ECE 656: Electronic Transport in Semiconductors Fall 2017

Ionized Impurity Scattering:

Brooks Herring Approach

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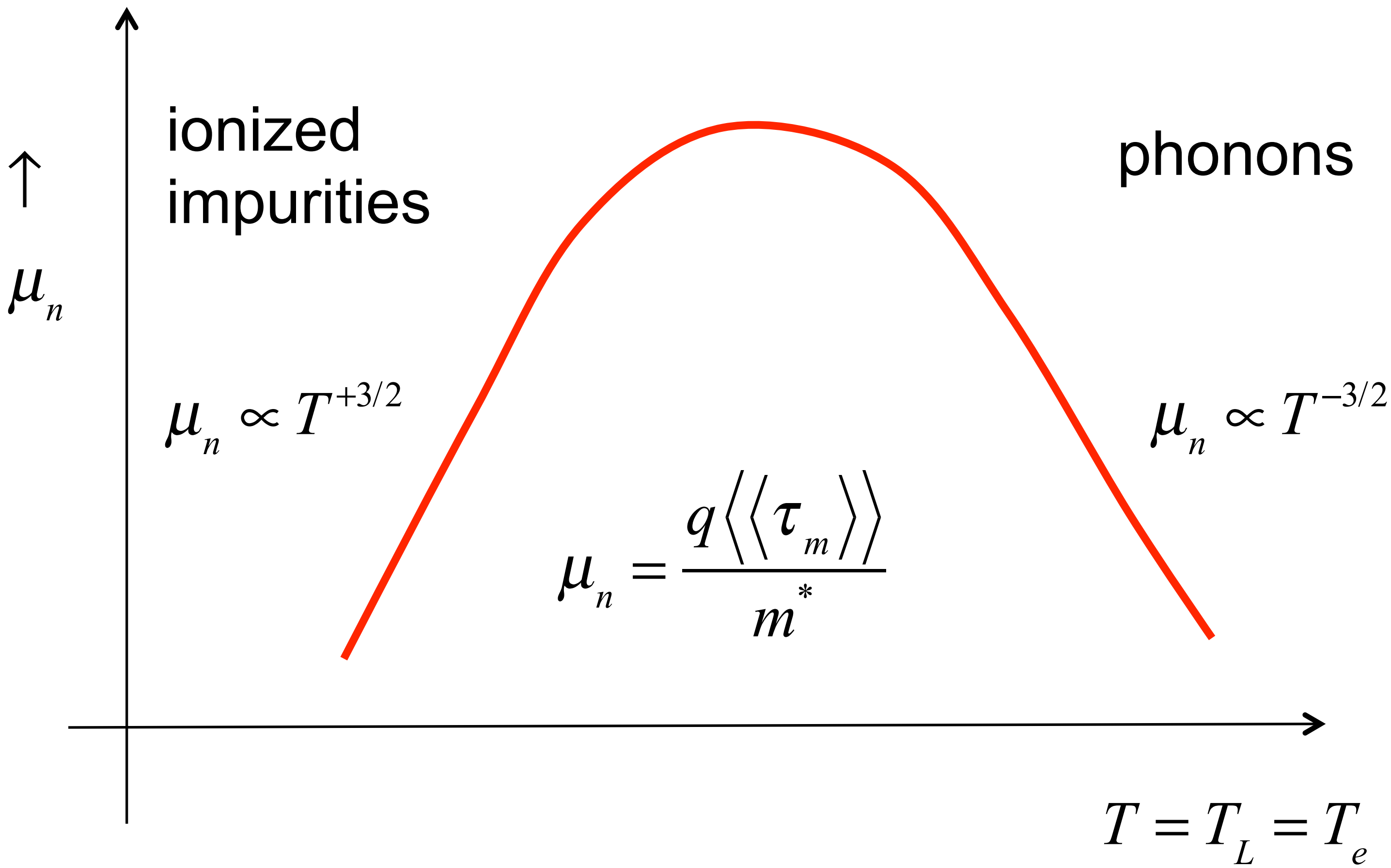
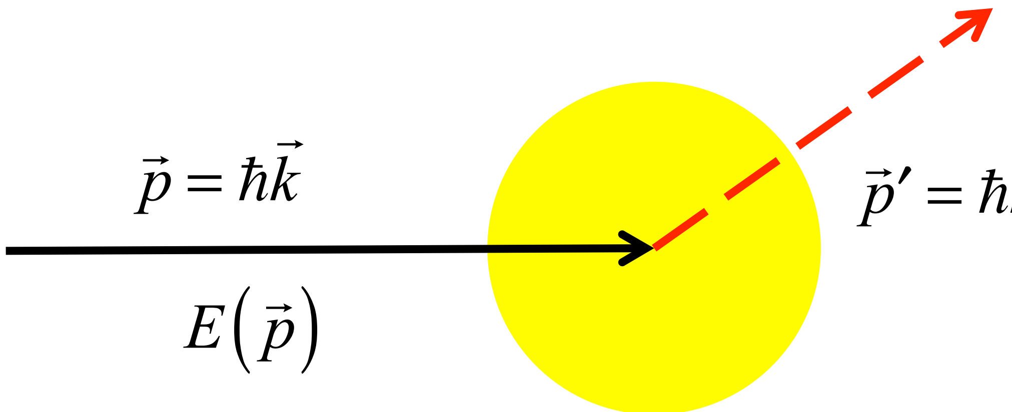
Mobility vs. temperature



Fermi's Golden Rule

$$S(\vec{p},\vec{p}\,') = S(\vec{p} \to \vec{p}\,')$$

$$E(\vec{p}\,')$$



$$S(\vec{p} \to \vec{p}\,') = \frac{2\pi}{\hbar}\left|H_{\vec{p}\,',\vec{p}}\right|^2 \delta(E' - E)$$

