ECE 656: Electronic Transport in Semiconductors Fall 2017

Quantum Confinement

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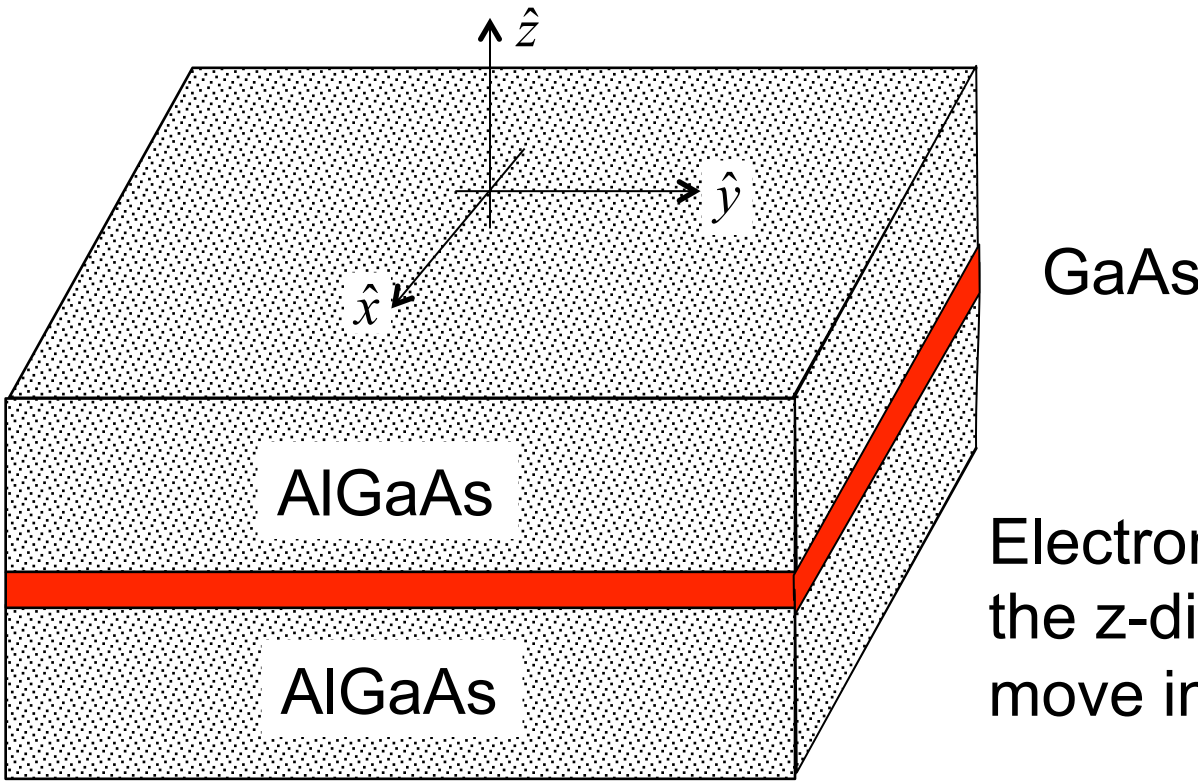
Outline

1) Introduction
2) Particle in a box
3) 3D wave equation
4) 2D electrons
5) 1D electrons
6) Summary

Quantum confinement with heterostrucrures



Electrons are confined in the z-direction, but free to move in the x-y plane.

"GaAs quantum well"

"Quasi-2D electrons"

