SOLUTIONS: ECE 656 Homework (Week 8)

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1) For electrons, the bandstructure is a plot of energy, $E(\vec{k})$, vs. wavevector, $\vec{k}$. For phonons, the dispersion is a plot of phonon energy, $\hbar\omega(\vec{q})$, vs. phonon wavector, $\vec{q}$. For electrons, we <u>often</u> approximate the bandstructure with simple, parabolic bands,

$$E(\vec{k}) = \frac{\hbar^2 k^2}{2m^*}$$

For phonons, we can <u>sometimes</u> approximate the phonon dispersion with the Debye approximation,

$$\hbar\omega = \hbar\upsilon_D q ,$$

where υ_D is the Debye velocity (an average of the longitudinal and transverse acoustic velocities.)

1a) Compute the density-of-states, $D_{ph}(\hbar\omega)$, for phonons in the Debye model.

Solution:

Equate the DOS in q-space to energy space:

$$\frac{1}{\Omega} N_q dq = D_{ph}(\hbar\omega) d(\hbar\omega) \qquad \text{(i)}$$

$$\frac{1}{\Omega} N_q dq = \frac{1}{8\pi^3} \times 3(4\pi q^2) dq = D_{ph}(\hbar\omega) d(\hbar\omega) \qquad \text{(ii)}$$

Note that there is no factor of 2 for spin in this case, but we have a factor of three because of the three polarizations, longitudinal and two transverse acoustic phonons.

From the dispersion,

$$\hbar\omega = \hbar\upsilon_D q \qquad \text{(iii)}$$

we have

$$q^2 = \left(\frac{\hbar\omega}{\hbar\upsilon_D}\right)^2 \qquad \text{(iv)}$$

and

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$$dq = \frac{d(\hbar\omega)}{\hbar\upsilon_D} \qquad \text{(v)}$$

Using (iv) and (v) in (ii), we find

$$D_{ph}(\hbar\omega)d(\hbar\omega) = \frac{3}{2\pi^2}q^2 dq = \frac{3}{2\pi^2}\left(\frac{\hbar\omega}{\hbar\upsilon_D}\right)^2 \frac{d(\hbar\omega)}{\hbar\upsilon_D} \qquad \text{(vi)}$$

so the final answer is

$$\boxed{D_{ph}(\hbar\omega) = \frac{3(\hbar\omega)^2}{2\pi^2(\hbar\upsilon_D)^3} \quad \left(\text{J-m}^3\right)^{-1}} \qquad \text{(vii)}$$

1b) Compute the distribution of channels, $M_{ph}(\hbar\omega)$, for phonons in the Debye model.

Solution:

Begin with the definition:

$$M_{ph}(\hbar\omega) = \frac{h}{2}\left\langle \upsilon_x^+ \right\rangle D_{ph}(\hbar\omega) \qquad \text{(viii)}$$

From the dispersion, (iii), we find

$$\upsilon(\hbar\omega) = \upsilon_D \qquad \text{(ix)}$$

and the average over angles in 3D gives

$$\left\langle \upsilon_x^+ \right\rangle = \frac{\upsilon_D}{2} \qquad \text{(x)}$$

Now using (x) in (viii), we find

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$$M_{ph}\left(\hbar\omega\right) = \frac{h}{2}\left\langle \upsilon_x^+ \right\rangle D_{ph}\left(\hbar\omega\right) = \frac{h}{2}\frac{\upsilon_D}{2}\frac{3\left(\hbar\omega\right)^2}{2\pi^2\left(\hbar\upsilon_D\right)^3} \qquad \text{(xi)}$$

$$\boxed{M_{ph}\left(\hbar\omega\right) = \frac{3\left(\hbar\omega\right)^2}{4\pi\left(\hbar\upsilon_D\right)^2}}$$