ECE 656: Electronic Transport in Semiconductors Fall 2017

Characteristic Times for Scattering

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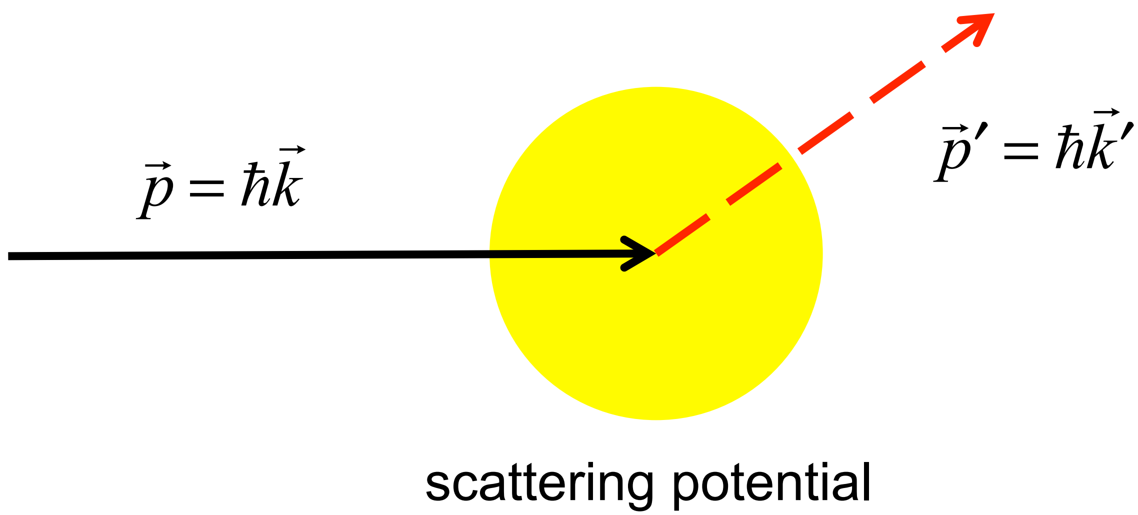
Mobility

$$\mu = \frac{q\tau}{m^*}$$

Scattering time, but what scattering time?

Transition rate

$$S\left(\vec{p},\vec{p}'\right) = S\left(\vec{p} \rightarrow \vec{p}'\right)$$

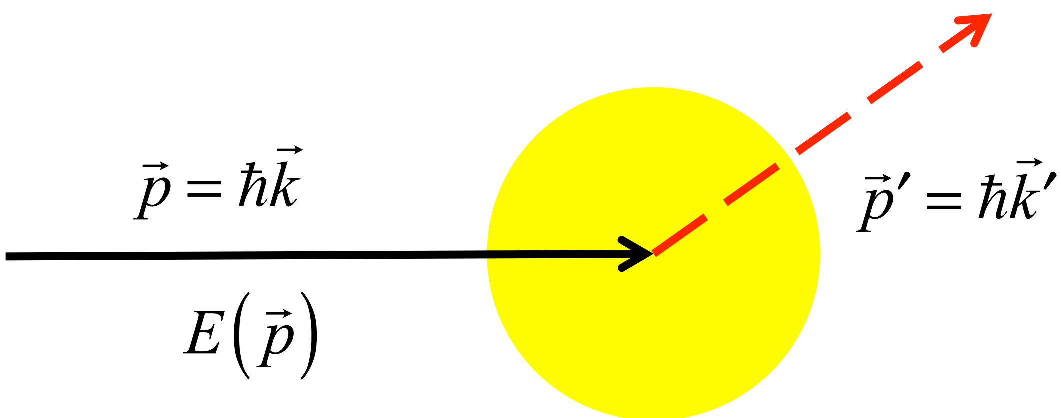


Probability per sec that an electron is scattered from a initial state to **one specific** final state.