Review: 1D particle in a box

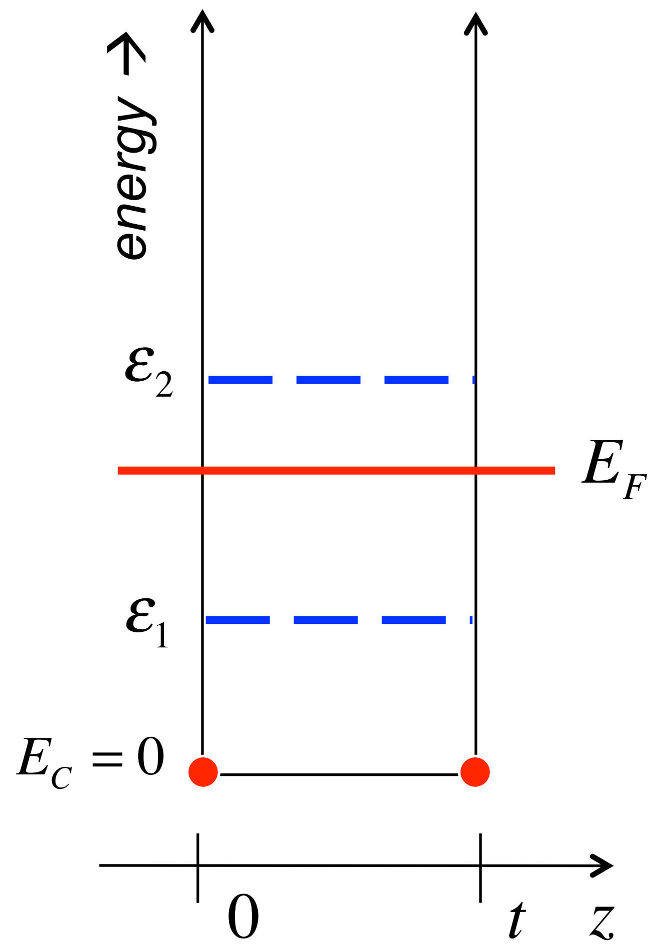
$$\frac{d^2\psi(z)}{dz^2} + k^2\psi = 0 \qquad k^2 = \frac{2m^*\left(E - E_C(z)\right)}{\hbar^2}$$

$$\psi(z) = \sin k_j z$$

$$\psi(0) = \psi(t) = 0 \qquad k_j = \frac{\pi}{t} j = 1,2,3\ldots$$

$$\varepsilon_j = \frac{\hbar^2 k_j^2}{2m^*} = \frac{\hbar^2 j^2 \pi^2}{2m^* t^2}$$

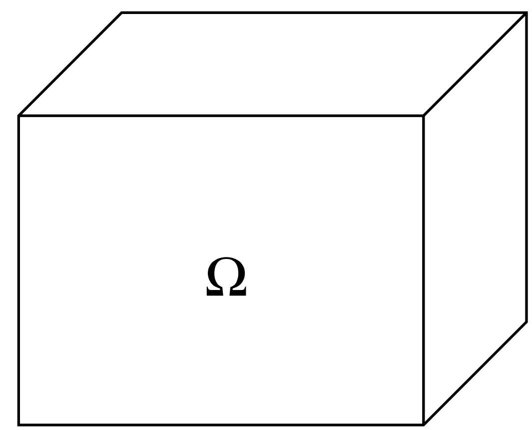
Light mass or narrow well → high subband energy, but specifics depend on the shape of the well.



Note: If electrons are free to move in the *x*-*z* plane. **2D electrons.**

3D wave equation

$$-\frac{\hbar^2}{2m^*}\nabla^2\psi(\vec{r})+U(\vec{r})\psi(\vec{r})=E\psi(\vec{r})$$



Separation of variables:

$$\psi(\vec{r})=X(x)Y(y)Z(z)$$

$$U(\vec{r})=0$$

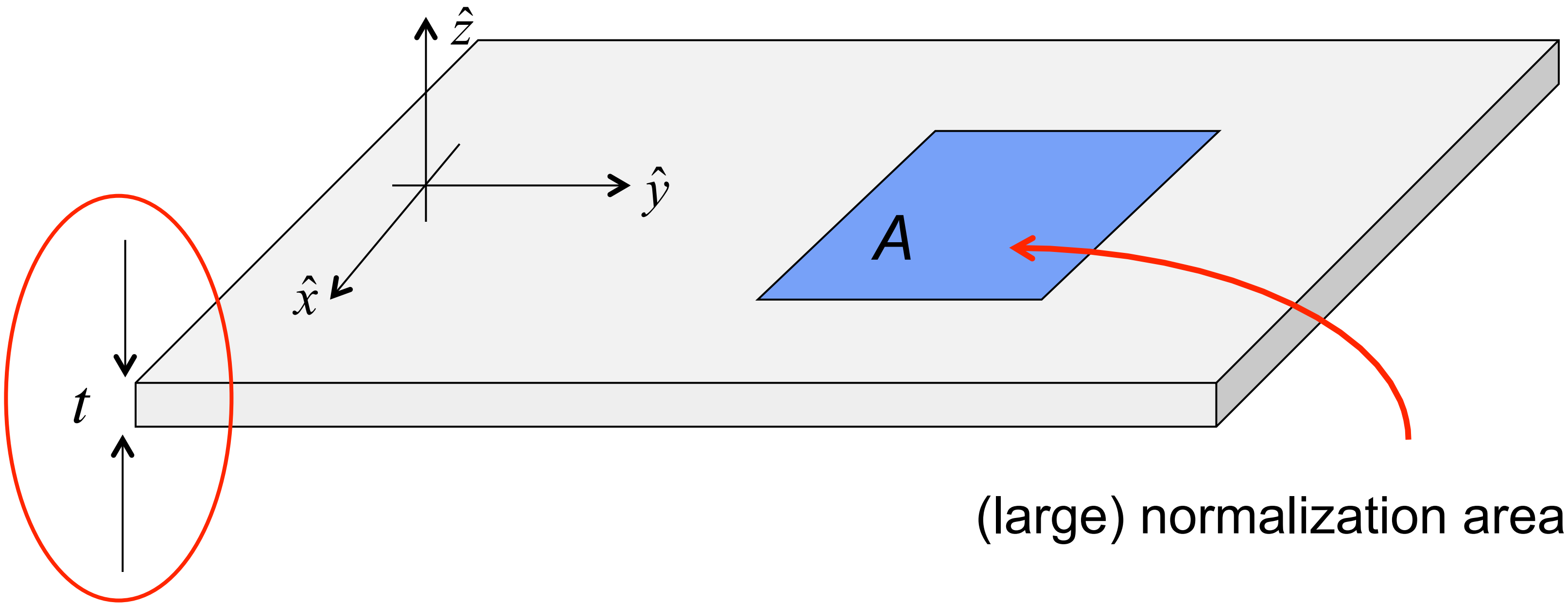
$$\psi(\vec{r})=A\,e^{ik_x x}\times e^{ik_y y}\times e^{ik_z z}$$

$$\int_{\Omega}\psi^*(\vec{r})\psi^*(\vec{r})\,d^3r=1$$

$$E=\frac{\hbar^2k_x^2}{2m^*}+\frac{\hbar^2k_y^2}{2m^*}+\frac{\hbar^2k_z^2}{2m^*}$$

$$\psi(\vec{r})=\frac{1}{\sqrt{\Omega}}e^{ik_x x}\times e^{ik_y y}\times e^{ik_z z}$$

Quasi-2D electrons



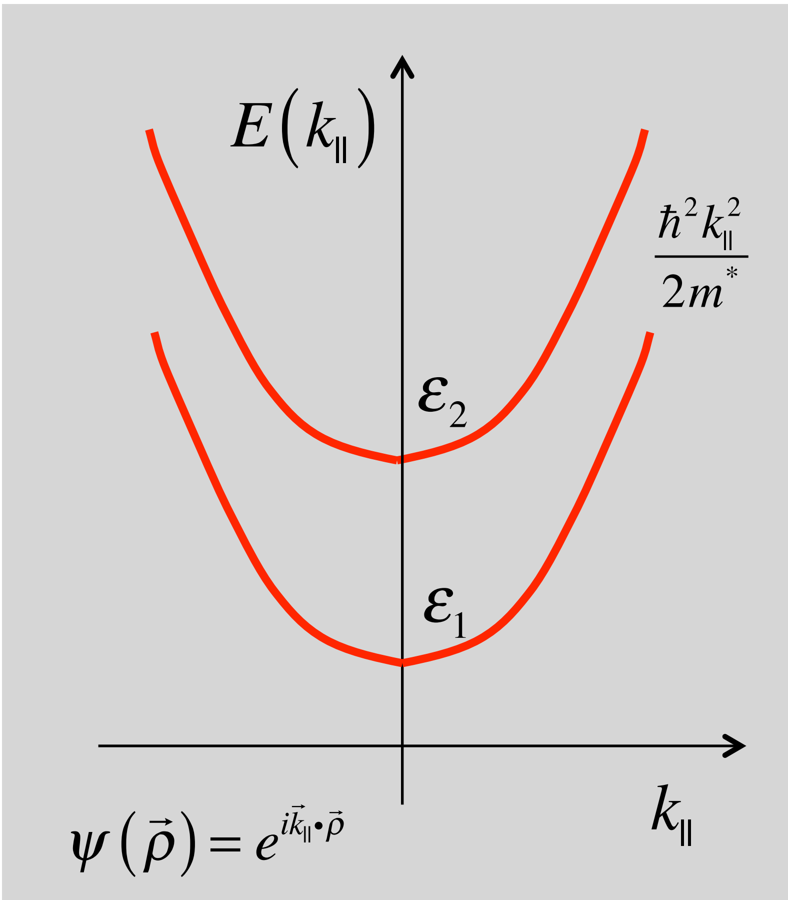
Semi-infinite in the x-y plane, but very thin in the z-direction.