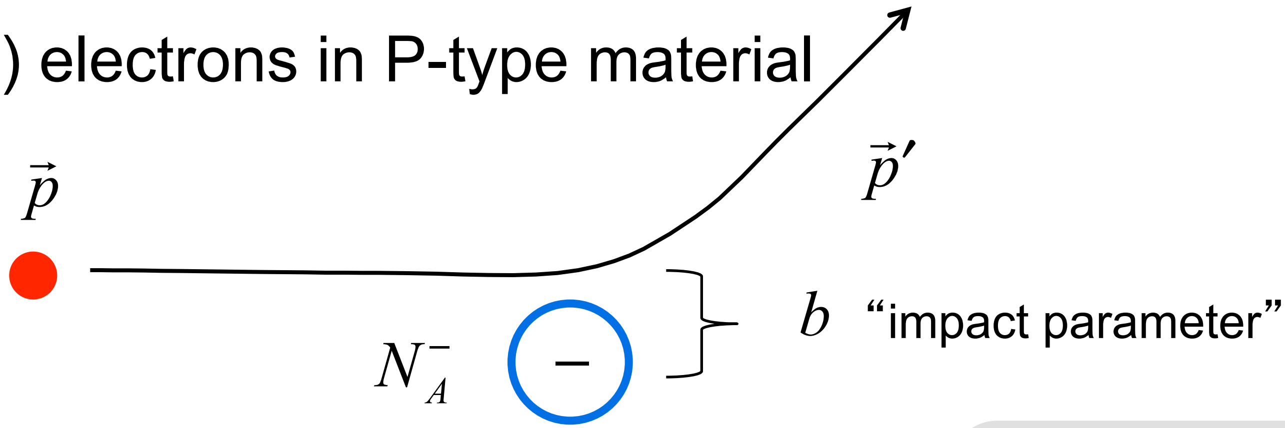
$$H_{\vec{p}\,',\vec{p}} = \int_{-\infty}^{+\infty} \psi_f^* U_S(\vec{r})\psi_i d\vec{r}$$

$$U_S(\vec{r}) = \pm\frac{q^2}{4\pi\kappa_S\varepsilon_0 r}$$

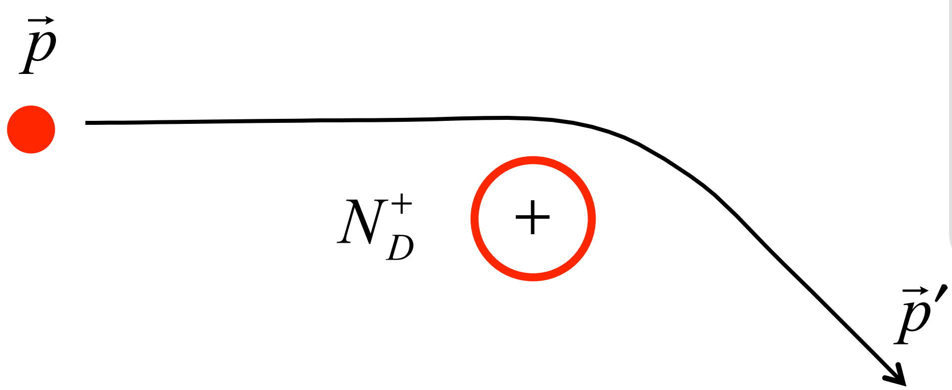
Coulomb potential

Sign of the scattering potential

i) electrons in P-type material



ii) electrons in N-type material



$$U_S(\vec{r}) = \pm \frac{q^2}{4\pi\kappa_S\varepsilon_0 r}$$

According to FGR, the transition rate is independent of the sign of the scattering potential.

In fact, attractive potentials scatter more strongly.

II scattering: Approaches

1) II scattering (Brooks-Herring - ***screened***)

2) II scattering (Conwell-Weisskopf - ***unscreened***)

Historically, the CW approach came first, but we will discuss the BH approach first, then CW.

Outline

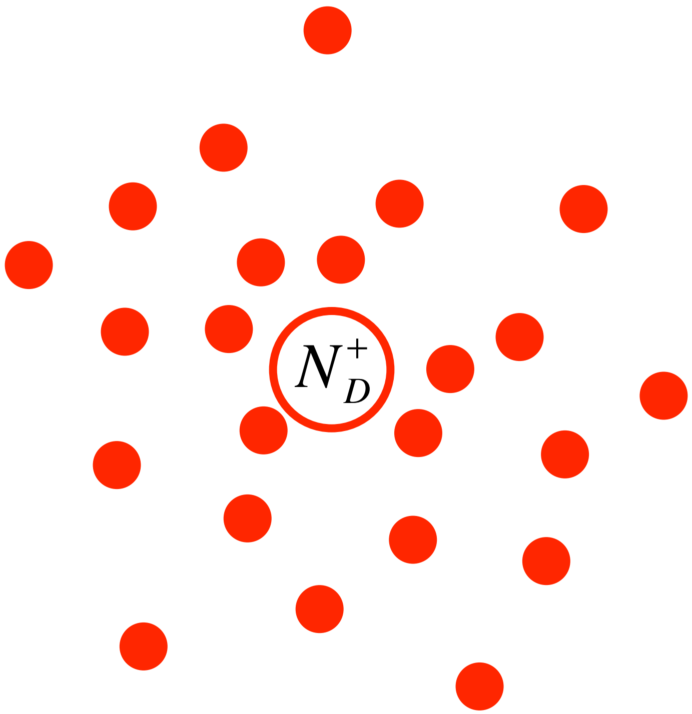
1) Introduction

2) Screening

3) Transition rate

4) Characteristic times

Screened Coulomb potential



Bare Coulomb potential:

$$U_S(\vec{r}) = \frac{q^2}{4\pi\kappa_S\varepsilon_0 r}$$

Screened Coulomb potential:

??

