ECE 656 Homework (Week 7)

Mark Lundstrom
Purdue University

1) This homework exercise will help you become familiar with how B –fields affect transport

Consider the equation of motion for an average electron,

$$\vec{F}_e = -q\vec{\mathcal{E}} - q\vec{\upsilon}\times\vec{B} = \frac{d\vec{p}}{dt} \quad . \tag{i}$$

Assume that the electron moves for a time, τ_m, then scatters, returning the average momentum to zero, so

$$\frac{dp}{dt} = -\frac{p}{\tau_m} \quad . \tag{ii}$$

Assuming that $\vec{p} = m^*\vec{\upsilon}$, we find an equation for the average velocity as

$$\vec{\upsilon} = -\frac{q\tau}{m^*}\vec{\mathcal{E}} - \frac{q\tau}{m^*}\vec{\upsilon}\times\vec{B} \, . \tag{iii}$$

This equation can be solved exactly for the velocity (see prob. 4.18, Lundstrom, *Fundamentals of Carrier Transport*, 2000), but let's take a different approach.

1a) Assume carriers move in 2D and that only a z-directed B-field is present. Evaluate eqn. (iii) and find two coupled equations for υ_x and υ_y .

1b) Solve the two equations for υ_x and υ_y in terms of the electric field and the B-field.

1c) Write the current densities as

$$J_x = -n_S q \upsilon_x \tag{iva}$$
$$J_y = -n_S q \upsilon_y \tag{ivb}$$

and use the results of problem 5b) to find the current densities as

ECE 656 Homework (Week 7) (continued)

$$J_x = -nq\upsilon_x = \frac{\sigma_n}{1+(\mu_n B_z)^2}\left(\mathcal{E}_x - \mu_n B_z \mathcal{E}_y\right) \tag{va}$$

$$J_y = -nq\upsilon_y = \frac{\sigma_n}{1+(\mu_n B_z)^2}\left(\mathcal{E}_y + \mu_n B_z \mathcal{E}_x\right), \tag{vb}$$

which can also be written as

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \frac{\sigma_n}{1+(\mu_n B_z)^2}\begin{bmatrix} 1 & -\mu_n B_z \\ \mu_n B_z & 1 \end{bmatrix}\begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix} \tag{via}$$

or as

$$J_i = \sigma_{ij}(B_z)\mathcal{E}_j . \tag{vib}$$

Note that the magnetic field affects both the diagonal and off-diagonal components of the magnetoconductivity tensor. **Explain** why there is no Hall factor, r_H , in the result.

1d) Solve eqn. (via) for the electric field and show that

$$\begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix} = \frac{1}{\sigma_n}\begin{bmatrix} 1 & \mu_n B_z \\ -\mu_n B_z & 1 \end{bmatrix}\begin{pmatrix} J_x \\ J_y \end{pmatrix} \tag{vii}$$

According to eqn. (vii), the longitudinal magnetoresistivity is independent of the *B*-field (while the longitudinal magnetoconductivity depends on *B* as shown in eqn. (via). Equation (vii) shows that the Hall voltage is proportional to *B*.

1e) Show that for small B-fields, eqn. (via) can be written as

$$\vec{J}_n = \sigma_n \vec{\mathcal{E}} - (\sigma_n \mu_n)\vec{\mathcal{E}} \times \vec{B} \tag{vii}$$

Note that while this analysis is simpler than solving the BTE, by beginning with an average electron with an average momentum, $\vec{p}$, we have missed the averaging of the distribution of momenta which leads to a non-unity Hall factor, r_H .

ECE 656 Homework (Week 7) (continued)

2) Hall factors are important to consider when analyzing experiments. Answer the following questions.

2a) Assuming parabolic energy bands, derive an expression for the Hall factor in 3D and show that for ionized impurity scattering, it gives $r_H = 1.93$.

2b) Assume parabolic energy bands and develop an expression for the Hall factor in 2D.

2c) Ionized impurity scattering in graphene is said to vary as *E*. What is the Hall factor for graphene?