Purdue University, Spring 2019

Solar Cell Fundamentals

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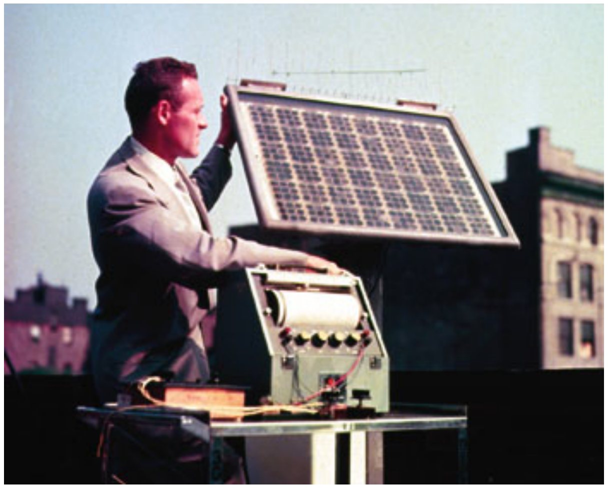
The modern solar cell

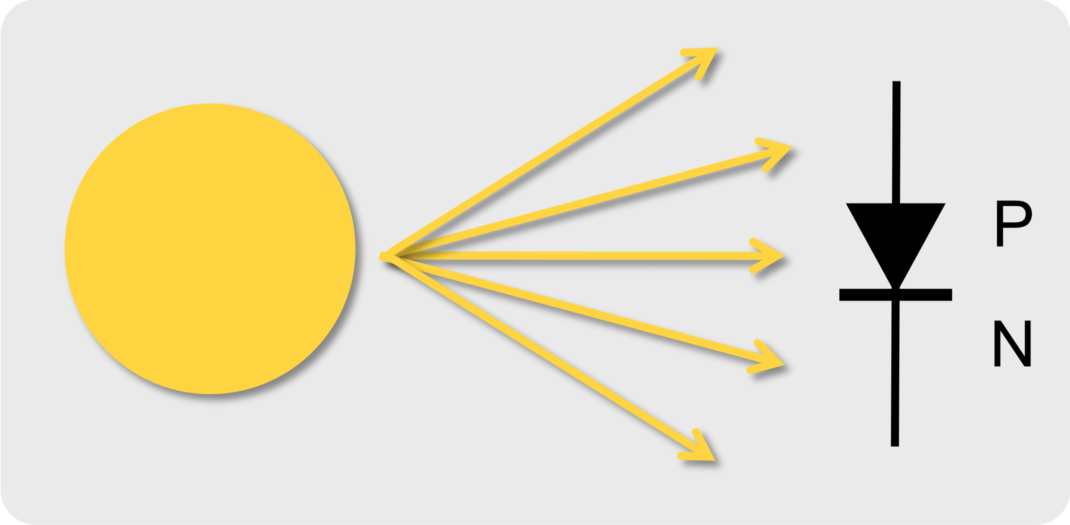


Chapin, Pearson, and Fuller, Bell Labs, 1954

http://www.bell-labs.com/org/physicalsciences/timeline/span10.html#

Solar cells

A solar cell is a junction (usually a PN junction) with sunlight shining on it.



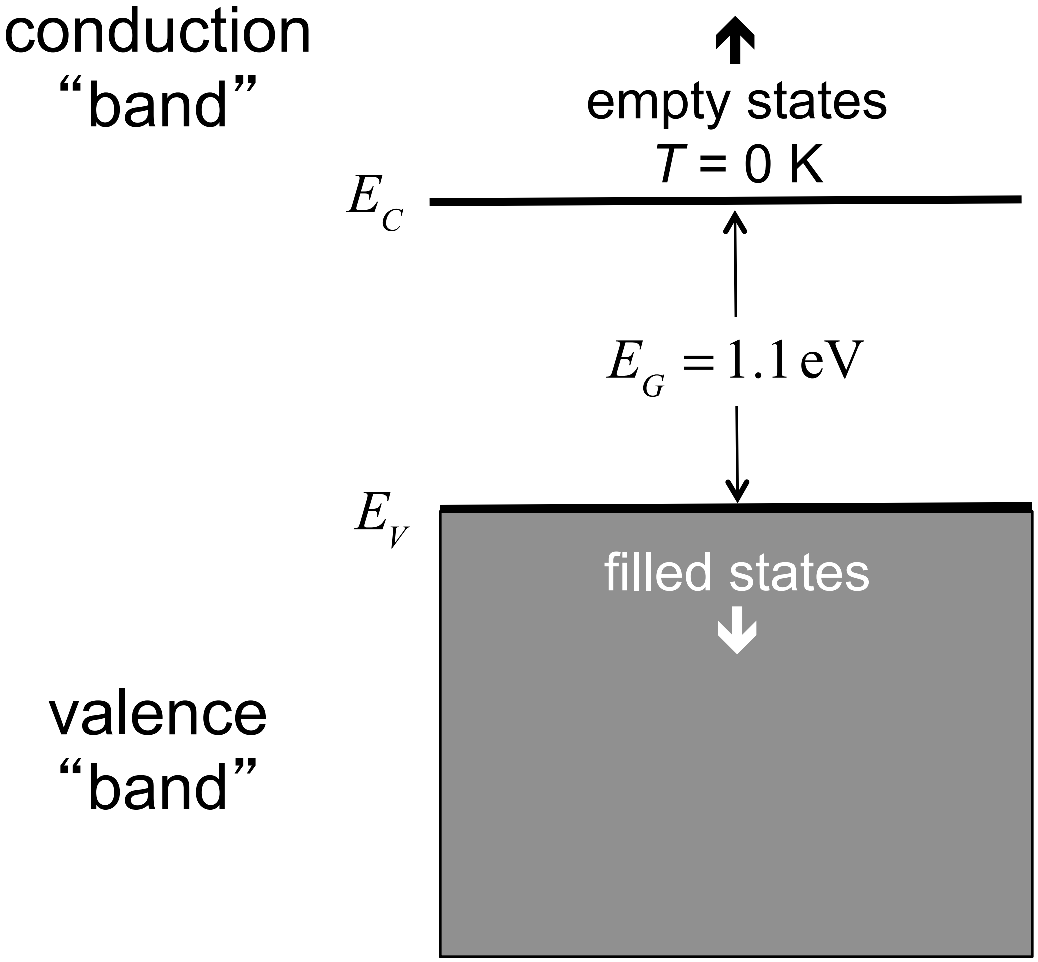
To understand how a solar cell works, we need to understand:

1) how a PN junction works (in the dark)
2) how light is absorbed in a semiconductor (without a PN junction)
3) what happens when we put the two together.

Outline

1) Introduction
2) **Review of semiconductor physics**
3) PN junctions in equilibrium
4) PN junctions under forward bias
5) Optical absorption / e-h generation
6) PN junctions under illumination
7) Solar cell parameters
8) Summary

Silicon energy bands



conduction
“band”

empty states

$T = 0$ K

E_C

$E_G = 1.1\,\text{eV}$

E_V

filled states

valence
“band”

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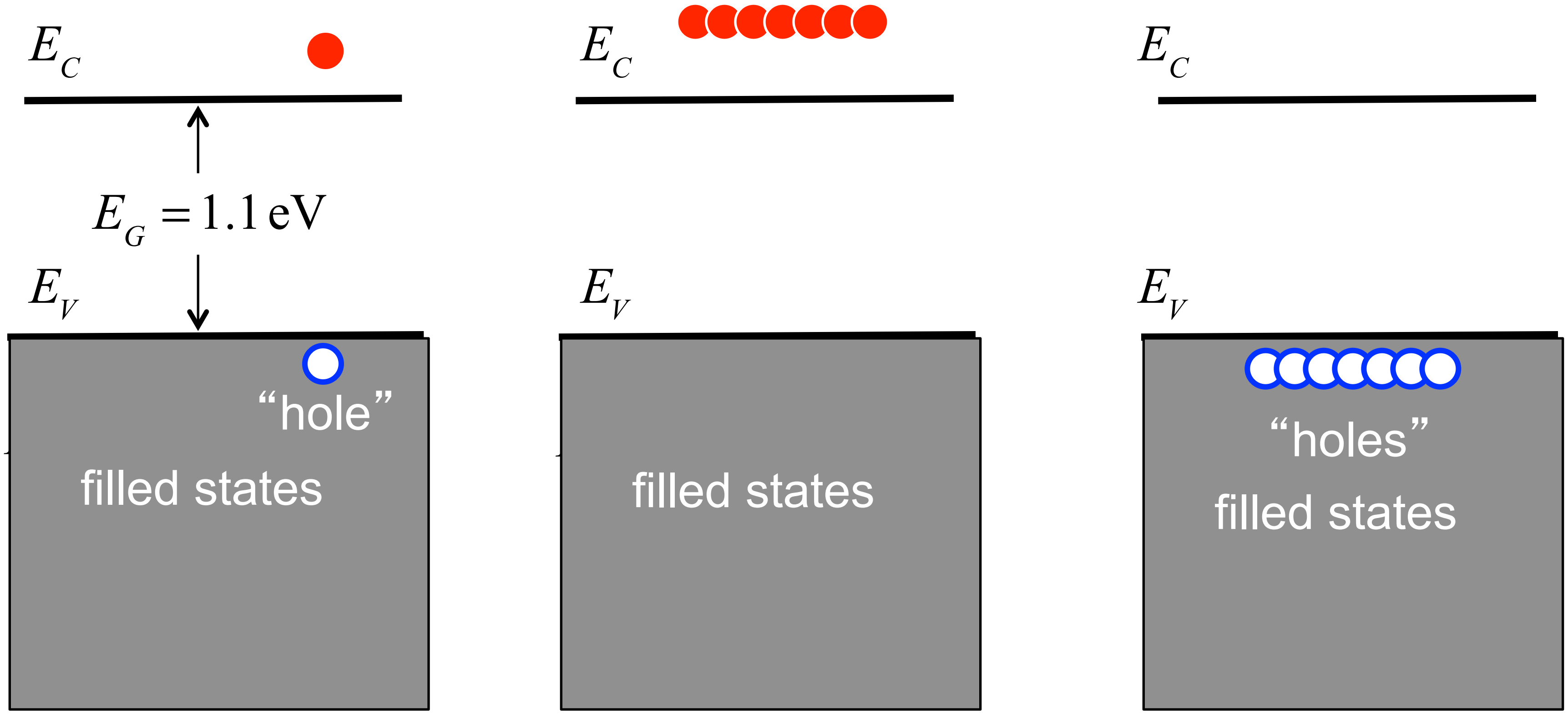
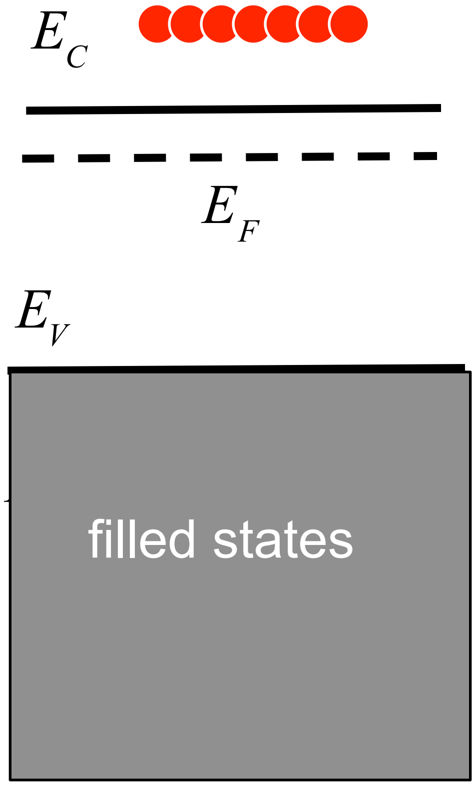
Intrinsic	N-type	P-type
$n_0 = p_0 = n_i$	$n_{0N} \approx N_D^+$	$p_{0P} \approx N_A^-$



Fermi level (electrochemical potential)