Mobile charges attracted to fixed charges "screen" out the fixed charge.

Screening in 3D

$$\nabla^2 V(\vec{r}) = -\frac{\rho}{\kappa_S \varepsilon_0} = -\frac{q\left[N_D^+ - n(\vec{r})\right]}{\kappa_S \varepsilon_0}$$

$$n(\vec{r}) \approx N_D^+ = n_0$$

$$V(\vec{r}) = V_0$$

$$\delta n(\vec{r}) \approx n(\vec{r}) - n_0$$

$$\delta V(\vec{r}) \approx V(\vec{r}) - V_0$$

$$\nabla^2 \delta V(\vec{r}) = -\frac{q}{\kappa_S \varepsilon_0} \frac{\partial n(\vec{r})}{\partial V} \delta V(\vec{r})$$

$$\frac{1}{L_D^2} \equiv \frac{q}{\kappa_S \varepsilon_0} \frac{\partial n(\vec{r})}{\partial V}$$

$$n(\vec{r}) = \frac{1}{\Omega} \sum_{\vec{k}} f_0(k)$$

$$f_0(k) = \frac{1}{1 + e^{\left(E_C(\vec{r}) + E(\vec{k}) - E_F\right)/k_B T}}$$

$$\frac{\partial n(\vec{r})}{\partial V} = q \frac{\partial n(\vec{r})}{\partial E_F}$$

$$\frac{1}{L_D^2} \equiv \frac{q^2}{\kappa_S \varepsilon_0} \frac{\partial n(\vec{r})}{\partial E_F}$$

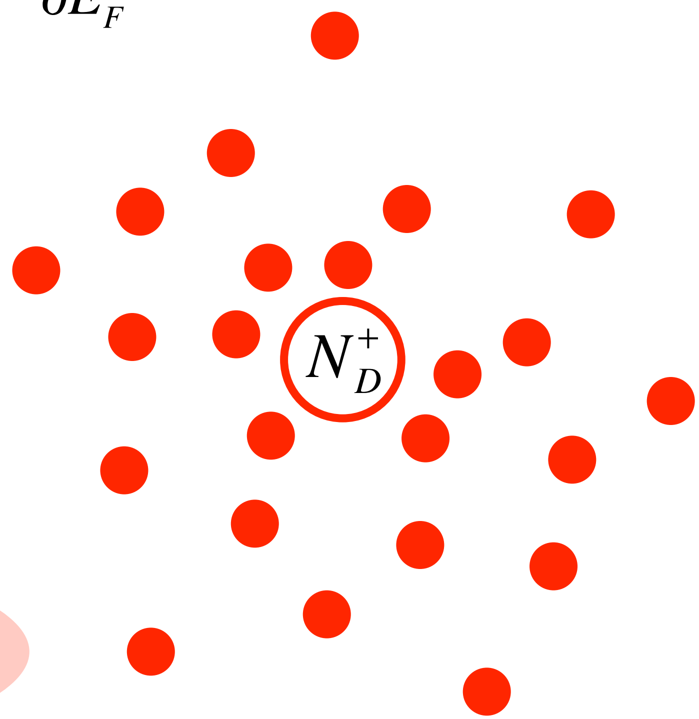
Screening in 3D

$$\nabla^2 \delta V(\vec{r}) = \frac{1}{L_D^2}\delta V(\vec{r}) \qquad \frac{1}{L_D^2} \equiv \frac{q^2}{\kappa_S \varepsilon_0}\frac{\partial n(\vec{r})}{\partial E_F}$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dV}{dr}\right) = \frac{1}{L_D^2}\delta V(\vec{r})$$

$$\delta V(r) = C\frac{e^{-r/L_D}}{r}$$

$$U_S(r) = q\delta V(r) = \frac{q^2}{4\pi\kappa_S\varepsilon_0 r}e^{-r/L_D}$$



Debye length in 3D

$$U_S(r) = \frac{q^2}{4\pi\kappa_S\varepsilon_0 r} e^{-r/L_D}$$

$$\frac{1}{L_D^2} \equiv \frac{q^2}{\kappa_S\varepsilon_0}\frac{\partial n(\vec{r})}{\partial E_F} = \frac{q^2}{\kappa_S\varepsilon_0 k_B T}\frac{\partial n(\vec{r})}{\partial \eta_F}$$

$$L_D = \sqrt{\frac{\kappa_S\varepsilon_0 k_B T}{q^2 n_0}}$$

Debye length

(non-degenerate)

$$n_{3D} = N_{3D}\mathcal{F}_{1/2}(\eta_F)$$

$$\frac{\partial n_{3D}}{\partial \eta_F} = N_{3D}\mathcal{F}_{-1/2}(\eta_F) = n_{3D}\frac{\mathcal{F}_{-1/2}(\eta_F)}{\mathcal{F}_{1/2}(\eta_F)} = n_{3D}$$ (non-degenerate)

