ECE 656 Homework (Week 3)
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1) Let \( \nu(\vec{p})E(\vec{p}) \) be the magnitude of the “energy flux” of a beam of electrons with initial momentum, \( \vec{p} = p_z \hat{z} \). Write down an expression for the energy flux relaxation time.

2) Assume that we have two independent scattering mechanisms, “one” and “two”. What is the average time between collisions?

3) For high energy electrons in semiconductors, the scattering rate may be on the order of \( 10^{14} \) per sec. Estimate the collisional broadening. \( \Delta E \).

4) In Lecture 8, we worked out the scattering rate for 3D electrons with a short range scattering potential of \( U_s(\vec{r}) = C\delta(0) \). Repeat the calculations for 2D electrons with a short range scattering potential.

5) Assume a small scattering potential as shown below and assume that electrons are free to move only in the \( z \)-direction.

   a) Work out an expression for the transition rate, \( S(\vec{p} \to \vec{p}') \), for 1D electrons using Fermi’s Golden Rule. Be sure to normalize the wavefunction over a length, \( L \).

   b) An incident electron with crystal momentum, \( \vec{p} \), can only make a transition to one different state, \( \vec{p}' \). What is that state?

   c) Explain what would happen if the sign of \( \Delta U \) were to change.

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\[ U_s(z) \]

\[ \vec{p} \]

\[ \Delta U \]

\[ -\frac{W}{2} \]

\[ +\frac{W}{2} \]

\[ z \]
6) For unscreened Coulomb scattering, the transition rate, $S(\vec{p} \rightarrow \vec{p}')$, goes to infinity for small scattering angles. Explain why this occurs physically. Also explain in words how Conwell and Weisskopf avoid the singularity when integrating the transition rate.

7) Answer the following questions about Conwell-Weisskopf scattering.
   a) Show that the Conwell-Weisskopf scattering rate is
      \[ \frac{1}{\tau_m(E - E_C)} = N_I \pi b_{max}^2 \sqrt{\frac{2m^*}{m^*}(E - E_C)} \]
      You may assume that $E_C = 0$
   b) Provide a simple, physical explanation for the Conwell-Weisskopf scattering rate in terms of the cross-section for scattering, $\pi b_{max}^2$.
   c) Evaluate and plot the scattering rate for electrons in GaAs with the thermal average energy. Compare the scattering rate with the momentum relaxation rate. You should plot $\tau_m/\tau$ vs. $N_I$ for $10^{14} < N_I < 10^{18}$ cm$^{-3}$. Explain in physical terms why $\tau_m$ and $\tau$ differ, and explain why the ratio, $\tau_m/\tau$, decreases with $N_I$.

8) Compute and compare the momentum relaxation times due to ionized impurity scattering under the following two circumstances. (Assume GaAs at $T_L = 300$ K and doped at $N_D = 10^{18}$ cm$^{-3}$.)
   a) Find $1/\tau_m$ for electrons the thermal average energy, $3k_B T_L/2$.
   b) Find $1/\tau_m$ for electrons the $E = 0.3$ eV. Such electrons can be produced by the heterojunction launching ramp shown in Fig. 3.2 of *Fundamentals of Carrier Transport*. 

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9) The screening length is an important parameter in the Brooks-Herring treatment of ionized impurity scattering. Assume n-type Si doped so that \( n_0 = 10^{20} \text{ cm}^{-3} \) and compare the screening lengths as computed from the non-degenerate expression (i.e. the Debye length) to the degenerate expression. You should assume \( T = 300 \text{ K} \) and a conduction band effective density-of-states of \( N_C = 3.23 \times 10^{19} \text{ cm}^{-3} \).