Computed by “Fermi’s Golden Rule”

Elastic vs. inelastic scattering

$$S(\vec{p},\vec{p}\,') = S(\vec{p} \rightarrow \vec{p}\,')$$



$E(\vec{p}\,') = E(\vec{p})$ elastic scattering

$E(\vec{p}\,') = E(\vec{p}) + \Delta E$ inelastic scattering

isotropic vs.
anisotropic
scattering

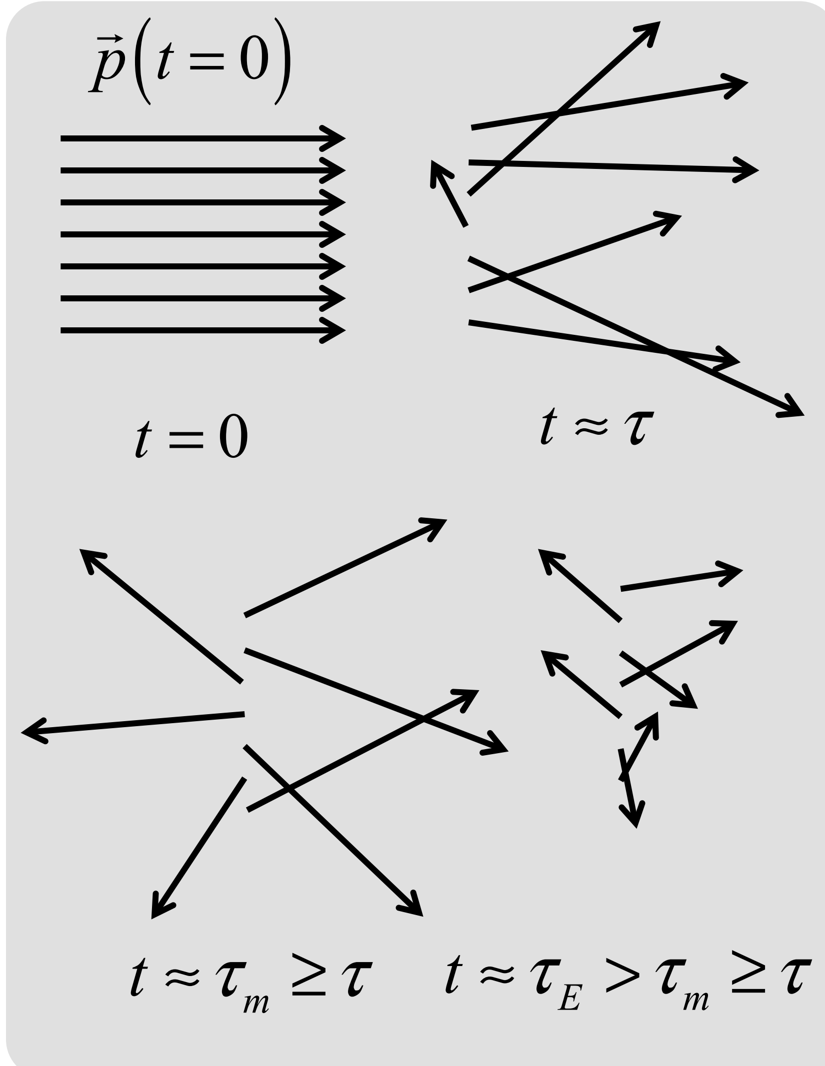
Characteristic times

$$\frac{1}{\tau(\vec{p})} = \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}')$$

(τ, single particle lifetime)

$$\frac{1}{\tau_m(\vec{p})} = \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}')\frac{\Delta p_z}{p_z}$$

