ECE 656 Homework (Week 9)

Mark Lundstrom
Purdue University

1) We have asserted that $\Delta_n = 2k_BT$ for a non-degenerate, 3D semiconductor with parabolic energy bands and an energy-independent mean-free-path for backscattering. This means that the average energy at which current flows is $2k_BT$ above the bottom of the conduction band. Repeat the calculation, but this time assume power law scattering,

$$\lambda(E) = \lambda_0\left[(E - E_C)/(k_BT)\right]^r .$$

What is Δ_n in this case?

2) Repeat prob. 1) in the strongly degenerate limit, and use the result to explain why the Seebeck coefficient of a metal approaches zero.

3) For practical TE devices, the semiconductor is doped so that $E_F \approx E_C$. Work out the four thermoelectric transport coefficients for n-type Ge doped at $N_D = 10^{19}$ cm^{-3}. You may assume that T = 300 K, that the dopants are fully ionized, and that the mean-free-path for backscattering, λ_0, is independent of energy.

Use the following material parameters:

$$T = 300 \text{ K}$$
$$N_C = 1.04\times 10^{19} \quad \text{cm}^{-3}$$
$$\mu_n = 330 \text{ cm}^2/\text{V-s}$$
$$m^* = 0.12 m_0$$

You may assume **non-degenerate carrier statistics** (but realize that this assumption may not well-justified for $E_F \approx E_C$, which is the case here, so we will only obtain estimates). Work out approximate, numerical values for λ_0, ρ, S, π, and κ_e.

ECE 656 Homework (Week 9) (continued)

4) Perhaps we should use Fermi-Dirac statistics for thermoelectric calculations when $E_F \approx E_c$. Repeat problem 3), but this time use Fermi-Dirac statistics to determine the approximate values of λ_0, ρ, S, π, and κ_e. You might find it useful to know that

$$\sigma_{3D} = \frac{2q^2}{h}\lambda_0\left(\frac{g_V m^* k_B T}{2\pi\hbar^2}\right)\mathcal{F}_0\left(\eta_F\right) \text{ and } S = -\left(\frac{k_B}{q}\right)\left\{\frac{2\mathcal{F}_1\left(\eta_F\right)}{\mathcal{F}_1\left(\eta_F\right)} - \eta_F\right\}$$

5) We have discussed two different electronic thermal conductivities – one measured under short circuit conditions, κ_0, and one measured under open circuit conditions, κ_e. The two are related according to:

$$\kappa_e = \kappa_0 - T\sigma S^2$$

Using the estimated TE transport coefficients for Ge doped such that $E_F \approx E_C$ (from prob. 4) find the numerical value of the ratio, κ_0/κ_e.

6) Using the results of prob. 4), estimate the thermoelectric FOM, *ZT* for n-type Ge at T = 300 K. You may assume that $\kappa_L = 58\ \text{W/m-K}$.

7) This problem concerns the Peltier coefficient for a 3D semiconductor with parabolic energy bands. Assuming that the MFP, λ_0, is independent of energy and show that the Peltier coefficient is:

$$\pi_{3D} = TS_{3D} = \left(\frac{k_B T}{-q}\right)\left(\frac{2\mathcal{F}_1\left(\eta_F\right)}{\mathcal{F}_0\left(\eta_F\right)} - \eta_F\right).$$

8) The expression for the short circuit (electronic) thermal conductivity is:

$$\kappa_0 = \int_{-\infty}^{+\infty}\frac{\left(E - E_F\right)^2}{q^2 T}\sigma'\left(E\right)dE$$

where $\sigma'(E)$, the differential conductivity, is given by

$$\sigma'\left(E\right) = \frac{2q^2}{h}\lambda\left(E\right)\left(M\left(E\right)/A\right)\left(-\frac{\partial f_0}{\partial E}\right).$$

ECE 656 Homework (Week 9) (continued)

Evaluate this expression assuming that the Fermi level is located above the middle of the gap, so that only the conduction band need be considered. You may assume that the mean-free-path for backscattering is independent of energy, $\lambda(E)=\lambda_0$, and parabolic energy bands so that in 3D:

$$M(E)/A=\frac{m^*}{2\pi\hbar^2}(E-E_C)H(E-E_C),$$

where $H(E-E_C)$ is the Heaviside step function.

Your answer should be expressed in terms of Fermi-Dirac integrals. Your final answer should be an expression for the short-circuit thermal conductivity of 3D electrons in a semiconductor with parabolic energy bands in terms of the normalized Fermi energy, $\eta_F=(E_F-E_C)/k_BT_L$.

9) An appreciation of the coupled current equations is necessary when experimentally characterizing electronic materials. The basic equations are:

$$\mathcal{E}_x=\rho J_x+S\frac{dT}{dx}\ \text{V/m} \qquad \text{(i)}$$

$$J_{Qx}=\pi J_x-(\kappa_e+\kappa_L)\frac{dT}{dx}\ \text{W/m}^2 \qquad \text{(ii)}$$

To measure the resistivity of the sample, we force a current, J_x, and measure the resulting voltage. In the first case, we are careful to maintain isothermal conditions, and in the second case, we are careful to maintain adiabatic (zero heat current) conditions. Answer the following questions.

9a) If we divide the measured voltage by the injected current, what "isothermal resistivity" do we measure. (First case.)

9b) If we divide the measured voltage by the injected current, what "adiabatic resistivity" do we measure. (Second case.)

9c) Using numbers for lightly doped Ge at room temperature:

$$\rho_n=2\quad \Omega\text{-cm}=0.02\quad \Omega\text{-m}$$

$$S_n=-970\quad \mu\text{V/K}$$

10) We have seen a lot of equations so far, but the course is not about memorizing equations. With a solid understanding of the physical concepts, only a few equations are needed. On one sheet (front only, font size 12) summarize the key equations describing near-equilibrium transport. The point is not to write down every equation, the point is to identify the few, really important results from which you can derive anything else you need.