$$\psi(\vec{r}) \sim e^{i\vec{k}\cdot\vec{r}} \rightarrow \sin(k_z z)\times e^{ik_x x}\times e^{ik_y y}$$

$$k_z = k_{zj} = \frac{\pi}{t} j = 1,2,3\ldots$$

$$\psi(\vec{r}) = \sqrt{\frac{2}{t}}\sin(k_z z)\times\frac{1}{\sqrt{A}}e^{i(k_x x + k_y y)}$$

"Subbands"



$$\varepsilon_j = \frac{\hbar^2 j^2 \pi^2}{2m^* t^2}$$

$$k_{\|} = \sqrt{k_x^2 + k_y^2}$$

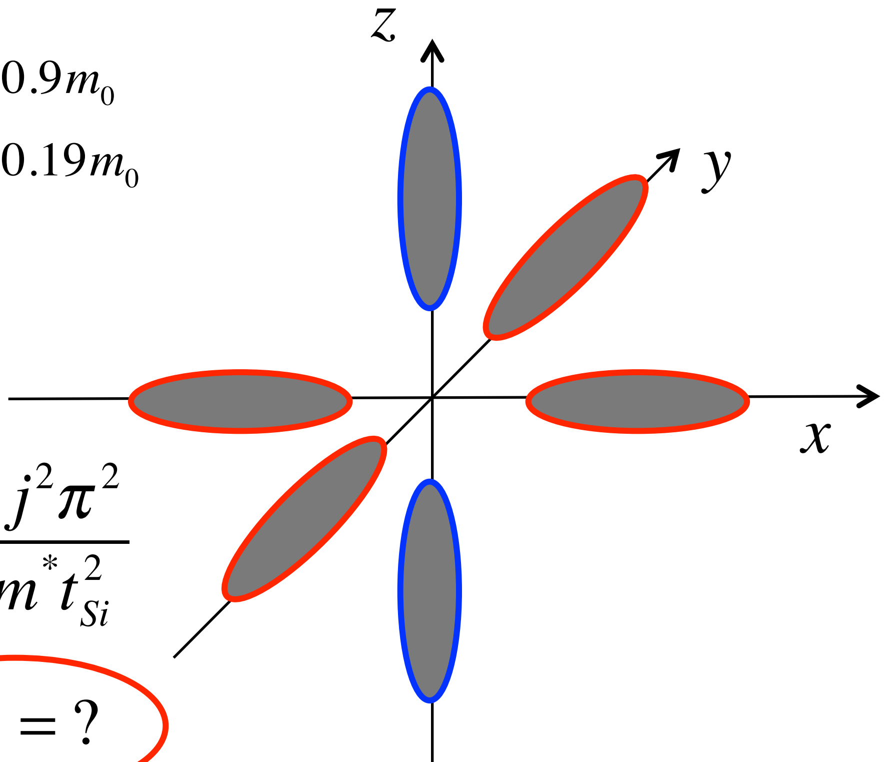
$$E = \varepsilon_j + \frac{\hbar^2 k_{\|}^2}{2m^*}$$

ex. Quantum confinement in Si cond. band.