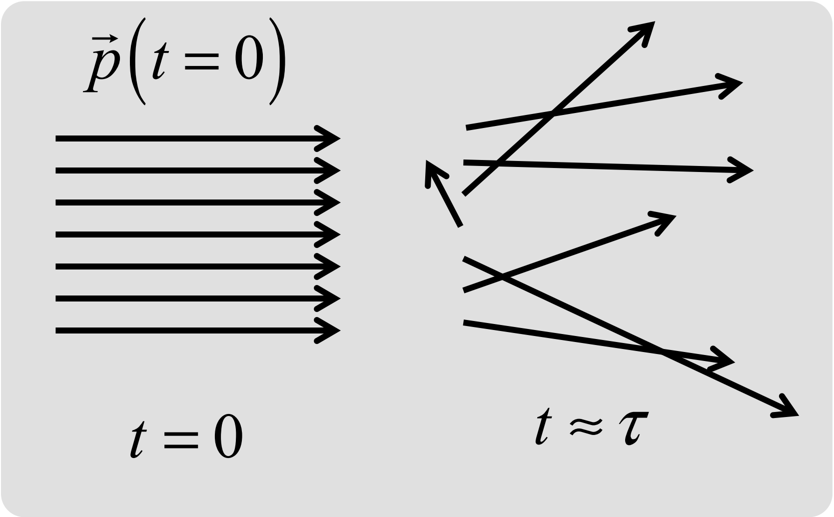
$$\frac{1}{\tau_E(\vec{p})} = \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}')\frac{\Delta E}{E}$$



Scattering rate

$$\frac{1}{\tau(\vec{p})} = \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}')$$

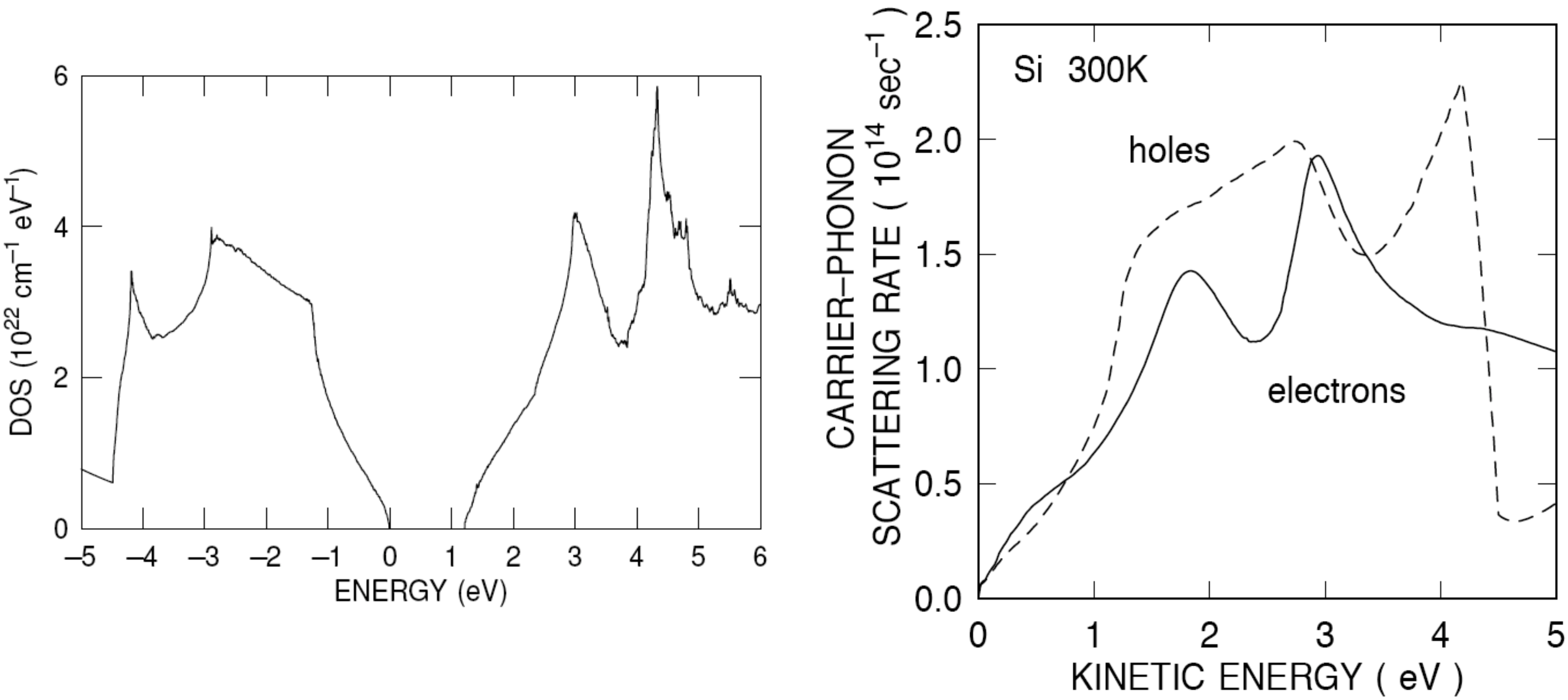
(τ, single particle lifetime)



