ECE 656 Homework (Week 12)

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1) Derive the current equation for 3D electrons using our balance equation approach.

2) When deriving balance equations in 3D, we obtained a tensor

$$W_{ij} = \frac{1}{\Omega}\sum_{\vec{p}} \frac{\upsilon_i p_j}{2} f\left(\vec{r},\vec{p},t\right).$$

In general,

$$W_{ij} = \begin{bmatrix} W_{xx} & W_{xy} & W_{xz} \\ W_{yx} & W_{yy} & W_{yz} \\ W_{zx} & W_{zy} & W_{zz} \end{bmatrix}.$$

Evaluate this tensor in <u>equilibrium</u> for a non-degenerate semiconductor and show that:

$$W_{ij}^0 = \frac{W_0}{3}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

What is W_0?

3) For 3D electrons with spatial variations along the x-direction alone, the energy flux balance equation is

$$\frac{\partial F_{Wx}\left(x,t\right)}{\partial t} = -\frac{dX_{xx}}{dx} - \frac{5}{3}\frac{q}{m^*}W\mathcal{E}_x - \frac{F_{Wx}}{\left\langle \tau_{F_W}\right\rangle}, \qquad \text{(A)}$$

which can be simplified to:

$$F_W = -\frac{5}{3}\mu_E W\mathcal{E}_x - \frac{10}{3}\mu_E \frac{d\left(W k_B T_e / q\right)}{dx} \qquad \text{(B)}$$

Verify that (A) and (B) are correct under the appropriate simplifying assumptions. Note that there is an error in the course text, *Fundamentals of Carrier Transport*, problem 5.7, p. 244.

ECE 656 Homework (Week 12) (continued)

4) In reading the literature, you will sometimes see the near-equilibrium drift-diffusion equation written as

$$\vec{J}_n = nq\mu_n\vec{\mathcal{E}} + qD_n\nabla n \qquad \text{(A)}$$

or as

$$\vec{J}_n = nq\mu_n\vec{\mathcal{E}} + q\nabla\left(D_n n\right) \qquad \text{(B)}$$

Which one is correct? Are both correct? If so, under what conditions is each of them correct?

You can address this question using the concepts and approaches that we have been discussing in ECE-656. AFTER you figure out the answer, you may want to consult:

[1] P.T. Landsberg, "D grad ν or grad(Dν)?," *J. Appl. Phys.*, **56**, pp. 1119-1122, 1984

[2] P.T. Landsberg and S.A. Hope, "Two Formulations of the Semiconductor Transport Equations," *Solid-State Electronics*, **20**, pp. 421-429, 1977.

5) Derive an energy balance equation for graphene, and then simplify it to terminate the balance equation hierarchy. (Consider only electrons in the conduction band with energies above the Dirac point, $E = E_D = 0$.

6) Use the balance equation approach and derive a current equation for a semiconductor nanowire. You may assume that only 1 subband is occupied, **but do not assume parabolic energy bands.**

7) When deriving the momentum balance equation in 3D, a tensor,

$$W_{ij} = \frac{1}{\Omega}\sum_{\vec{p}}\frac{\upsilon_i p_j}{2} f\left(\vec{r},\vec{p},t\right)$$

occurs. This problem asks you to write a balance equation for W_{xx}. You may assume parabolic energy bands and an electric field in the x-direction.

8) Derive a balance equation for nk_BT_e, where T_e is the carrier temperature (not the carrier energy).

9) Using the balance equation approach, derive a set of coupled current equations for near-equilibrium transport and compare them to the results obtain from the Landauer approach (or the near-equilibrium BTE).