Si conduction band

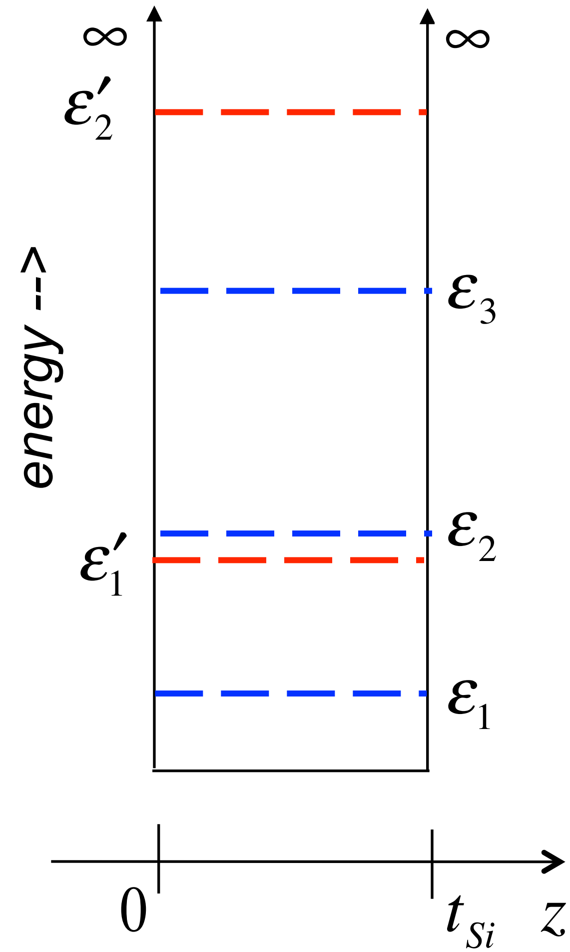
$m_\ell^* = 0.9 m_0$

$m_t^* = 0.19 m_0$



$$\varepsilon_j = \frac{\hbar^2 j^2 \pi^2}{2 m^* t_{Si}^2}$$

$$m^* = ?$$

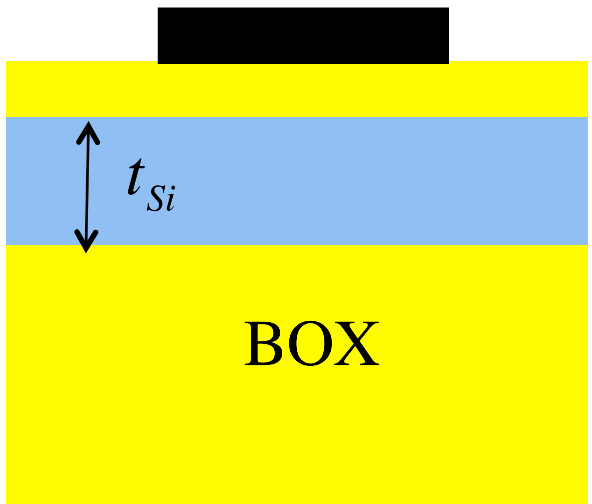


unprimed ladder: $m^* = m_\ell^* \quad g_V = 2$

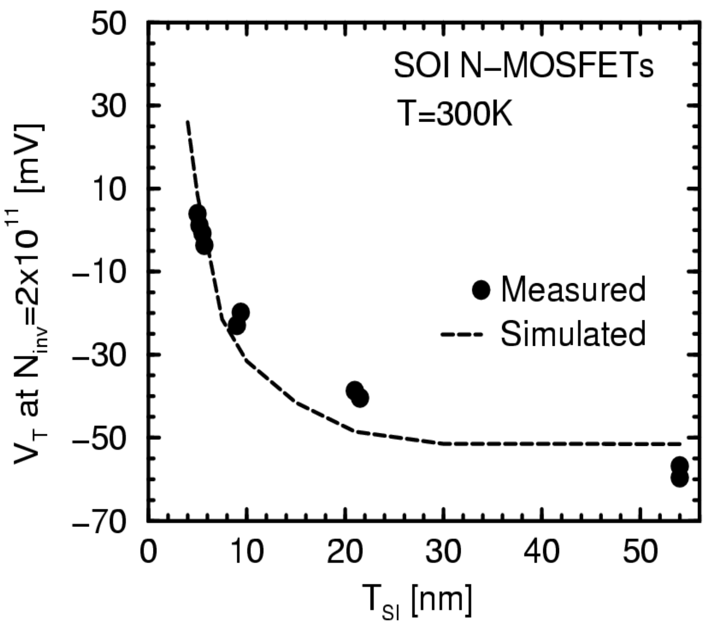
primed ladder: $m^* = m_t^* \quad g_V = 4$

