ECE-656 Key Equations (Weeks 1-4)

Physical constants:

$\hbar = 1.055 \times 10^{-34}$ [J-s] $m_0 = 9.109 \times 10^{-31}$ [kg]

$k_B = 1.380 \times 10^{-23}$ [J/K] $q = 1.602 \times 10^{-19}$ [C] $\varepsilon_0 = 8.854 \times 10^{-14}$ [F/cm]

Density of states in k-space:

1D: $N_k = 2 \times (L/2\pi) = L/\pi$ 2D: $N_k = 2 \times (A/4\pi^2) = A/2\pi^2$ 3D: $N_k = 2 \times (\Omega/8\pi^2) = \Omega/4\pi^3$

Density of states in energy (parabolic bands, per length, area, or volume):

$$D_{1D}(E) = \frac{g_v}{\pi\hbar}\sqrt{\frac{2m^*}{(E-\varepsilon_1)}} \qquad D_{2D}(E) = g_V \frac{m^*}{\pi\hbar^2} \qquad D_{3D}(E) = g_v \frac{m^*\sqrt{2m^*(E-E_C)}}{\pi^2\hbar^3}$$

Fermi function and Fermi-Dirac Integrals:

$$f_0(E) = \frac{1}{1+e^{(E-E_F)/k_BT}}$$

$$\mathcal{F}_j(\eta_F) = \frac{1}{\Gamma(j+1)}\int_0^\infty \frac{\eta^j d\eta}{1+e^{\eta-\eta_F}} \qquad \mathcal{F}_j(\eta_F) \to e^{\eta} \quad \eta_F << 0 \qquad \frac{d\mathcal{F}_j}{d\eta_F} = \mathcal{F}_{j-1}$$

$\Gamma(n) = (n-1)!$ (n an integer) $\Gamma(1/2) = \sqrt{\pi}$ $\Gamma(p+1) = p\Gamma(p)$

Scattering:

$$S(\vec{p},\vec{p}') = \frac{2\pi}{\hbar}\left|H_{\vec{p}',\vec{p}}\right|^2 \delta(E'-E-\Delta E) \qquad H_{\vec{p}',\vec{p}} = \frac{1}{\Omega}\int_{-\infty}^{+\infty} e^{-i\vec{p}'\bullet r/\hbar} U_S(\vec{r}) e^{i\vec{p}\bullet\vec{r}/\hbar} d\vec{r}$$

$$\frac{1}{\tau(\vec{p})} == \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}') \qquad \frac{1}{\tau_m(\vec{p})} = \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}')\frac{\Delta p_z}{p_{z0}} \qquad \frac{1}{\tau_E(\vec{p})} = \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}')\frac{\Delta E}{E_0}$$

ADP: $\left|K_q\right|^2 = q^2 D_A^2$ ODP: $\left|K_q\right|^2 = D_0^2$ PZ: $\left|K_q\right|^2 = (ee_{PZ}/\kappa_S\varepsilon_0)^2$ POP: $\left|K_\beta\right|^2 = \frac{\rho e^2\omega_0^2}{q^2\kappa_0\varepsilon_0}\left(\frac{\kappa_0}{\kappa_\infty} - 1\right)$

$$S(\vec{p},\vec{p}') = \frac{\pi}{\Omega\rho\,\omega}\left|K_q\right|^2\left(N_\omega + \frac{1}{2} \mp \frac{1}{2}\right)\delta(\vec{p}'-\vec{p}\mp\hbar\vec{q})\delta(E'-E\mp\hbar\omega)$$

$$\delta(\vec{p}'-\vec{p}\mp\hbar\vec{q})\delta(E'-E\mp\hbar\omega_\beta) \to \frac{1}{\hbar\upsilon q}\delta\left(\pm\cos\theta + \frac{\hbar q}{2p} \mp \frac{\omega_q}{\upsilon q}\right)$$

$$\frac{1}{\tau} = \frac{1}{\tau_{abs}} + \frac{1}{\tau_{ems}} = \frac{2\pi}{\hbar}\left(\frac{D_A^2 k_B T}{c_l}\right)\frac{D_{3D}(E)}{2} \text{ (ADP)} \qquad L_D = \sqrt{\frac{\kappa_S\varepsilon_0 k_B T}{q^2 n_0}}$$

$$\frac{1}{\tau} = \frac{1}{\tau_m} = \frac{2\pi}{\hbar}\left(\frac{\hbar D_O^2}{2\rho\omega_0}\right)\left(N_0 + \frac{1}{2} \mp \frac{1}{2}\right)\frac{D_{3D}(E\pm\hbar\omega_0)}{2} \qquad N_0 = \frac{1}{e^{\hbar\omega_o/k_BT} - 1} \text{ (ODP)}$$