$$n_{0N} \approx N_D^+$$

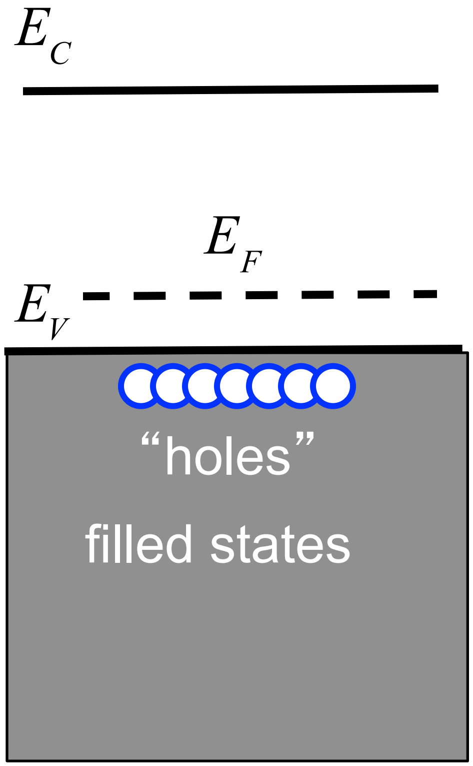


$$f_0(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}}$$

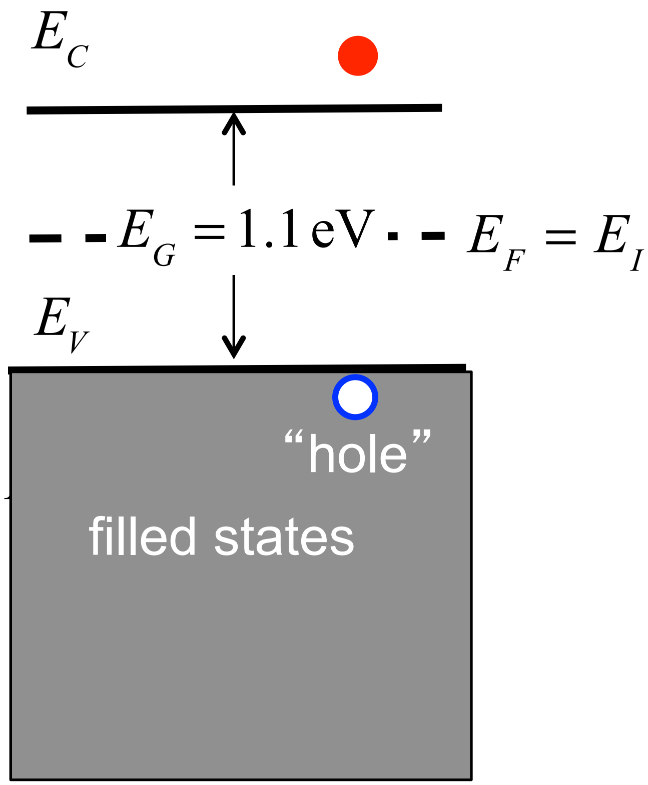
$$E << E_F \rightarrow f_0(E) = 1$$

$$E >> E_F \rightarrow f_0(E) = 0$$

$$p_{0P} \approx N_A^-$$



Intrinsic semiconductor



$$n_0 = p_0 = n_i$$

$$n_0 p_0 = n_i^2$$

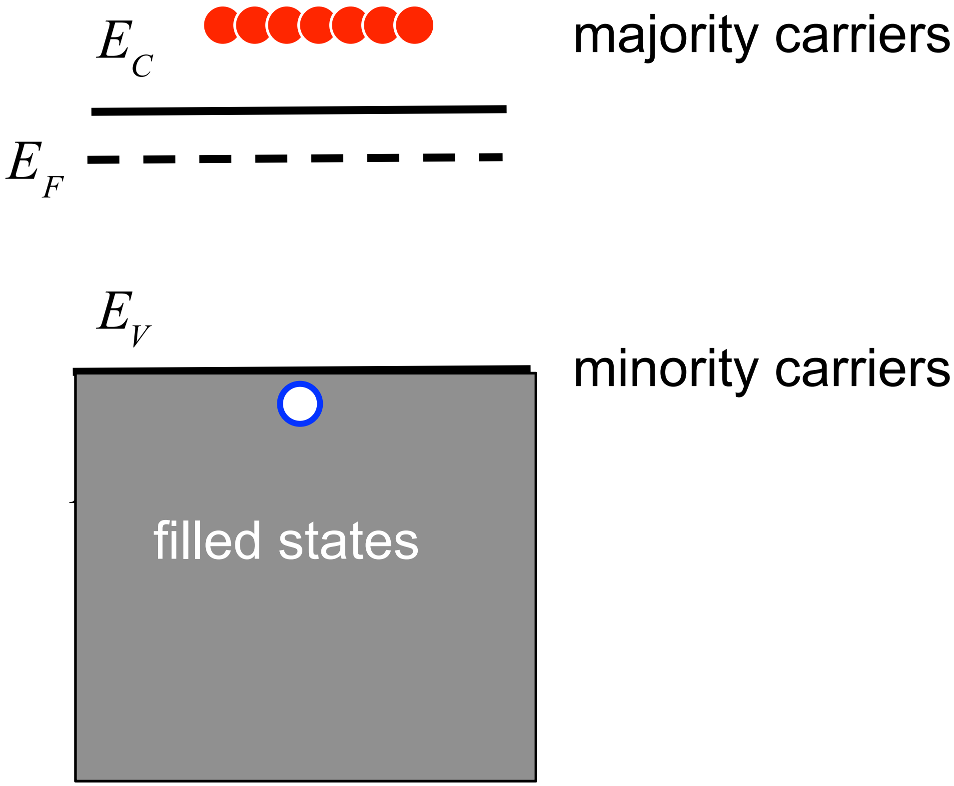
$$n_i^2 \propto e^{-E_G/k_B T}$$

silicon

$$n_i(300\text{ K}) \approx 10^{10}\text{ cm}^{-3}$$

N-type semiconductor

$$n_{0N} \approx N_D^+$$



$$n_0 \approx N_D^+$$

$$n_0 p_0 = n_i^2$$

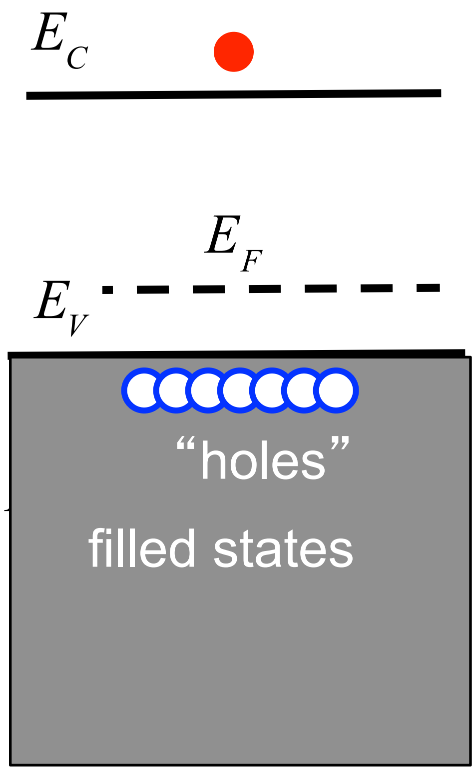
$$p_0 = n_i^2 / N_D^+$$

$$n_0 = N_D \approx 10^{17}\ \mathrm{cm}^{-3}$$

$$n_i(300\ \mathrm{K}) \approx 10^{10}\ \mathrm{cm}^{-3}$$

$$p_0 \approx 10^{3}\ \mathrm{cm}^{-3}$$

P-type semiconductor



minority carriers

majority carriers

$$p_0 \approx N_A^-$$

$$n_0 p_0 = n_i^2$$

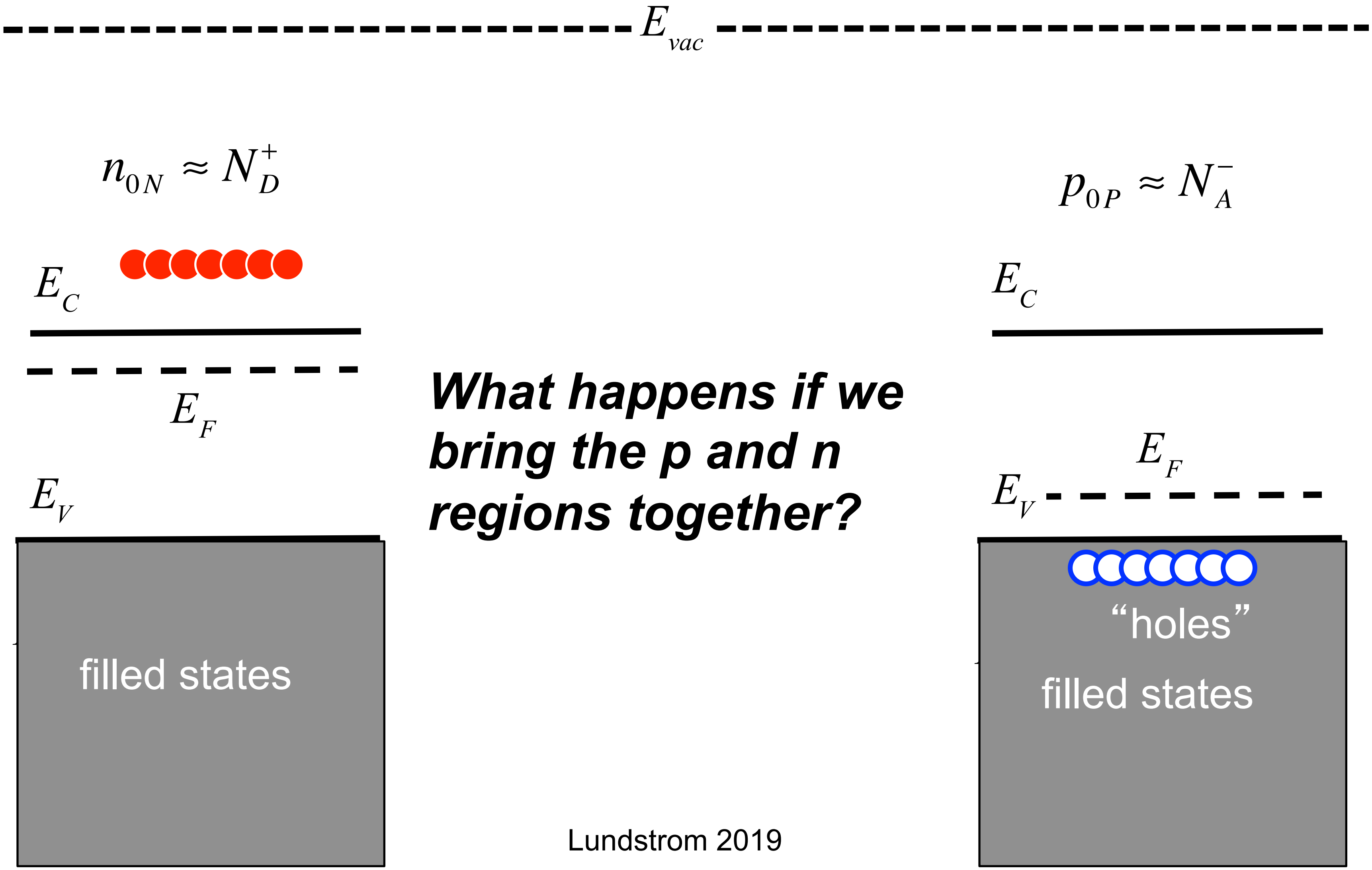
$$n_0 = n_i^2 / N_A^-$$

$$p_0 = N_A \approx 10^{17}\ \text{cm}^{-3}$$

$$n_i(300\ \text{K}) \approx 10^{10}\ \text{cm}^{-3}$$

$$n_0 \approx 10^{3}\ \text{cm}^{-3}$$

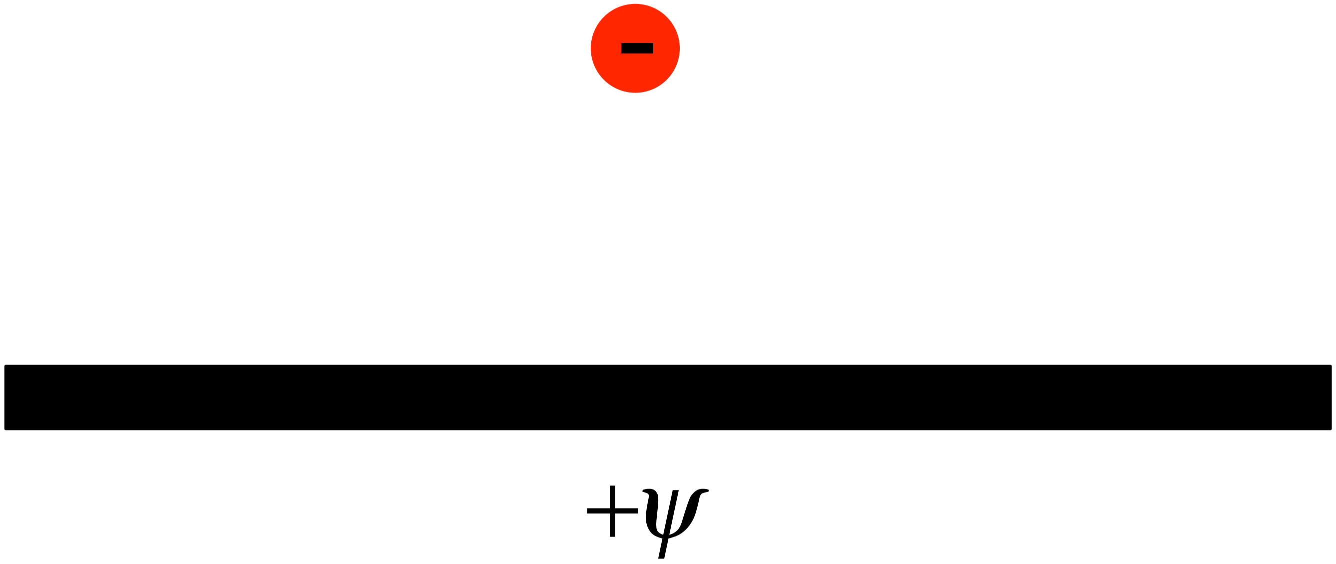
PN junction



What happens if we bring the p and n regions together?

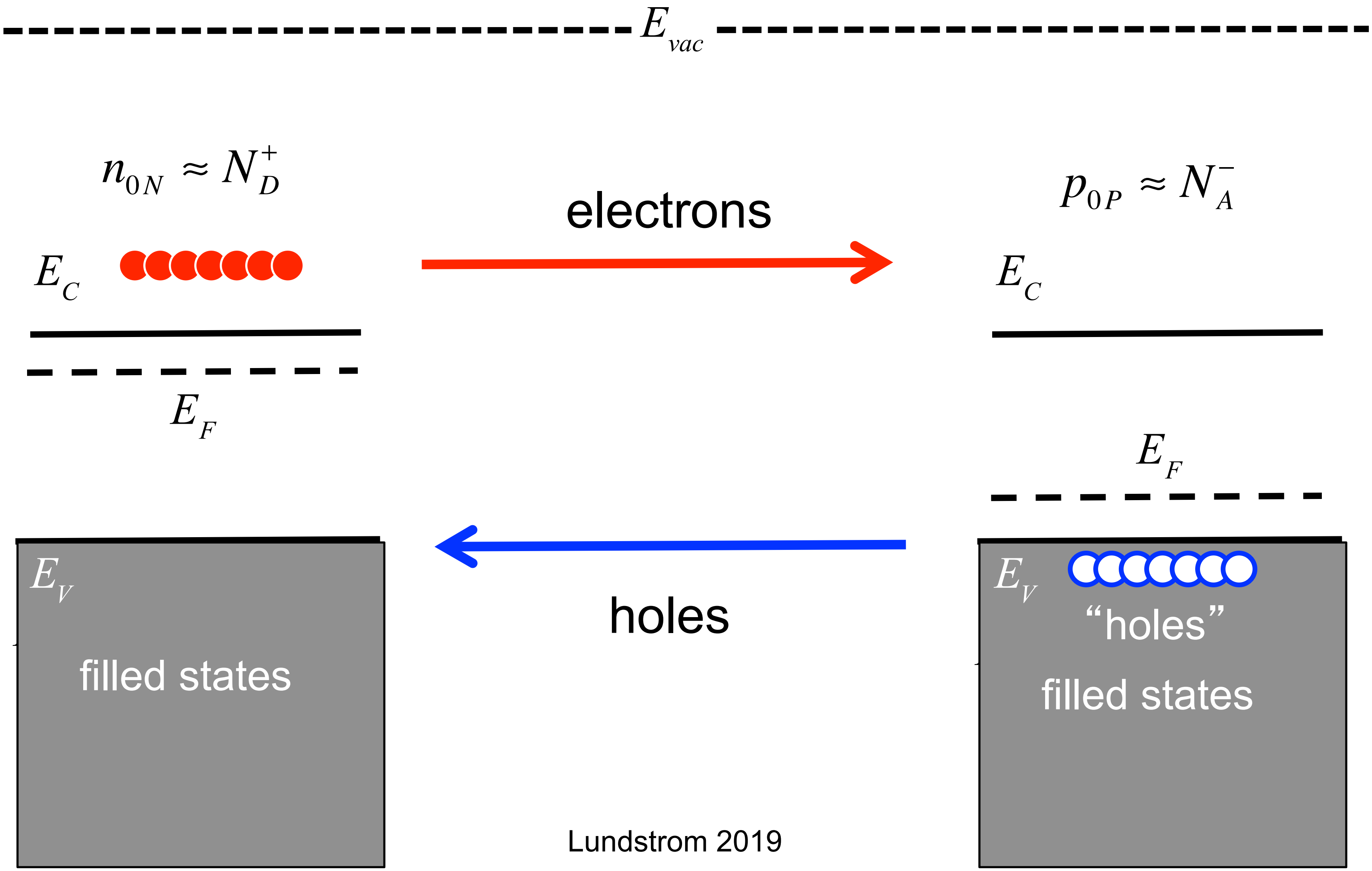
Electrostatic potential and electron energy

$$E = -q\psi$$



A positive potential **lowers** the energy of an electron.

PN junction: charge transfer



E_{vac}

$n_{0N} \approx N_D^+$

electrons

$p_{0P} \approx N_A^-$

E_C

E_C

E_F

E_F

E_V

E_V

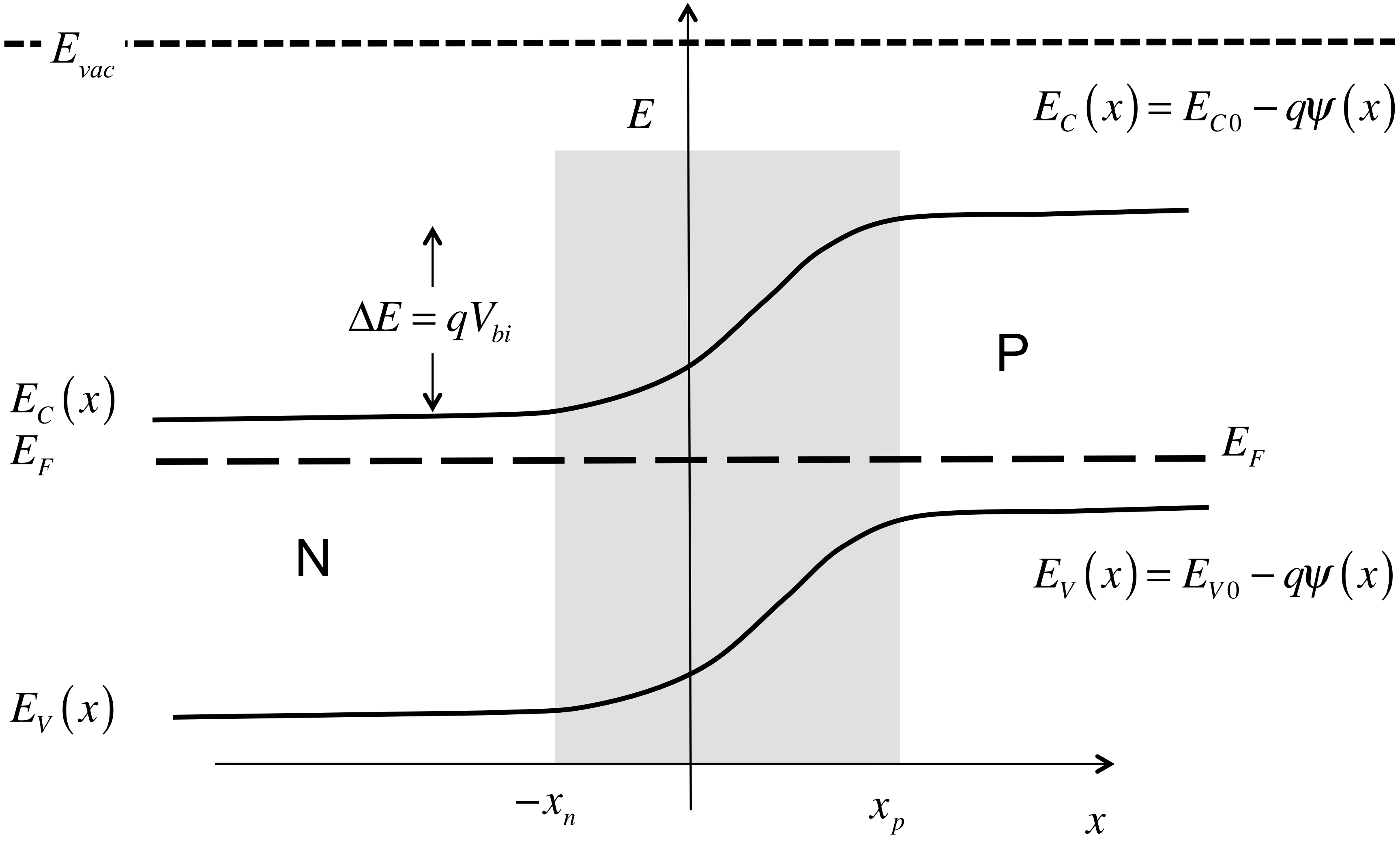
“holes”

holes

filled states

filled states

Equilibrium PN junction E-band diagram



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“Reading” an E-band diagram

$$E_C(x) = E_{C0} - q\psi(x)$$

$$E_V(x) = E_{V0} - q\psi(x)$$

$$\psi(x) \propto -E_C(x)$$

$$\psi(x) \propto -E_V(x)$$

$$\mathcal{E}(x) \propto dE_C(x)/dx$$

$$\mathcal{E}(x) \propto dE_V(x)/dx$$

$$n_0(x) \propto e^{(E_F - E_C(x))/k_B T}$$

$$p_0(x) \propto e^{(E_V(x) - E_F)/k_B T}$$

Energy band diagrams

Kroemer’s lemma of proven ignorance:

“If, in discussing a semiconductor problem, you cannot draw an **Energy Band Diagram**, this shows that you don’t know what you are talking about.”

corollary:

“If you can draw one, but don’t, then your audience won’t know what you are talking about.”

