/\langle\tau_E\rangle = \frac{2}{3}\frac{C}{k_B T_L}\sqrt{T_L/T_e}$$

$$C \approx 10^{-8} \text{ W}$$

The procedure

1) Identify the scattering mechanism that controls momentum relaxation

e.g. ADP scattering in Si $\mu_n(T_e) = \mu_{n0}\sqrt{T_L/T_e}$

2) Identify the scattering mechanism that controls energy relaxation

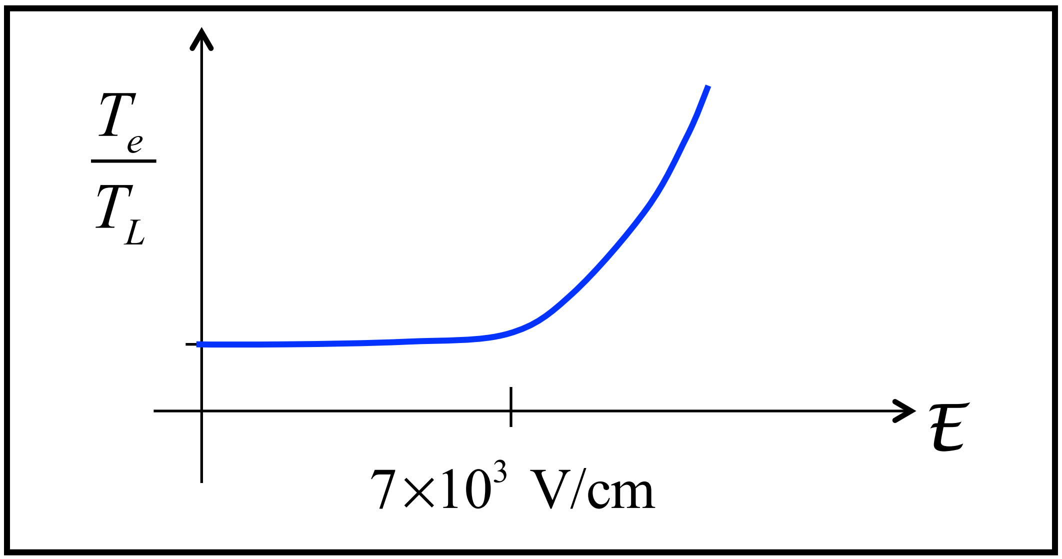
e.g. IV scattering in Si $1/\langle\tau_E\rangle = \frac{2}{3}\frac{C}{k_B T_L}\sqrt{T_L/T_e}$

3) Solve the energy balance equation for T_e

$$\frac{T_e}{T_L} = 1 + \frac{2\mu_n\langle\tau_E\rangle}{3k_B T_L}\mathcal{E}_x^{\,2}$$

Result (for silicon)

$$\frac{T_e}{T_L} = 1 + \frac{q\mu_{n0}}{C}\mathcal{E}_x^{\,2} = 1 + \left(\mathcal{E}/\mathcal{E}_C\right)^2 \qquad \mathcal{E}_C \approx 7\times10^3 \text{ V/cm}$$



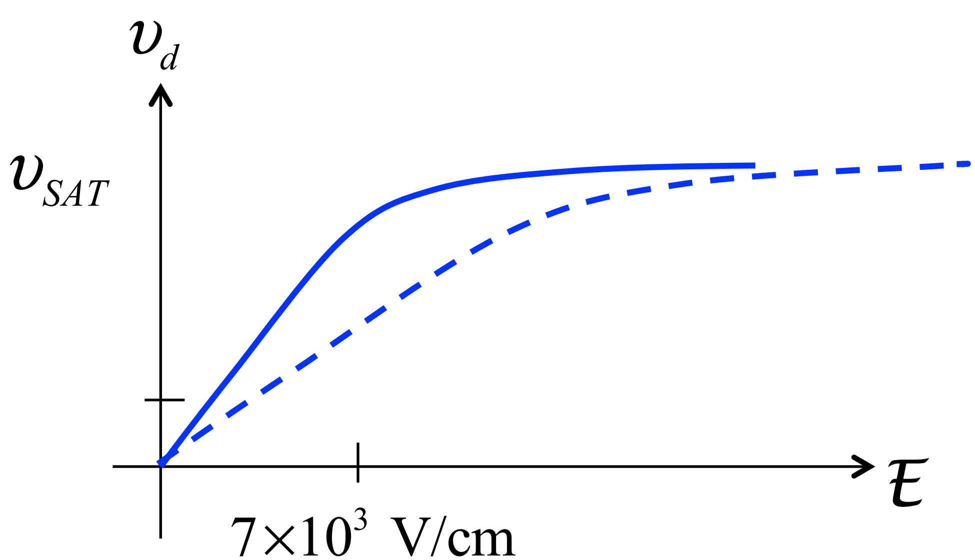
$$\mu_n\left(T_e\right) = \mu_{n0}\sqrt{T_L/T_e} = \frac{\mu_{n0}}{\sqrt{1+\left(\mathcal{E}/\mathcal{E}_C\right)^2}}$$

