**Notes on Thermal Velocities**

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There are different ways that one can define a "thermal velocity." It should not be surprising that these different ways give different thermal velocities – they are, after all defined differently, but one needs to be sure that one is using the correct thermal velocity for the problem of interest.

**1) Root mean square thermal velocity:**
Recall that the kinetic energy of a dilute gas (e.g. the electrons in a non-degenerate n-type semiconductor) is $k_BT/2$ per degree of freedom, so the average energy per electron is

$$\begin{aligned} u &= \frac{k_BT}{2} && \text{(1D)} \\ u &= k_BT && \text{(2D)} \\ u &= \frac{3k_BT}{2} && \text{(3D)} \end{aligned} \tag{1}$$

The kinetic energy of electrons in a parabolic band semiconductor is

$$KE = \frac{1}{2}m^*\upsilon^2 , \tag{2}$$

where $\upsilon$ is the magnitude of the velocity. In 1D, the magnitude is $\upsilon$, in 2D, $\sqrt{\upsilon_x^2+\upsilon_y^2}$, and in 3D, $\sqrt{\upsilon_x^2+\upsilon_y^2+\upsilon_z^2}$. By equating the electron kinetic energy to the thermal energy, we find

$$\begin{aligned} \frac{1}{2}m^*\upsilon^2 &= \frac{k_BT}{2} && \text{(1D)} \\ \frac{1}{2}m^*\upsilon^2 &= k_BT && \text{(2D)} \\ \frac{1}{2}m^*\upsilon^2 &= \frac{3k_BT}{2} && \text{(3D)} \end{aligned} \tag{3}$$

Now it is a simple matter to solve for the rms thermal velocity as follows.

$$\sqrt{\upsilon^2} = \upsilon_{rms} = \sqrt{\frac{k_B T}{m^*}} \qquad \text{(1D)}$$
$$\sqrt{\upsilon^2} = \upsilon_{rms} = \sqrt{\frac{2k_B T}{m^*}} \qquad \text{(2D)} \qquad (4)$$
$$\sqrt{\upsilon^2} = \upsilon_{rms} = \sqrt{\frac{3k_B T}{m^*}}\,, \qquad \text{(3D)}$$

so the rms thermal velocity depends on dimensionality.

**2) Uni-directional thermal velocity:**
The unidirectional thermal velocity is the velocity along one direction. It is defined as follows.

$$\upsilon_T = \frac{\sum\limits_{k_x>0} \upsilon_x f_0\left(k_x\right)}{\sum\limits_{k_x>0} f_0\left(k_x\right)} \qquad \text{(1D)} \qquad (5a)$$

$$\upsilon_T = \frac{\sum\limits_{k_x>0,k_y} \upsilon_x f_0\left(k_x,k_y\right)}{\sum\limits_{k_x>0,k_y} f_0\left(k_x,k_y\right)} \qquad \text{(2D)} \qquad (5b)$$

$$\upsilon_T = \frac{\sum\limits_{k_x>0,k_y,k_z} \upsilon_x f_0\left(k_x,k_y,k_z\right)}{\sum\limits_{k_x>0,k_y,k_z} f_0\left(k_x,k_y,k_z\right)}\,. \qquad \text{(3D)} \qquad (5c)$$

For a non-degenerate semiconductor with parabolic bands, the Fermi function simplifies to:

$$f_0\left(E\right) = e^{\left(E_F - E_C - \frac{\hbar^2 k^2}{2m^*}\right)/k_B T} = e^{\left(E_F - E_C\right)/k_B T} \times e^{-\hbar^2 k^2/\left(2m^* k_B T\right)}\,.$$

Now let's work it out in 3D.

$$\upsilon_T = \frac{\sum_{k_x>0,k_y,k_z} \upsilon_x e^{-\hbar^2 k^2/\left(2m^* k_B T\right)}}{\sum_{k_x>0,k_y,k_z} e^{-\hbar^2 k^2/\left(2m^* k_B T\right)}} = \frac{\int_0^\infty \frac{\hbar k_x}{m^*} e^{-\hbar^2 k_x^2/\left(2m^* k_B T\right)}\, dk_x}{\int_0^\infty e^{-\hbar^2 k_x^2/\left(2m^* k_B T\right)}\, dk_x} \times \frac{\int_{-\infty}^\infty e^{-\hbar^2 k_y^2/\left(2m^* k_B T\right)}\, dk_y}{\int_{-\infty}^\infty e^{-\hbar^2 k_y^2/\left(2m^* k_B T\right)}\, dk_y} \times \frac{\int_{-\infty}^\infty e^{-\hbar^2 k_z^2/\left(2m^* k_B T\right)}\, dk_z}{\int_{-\infty}^\infty e^{-\hbar^2 k_z^2/\left(2m^* k_B T\right)}\, dk_z}$$

$$\upsilon_T = \frac{\int_0^\infty \frac{\hbar k_x}{m^*} e^{-\hbar^2 k_x^2/\left(2m^* k_B T\right)}\, dk_x}{\int_0^\infty e^{-\hbar^2 k_x^2/\left(2m^* k_B T\right)}\, dk_x} = \sqrt{\frac{2k_B T}{\pi m^*}} \qquad \text{(1D), (2D), and (3D)} \qquad (6)$$

We can see that if we do this in 1D, 2D, or 3D, we will get the same result, so there unidirectional thermal velocity is the same in 1D, 2D, and 3D. This makes sense because it is always the average velocity in one (positive) direction.

**3) Richardson velocity:**

In Schottky barrier problems, one often encounters the "Richardson velocity", $\upsilon_R$, which is defined as

$$\upsilon_T = \frac{\sum_{k_x>0} \upsilon_x f_0\left(k_x\right)}{\sum_{k_x} f_0\left(k_x\right)}. \qquad (7)$$

The Richardson velocity is simply

$$\upsilon_R = \frac{\upsilon_T}{2}. \qquad (8)$$

We see that there are different thermal velocities depending on how we choose to define them. Since we are interested in current flow along one direction, the uni-directional thermal velocity is relevant for us in this course.

The bottom line is that if someone tells you that they used the "thermal velocity" for a problem, you should ask: "What thermal velocity?"