Herbert Kroemer, Nobel Lecture, Dec. 8, 2000.

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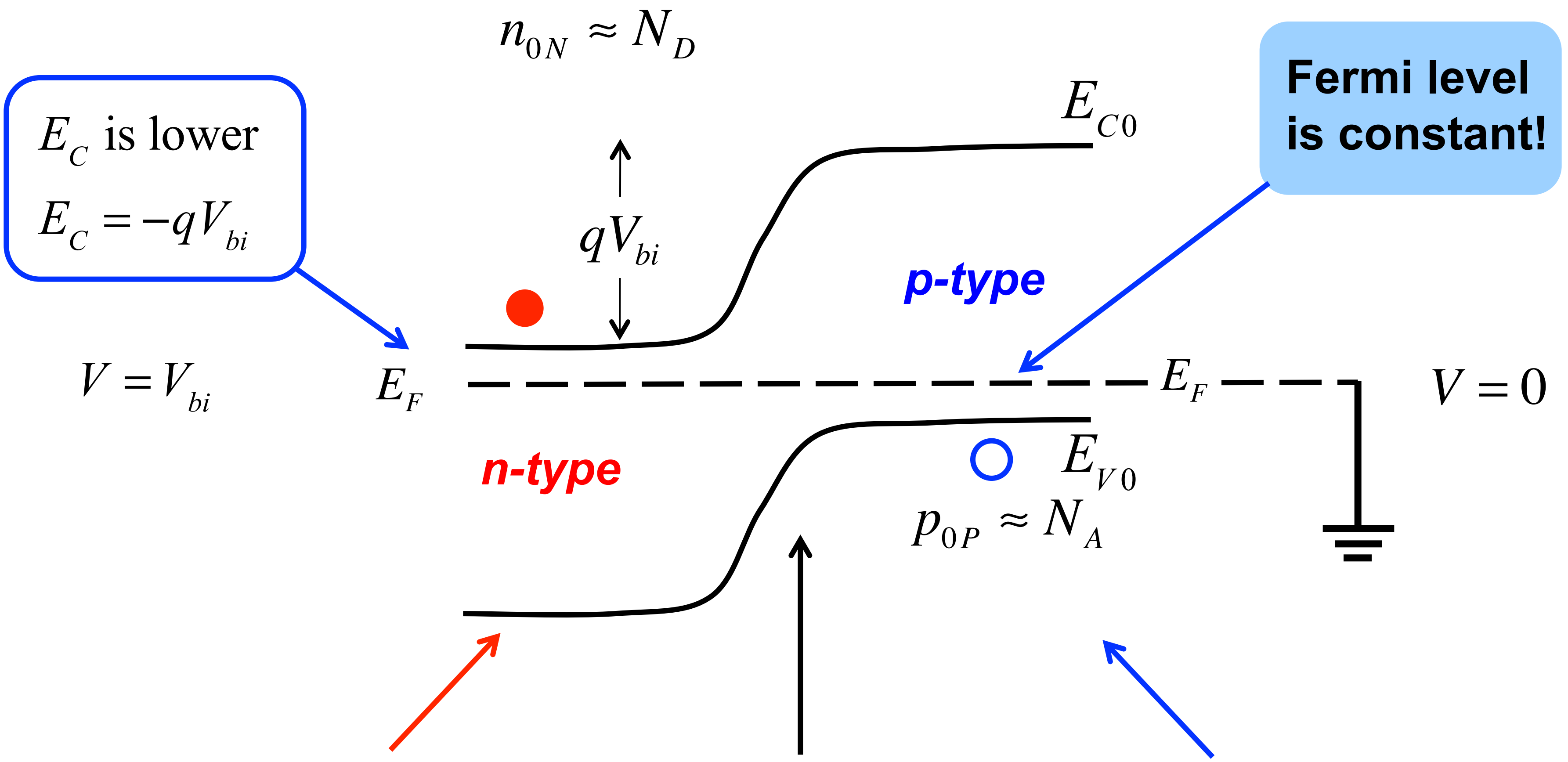
PN junction in equilibrium



E_C is lower

$E_C = -qV_{bi}$

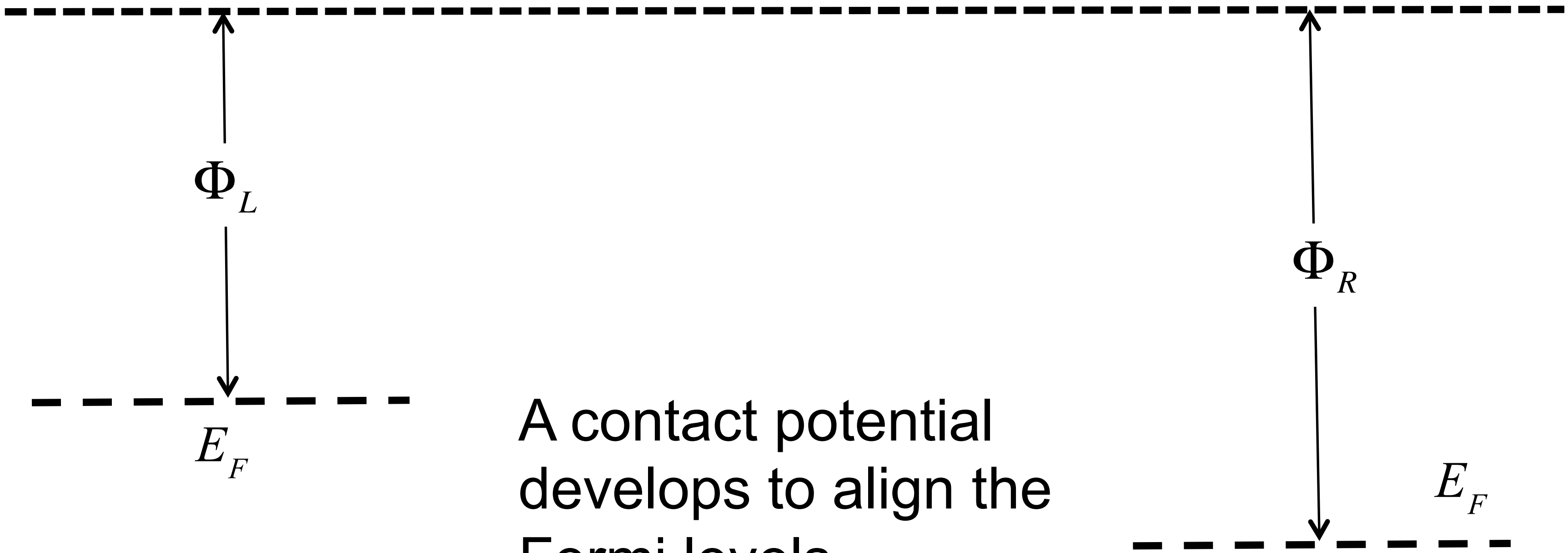
Fermi level is constant!

no electric field

strong electric field

no electric field

Contact potentials

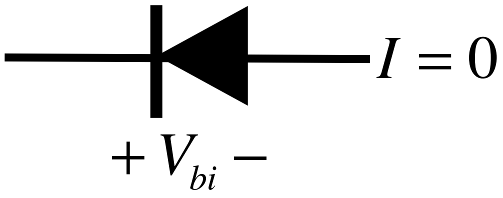


A contact potential develops to align the Fermi levels

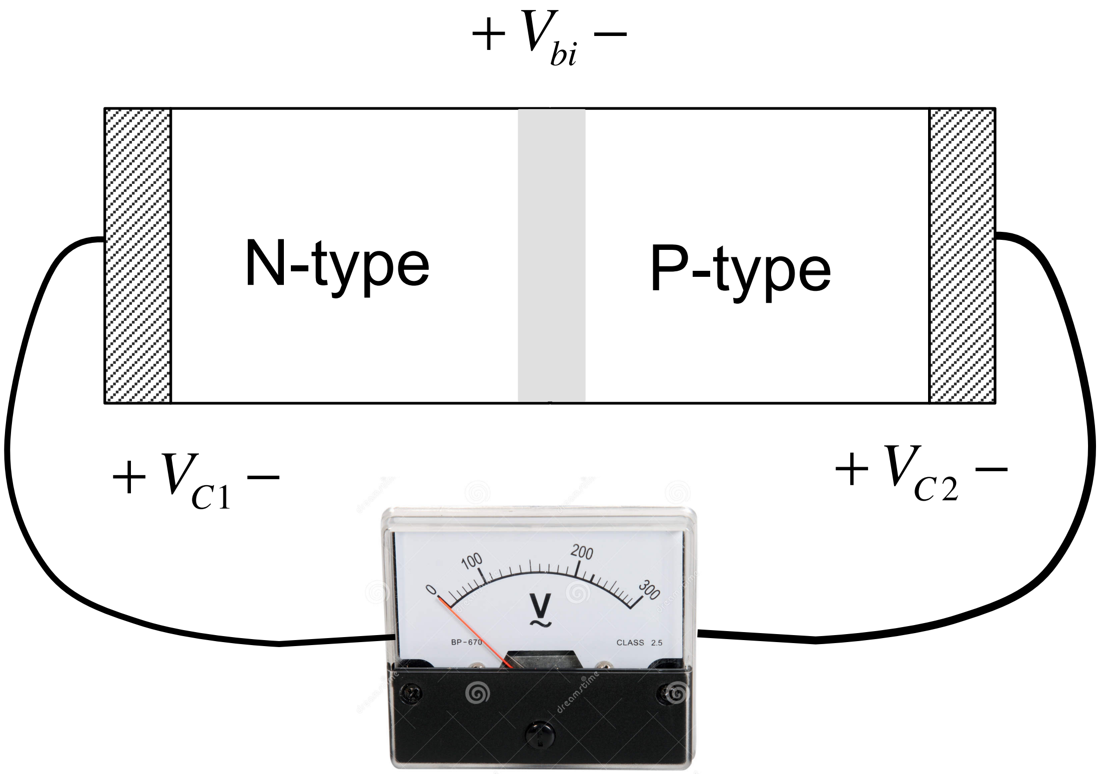
small workfunction

large workfunction

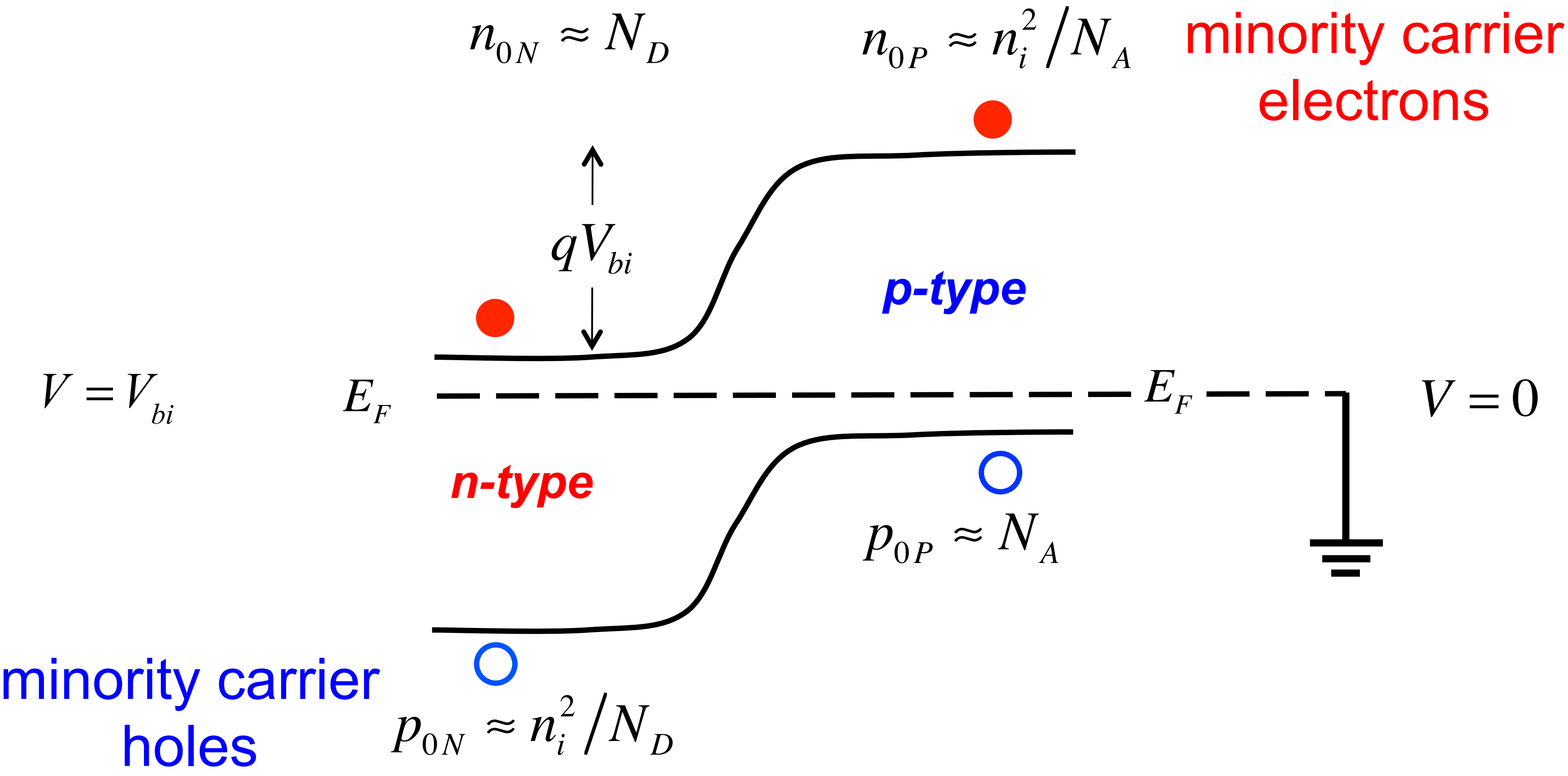
Built-in potential



$$\boxed{V_{bi} = \frac{k_B T}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)}$$



