# ECE 656 Homework (Week 6)

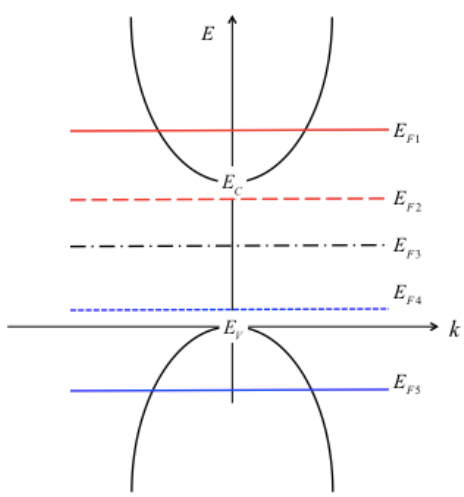
Mark Lundstrom
Purdue University

1) We have discussed $M(E)$ for a 3D semiconductor with parabolic energy bands. Answer the following two questions about a 3D semiconductor with non-parabolic energy bands.

   a) Assume that the non-parabolicity can be described by
   $$E(1+\alpha E)=\frac{\hbar^2 k^2}{2m^*(0)} .$$
   Derive an expression for the corresponding $M(E)$.

   b) Using the following numbers for GaAs,
   $m^*(0)=0.067\,m_0$
   $\alpha=0.64$,
   plot $M(E)$ from the bottom of the $\Gamma$ valley to $E=0.3$ eV comparing results from the non-parabolic expression derived in part a) to the parabolic expression.

2) The figure below shows a semiconductor with the Fermi level located in five different locations. If we use the Landauer expression to compute the current:
$$I=(2q/h)\int_{E_1}^{E_2}\mathcal{T}(E)M(E)(f_1-f_2)dE$$
what are appropriate limits of integration, $E_1$ and $E_2$, for each case? You may assume room temperature and a bandgap of 1 eV and that $E_{F1}\approx E_{F2}\approx E_F$.



**ECE 656 Homework (Week 6) (continued)**

3) Determine the limits of integration, $E_1$ and $E_2$, for the integral in the Landauer expression:

$$I = (2q/h)\int_{E_1}^{E_2} \mathcal{T}(E)M(E)(f_1 - f_2)dE$$

for the case of $T$ = 0 K. Assume that contact one is grounded and that a positive voltage (not necessarily small) has been applied to contact 2.

4) The ballistic conductance is often derived from a k-space treatment, which writes the current from left to right as

$$I^+ = \frac{1}{L}\sum_{k>0} q\upsilon_x f_0(E_{F1})$$

and the current from right to left as

$$I^- = \frac{1}{L}\sum_{k<0} q\upsilon_x f_0(E_{F2})$$

The net current is the difference between the two. In the ballistic limit, the Landauer expression for the current is

$$I = (2q/h)\int_{E_1}^{E_2} M(E)(f_1 - f_2)dE$$

4a) Assume parabolic energy bands, evaluate the net current from the k-space approach, and show that it is the same as the Landauer expression.

4b) Assume parabolic energy bands, but now assume **2D electrons**. Evaluate the net current from the k-space approach, and show that it is the same as the Landauer expression.

4c) Assume parabolic energy bands, but now assume 3D electrons. Evaluate the net current from the k-space approach, and show that it is the same as the Landauer expression.

**ECE 656 Homework (Week 6) (continued)**

5) The quantity,

$$\langle M \rangle = \int_{E_C}^{\infty} M(E)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

is the number of conduction band channels in the Fermi-Window. Answer the following questions.

5a) Evaluate $\langle M \rangle$ for an arbitrary temperature and location of the Fermi level assuming a 2D semiconductor with parabolic energy bands.

5b) Evaluate $\langle M \rangle$ for an arbitrary temperature and location of the Fermi level above the Dirac point, $E_D$, assuming graphene.

5c) Assume that $E_F = E_C + 0.1\,\text{eV} = E_D + 0.1\,\text{eV}$. For 5a), assume Si with $m^* = 0.19 m_0$ and a valley degeneracy of 2. For 5b), assume graphene parameters, $\upsilon_F = 10^8$ cm/s and a valley degeneracy of 2. Compare the numerical values of $\langle M \rangle$ for these two cases assuming $T$ = 300 K.

6) For a 3D diffusive resistor, we relate the current density to the electric field by

$\mathcal{E}_x = \rho_{3D} J_x$ V/m,

where $\mathcal{E}_x$ is the electric field in V/m and $J_x$ is the current density in A/m$^2$. Write the corresponding equations in 1D and 2D and determine the units of $\rho_{1D}$, $\rho_{2D} = \rho_S$, and $\rho_{3D}$.