Comments

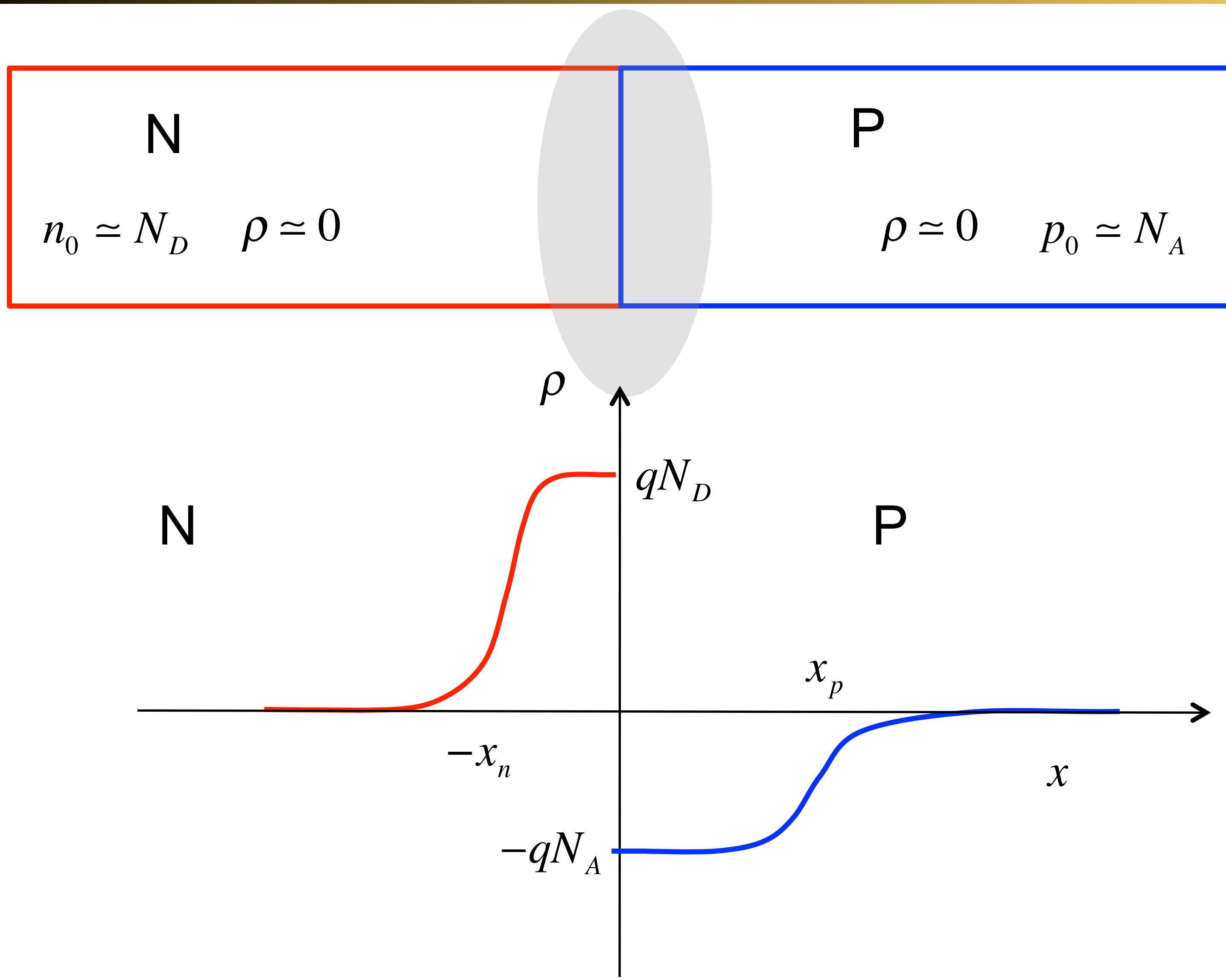
1) Our semi-classical approach assumes that the potential is slowly varying on the scale of the electron's wavelength. For rapidly varying potentials, a more sophisticated approach is needed. (See Ashcroft and Mermin, pp. 340-343 for a discussion of the Lindhard theory.)

2) Our semi-classical approach also assumes that the potential is slowly in time. (See Ashcroft and Mermin, p. 344 for a brief discussion.)

3) For potentials that vary rapidly in space and time, a "dynamic screening" treatment is needed. (See chapter 9 in Ridley, *Quantum Processes in Semiconductors*, 4th Ed. and Chapter 10 in Ridley, *Electrons and Phonons in Semiconductor Multilayers*.)

4) Screening is generally less effective in 2D and in 1D. (See J.H. Davies, *The Physics of Low-Dimensional Structures*, pp. 350-356

Example: Rapid variation in space



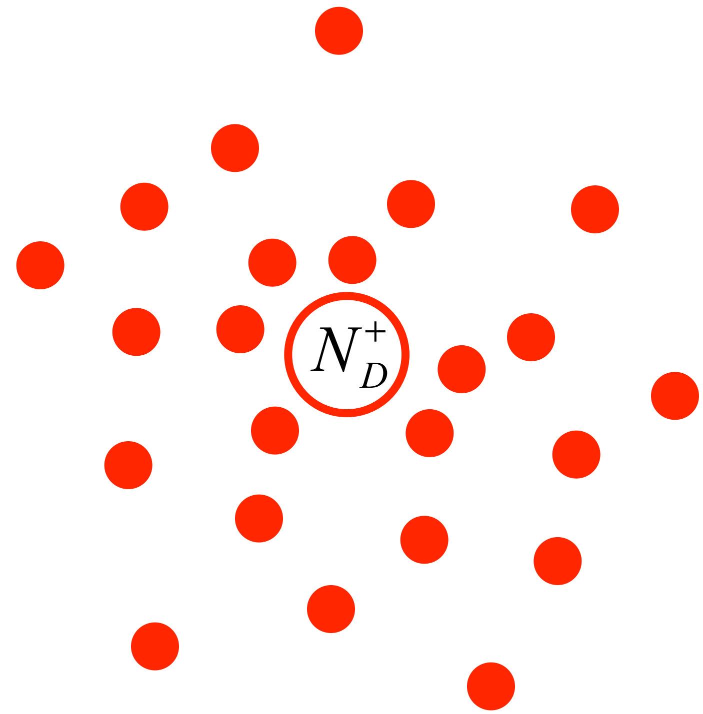
Screened Coulomb potential

$$U_S(r) = -\frac{q^2}{4\pi\kappa_S\varepsilon_0 r} e^{-r/L_D}$$

$$L_D = \sqrt{\frac{\kappa_S\varepsilon_0 k_B T_L}{q^2 n_0}}$$

Debye length

(non-degenerate)