Velocity vs. field characteristic

$$\mu_n(T_e) = \frac{\mu_{n0}}{\sqrt{1+(\mathcal{E}/\mathcal{E}_C)^2}}$$

$$\upsilon_d = \mu_n(T_e)\mathcal{E} = \frac{\mu_{n0}\mathcal{E}}{\sqrt{1+(\mathcal{E}/\mathcal{E}_C)^2}}$$

$$\upsilon_{SAT} = \mu_{n0}\mathcal{E}_C = 1\times10^7 \text{ cm/s}$$



High- field diffusion coefficient

$$D_n = \frac{k_B T_e}{q}\mu_n(T_e) \;=\; D_{n0}\sqrt{1+(\mathcal{E}/\mathcal{E}_C)^2}$$

but….in practice, $D_n(\mathcal{E}) \approx D_{n0}$ $\quad \left(\dfrac{D_n}{\mu_n} = \dfrac{2u_{xx}}{q}\right)$

a failure of the electron temperature model!

Outline

Summary

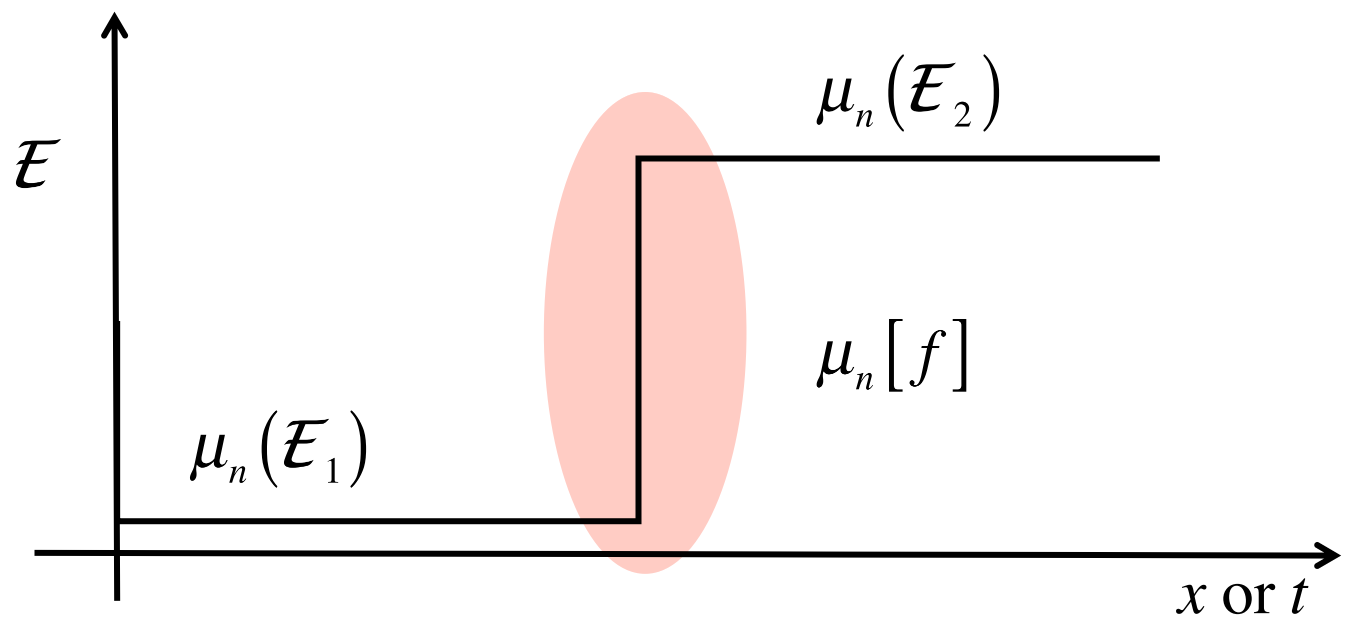
1) High-field transport leads to field-dependent mobilities and diffusion coefficients (when the field varies slowly in space and time).

2) The electron temperature approach provides a qualitative (and sometimes quantitative) way to view high-field (hot carrier) transport.

3) Rapidly varying electric fields lead to "off-equilibrium", "non-local" or "non-stationary" transport effects that cannot be described with (local) field-dependent field dependent transport parameters.

Non-local transport

Rapidly varying electric fields lead to "off-equilibrium", "non-local" or "non-stationary" transport effects that cannot be described with (local) field-dependent field dependent transport parameters.



Questions

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