Characteristic Times
for Scattering

Mark Lundstrom

Electrical and Computer Engineering
Purdue University
West Lafayette, IN USA

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Mobility

\[ \mu = \frac{q \tau}{m^*} \]

Scattering time, but what scattering time?
Transition rate

\[ S(\vec{p}, \vec{p}') = S(\vec{p} \rightarrow \vec{p}') \]

Probability per sec that an electron is scattered from a initial state to one specific final state. Computed by “Fermi’s Golden Rule”
Elastic vs. inelastic scattering

\[ S(\vec{p}, \vec{p}') = S(\vec{p} \rightarrow \vec{p}') \]

\[ \vec{p} = \hbar \vec{k} \]

\[ \vec{p}' = \hbar \vec{k}' \]

\[ E(\vec{p}) \]

\[ E(\vec{p}') = E(\vec{p}) \quad \text{elastic scattering} \]

\[ E(\vec{p}') = E(\vec{p}) + \Delta E \quad \text{inelastic scattering} \]

isotropic vs. anisotropic scattering
Characteristic times

\[ \frac{1}{\tau(\vec{p})} = \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') \]

(\( \tau \), single particle lifetime)

\[ \frac{1}{\tau_m(\vec{p})} = \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') \frac{\Delta p_z}{p_z} \]

\[ \frac{1}{\tau_E(\vec{p})} = \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') \frac{\Delta E}{E} \]
Scattering rate

\[
\frac{1}{\tau(\vec{p})} = \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}')
\]

(\(\tau\), single particle lifetime)

\[
\frac{1}{\tau(E)} \propto D_f(E + \Delta E)
\]

Scattering rate is often proportional to the density of final states.
Phonon scattering in Si

Phonon scattering in GaAs

Scattering rate and momentum relaxation rate

\[
\frac{1}{\tau_m(\vec{p})} = \sum_{\vec{p}',\uparrow} S(\vec{p}, \vec{p}') \frac{\Delta p_z}{p_z}
\]

\[
\vec{p} = p\hat{z}
\]

\[
\Delta p_z = \frac{p - p' \cos \theta}{p} = \left(1 - \frac{p'}{p} \cos \theta\right)
\]

\[
\frac{1}{\tau_m(\vec{p})} = \sum_{\vec{p}',\uparrow} S(\vec{p}, \vec{p}') - \sum_{\vec{p}',\uparrow} S(\vec{p}, \vec{p}') \frac{p'}{p} \cos \theta
\]

\[
\frac{1}{\tau_m(\vec{p})} = \frac{1}{\tau(\vec{p})} - A
\]

\[t \approx \tau_m \geq \tau\]
Mobility

\[ \mu = \frac{q \tau_m}{m^*} \]

Momentum relaxation time
Isotropic scattering

\[ \frac{1}{\tau_m(\vec{p})} = \frac{1}{\tau(\vec{p})} - A \]

\[ A = \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') \frac{p'}{p} \cos \theta \]

\[ A = \frac{\Omega}{8\pi^3} \int_{0}^{\infty} S(\vec{p}, \vec{p}') \frac{p'}{p} p^2 dp \int_{0}^{\pi} \cos \theta \sin \theta d\theta \int_{0}^{2\pi} d\phi \]

\[ \frac{1}{\tau_m(\vec{p})} = \frac{1}{\tau(\vec{p})} \]

For isotropic scattering

Lundstrom ECE-656 F17
Scattering rate and energy relaxation rate

\[ \frac{1}{\tau_E(\vec{p})} = \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') \frac{\Delta E}{E} \]

i) Elastic scattering:

\[ \Delta E = 0 \]

\[ \frac{1}{\tau_E(\vec{p})} = 0 \]

\[ \tau_E(\vec{p}) = \infty \]
Energy relaxation by phonon emission

\[
\frac{1}{\tau_E(\vec{p})} = \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') \frac{\Delta E}{E}
\]

ii) Phonon emission:

\[
\Delta E = \hbar \omega
\]

\[
\frac{1}{\tau_E(\vec{p})} = \frac{\hbar \omega}{E} \frac{1}{\tau(\vec{p})}
\]
Typical phonons involved in acoustic phonon scattering.

\[ \hbar \omega \approx 0 \]  
(at 300 K)

Typical phonons involved in optical phonon scattering.

\[ \hbar \omega \approx k_B T \]  
(at 300 K)
Example

\[ \frac{1}{\tau_E(\bar{p})} = \frac{\hbar \omega_0}{E \tau(\bar{p})} \]

Silicon:

\[ \hbar \omega_0 = 0.060 \text{ eV} \]

\[ \tau_E(\bar{p}) \approx \frac{E_{\text{inj}}}{\hbar \omega_0} \tau(\bar{p}) \approx 4 \tau(\bar{p}) \]

\[ \bar{p}(t=0) \]

\[ t = 0 \]

\[ E_{\text{inj}} \approx 10k_B T \approx 0.26 \text{ eV} \]
Scattering mechanisms

1) Electron-phonon scattering
2) Electron-ionized impurity scattering
3) plus several more…
Summary

1) Characteristic times are derived from the transition rate, \( S(\rho,\rho') \)

2) \( S(\rho,\rho') \) is obtained from Fermi’s Golden Rule

3) The scattering rate is often proportional to the final DOS

4) Our goal is to understand the general features of scattering in common semiconductors.