Minority carriers



$n_{0N} \approx N_D$

$n_{0P} \approx n_i^2 / N_A$

minority carrier electrons

qV_{bi}

p-type

$V = V_{bi}$

E_F

E_F

$V = 0$

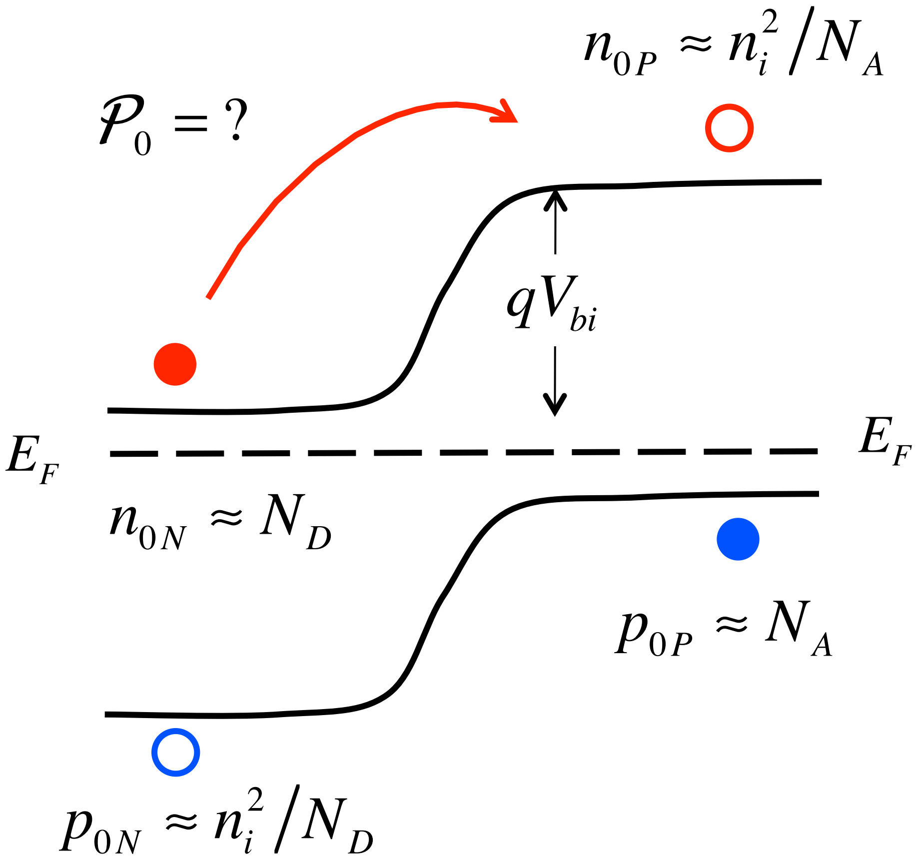
n-type

$p_{0P} \approx N_A$

minority carrier holes

$p_{0N} \approx n_i^2 / N_D$

Injection across a barrier



$$n_{0N}\mathcal{P}_0 = n_{0P}1$$

$$\mathcal{P}_0 = \frac{n_{0P}}{n_{0N}}$$

$$\mathcal{P}_0 = \frac{e^{(E_F - E_{CP})/k_BT}}{e^{(E_F - E_{CN})/k_BT}}$$

$$\mathcal{P}_0 = e^{-qV_{bi}/k_BT}$$

$$\boxed{\mathcal{P}_0 = e^{-\Delta E/k_BT}}$$

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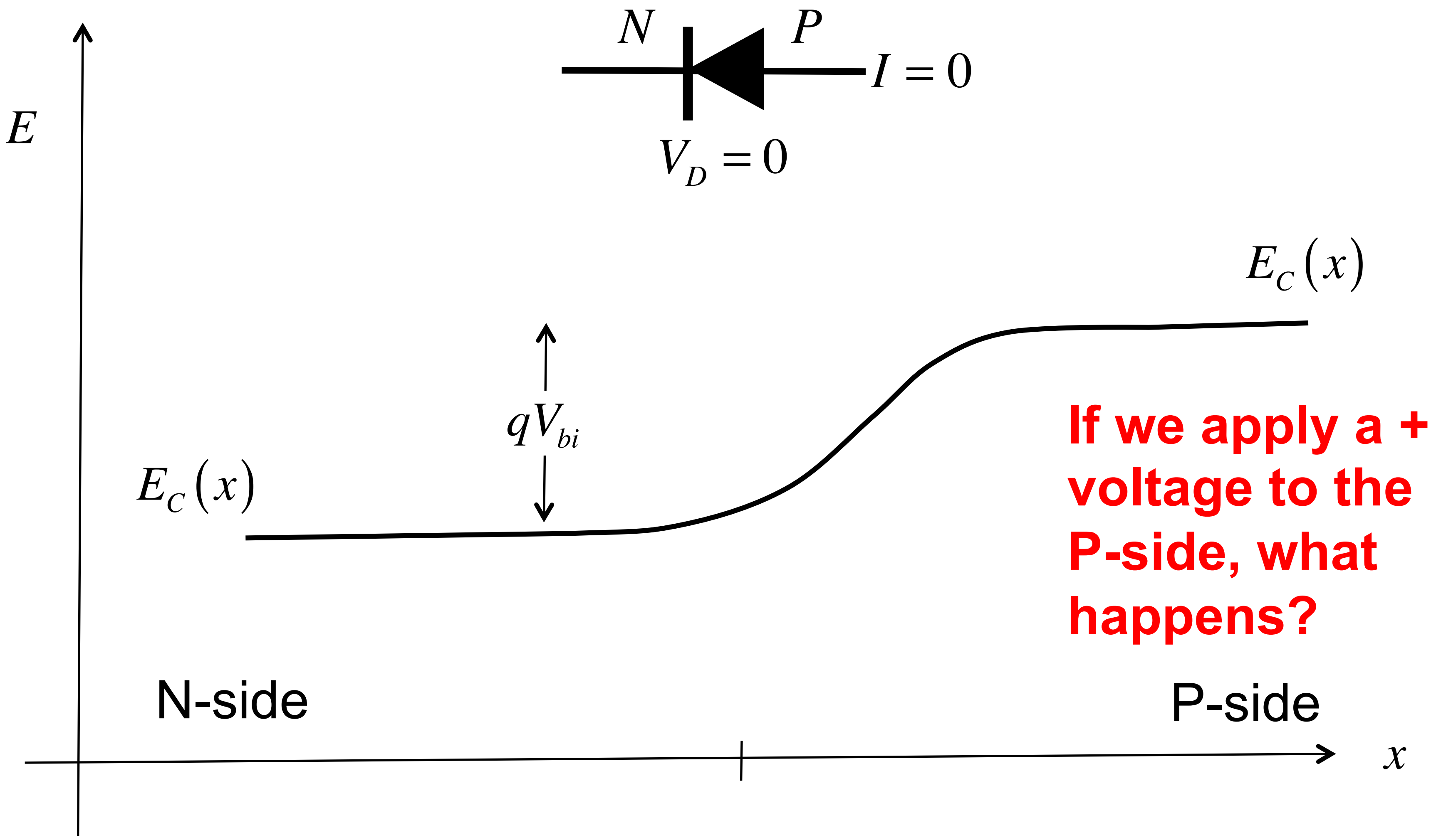
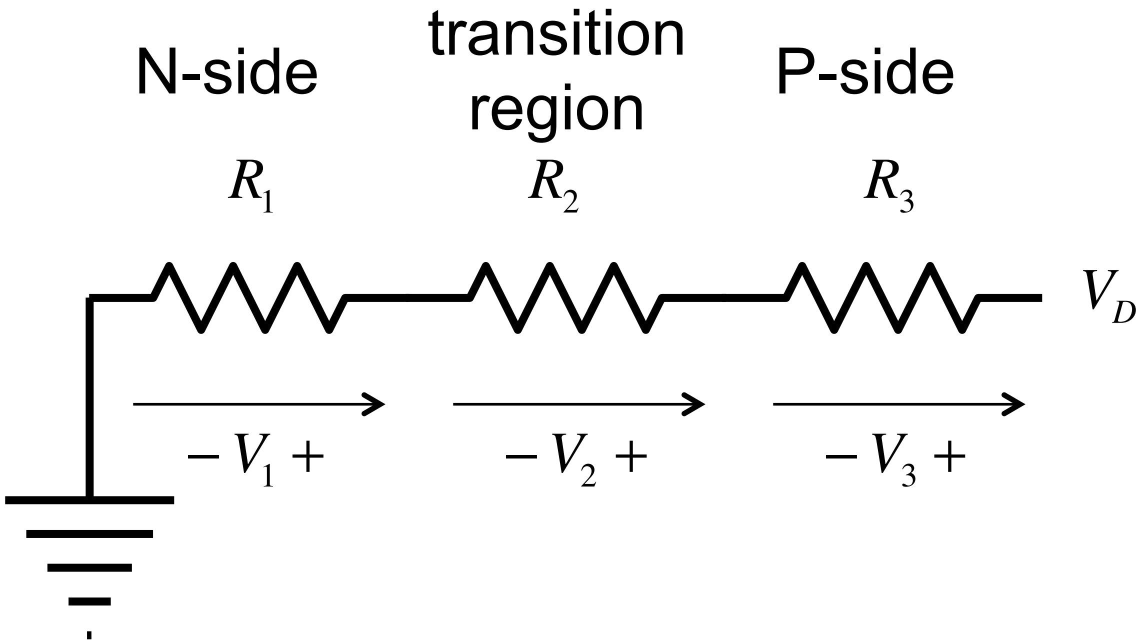
PN junction in equilibrium



If we apply a + voltage to the P-side, what happens?

Where does the voltage drop?

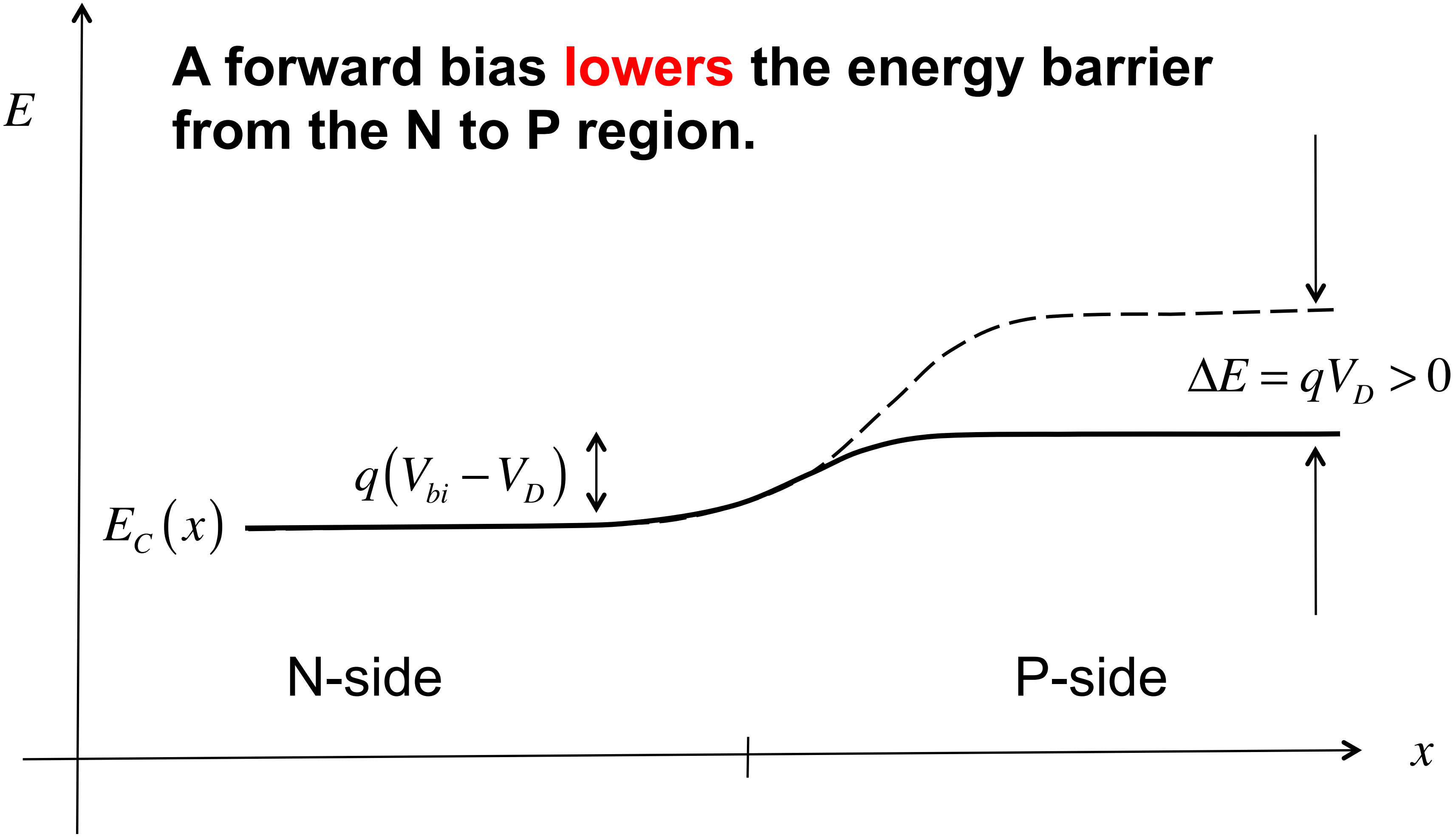


$V_2 \approx ?$

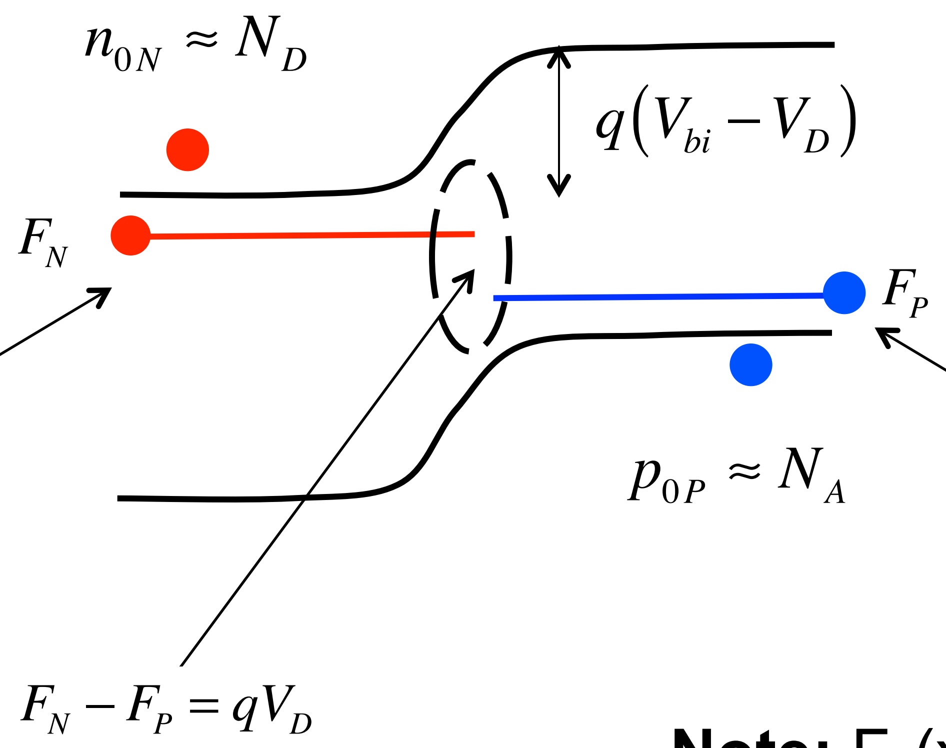
$R_2 >> R_1, R_2$

Forward bias

A forward bias lowers the energy barrier from the N to P region.