$$\frac{1}{\tau(E)} \propto D_f(E+\Delta E)$$

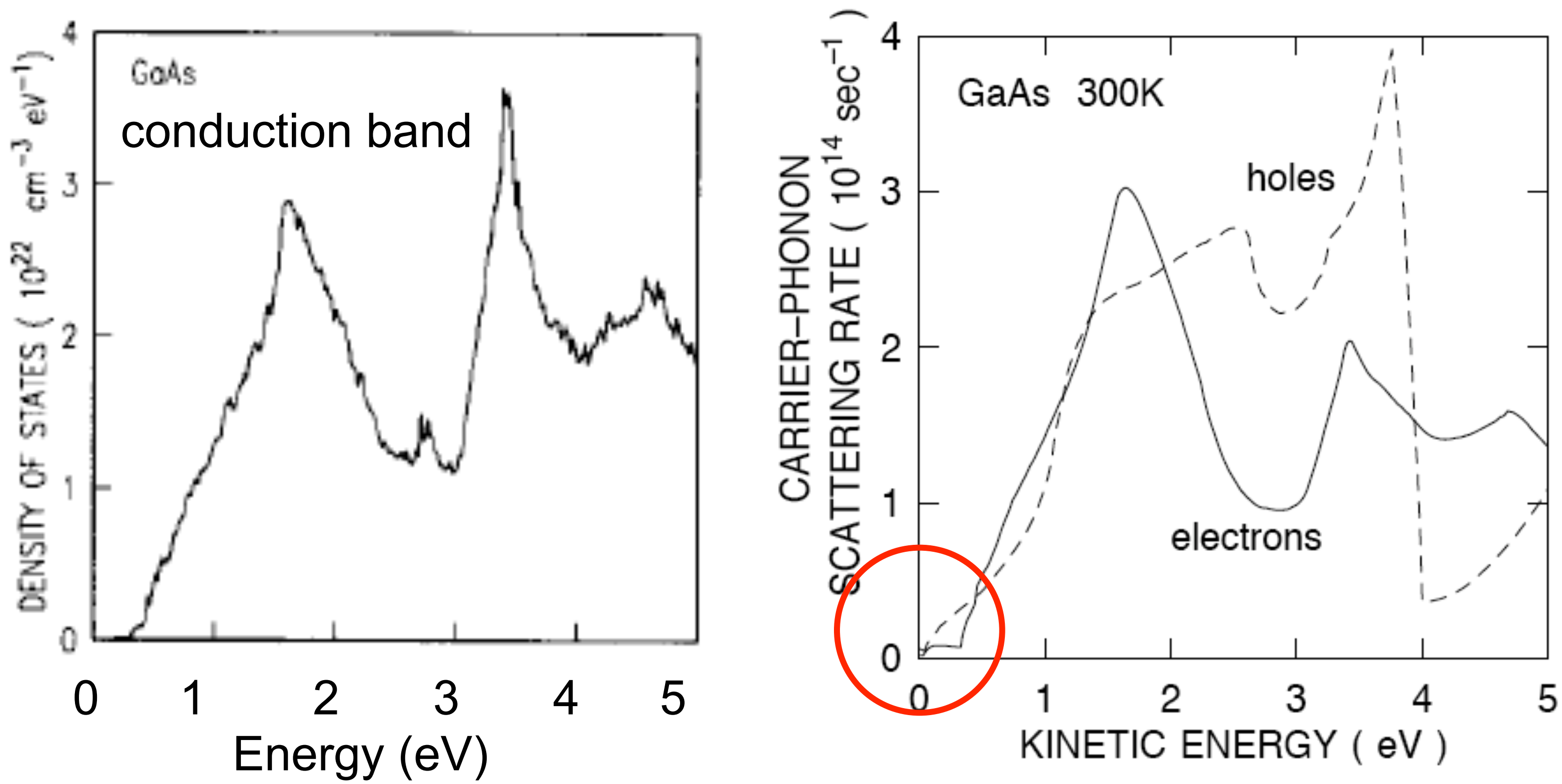
Scattering rate is often proportional to the density of final states.

