ECE 656 Homework SOLUTIONS (Week 10)

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1) Show that the following relation (eqn. (4.119) in *Fundamentals of Carrier Transport*) is true:

$$e^{-\frac{\pi}{\rho_S}R_{MN,OP}} + e^{-\frac{\pi}{\rho_S}R_{NO,PM}} = 1$$

Solution:

From eqns. (4.117) and (4.118) of FCT:

$$R_{MN,OP} = \frac{\rho_S}{\pi}\ln\left(\frac{(a+b)(b+c)}{b(a+b+c)}\right) \rightarrow \frac{\pi R_{MN,OP}}{\rho_S} = \ln\left(\frac{(a+b)(b+c)}{b(a+b+c)}\right) \qquad \text{(i)}$$

$$R_{NO,PM} = \frac{\rho_S}{\pi}\ln\left(\frac{(a+b)(b+c)}{ac}\right) \rightarrow \frac{\pi R_{NO,PM}}{\rho_S} = \ln\left(\frac{(a+b)(b+c)}{ac}\right) \qquad \text{(ii)}$$

From (i) and (iii)

$$e^{-\frac{\pi}{\rho_S}R_{MN,OP}} + e^{-\frac{\pi}{\rho_S}R_{NO,PM}} = \frac{b(a+b+c)+ac}{(a+b)(b+c)} = \frac{ab+b^2+bc+ac}{ab+ac+b^2+bc} = 1$$

$$\boxed{e^{-\frac{\pi}{\rho_S}R_{MN,OP}} + e^{-\frac{\pi}{\rho_S}R_{NO,PM}} = 1}$$

2) For n-type, bulk silicon doped at $N_D = 10^{17}$ cm^{-3} the room temperature mobility is 800 cm^2/V-s. Answer the following questions. Some potentially useful information is:

$N_C = 3.23\times10^{19}$ cm^{-3} $\quad N_V = 1.83\times10^{19}$ cm^{-3} $\quad E_G = 1.11$ eV $\quad \upsilon_T = 1.05\times10^7$ cm/s.

Estimate the Seebeck coefficient. Make reasonable assumptions, but clearly state them.

Solution:

$$S = -\left(\frac{k_B}{q}\right)\left[\frac{\Delta_n}{k_BT_L} - \eta_F\right]$$

$$n_0 = N_C e^{\eta_F} \rightarrow \eta_F = \ln(n_0/N_C) = -5.78 \qquad \text{(non-degenerate semiconductor)}$$

Assume $\Delta_n = 2k_BT_L$ (non-degenerate, constant MFP)

$$S = -(86)[2+5.78] = -669\ \mu\text{V/K}$$

$$\boxed{S = -669\ \mu\text{V/K}}$$

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3) A Hall effect experiment is performed on a n-type semiconductor with a length of 2.65 cm, a width of 1.70 cm, and a thickness of 0.0520 cm, in a magnetic field of 0.5 T. The current in the sample along its length is 200 mA. The potential difference along the length of the sample is 195 mV and across the width is 21.4 mV.

3a) What is the carrier concentration of the sample?

Solution:

Recall that in 2D:

$$\vec{\mathcal{E}} = \rho_S \vec{J}_n + \left(\rho_S \mu_n r_H\right) \vec{J}_n \times \vec{B}$$

$$\mathcal{E}_y = \rho_S J_y - \left(\rho_S \mu_n r_H\right) J_x B_z = -\left(\rho_S \mu_n r_H\right) J_x B_z$$

$$V_H = -W\mathcal{E}_y = \left(\rho_S \mu_n r_H\right) I_x B_z = \left(\frac{1}{n_S q \mu_n} \mu_n r_H\right) I_x B_z = \left(\frac{r_H}{q n_S}\right) I_x B_z$$

which is the same as eqn. (4.110) in FCT

$$V_H = \frac{r_H}{q n_S} B_z I$$

$$\frac{n_S}{r_H} = \frac{B_z I}{q V_H} = \frac{0.5 \times 200 \times 10^{-6}}{1.6 \times 10^{-19}\left(21.4 \times 10^{-3}\right)} = 2.92 \times 10^{16}\,\text{cm}^{-2}$$

$$\frac{n_S / r_H}{t} = n_H = \frac{2.92 \times 10^{16}\,\text{cm}^{-2}}{0.0520\ \ \text{cm}} = 5.62 \times 10^{17}\,\text{cm}^{-3}$$

$$\boxed{n_H = \frac{n}{r_H} = 5.62 \times 10^{17}\,\text{cm}^{-3}}$$

Not the carrier concentration, but the "Hall carrier concentration."

3b) What is the mobility?

Solution:

$$R_{xx} = \frac{195 \times 10^{-3}}{200 \times 10^{-6}} = 975\ \Omega$$

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$$R_{xx} = \rho_S \frac{L}{W} = 975\ \Omega$$

$$\rho_S = \frac{W}{L} 975\ \Omega = \frac{1.70}{2.65} \times 975 = 625 = \frac{1}{n_S q \mu_n}$$

$$\mu_n = \frac{1}{n_S q \rho_S} = \frac{1}{r_H \times 2.92 \times 10^{16} \times 1.6 \times 10^{-19} \times 625}$$

$$r_H \mu_n = \mu_H = \frac{1}{2.92 \times 10^{16} \times 1.6 \times 10^{-19} \times 625} = 0.3\,\mathrm{cm^2/V\text{-}s}$$

$$\boxed{\mu_H = 0.3\,\mathrm{cm^2/V\text{-}s}}$$

Not the mobility, but the "Hall mobility."