Quasi-Fermi Levels (QFL's)



$n_{0N} \approx N_D$

$q\left(V_{bi} - V_D\right)$

F_N

F_P

$p_{0P} \approx N_A$

QFL for electrons

QFL for holes

$F_N - F_P = qV_D$

Note: $F_n(x)$ and $F_p(x)$ defined everywhere

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Fermi level vs. Quasi-Fermi Level

Equilibrium

$$n_0(x) \propto e^{\left(E_F - E_C(x)\right)/k_B T}$$

$$p_0(x) \propto e^{\left(E_V(x) - E_F\right)/k_B T}$$

$$n_0 p_0 = n_i^2$$

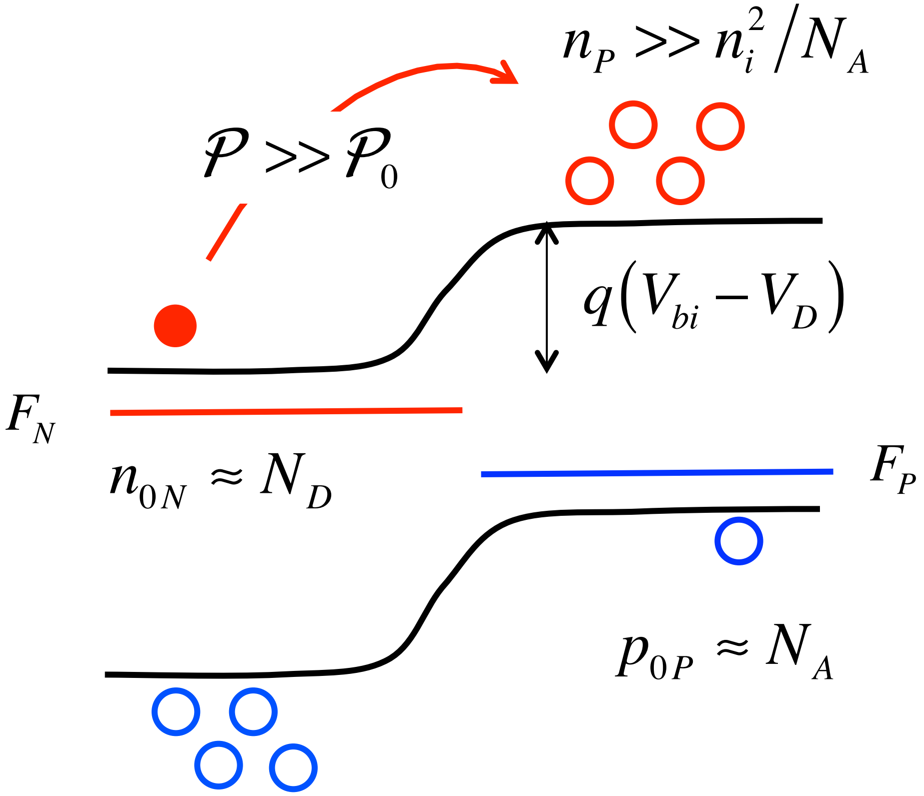
Non-equilibrium

$$n(x) \propto e^{\left(F_n(x) - E_C(x)\right)/k_B T}$$

$$p(x) \propto e^{\left(E_V(x) - F_p(x)\right)/k_B T}$$

$$n(x)p(x) = n_i^2 e^{\left(F_n(x) - F_p(x)\right)/k_B T}$$

Minority carrier injection



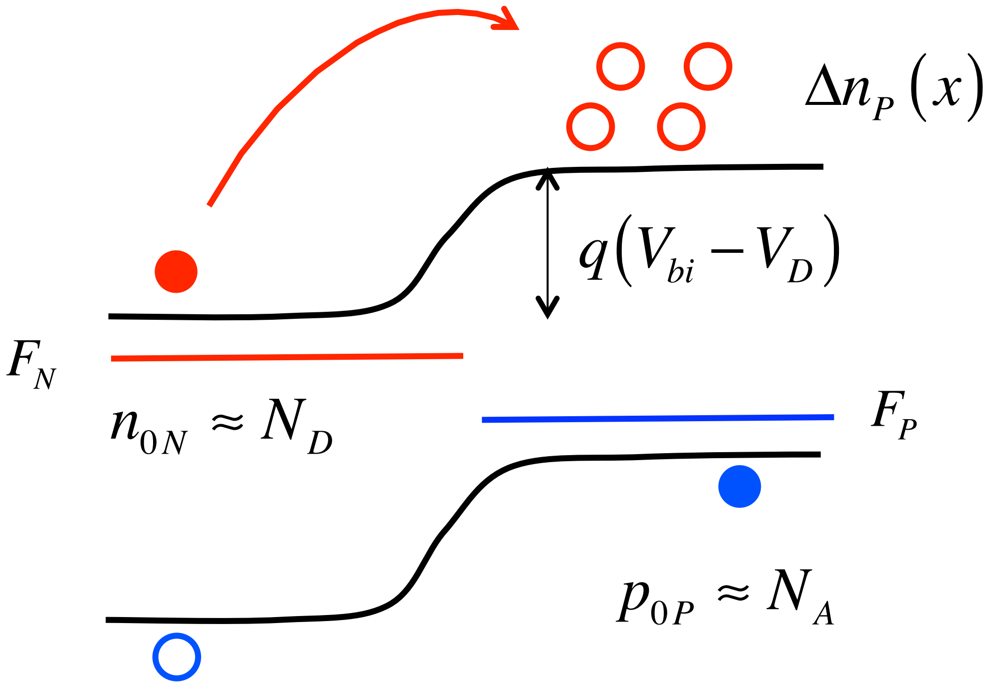
$$\mathcal{P} = e^{-q(V_{bi} - V_D)/k_B T}$$

$$\mathcal{P} = \mathcal{P}_0 e^{qV_D/k_B T}$$

“excess minority carriers”

$p_N >> n_i^2 / N_D$

Excess carriers and current



Excess electron concentration on the p-side.

$$Q_n = q\int_0^{W_P} \Delta n_P(x)\,dx \ \ \mathrm{C/cm^2}$$

$$\boxed{I_D = \frac{Q_n}{t_n}}$$

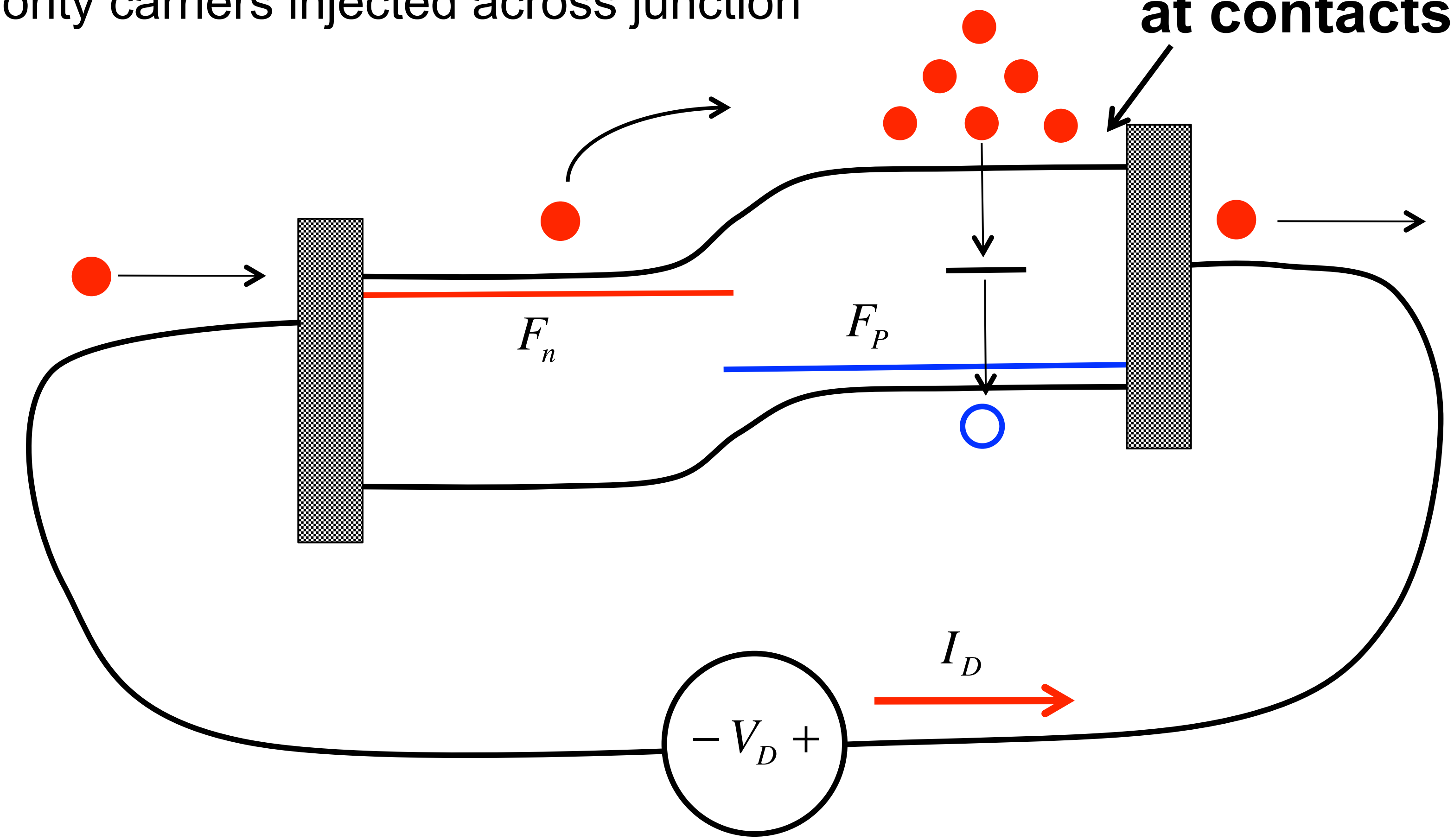
The time, *t*$_n$ is the average time for an electron on the p-side to “recombine” or to diffuse to the contact and recombine.

Recombination leads to current.

Recombination leads to current

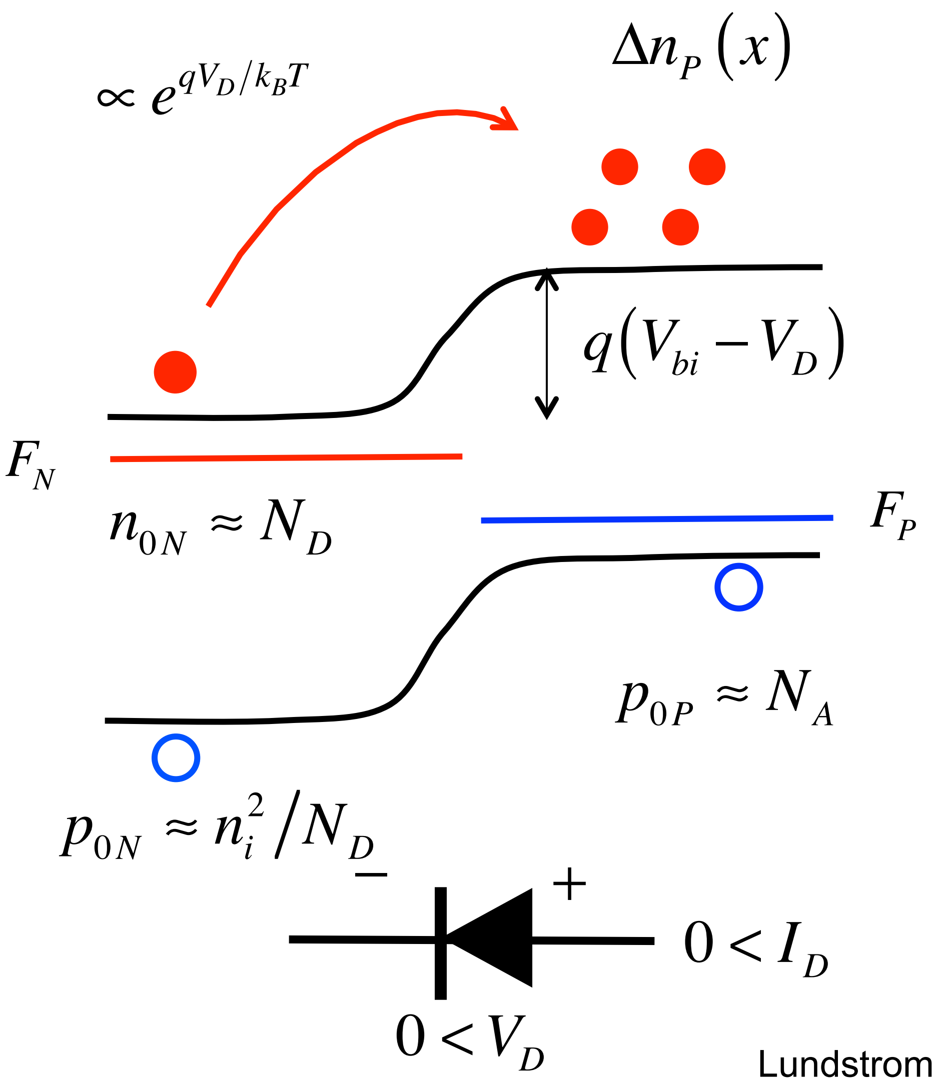
minority carriers injected across junction

at contacts too!



Every time a minority electron recombines on the p-side, one electron flows in the external current.

Forward bias summary



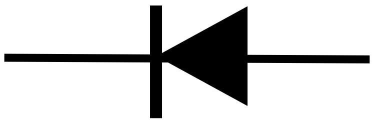
1) Injected current produces a population of “excess” electrons in the P-type region...

2) Excess electrons in the P-type region recombine…

3) Every time an electron and hole recombine, an electron flows in the external circuit.

$$I_D = qR(V_D)$$

Ideal diode equation



$0 < I_D$

$- V_D +$

$$I_D = \frac{Q_n}{t_n} + \frac{Q_p}{t_p}$$

Holes that are injected into the N-side and recombine

$$Q_n \propto \frac{n_i^2}{N_A}\left(e^{qV_D/k_BT} - 1\right)$$

$$Q_p \propto \frac{n_i^2}{N_D}\left(e^{qV_D/k_BT} - 1\right)$$

$$I_D = I_0\left(e^{qV_D/k_BT} - 1\right)$$

"ideal diode equation"

$$I_D = I_0\left(e^{qV_D/nk_BT} - 1\right)$$

$$n = 1$$

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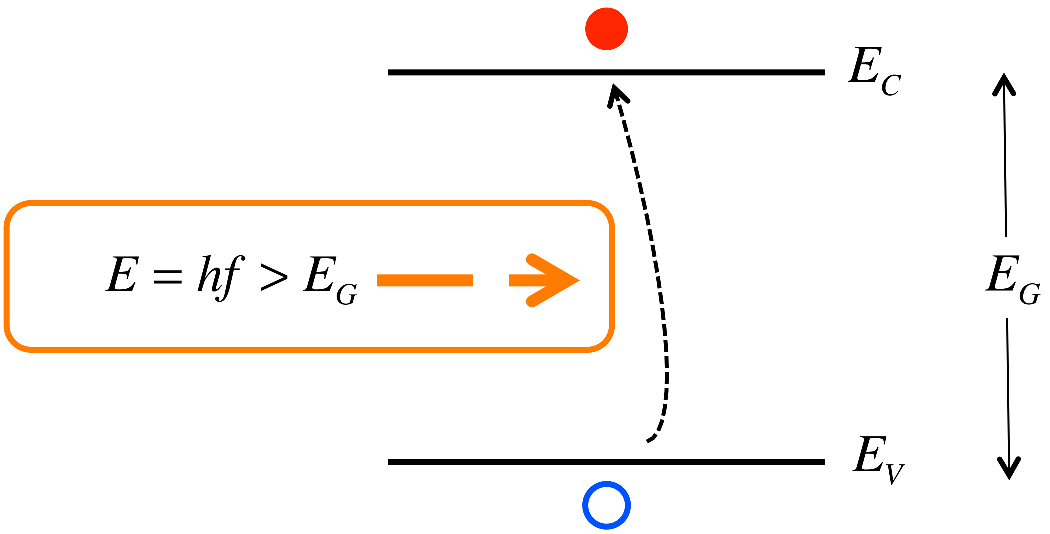
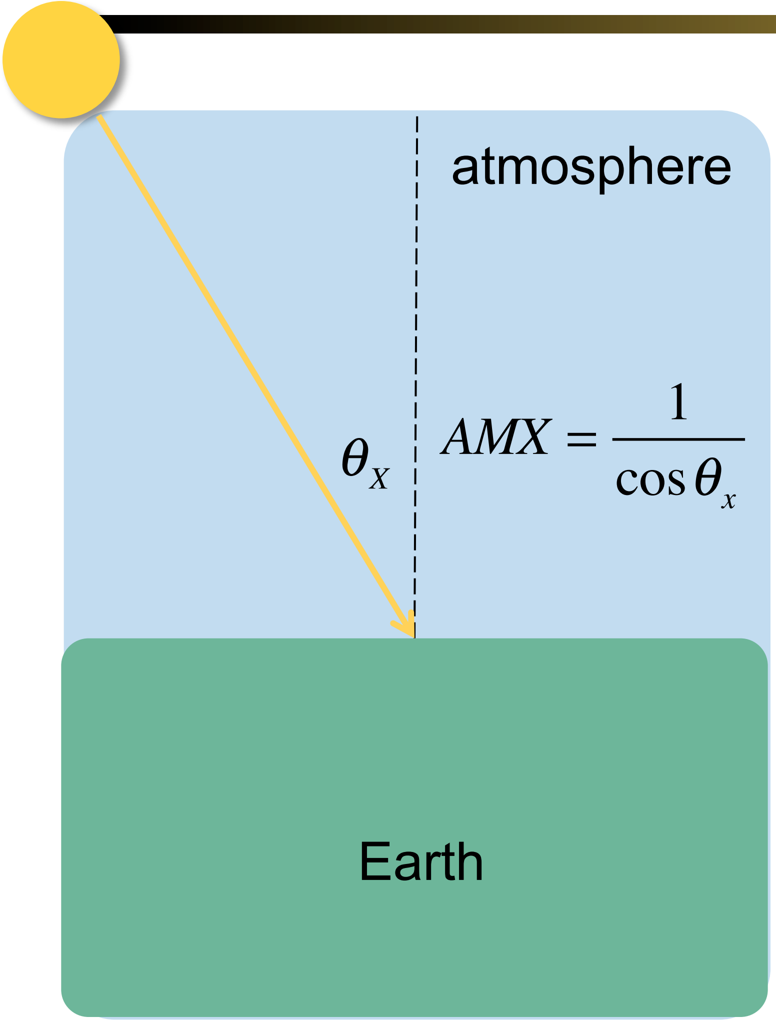
Absorption of light