Phonon scattering in Si



[2] Figures provided by Massimo V. Fischetti, October, 2009.

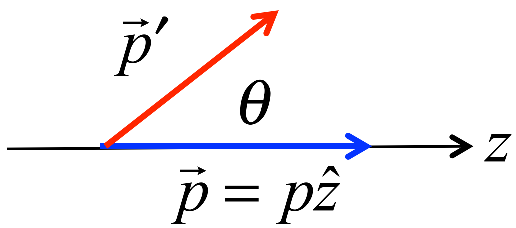
Phonon scattering in GaAs



DOS: [1] M. V. Fischetti," *IEEE Trans. Electron Dev.*, **38**, pp. 634-649, 1991
Scattering rate: [2] Provided by M. V. Fischetti, October, 2009.

Scattering rate and **momentum relaxation rate**

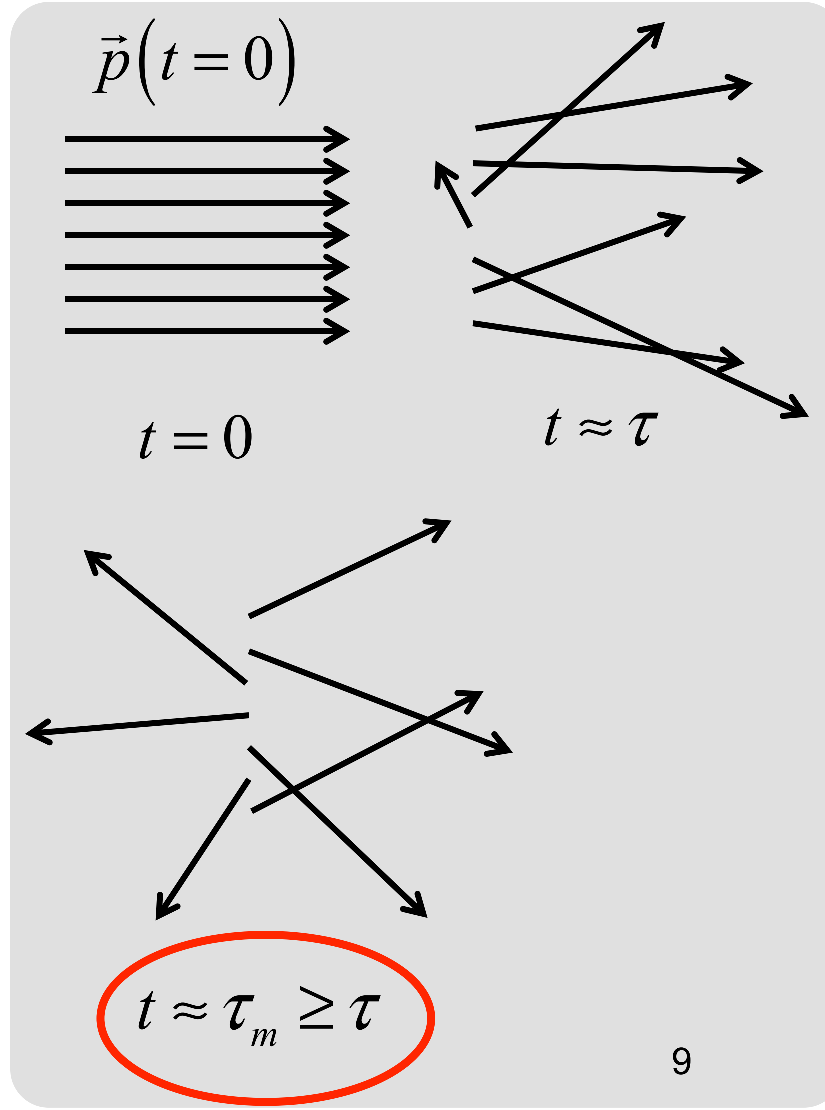
$$\frac{1}{\tau_m(\vec{p})} = \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}')\frac{\Delta p_z}{p_z}$$