**ECE 656 Homework (Week 6) (continued)**

7) The general expression for conductance,

$$G = \frac{2q^2}{h}\int \mathcal{T}(E) M(E)\left(-\frac{\partial f_0}{\partial E}\right) dE,$$

can be written as

$$G = \frac{2q^2}{h}\langle\langle \mathcal{T}(E)\rangle\rangle\langle M(E)\rangle.$$

For a 3D resistor in the diffusive limit,

$$G_{3D} = \frac{2q^2}{h}\langle\langle \lambda(E)\rangle\rangle\langle M(E)/A\rangle\frac{A}{L}.$$

**Derive** the general expressions for $\langle M(E)/A\rangle$ and for $\langle\langle \lambda(E)\rangle\rangle$ in terms of their energy-dependent quantities, $M(E)$ and $\lambda(E)$. HINT: Begin at the ballistic limit and determine $\langle M(E)/A\rangle$ first.

8) According to equ. (6.9) on p. 182 of *Advanced Semiconductor Fundamentals*, 2nd Ed., (R.F. Pierret, 2003) the mobility of electrons in bulk Si for $N_D = 10^{14}$ cm$^{-3}$ at $T$ = 300 K is 1268 cm$^2$/V-s and at $N_D = 10^{20}$ cm$^{-3}$ it is 95 cm$^2$/V-s. Assume an energy independent mean-free-path and **determine the mean-free-path**, $\lambda_0$ in both cases.

9) For solving problems involving 2D electrons in parabolic band semiconductors (i.e. as in the channel of a transistor), we can use the result in Lundstrom and Jeong, Appendix, p. 218, eqn. (A.30)

$$\sigma_S = \frac{2q^2}{h}\lambda_0 \frac{\sqrt{2m^* k_B T}}{\pi\hbar}\Gamma\left(r+\frac{3}{2}\right)\mathcal{F}_{r-1/2}(\eta_F)$$

where *r* is the characteristic exponent for scattering,

$$\lambda(E) = \lambda_0\left[(E - E_C)/k_B T\right]^r.$$

## ECE 656 Homework (Week 6) (continued)

Use this expression and answer the following two questions.

9a) Assume a constant mfp,
$\lambda(E) = \lambda_0$ (i.e. $r = 0$)

and **work out** an expression for the 2D mobility in terms of the mfp. Your results should be valid for any level of carrier degeneracy. **Simplify** your results for $T = 0$K and for non-degenerate conditions.

9b) Assume a "power law" mfp describe by
$\lambda(E - E_C) = \lambda_0 \left[ \left( E - E_C / k_B T \right) \right]^r$,

where "$r$" is a characteristic exponent that describes scattering. Repeat problem 4a) for this energy-dependent mfp. **Note:** for $T = 0$ K, only one energy matters, so it is best just to write $\lambda(E)$ and not use the power law form.

10) Assume an n-channel MOSFET at $T = 300$ K with $n_S = 10^{13}$ cm$^{-3}$. Assume that only the lowest subband is occupied and compute $\langle M_{2D} \rangle$, the average number of modes in the Fermi window per micrometer of channel width.

11) In 1D, we express the resistance of a long (diffusive) resistor by $R_{1D} = (1/\sigma_{1D}) L$. In 2D, we write $R_{2D} = (1/\sigma_{2D}) L/W$ and in 3D $R_{3D} = (1/\sigma_{3D}) L/A$. Assuming a degenerate conductor (i.e. $T = 0$ K), begin with $G_{ball} = \dfrac{2q^2}{h} M(E_F)$ and develop expressions for the 1D, 2D, and 3D "ballistic conductivities."

12) One can derive a near-equilibrium current equation for a 2D, n-type conductor in the diffusive limit and write it as $J_n = \sigma_S \, d(F_n/q)/dx$ A/m. Derive the corresponding equation for a p-type semiconductor.

13) Begin with $J_{nx} = \sigma_n \, d(F_n/q)/dx$ and derive the drift-diffusion equation for a 3D n-type semiconductor with parabolic energy bands. **Do not** assume Maxwell-Boltzmann statistics.

**ECE 656 Homework (Week 6) (continued)**

14) When we write the resistance as $R = R_{\text{ball}}\left(1 + L/\lambda_0\right)$, we assume a constant (energy-independent) mean-free-path. What is the corresponding expression for an energy dependent mean-free-path, $\lambda(E)$? Is a plot of resistance vs. length of the resistor a straight line?