Theory and Practice of Solar Cells: A Cell to System Perspective

# PV Performance Limits by Shockley-Queisser (SQ) Triangle

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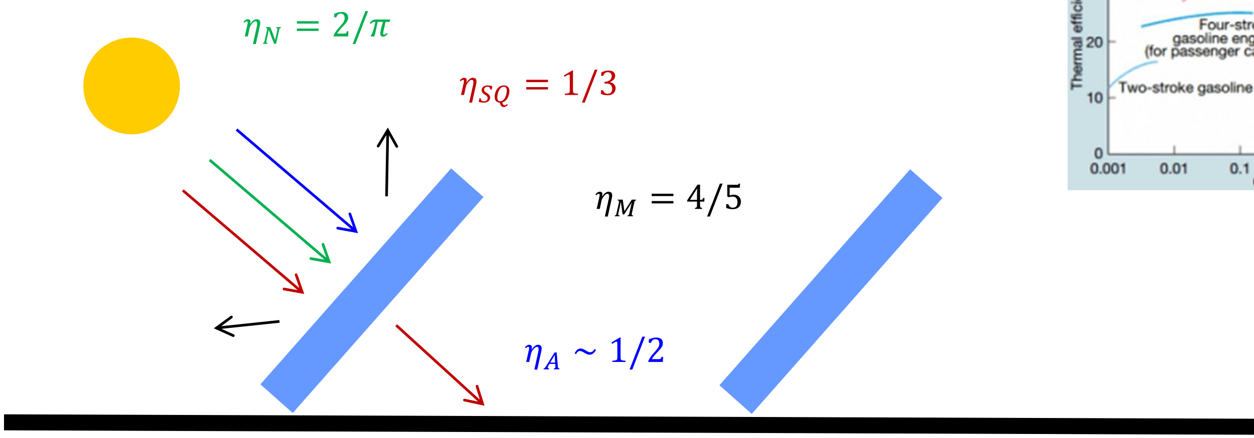
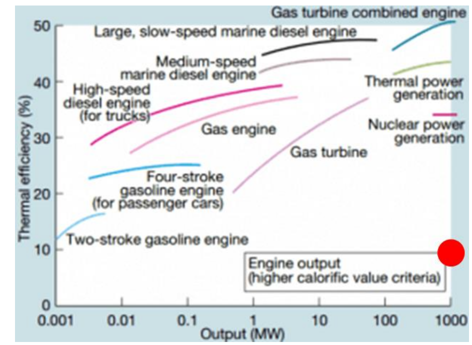
# Outline

1) Motivation
2) Derivation of the SQ-Triangle
3) Applications of SQ-Triangle
4) Conclusions
5) Appendix: Self-test questions

# An Inefficient Machine!



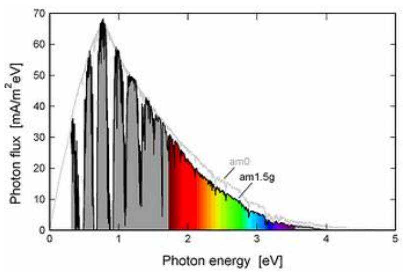




$$\eta = \eta_N \times \eta_{SQ} \times \eta_M \times \eta_A = \frac{2}{\pi} \times \frac{1}{3} \times \frac{5}{6} \times \frac{1}{2} \sim \frac{1}{10}$$

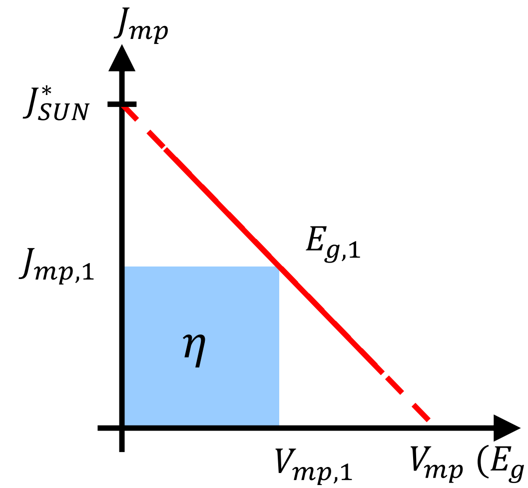
↑ Tandem ($\eta_{SQ}$)

