ECE 656 Homework SOLUTIONS (Week 15)

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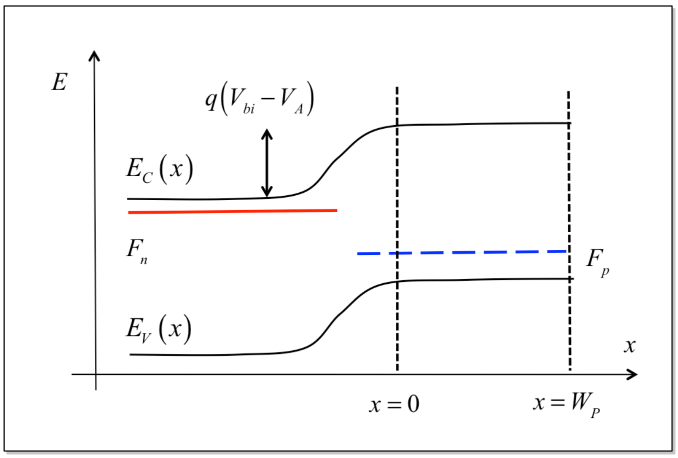
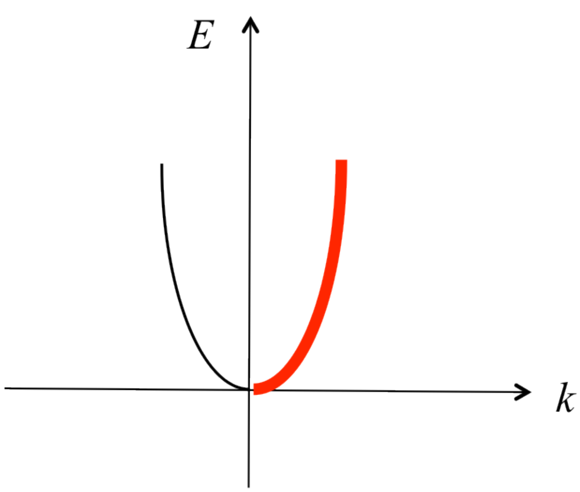
1) Consider the forward biased n⁺p junction sketched below. The classic, short-base theory of the diode gives the current as $I_D = qA\frac{n_i^2}{N_A}\frac{D_n}{W_P}\left(e^{qV_A/k_BT}-1\right)$. (Non-degenerate carrier statistics have been assumed).



1a) Treat the n⁺ region as a Landauer contact and assume that the depletion region is ballistic. Also assume an absorbing contact at $x=W_P$. On the sketch below, which shows the k-states at the beginning of the p-type quasi-neutral region, indicate which k-states are populated from the n⁺ region.

Solution:

All of the +k-states are occupied by electrons coming from the n⁺ region.



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1b) Using the results of problem, 1a), present an expression for the magnitude of the electron current injected into the p-type quasi-neutral region, $I_n^+(0)$. (You do not need to solve this expression).

Solution:

$$\boxed{I_n^+(0) = qA\frac{1}{\Omega}\sum_{\vec{k},k_x>0} \upsilon_x f_0(F_n)}$$

where F_n is the electron quasi-Fermi level in the neutral n^+ region.

1c) The diode current is all carried by electrons in an n^+p diode, so $I_D = \mathcal{T} I_n^+(0)$. Evaluate this expression in the classic, diffusive limit and use it to solve for $I_n^+(0)$. HINT: You could get the same answer by evaluating the expression in 1b), but this approach is easier.

Solution:

$$I_D = \mathcal{T} I_n^+(0) = qA\frac{n_i^2}{N_A}\frac{D_n}{W_P}\left(e^{qV_A/k_BT} - 1\right)$$

$$I_n^+(0) = \frac{1}{\mathcal{T}} qA\frac{n_i^2}{N_A}\frac{D_n}{W_P}\left(e^{qV_A/k_BT} - 1\right)$$

In the diffusive limit:

$$\mathcal{T} = \frac{\lambda}{\lambda + W_P} \rightarrow \frac{\lambda}{W_P}$$

so

$$I_n^+(0) = \frac{W_P}{\lambda} qA\frac{n_i^2}{N_A}\frac{D_n}{W_P}\left(e^{qV_A/k_BT} - 1\right) = \frac{D_n}{\lambda} qA\frac{n_i^2}{N_A}\left(e^{qV_A/k_BT} - 1\right)$$

$$I_n^+(0) = \frac{\upsilon_T \lambda}{2\lambda} qA\frac{n_i^2}{N_A}\left(e^{qV_A/k_BT} - 1\right) = qA\frac{n_i^2}{2N_A}\upsilon_T\left(e^{qV_A/k_BT} - 1\right)$$

$$\boxed{I_n^+(0) = qA\frac{n_i^2}{2N_A}\upsilon_T\left(e^{qV_A/k_BT} - 1\right)}$$

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1d) Use the result of 3c) to develop an expression for the diode current, I_D, that is valid from the ballistic to diffusive limit. Compare your answer to the classic, short base result and discuss the differences.

Solution:

$$I_D = \mathcal{T} I_n^+(0)$$

$$I_D = \frac{\lambda}{\lambda + W_P} I_n^+(0)$$

$$I_D = \frac{\lambda}{\lambda + W_P} qA \frac{n_i^2}{2N_A} \upsilon_T \left(e^{qV_A/k_BT} - 1\right)$$

$$I_D = \frac{\lambda \upsilon_T}{2(\lambda + W_P)} qA \frac{n_i^2}{N_A} \left(e^{qV_A/k_BT} - 1\right) = \frac{D_n}{(\lambda + W_P)} qA \frac{n_i^2}{N_A} \left(e^{qV_A/k_BT} - 1\right)$$

$$I_D == \frac{W_P}{(\lambda + W_P)} qA \frac{n_i^2}{N_A} \frac{D_n}{W_P} \left(e^{qV_A/k_BT} - 1\right)$$

$$\boxed{I_D = \frac{1}{(1 + \lambda/W_P)} \times \left\{ qA \frac{n_i^2}{N_A} \frac{D_n}{W_P} \left(e^{qV_A/k_BT} - 1\right) \right\}}$$

i) diffusive limit: $W_P >> \lambda$: $I_D = qA \frac{n_i^2}{N_A} \frac{D_n}{W_P} \left(e^{qV_A/k_BT} - 1\right)$ (classic result)

ii) ballistic limit: $W_P << \lambda$:

$$I_D = \frac{W_P}{\lambda} qA \frac{n_i^2}{N_A} \frac{D_n}{W_P} \left(e^{qV_A/k_BT} - 1\right)$$

$$I_D = qA \frac{n_i^2}{2N_A} \upsilon_T \left(e^{qV_A/k_BT} - 1\right) = I_n^+(0)$$