Screened Coulomb potential: $U_S(r) = -\frac{q^2}{4\pi\kappa_S\varepsilon_0 r} e^{-r/L_D}$

Outline

1) Introduction

2) Screening

3) Transition rate

4) Characteristic times

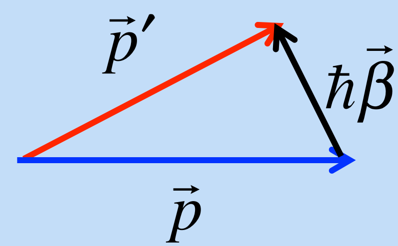
Transition rate

$$S(\vec{p},\vec{p}\,') = \frac{2\pi}{\hbar}\left|H_{p',p}\right|^2 \delta(E'-E)$$

$$H_{p',p} = \frac{1}{\Omega}\int_{-\infty}^{+\infty} e^{-i\vec{p}\,'\bullet r/\hbar}\,U_S(r)\,e^{i\vec{p}\bullet r/\hbar}dr = \frac{1}{\Omega}\int_{-\infty}^{+\infty} U_S(r)\,e^{-i(\vec{p}\,'-\vec{p})\bullet\vec{r}/\hbar}\,dr$$

$$= \frac{1}{\Omega}\int_{-\infty}^{+\infty} U_S(r)\,e^{-i\vec{\beta}\bullet\vec{r}}\,d\vec{r} \equiv \frac{1}{\Omega}\tilde{U}_S\left(\vec{\beta}\right)$$

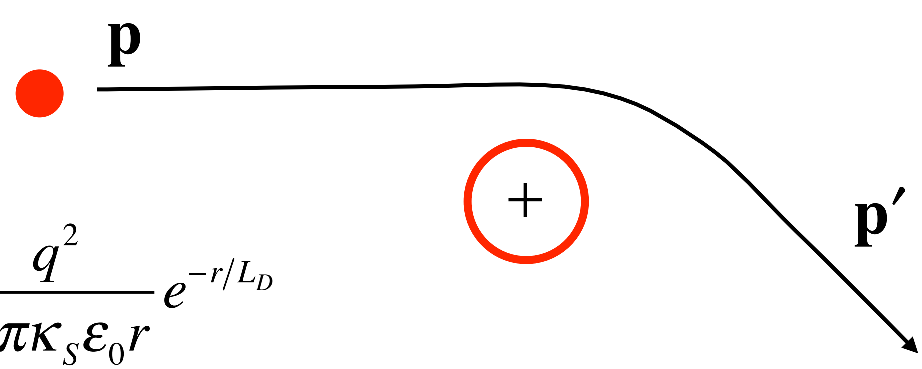
$$\vec{p}\,' = \vec{p} + \hbar\vec{\beta}$$



$$S(\vec{p},\vec{p}\,') = \frac{2\pi}{\hbar}\frac{1}{\Omega^2}\left|\tilde{U}_S(\beta)\right|^2 \delta(E'-E)$$

Transition rate is proportional to the square of the Fourier transform of the screened Coulomb potential

Recap

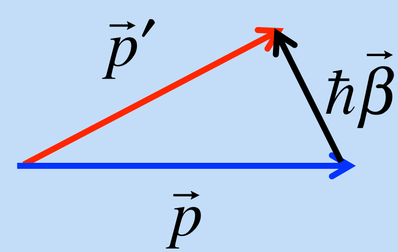


$$U_S(r) = \frac{q^2}{4\pi\kappa_S\varepsilon_0 r} e^{-r/L_D}$$

$$L_D = \sqrt{\frac{\kappa_S\varepsilon_0 k_B T}{q^2 n_0}}$$ Debye length

$$\tilde{U}_S(\beta) = \int_{-\infty}^{+\infty} U_S(r) e^{-i\vec{\beta}\cdot\vec{r}}\, d\vec{r}$$

$$\vec{p}\,' = \vec{p} + \hbar\vec{\beta}$$



Fourier transform

$$\tilde{U}_S(\beta) = \int_{-\infty}^{+\infty} \frac{q^2}{4\pi\kappa_S\varepsilon_0 r} e^{-r/L_D} e^{-i\vec{\beta}\bullet\vec{r}}\, d\vec{r}$$

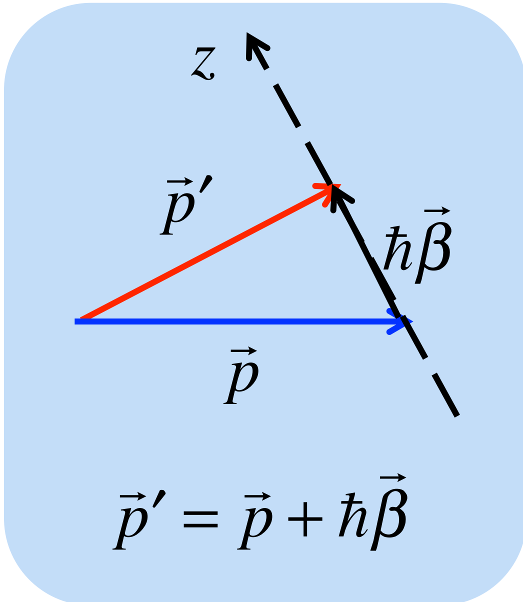
$$\tilde{U}_S(\beta) = \frac{q^2}{4\pi\kappa_S\varepsilon_0} \int_0^{2\pi} d\phi \int_0^{\pi} \int_0^{\infty} e^{-r/L_D} e^{-i\vec{\beta}\bullet\vec{r}}\, r \sin\theta d\theta dr$$

$$\vec{\beta}\bullet\vec{r} = \beta r \cos\theta$$

$$\sin\theta d\theta = -d(\cos\theta)$$

choose z-axis along *β:*

$$\tilde{U}_S(\beta) = \frac{q^2}{2\kappa_S\varepsilon_0} \int_0^{\infty} e^{-r/L_D} r\, dr \underbrace{\int_{-1}^{+1} e^{-i\beta r\cos\theta}\, d(\cos\theta)}_{\frac{2\sin(\beta r)}{\beta r}}$$