$$\frac{\Delta p_z}{p_z} = \frac{p - p'\cos\theta}{p} = \left(1 - \frac{p'}{p}\cos\theta\right)$$

$$\frac{1}{\tau_m(\vec{p})} = \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}') - \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}')\frac{p'}{p}\cos\theta$$

$$\frac{1}{\tau_m(\vec{p})} = \frac{1}{\tau(\vec{p})} - A$$



Mobility

$$\mu = \frac{q\tau_m}{m^*}$$

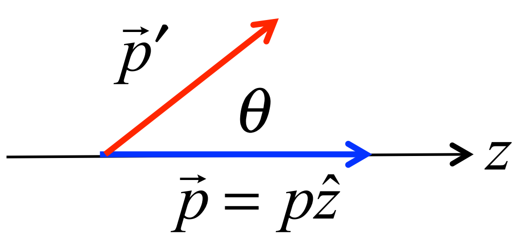
Momentum relaxation time

Isotropic scattering

$$\frac{1}{\tau_m(\vec{p})} = \frac{1}{\tau(\vec{p})} - A$$

$$A = \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}') \frac{p'}{p} \cos\theta$$

$$A = \frac{\Omega}{8\pi^3} \int_0^\infty S(\vec{p},\vec{p}') \frac{p'}{p} p^2 \, dp \int_0^\pi \cos\theta \sin\theta \, d\theta \int_0^{2\pi} d\phi$$



$$\frac{1}{\tau_m(\vec{p})} = \frac{1}{\tau(\vec{p})}$$

For isotropic scattering

Scattering rate and **energy relaxation rate**

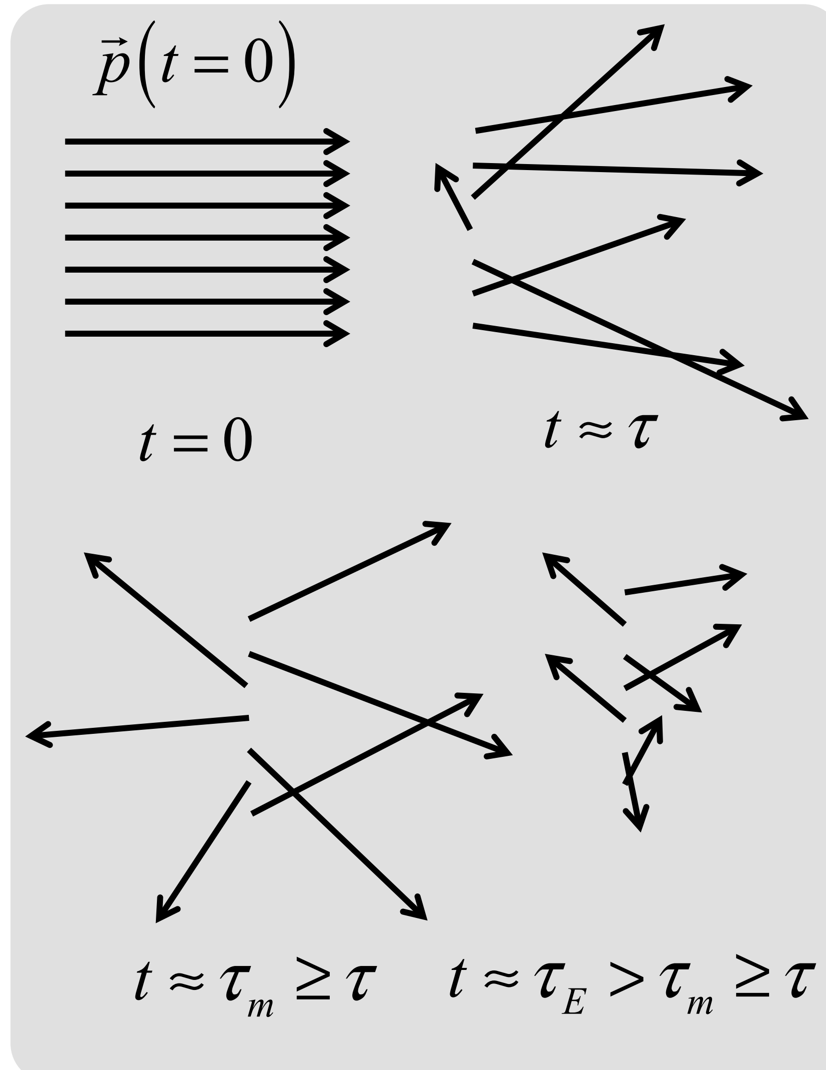
$$\frac{1}{\tau_E(\vec{p})} = \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}')\frac{\Delta E}{E}$$

