ECE 656 Homework SOLUTIONS (Week 14)

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1) Monte Carlo simulations of high-field transport in bulk silicon with an electric field of 100,000 V/cm show the following results for the average velocity and kinetic energy:

$$\upsilon_d = \langle \upsilon \rangle = 1.04\times 10^7 \text{ cm/s}$$
$$u = \langle KE \rangle = 0.364 \text{ eV}$$

Estimate the average momentum relaxation time, $\langle \tau_m \rangle$ and energy relaxation time, $\langle \tau_E \rangle$.

Solution:

$$\mu_n = \langle \upsilon \rangle / \mathcal{E} = 104 \text{ cm}^2/\text{V-s}$$
$$\mu_n = \frac{q\langle \tau_m \rangle}{m_c^*},$$

where m_c^* is the conductivity effective mass. Use MKS units for the calculation:

$$\langle \tau_m \rangle = \frac{q}{\mu_n m_c^*} = \frac{\mu_n m_c^*}{q} = \frac{104\times 10^{-4}\times 0.23\times 9.11\times 10^{-31}}{1.6\times 10^{-19}} 1.36\times 10^{-14}$$

$$\boxed{\langle \tau_m \rangle = 1.36\times 10^{-14} \text{ s}}$$

From the energy balance equation:

$$J_x\mathcal{E}_x = nq\langle \upsilon \rangle \mathcal{E}_x = \frac{n(u-u_0)}{\langle \tau_E \rangle}$$
$$\langle \tau_E \rangle = \frac{(u-u_0)}{q\langle \upsilon \rangle \mathcal{E}_x}$$
$$\frac{u_0}{q} = 1.5\frac{k_B T_L}{q} = 0.039$$
$$\langle \tau_E \rangle = \frac{(u-u_0)}{q\langle \upsilon \rangle \mathcal{E}_x} = \frac{(0.364-0.039)}{1.04\times 10^7\times 10^5} = 0.313\times 10^{-12}$$

$$\boxed{\langle \tau_E \rangle = 0.3 \text{ ps}}$$

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2) In this problem, we will try to estimate the average kinetic energy and the energy relaxation time of electrons in bulk, <111> oriented Si under an electric field of 20 kV/cm. Assume that the velocity is saturated at 10^7 cm/s and that the conductivity effective mass is $m_C^* = 0.26 m_0$. You should also assume that the dominant scattering mechanism is optical (or intervalley) phonon scattering with $\hbar\omega_0 = 0.063$ eV and that momentum relaxation is dominated by ADP scattering with $\mu_n = \mu_{n0}\sqrt{T_L/T_e}$, where $\mu_{n0} = 1400$ cm²/V-s, $T_L = 300$ K is the lattice temperature and T_e is the electron temperature.

HINT: <u>Some</u> of this information may be useful in solving this problem.

2a) Determine the electron temperature.

Solution:

$$\mu_n = \frac{\langle \upsilon \rangle}{\mathcal{E}} = \frac{10^7}{2\times 10^5} = 500\ \text{cm}^2/\text{V-s}$$

$$\mu_n = \mu_{n0}\sqrt{T_L/T_e}$$

$$500 = 1400\sqrt{300/T_e}$$

$$T_e = 300\left(\frac{1400}{500}\right)^2 = 2352\ \text{K}$$

$$\boxed{T_e = 2352\ \text{K}}$$

2b) Develop an <u>expression</u> for the energy relaxation time of an average electron in terms of the momentum relaxation time (you will not be able to get a numerical answer at this point).

Solution:

Input energy from the field = power dissipated by scattering

$$J_{nx}\mathcal{E}_x = nq\langle \upsilon \rangle \mathcal{E}_x = \frac{n(u-u_0)}{\langle \tau_E \rangle} = \frac{n\frac{3}{2}k_B(T_e - T_L)}{\langle \tau_E \rangle}$$

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$$\langle \tau_E \rangle = \frac{3}{2}\frac{k_B\left(T_e - T_L\right)}{q\langle \upsilon \rangle \mathcal{E}_x} = 1.5 \times \frac{1.38\times10^{-23}}{1.6\times10^{-19}} \times \frac{2352-300}{10^7\times2\times10^4} = 1.33\times10^{-12}$$

$$\boxed{\langle \tau_E \rangle = 1.33 \text{ ps}}$$

2c) Compare (quantitatively) the energy relaxation time to the momentum relaxation time for this example.

Solution:

$$\mu_n = \frac{\langle \upsilon \rangle}{\mathcal{E}} = \frac{10^7}{2\times10^5} = 500 \text{ cm}^2/\text{V-s}$$

$$\mu_n = \frac{q\langle \tau_m \rangle}{m^*}$$ (use MKS units for the calculation below)

$$\langle \tau_m \rangle = \frac{m^*\mu_n}{q} = \frac{0.26\times9.11\times10^{-31}}{1.6\times10^{-19}}500\times10^{-4} = 7.4\times10^{-14}$$

$$\frac{\langle \tau_E \rangle}{\langle \tau_m \rangle} = \frac{133\times10^{-14}}{7.4\times10^{-14}} = 18$$

$$\boxed{\frac{\langle \tau_E \rangle}{\langle \tau_m \rangle} = 18}$$