Fourier transform (ii)

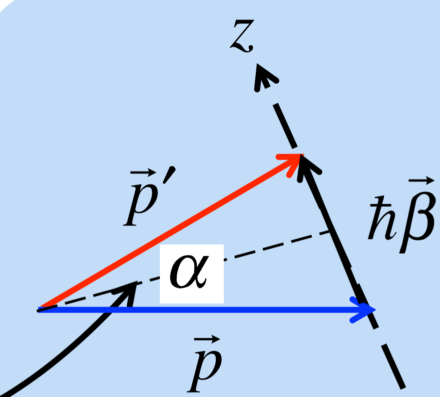
$$\tilde{U}_S(\beta) = \frac{q^2}{\kappa_S \varepsilon_0} \int_0^\infty \frac{e^{-r/L_D} \sin(\beta r)}{\beta} dr$$

$$\vec{\beta} \bullet \vec{r} = \beta r \cos\theta$$

$$\sin\theta d\theta = -d(\cos\theta)$$

$$\tilde{U}_S(\beta) = \frac{q^2}{\kappa_S \varepsilon_0 \Omega} \left(\frac{1}{\beta^2 + 1/L_D^2} \right)$$

$$\tilde{U}_S(\beta) = \frac{q^2}{\kappa_S \varepsilon_0 \Omega} \left(\frac{1}{4(p/\hbar)^2 \sin^2(\alpha/2) + 1/L_D^2} \right)$$

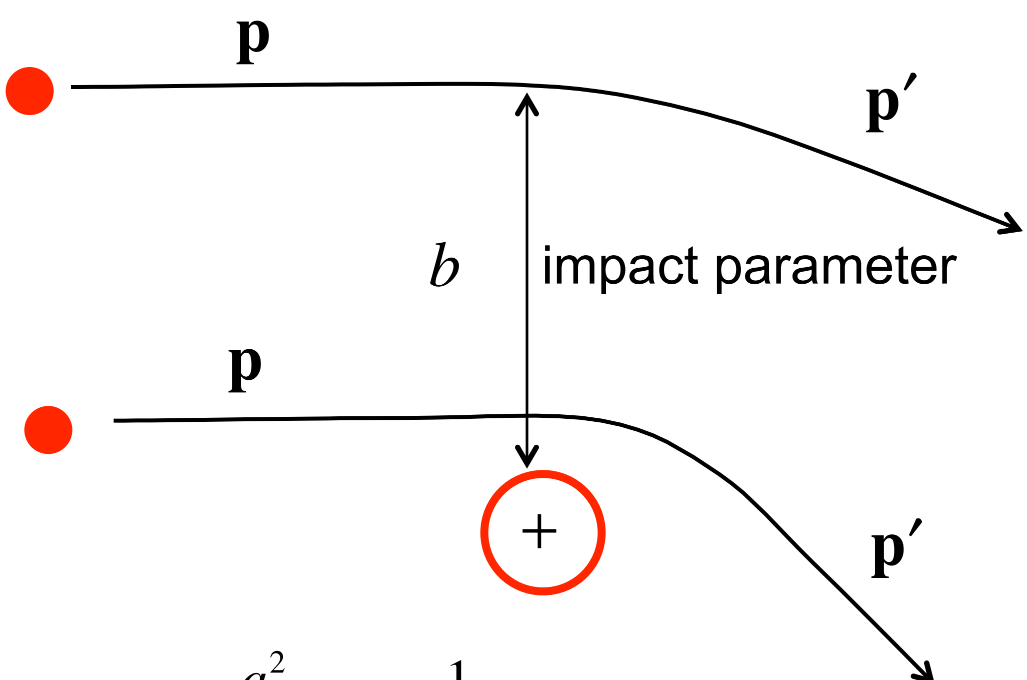


$$\hbar\beta/2 = p\sin(\alpha/2)$$

$$\vec{p}\,' = \vec{p} + \hbar\vec{\beta}$$

small angle scattering preferred!!

Relation to impact parameter



A small angle scattering event implies a large impact parameter.