i) Elastic scattering:

$$\Delta E = 0$$

$$\frac{1}{\tau_E(\vec{p})} = 0$$

$$\tau_E(\vec{p}) = \infty$$



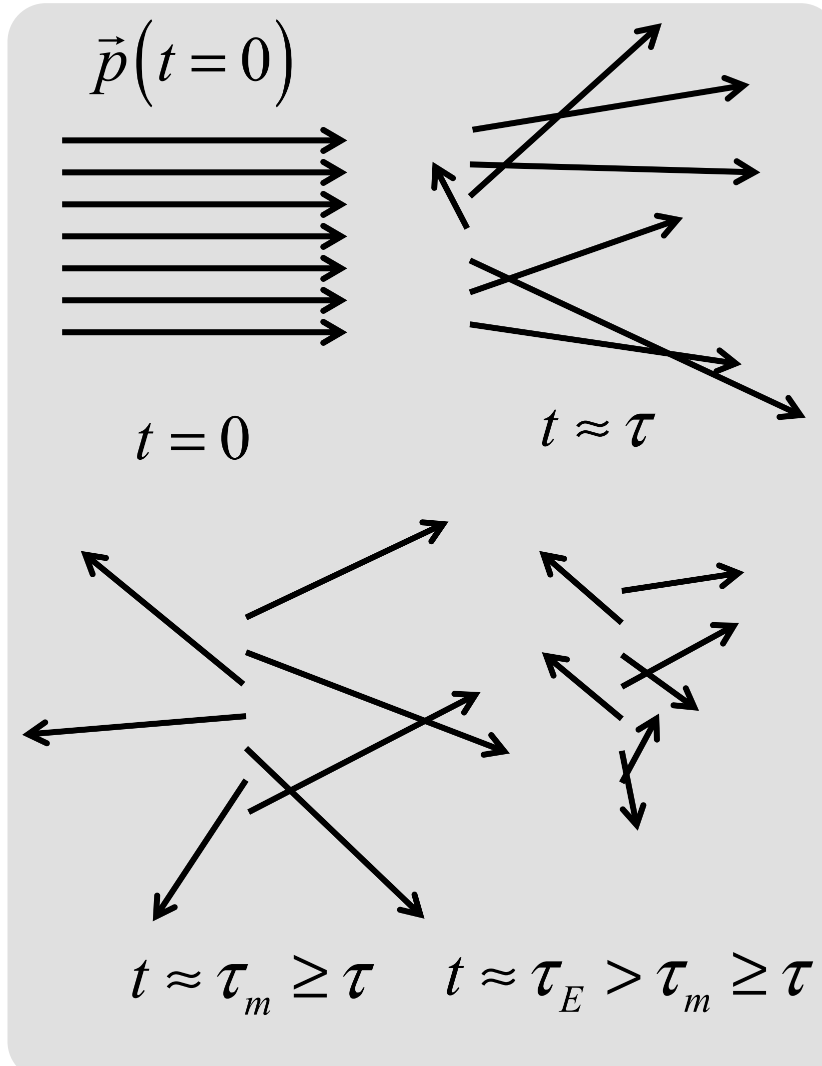
Energy relaxation by phonon emission

$$\frac{1}{\tau_E(\vec{p})} = \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}')\frac{\Delta E}{E}$$

ii) Phonon emission:

$$\Delta E = \hbar\omega$$

$$\frac{1}{\tau_E(\vec{p})} = \frac{\hbar\omega}{E}\frac{1}{\tau(\vec{p})}$$

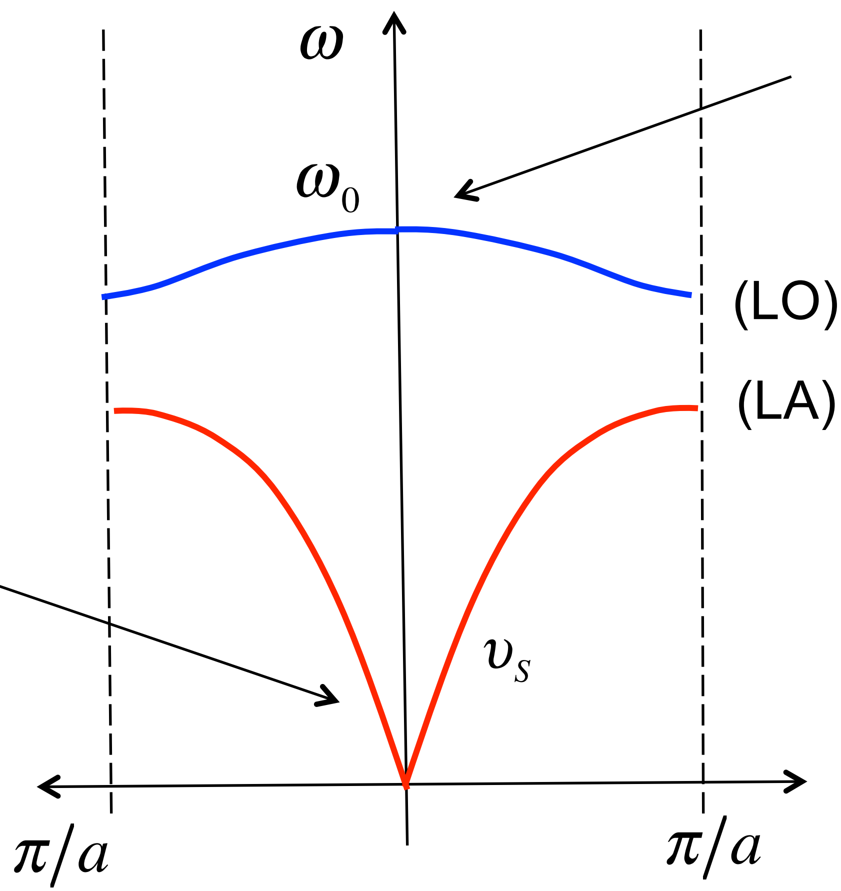


Phonons

Typical phonons involved in acoustic phonon scattering.

$$\hbar\omega \approx 0$$

(at 300 K)



Typical phonons involved in optical phonon scattering.

$$\hbar\omega \approx k_B T$$

(at 300 K)

Example

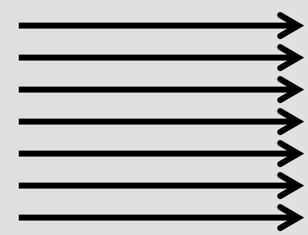
$$\frac{1}{\tau_E(\vec{p})} = \frac{\hbar\omega_0}{E}\frac{1}{\tau(\vec{p})}$$

Silicon:

$$\hbar\omega_0 = 0.060\ \text{eV}$$

$$\tau_E(\vec{p}) \approx \frac{E_{inj}}{\hbar\omega_0}\tau(\vec{p}) \approx 4\tau(\vec{p})$$

$$\vec{p}(t=0)$$



$$t = 0$$

$$E_{inj} \approx 10 k_B T \approx 0.26\ \text{eV}$$

Scattering mechanisms

1) Electron-phonon scattering

2) Electron-ionized impurity scattering

3) plus several more…

Summary

1) Characteristic times are derived from the transition rate, $S(p,p')$
2) $S(p,p')$ is obtained from Fermi's Golden Rule
3) The scattering rate is often proportional to the final DOS
4) Our goal is to understand the general features of scattering in common semiconductors.