Quantum confinement in SOI

SOI

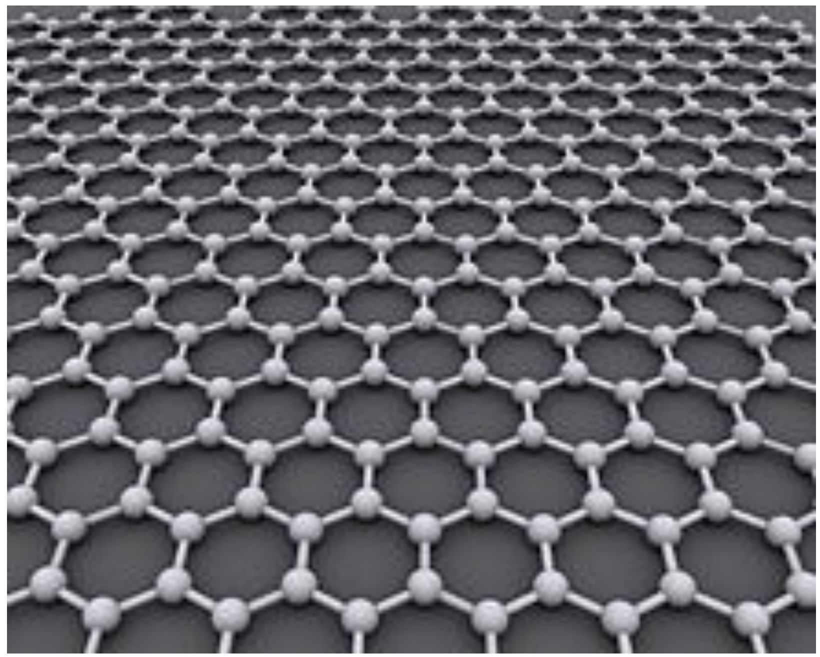


$$\Delta\psi_S^{QM} = \frac{\varepsilon_1}{q}$$



(D. Esseni et al. IEDM 2000 and TED 2001)

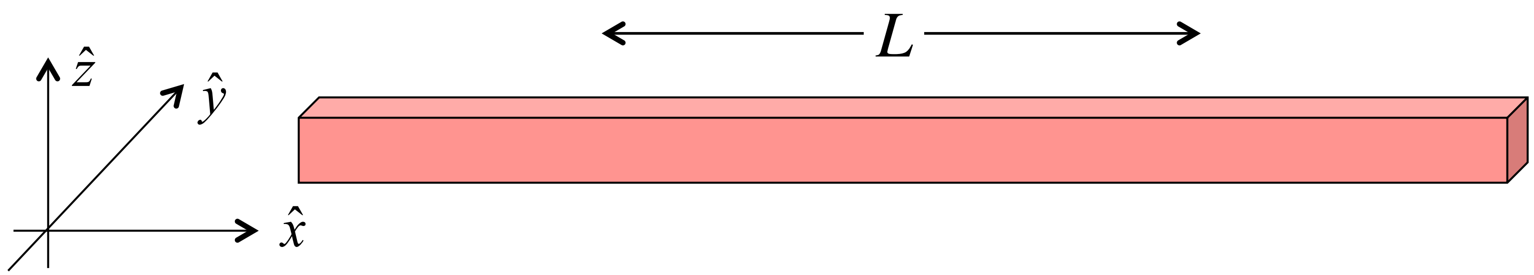
Graphene and 2D materials



A 2D hexagonal lattice of carbon atoms

https://en.wikipedia.org/wiki/Graphene

Quantum confinement in 1D



Semi-infinite in the x direction, but very thin in the y and z-directions.

$$\psi(\vec{r}) \sim e^{i\vec{k}\bullet\vec{r}} \rightarrow \sin(k_y y)\sin(k_z z)\times e^{ik_x x}$$

$$\psi(\vec{r}) = \sqrt{\frac{2}{t_y}}\sin(k_y y)\sqrt{\frac{2}{t_z}}\sin(k_z z)\times\frac{1}{\sqrt{L}}e^{ik_x x}$$

$$\varepsilon_{m,n} = \frac{\hbar^2 m^2 \pi^2}{2m^* t_y^2} + \frac{\hbar^2 n^2 \pi^2}{2m^* t_z^2}$$

$$E = \varepsilon_{m,n} + \frac{\hbar^2 k_x^2}{2m^*}$$

Summary

1) Quantum confinement introduces subbands.

2) The number of subbands that are occupied depends on the location of the Fermi level.

3) If one subband is occupied, electrons are “quasi-1D (2D)” electrons.

4) If many subbands are occupied, then electrons are 3D-like.