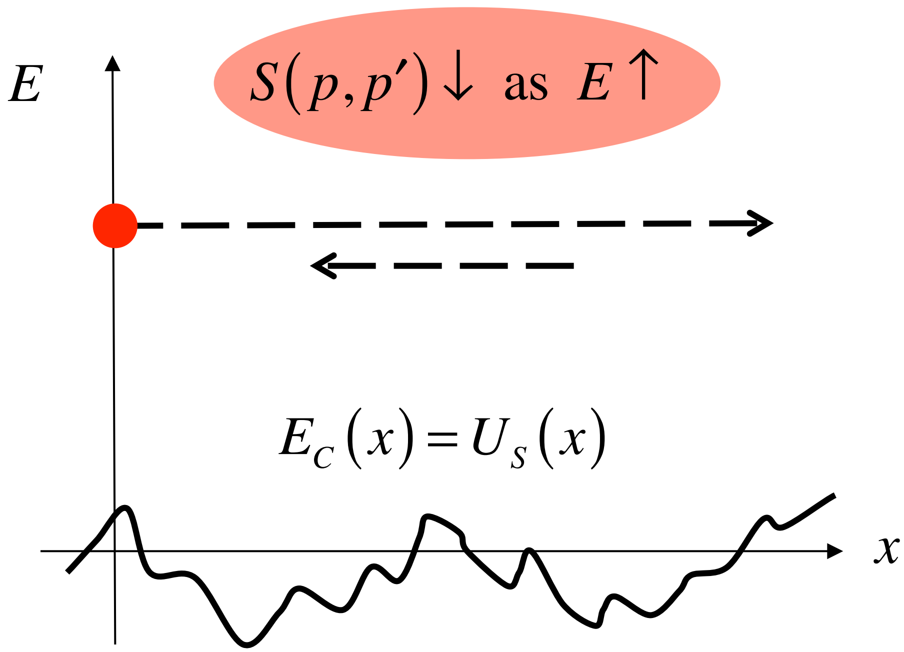
$$\tilde{U}_S(\beta) = \frac{q^2}{\kappa_S \varepsilon_0} \frac{1}{\left(\beta^2 + 1/L_D^2\right)}$$

$$U_S(r) = \frac{q^2}{4\pi\kappa_S\varepsilon_0 r} e^{-r/L_D}$$

II scattering of high energy electrons

$$\tilde{U}_S(\beta) = \frac{q^2}{\kappa_S \varepsilon_0}\left(\frac{1}{4(p/\hbar)^2 \sin^2(\alpha/2) + 1/L_D^2}\right)$$

For a given deflection angle, higher energies scatter less.



Random charges introduce random fluctuations in E_C, which act a scattering centers.

High energy electrons don't "see" these fluctuations and are not scattered as strongly.

Recap

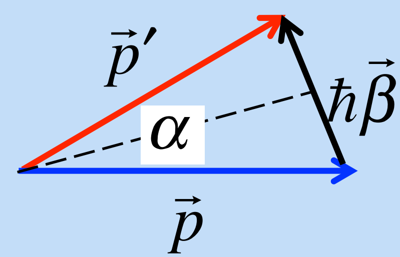
$$S(\vec{p},\vec{p}\,') = \frac{2\pi}{\hbar}\left|H_{p',p}\right|^2 \delta(E'-E) \quad H_{p,p'} = \frac{1}{\Omega}\tilde{U}_S(\beta) \quad \tilde{U}_S(\beta) = \frac{q^2}{\kappa_S \varepsilon_0}\frac{1}{\left(\beta^2 + 1/L_D^2\right)}$$

Need to multiple by the total number of ionized impurities in the volume, Ω.

$$S(\vec{p},\vec{p}\,') = \frac{2\pi q^4 N_I}{\hbar \kappa_S^2 \varepsilon_0^2 \Omega}\frac{\delta(E'-E)}{\left(\beta^2 + 1/L_D^2\right)^2}$$

$$S(\vec{p},\vec{p}\,') = \frac{2\pi q^4 N_I}{\hbar \kappa_S^2 \varepsilon_S^2 \Omega}\frac{\delta(E'-E)}{\left(\dfrac{4p^2}{\hbar^2}\sin^2\alpha/2 + 1/L_D^2\right)^2}$$

$$\vec{p}\,' = \vec{p} + \hbar\vec{\beta}$$



$$\frac{\hbar\beta}{2} = p\sin\alpha/2$$

Examine result

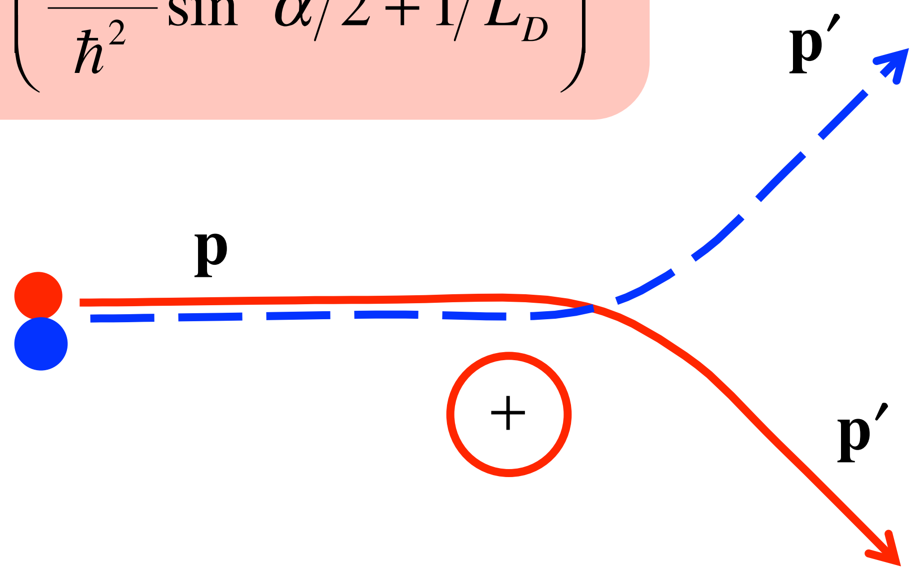
$$S(\vec{p},\vec{p}') = \frac{2\pi q^4 N_I}{\hbar \kappa_S^2 \varepsilon_S^2 \Omega} \frac{\delta(E'-E)}{\left(\frac{4p^2}{\hbar^2}\sin^2\alpha/2 + 1/L_D^2\right)^2}$$

