ECE 656 Homework (Week 2)
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1) Assume $T = 0 K$ and work out the electron density per unit area for two cases:

i) A 2D semiconductor with parabolic energy bands and an effective mass of $m^*$. (Assume a valley degeneracy of 2.)

ii) Graphene, where we consider $E > 0$ to be the conduction band. ($E = 0$ is where the bands cross, the so-called Dirac point.) (Assume a valley degeneracy of 2.)

1a) Express your two answers in terms of the Fermi energy, and show that they are different.

1b) Express your two answers in terms of the Fermi wavevector and show that they are the same.

2) Assume a finite temperature and work out the sheet carrier densities, $n_s$, for:

2a) Electrons in the conduction band of a 2D parabolic band semiconductor

2b) Electrons in the conduction band ($E > 0$) of graphene.

Your answers to these questions should be in terms of material parameters and the Fermi level.

3) Assume $T = 0 K$ and work out the average $+x$-directed velocity for electrons in:

3a) A 2D semiconductor with a parabolic conduction band and

3b) The conduction band ($E > 0$) of graphene.

Your answer should be in terms of the Fermi energy, $E_F$.

4) Assume a nonparabolic, 1D energy bandstructure described by:

$$E(k_x)\left[1 + \alpha E(k_x)\right] = \frac{\hbar^2 k_x^2}{2m^*(0)}.$$

where

$$\frac{1}{m^*(0)} = \frac{1}{\hbar^2} \left. \frac{d^2 E(k_x)}{dk_x^2} \right|_{k_x=0}.$$
4a) Sketch (or produce a Matlab plot) of $E(k)$ vs. $k$ for two cases: i) $\alpha = 0$ and ii) $\alpha > 0$. If you are producing a Matlab plot, the energy range should be from 0 to 1 eV, and you can assume $\alpha = 0.5$ eV.

4b) For this bandstructure, derive an expression for the velocity, $v_x(k_x)$ as a function of $k_x$.

5) For parabolic energy bands, the 2D density of states is

$$D_{2D}(E) = \frac{m^*}{\pi \hbar^2} \Theta(E - \epsilon_1).$$

Assume a non-parabolic band described by the so-called Kane dispersion,

$$E(k)[1 + \alpha E(k)] = \frac{\hbar^2 k^2}{2m^*(0)},$$

and derive the 2D density of states.

6) Derive an expression for the 2D density of states for one of the conduction band ellipsoids in silicon.

HINT: You may find the discussion in Pierret (Advanced Semiconductor Fundamentals) on pp. 94-95 helpful.
ECE 656 Homework (Week 2) (continued)

7) Assume an ultra thin body (100) silicon structure with a thickness of 3 nm. Assume no bandbending within the structure and infinitely high energy barriers at the oxide-silicon interfaces. Compute and plot the 2D density of states vs. energy.