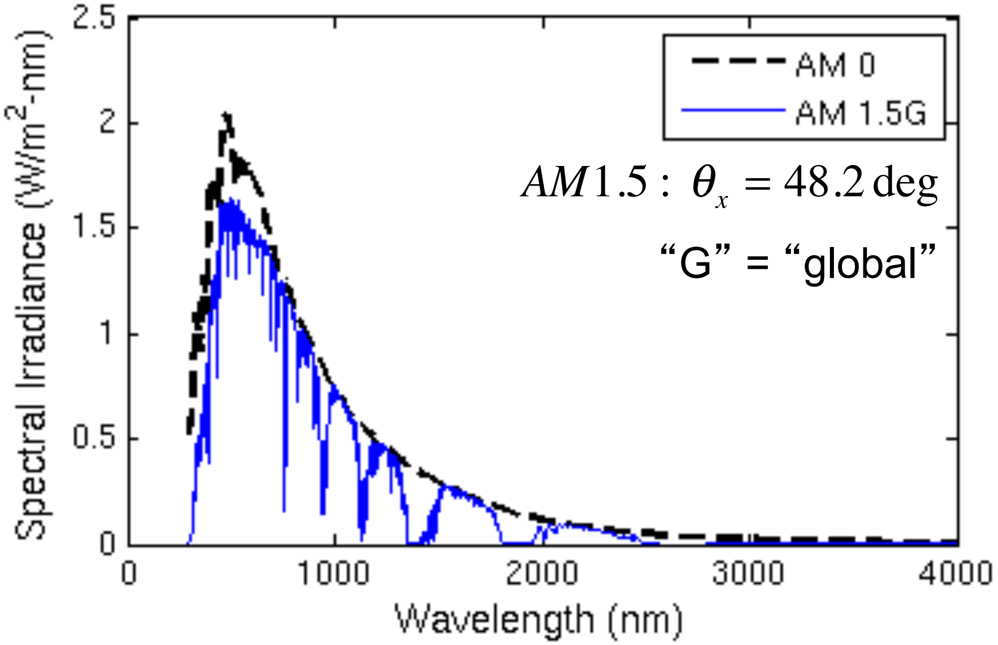
$$\lambda < \frac{hc}{E_G}$$



Solar spectrum (terrestrial)



$$AMX = \frac{1}{\cos\theta_x}$$



$$AM1.5: \ \theta_x = 48.2 \deg$$

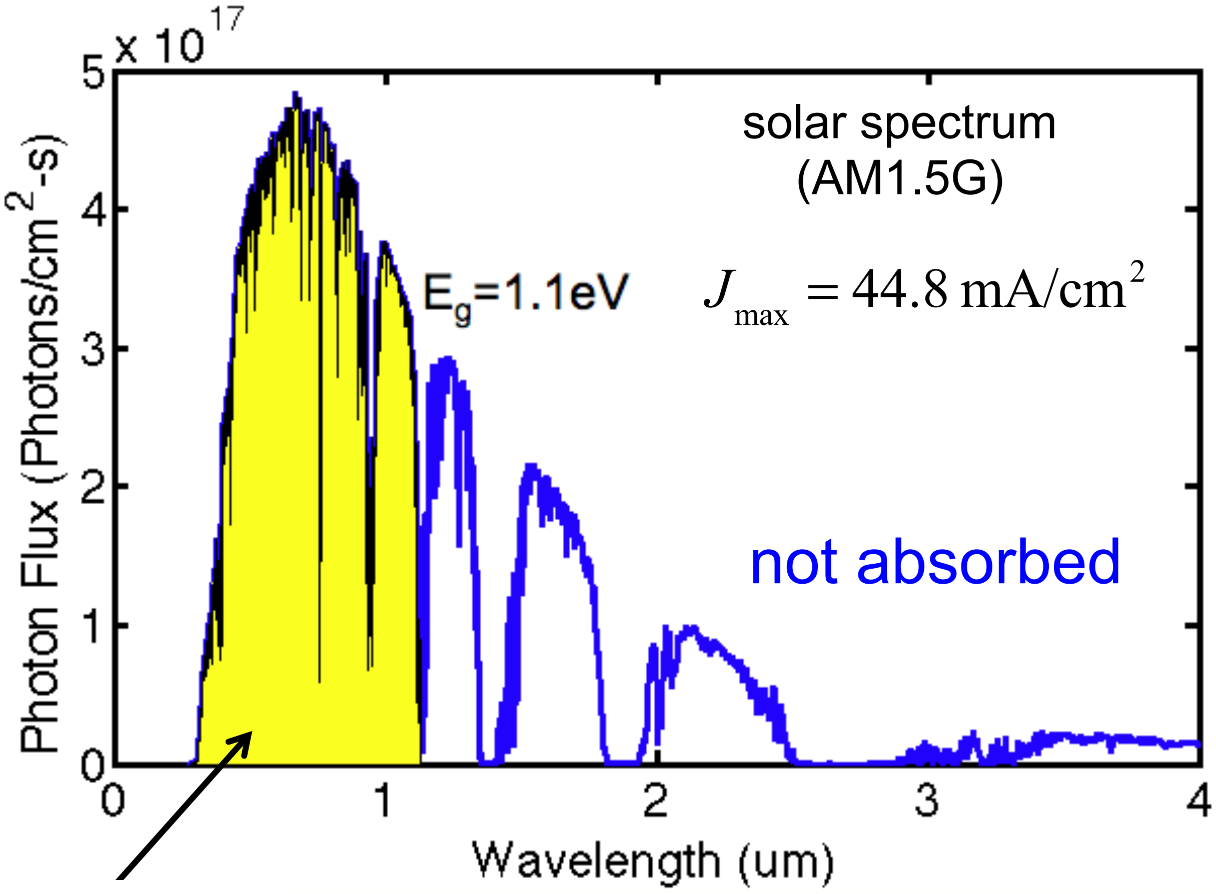
“G” = “global”

Integrated power = 100 mW/cm^2)

http://en.wikipedia.org/wiki/Air_mass_(solar_energy)

How many photons can be absorbed?

Example: Si E_G = 1.1eV. Only photons with a wavelength shorter than 1.13 µm will be absorbed.



$$J_{\max} = 44.8\ \mathrm{mA/cm^2}$$

excess energy

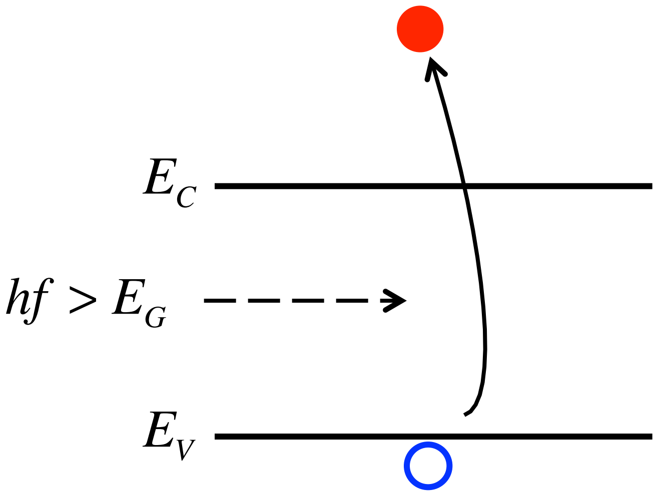
$$f\lambda = c$$

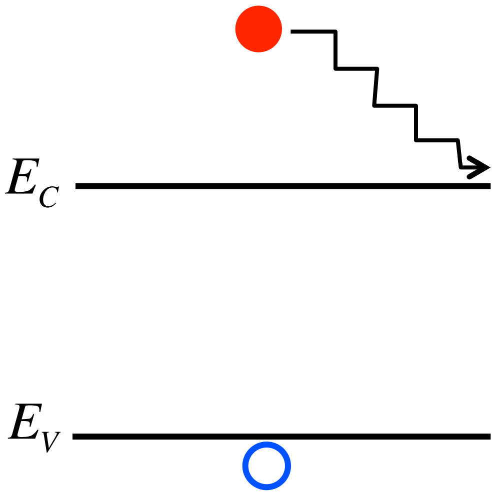
$$E = hf$$

$$E = \frac{hc}{\lambda}$$

Wasted energy for $E > E_G$

Energy is lost for photons with energy greater than E_G.





Electron is excited above the conduction band.

However, extra energy is lost due to thermalization as electron relaxes back to the band edge.

How many photons are absorbed in a finite thickness?

Incident flux: $\Phi_0 \ \mathrm{cm^{-2} s^{-1}}$

Flux at position, *x* : $\Phi(x) = \Phi_0 e^{-\alpha(\lambda)x}$

Optical absorption coefficient: $\alpha(\lambda) > 0 \quad (E > E_G)$

Generation rate at position, *x* :

$$G(x) = -\frac{d\Phi(x)}{dx} = \Phi_0 \alpha(\lambda) e^{-\alpha(\lambda)x}$$

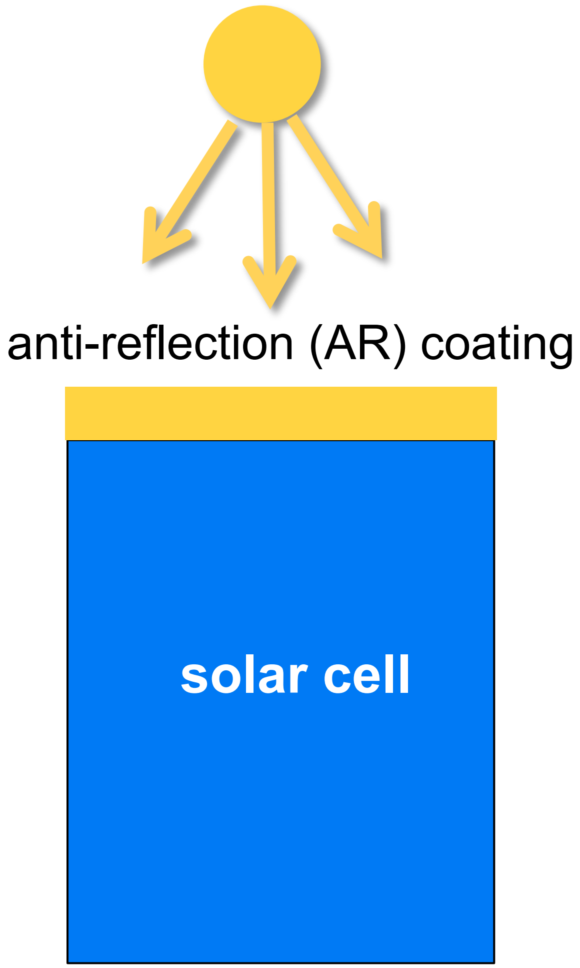
$$G_{TOT} = \int \left\{ \int_0^L G(x,\lambda)\, dx \right\} d\lambda$$

$$\boxed{\alpha(\lambda)\ \mathrm{cm^{-1}}}$$

Maximizing light absorption / generation

1) Maximize the number of phonons that get into the solar cell (AR coating, texturizing).

2) Maximize the "effective" thickness of the absorber (light trapping).



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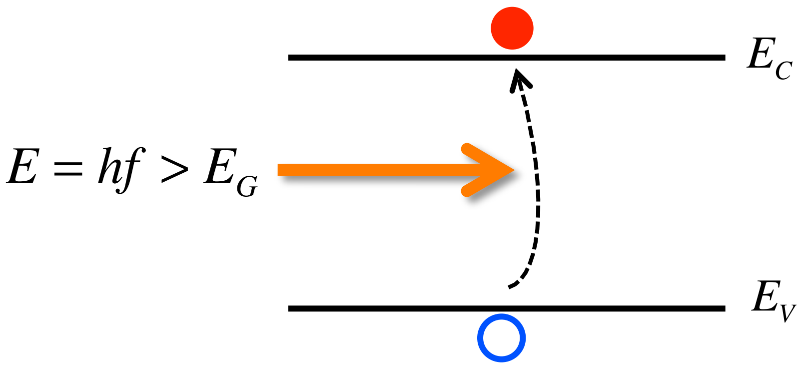
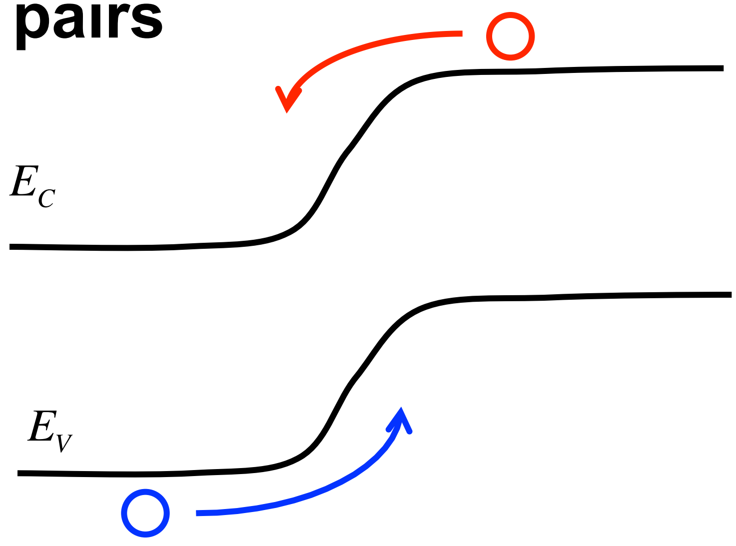
Collection of e-h pairs

1) Light generates electron-hole pairs



2) PN junction collects e-h pairs



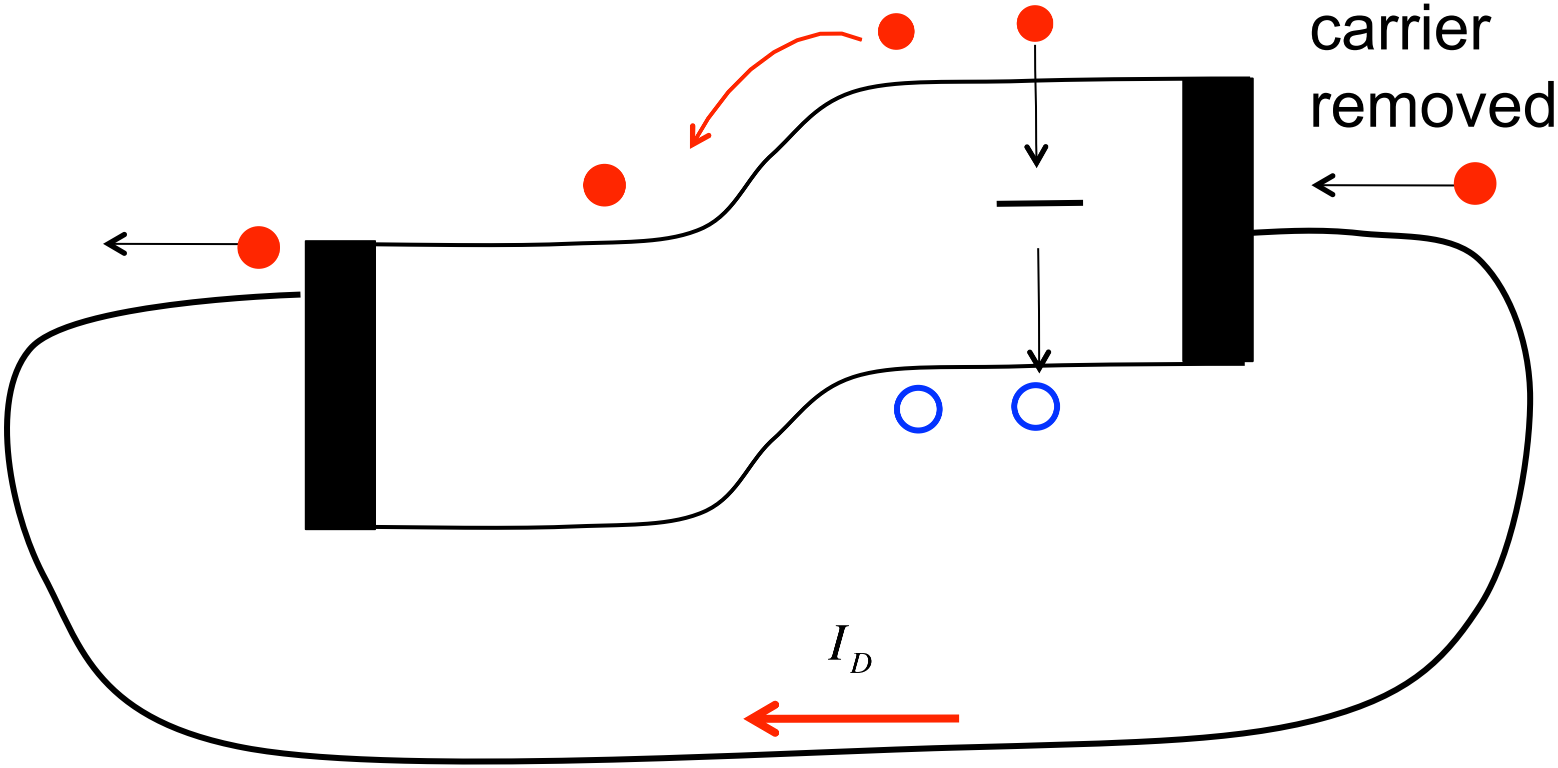
N-region collects the minority electrons.