1) $S(\vec{p},\vec{p}') \sim N_I$

2) $S(\vec{p},\vec{p}') \sim q^4$

3) $S(\vec{p},\vec{p}') \sim 1/E^2$

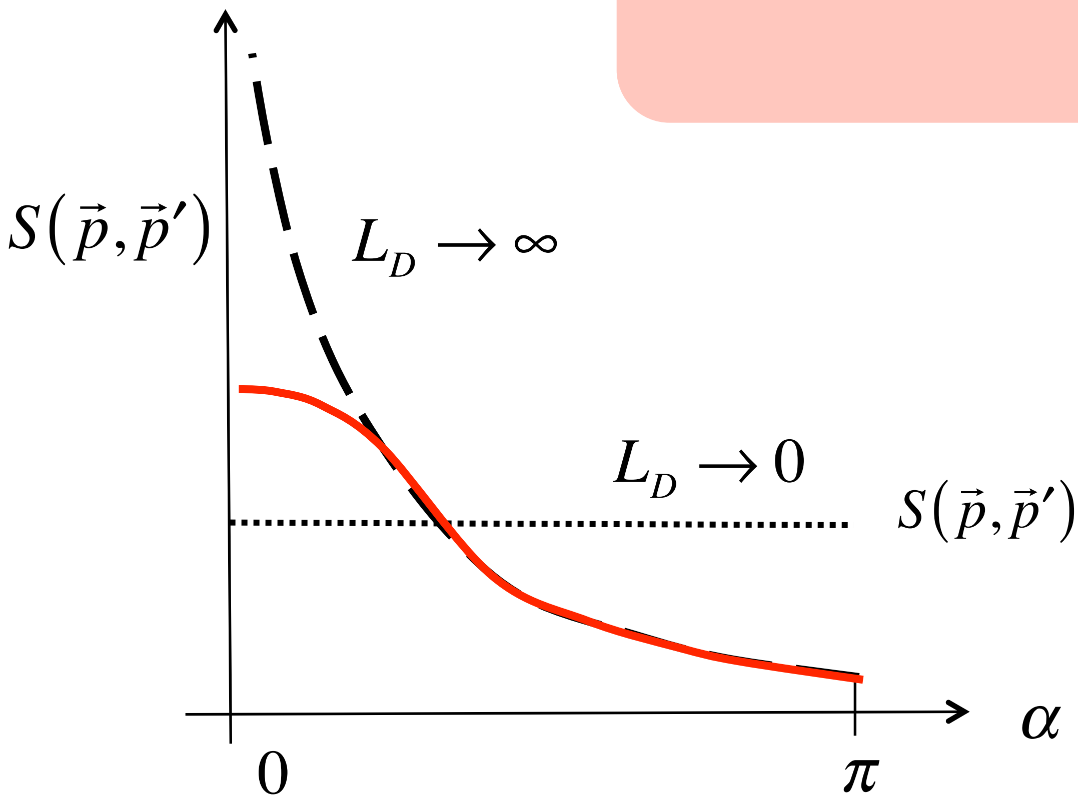
4) Favors small angle scattering



Examine result

4) angular dependence

$$S(\vec{p},\vec{p}\,') = \frac{2\pi q^4 N_I}{\hbar \kappa_S^2 \varepsilon_S^2 \Omega} \frac{\delta(E'-E)}{\left(\frac{4p^2}{\hbar^2}\sin^2\alpha/2 + 1/L_D^2\right)^2}$$



$$S(\vec{p},\vec{p}\,') \to \frac{2\pi q^4 N_I L_D^4}{\hbar \kappa_S^2 \varepsilon_0^2 \Omega}\delta(E'-E)$$

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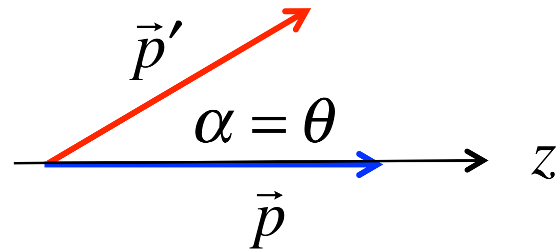
Characteristic times

$$\frac{1}{\tau_m} = \sum_{\vec{p}'} S(\vec{p},\vec{p}')\left(1 - \frac{p'}{p}\cos\alpha\right)$$



$$\frac{1}{\tau_m} = \sum_{\vec{p}'} S(\vec{p},\vec{p}')(1 - \cos\alpha)$$

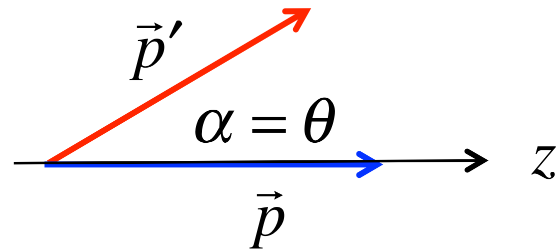
$S(\vec{p},\vec{p}')$ favors small angles

expect: $1/\tau_m < 1/\tau \quad \tau_m > \tau$

$1/\tau_E = 0 \quad \tau_E > \tau$

Momentum relaxation time

$$\frac{1}{\tau_m} = \sum_{\vec{p}'} S(\vec{p},\vec{p}')(1-\cos\alpha)$$



$$\tau_m(E) = \frac{16\sqrt{2m^*}\pi\kappa_S^2\varepsilon_0^2}{N_I q^4}\left[\ln\left(1+\gamma^2\right) - \frac{\gamma^2}{1+\gamma^2}\right]E^{3/2}$$

$$\gamma^2 = 8m^* E L_D^2 / \hbar^2$$

See Lundstrom (FCT) pp. 69-70

$$\tau_m(E) \sim E^{3/2}$$

$$\tau_m(E) \approx \tau_0\left(E/k_B T\right)^s \qquad \tau_0 \sim T^{3/2} \qquad s = 3/2$$

Questions?

1) Introduction

2) Screening

3) Transition rate

4) Characteristic times