3c) If the scattering time is 1ps, find the magnetic field for which this classical analysis of Hall effect is no longer valid?

Solution:

We require:

$$\omega_c \tau << 1$$

$$\frac{qB_z}{m^*} \tau << 1 \qquad \text{(i)}$$

$$\mu_n = \frac{q\tau}{m^*} \qquad \tau = \frac{m^* \mu_n}{q} \qquad \text{(ii)}$$

With (ii), (i) becomes

$$\frac{qB_z}{m^*} \frac{m^* \mu_n}{q} << 1 \rightarrow B_z \mu_n << 1$$

Using the Hall mobility as an estimate of the real mobility

$$B_z \mu_n = 0.5 \times 0.3 = 0.15 << 1$$

so this experiment is in the low B-field regime. The largest the B-field can be to be in the low field regime is

$$B_z = \frac{1}{\mu_n} = \frac{1}{0.3} = 3.33\,\mathrm{T}$$

$$\boxed{B_z = 3.33\,\mathrm{T}}$$

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4) Contact resistances are important. They can complicate measurements of semiconductor transport parameters, and they can degrade device performance. The constant resistance is specified by the interfacial contact resistivity, ρ_C, in $\Omega\text{-cm}^2$. A very good value is $\rho_C \approx 10^{-8}\,\Omega\text{-cm}^2$. Consider n+ Si at room temperature and doped to $N_D = 10^{20}\text{ cm}^{-3}$. What is the lower limit to ρ_C? (Assume a fully degenerate semiconductor and use appropriate effective masses for the conduction band of Si.)

Solution:

The lower limit resistance must be the ballistic contact resistance:

$$R_B = \frac{1}{G_B} = \frac{1}{\left(2q^2/h\right)\langle\langle T\rangle\rangle\langle M\rangle}$$

Assuming that one-half of the ballistic resistance is associated with each of the two contacts:

$$\rho_C = \frac{R_B A}{2} = \frac{h}{4q^2}\frac{1}{\langle\langle \mathcal{T}\rangle\rangle\langle M/A\rangle}$$

Assume a strongly degenerate semiconductor:

$$\rho_C = \frac{h}{4q^2}\frac{1}{\mathcal{T}\left(E_F\right)M\left(E_F\right)/A}$$

The lower limit occurs when the transmission is one

$$\rho_C^{\min} = \frac{h}{4q^2}\frac{1}{M\left(E_F\right)/A}$$

Need to find the Fermi level. Recall that at 0 K,

$$n_0 = \int_{E_C}^{E_F} D_{3D}\left(E\right)dE$$

$$D_{3D}\left(E\right) = \frac{\left(m_{DOS}^*\right)^{3/2}\sqrt{2\left(E-E_C\right)}}{\pi^2\hbar^3}$$

$$n_0 = \int_{E_C}^{E_F} D_{3D}\left(E\right)dE = \int_{E_C}^{E_F}\frac{\left(m_{DOS}^*\right)^{3/2}\sqrt{2\left(E-E_C\right)}}{\pi^2\hbar^3}dE = \frac{\sqrt{2}\left(m_{DOS}^*\right)^{3/2}}{\pi^2\hbar^3}\int_{E_C}^{E_F}\left(E-E_C\right)^{1/2}dE$$

$$n_0 = \frac{2\sqrt{2}\left(m_{DOS}^*\right)^{3/2}}{3\pi^2\hbar^3}\left(E_F-E_C\right)^{3/2}$$

$$\left(E_F-E_C\right) = \frac{1}{m_{DOS}^*}\left(\frac{3\pi^2\hbar^3}{2\sqrt{2}}\right)^{2/3}\left(n_0\right)^{2/3}$$

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$$M_{3\text{D}}\left(E_F\right)=\frac{m_{DOM}^{*}}{2\pi\hbar^2}\left(E_F-E_C\right)$$

$$M_{3\text{D}}\left(E_F\right)=\frac{m_{DOM}^{*}}{m_{DOS}^{*}}\frac{1}{2\pi\hbar^2}\left(\frac{3\pi^2\hbar^3}{2\sqrt{2}}\right)^{2/3}\left(n_0\right)^{2/3}=\frac{m_{DOM}^{*}}{m_{DOS}^{*}}\left(\frac{3\sqrt{\pi}}{8}\right)^{2/3}n_0^{2/3}$$

For Si, we have to consider the ellipsoidal bandstructure:

$$m_{DOS}^{*}=\left(6\right)^{2/3}\left(m_t^2 m_\ell\right)^{1/3}=1.06m_0$$

$$m_{DOM}^{*}=2m_t^{*}+4\sqrt{m_t^{*}m_\ell^{*}}=2.04m_0$$

(See: Jeong, Changwook; Kim, Raseong; Luisier, Mathieu; Datta, Supriyo; and Lundstrom, Mark S., "On Landauer versus Boltzmann and full band versus effective mass evaluation of thermoelectric transport coefficients*," J. Appl. Phys.*, **107**, 023707, 2010.)