P-region collects the minority holes.

Current = recombination - generation

1) minority carriers collected

2) majority carrier removed



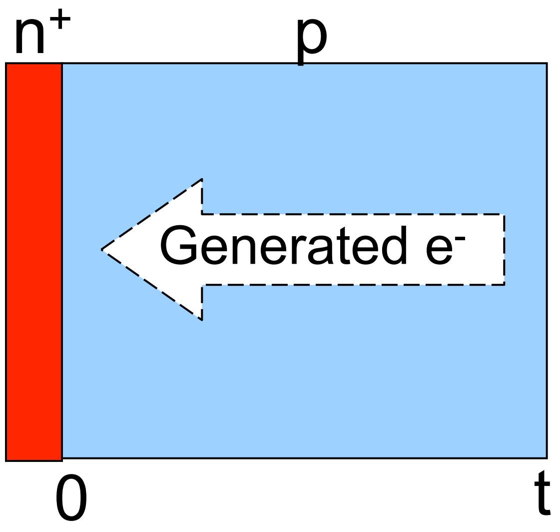
Every time a minority electron generated on the P-side is collected by the N-side, one electron flows in the external current.

Current collection

The generated carriers are only useful if they are collected before they recombine.

Electron diffusion length:

Distance electrons travel before they recombine.



$$t < L_n = \sqrt{D_n \tau_n}$$

Ex. (Si):

$$\mu_n \approx 1000 \ \ \text{cm}^2/\text{V-s}$$

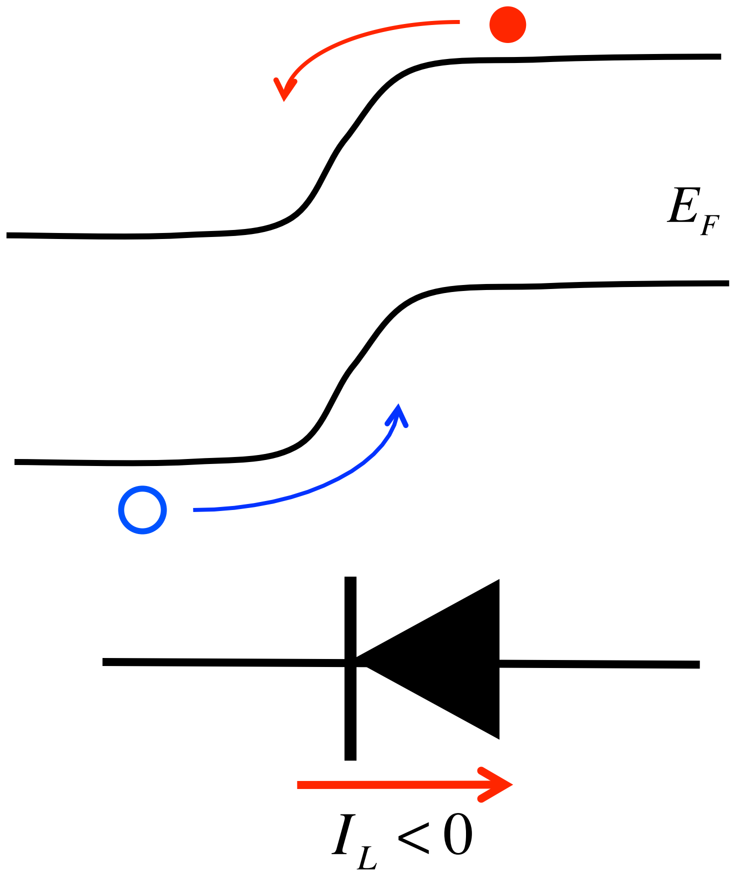
$$\tau_n \approx 50 \ \ \mu\text{s}$$

$$L_n \approx 700 \ \ \mu\text{m}$$

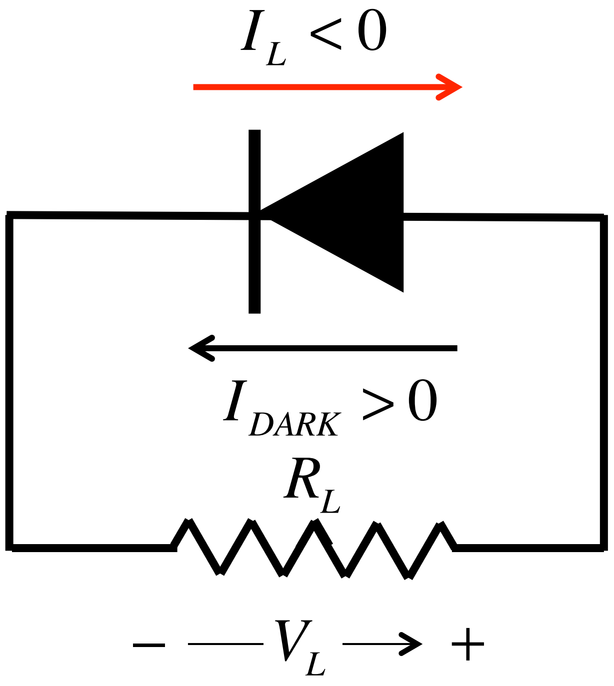
For Si, we cannot make the absorbing layer thick enough to absorb all of the above gap photons.

Diode current under illumination

1) PN junction collects e-h pairs



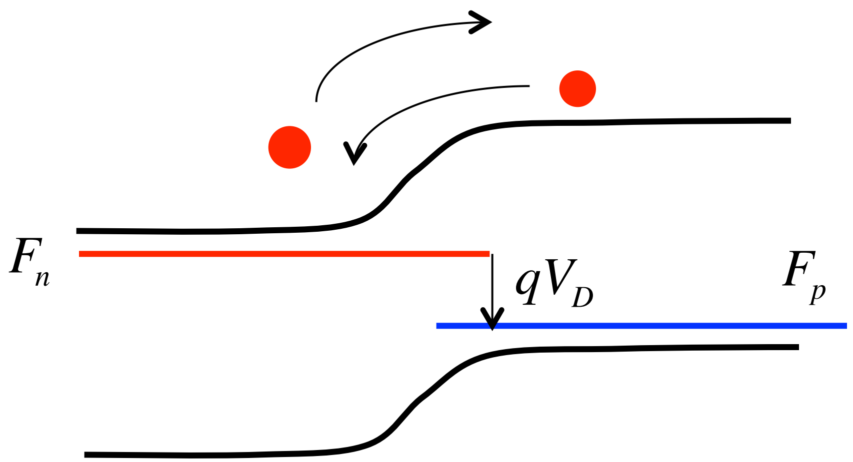
2) Current flows through load



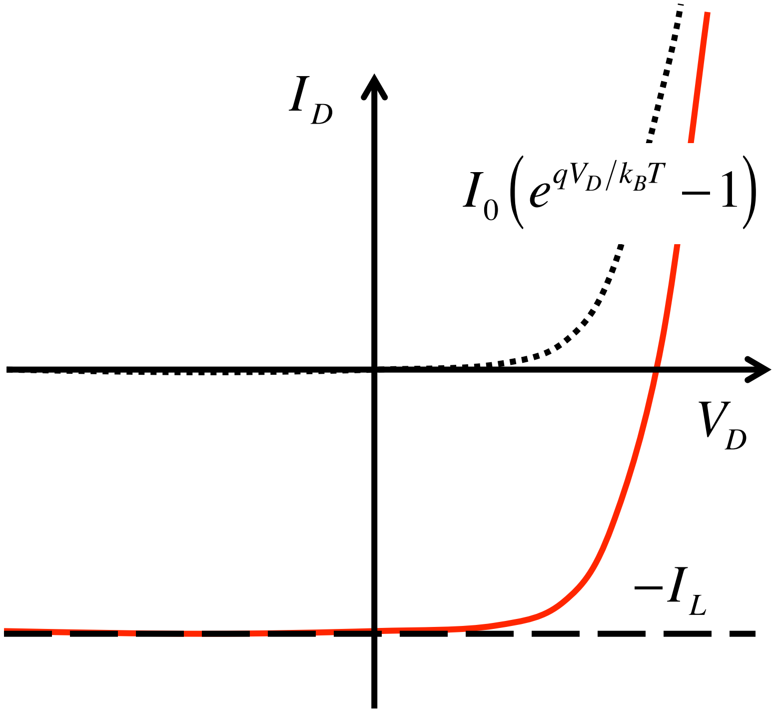
Forward bias across PN junction develops.

Net current

3) Forward bias reduces current



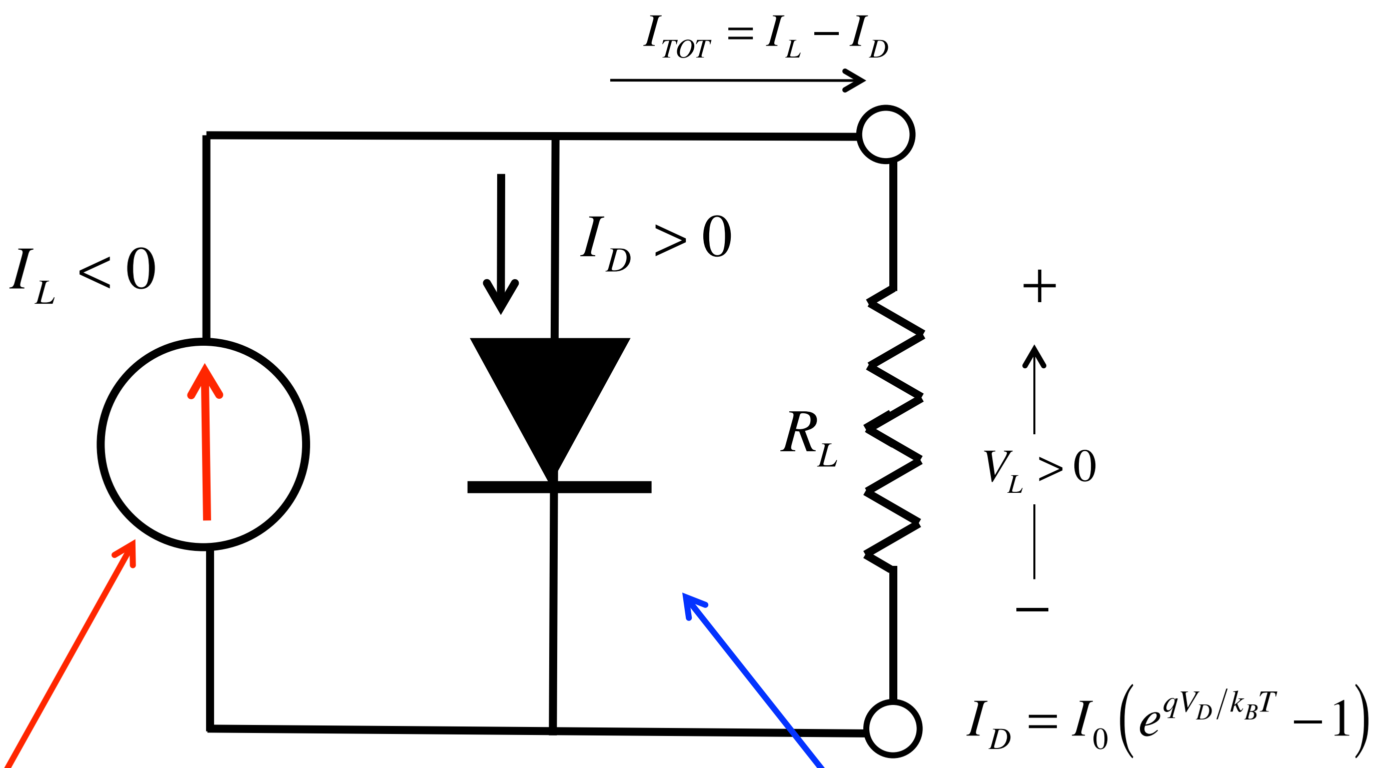
4) IV characteristic is a superposition



$$I_D = I_0\left(e^{qV_D/k_BT}-1\right) - I_L$$

dark current + photocurrent

“Superposition”



collection current due to optical generation (assumed to be bias independent)

recombination current due to minority carrier injection (assumed to be the dark current.

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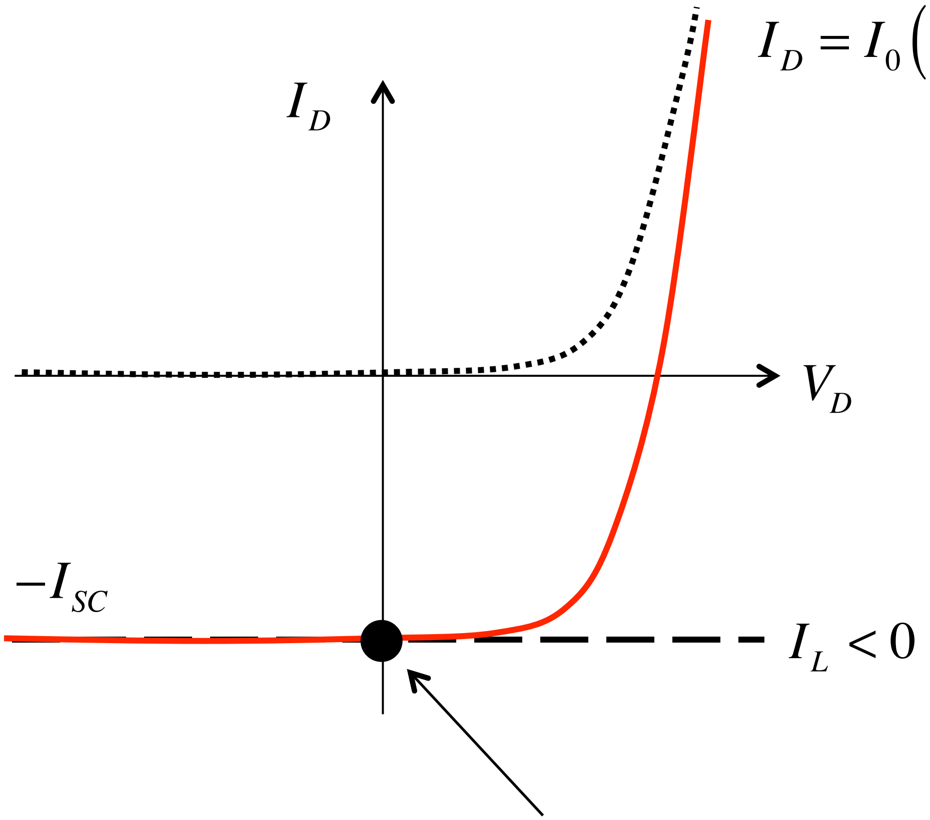
8) Summary

Short-circuit current



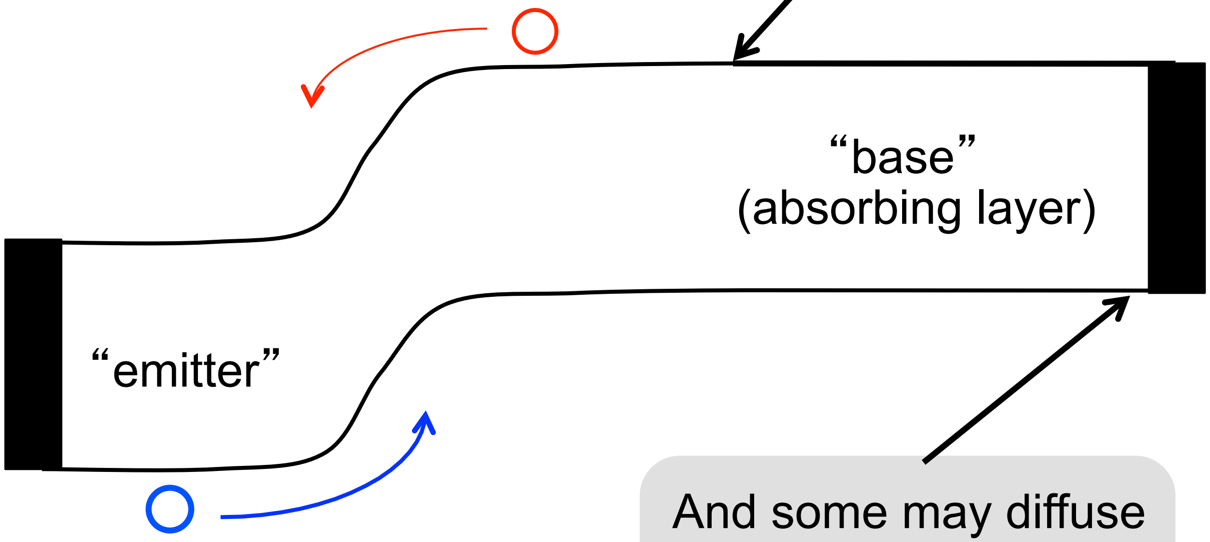
$$I_D = I_0\left(e^{qV_D/k_BT} - 1\right) - I_L$$

$I_{SC} = I_D\left(V_D = 0\right) \approx I_L$ (for a good solar cell)

Collection efficiency

Photo-generated carriers should diffuse to the junction and be collected.

But some may recombine in the base.

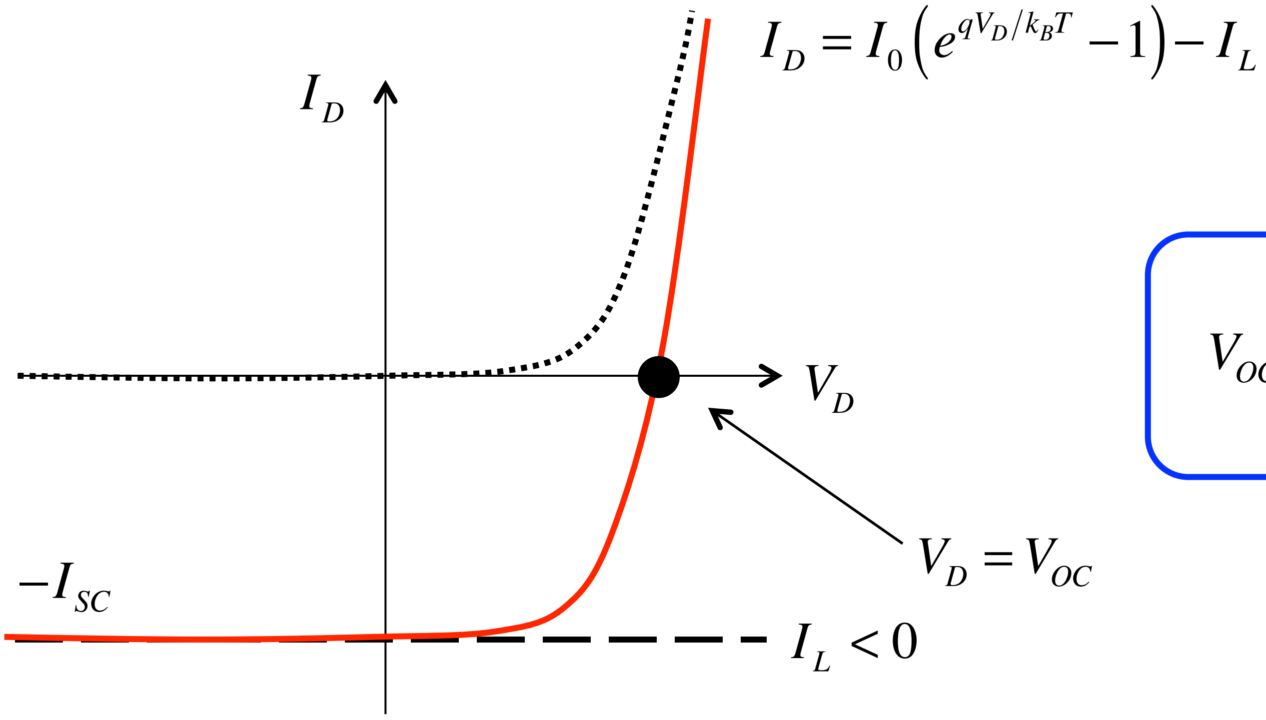


And some may diffuse to the contact and recombine.

$$CE \equiv \frac{I_{SC}}{qG_{TOT}}$$

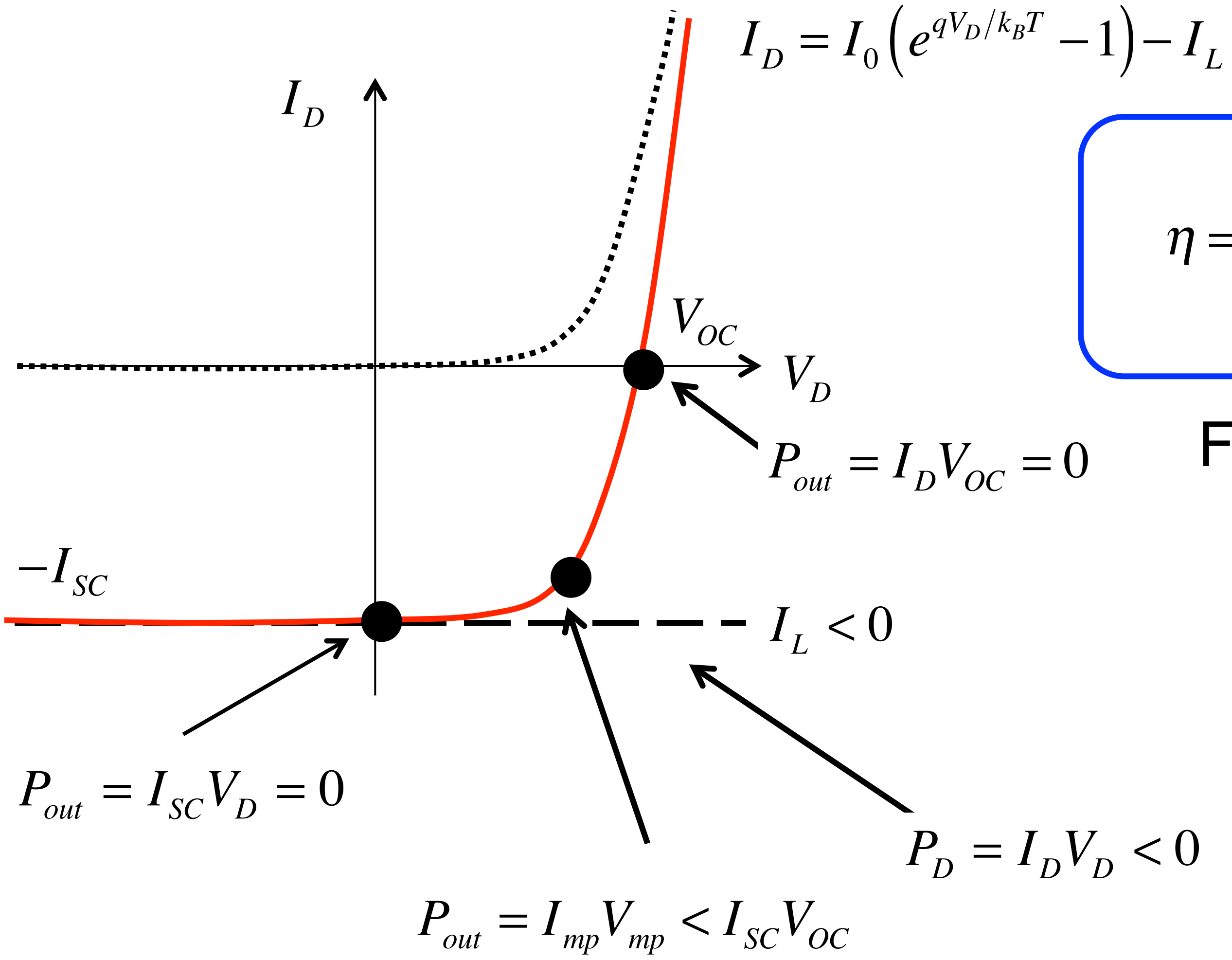
“collection efficiency”

Open-circuit voltage



$$V_{OC} = \frac{k_BT}{q}\ln\frac{|I_L|}{I_0}$$

Maximum power point



$$I_D = I_0\left(e^{qV_D/k_BT} - 1\right) - I_L$$

I_D

V_{OC}

V_D

$-I_{SC}$

$I_L < 0$

$P_{out} = I_D V_{OC} = 0$

$P_{out} = I_{SC} V_D = 0$

$P_D = I_D V_D < 0$

$P_{out} = I_{mp} V_{mp} < I_{SC} V_{OC}$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{I_{SC} V_{OC} FF}{P_{in}}$$

FF = fill factor

Fill factor

$$P_{out} = I_D V_D = \left[I_0 \left(e^{qV_D/k_BT} - 1 \right) - I_L \right] V_D \qquad \frac{dP_{out}}{dV_D} = 0$$

The FF is determined by the diode characteristic and by series and shunt resistances. FF ≈ 0.75-0.85 for Si.

$$FF \approx \frac{V_{OC}/\left(k_BT/q\right)}{4.7 + V_{OC}/\left(k_BT/q\right)}$$

< 2% error for $V_{OC}/\left(k_BT/q\right) > 10$

Martin A. Green, "Accuracy of analytical expressions for solar cell fill factors," *Solar Cells*, **7**, 337-340, (1982-83)

Solar cell efficiency

$$\boxed{\eta = \frac{P_{out}}{P_{in}} = \frac{I_{SC} V_{oc} FF}{P_{in}}}$$

It’s not enough to have a high short-circuit current, we also need a high open-circuit voltage.

Concentration increases efficiency

$$P_{in} = XP'_{in}$$

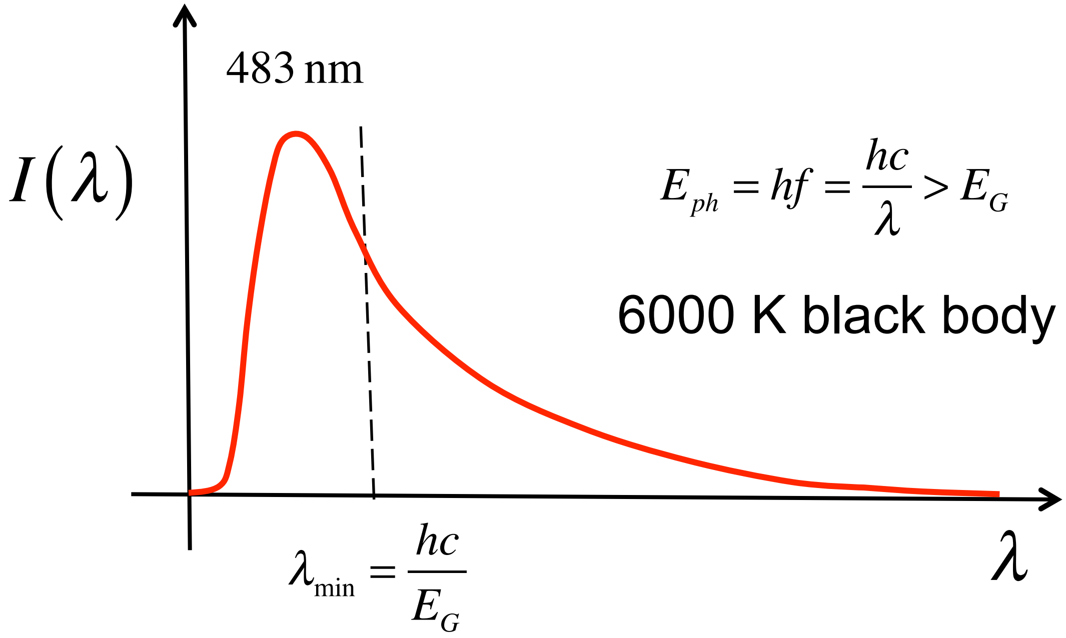
$$I_{SC} = XI'_{SC}$$

$$V_{OC} = \frac{k_B T}{q} \ln \frac{|I_{SC}|}{I_0} = \frac{k_B T}{q} \ln \frac{|XI'_{SC}|}{I_0}$$

$$\boxed{\eta = \frac{P_{out}}{P_{in}} > X\eta'}$$

I_{SC} and V_{OC} vs. bandgap

$$I_{SC} \sim 1/E_G$$



$$E_{ph} = hf = \frac{hc}{\lambda} > E_G$$

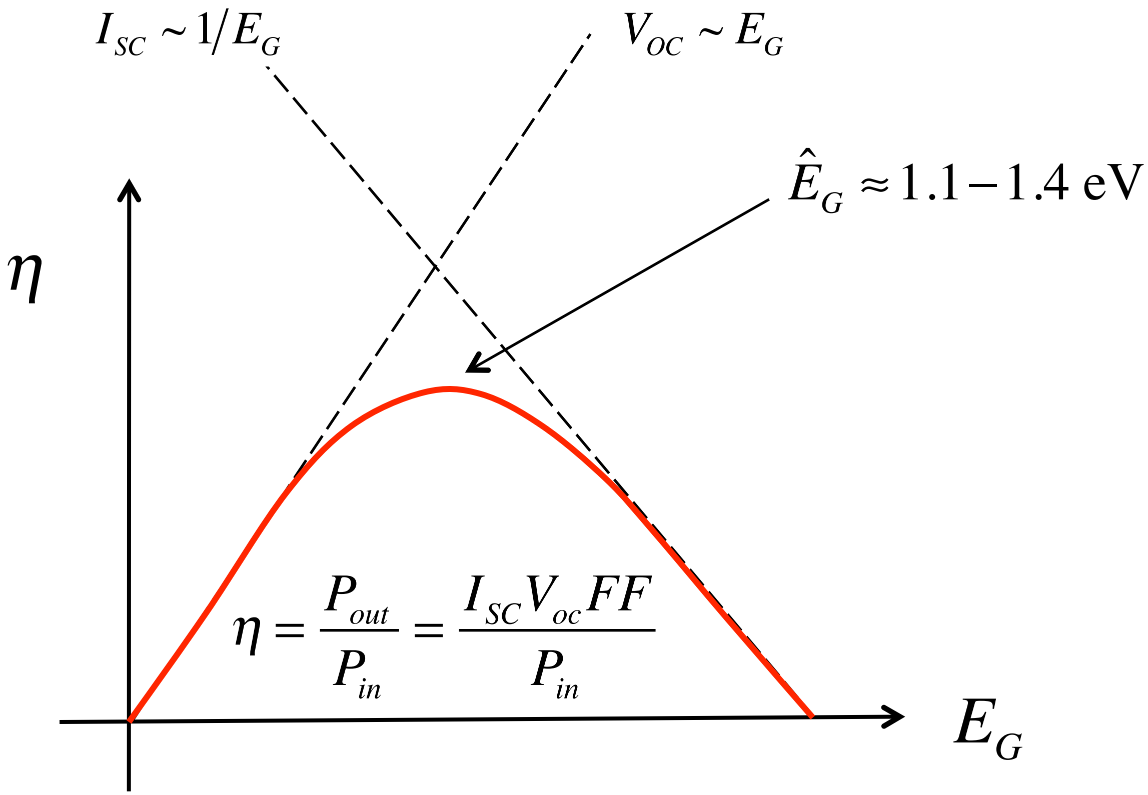
6000 K black body

$$V_{OC} = \frac{k_B T}{q} \ln\left(\frac{I_{SC}}{I_0}\right)$$

$$I_0 \propto n_i^2 \propto e^{-E_G/k_B T}$$

$$V_{OC} \sim E_G$$

I_{SC} and V_{OC} vs. bandgap



There is an optimum bandgap for the solar spectrum

Vocabulary

Short-circuit current
Collection efficiency
Open-circuit voltage
I_{SC}-V_{OC} trade-off
Optimum bandgap
Fill factor
Maximum power point
Solar cell efficiency

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Solar cell summary

1) Light is absorbed and produces e-h pairs

2) Junctions separate e-h pairs and collect the carriers.

3) Current flow in external circuit produces a FB voltage and the FB diode current reduces the total current.

4) Current is the difference between recombination and generation

4) Power out is $I_{SC}V_{OC}FF$.

5) Efficiency is critical because it lowers cost.

Next

1) How do we design a solar cell to maximize the short-circuit current?

2) How do we design a solar cell to maximize the open-circuit voltage?

3) How do we design a solar cell to preserve the fill factor?

Questions

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