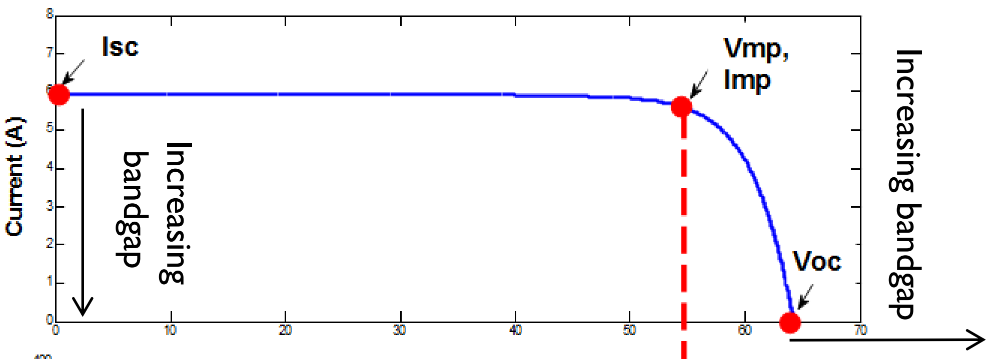
↑ Bifacial ($\eta_A$)

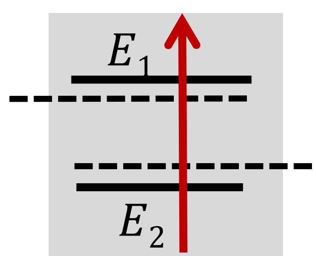
# Power and efficiency



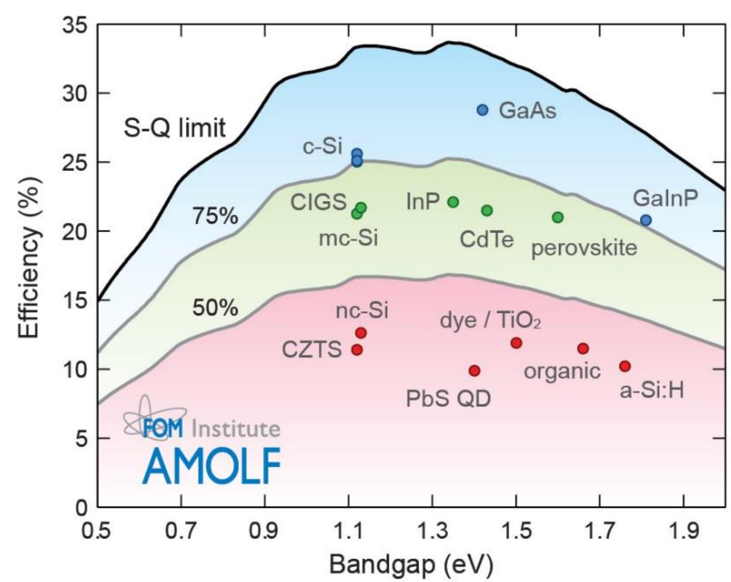

$$J_{mp} = 82(1 - 0.428E_g)$$




$$qV_{mp} = 0.95E_g - 0.3$$



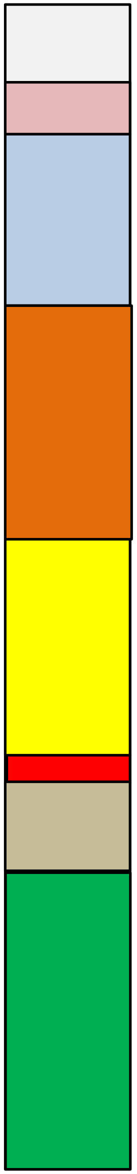
# Counting the photons – a roadmap

Carnot

Angle Entropy

Above bandgap

Yablonovitch



Radiative

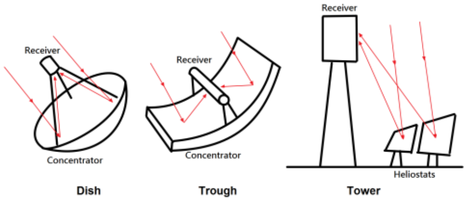
Below bandgap

Shockley-Queisser

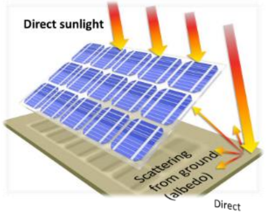
Partition,
Cell-Module

LED

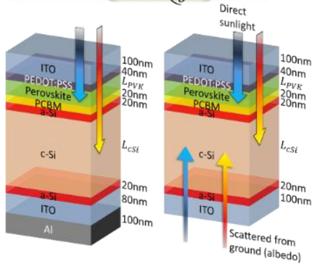
CPV




Tandem




MEG




$>4n^2$

Striping

M. A. Alam, PV Lecture Notes

# Outline

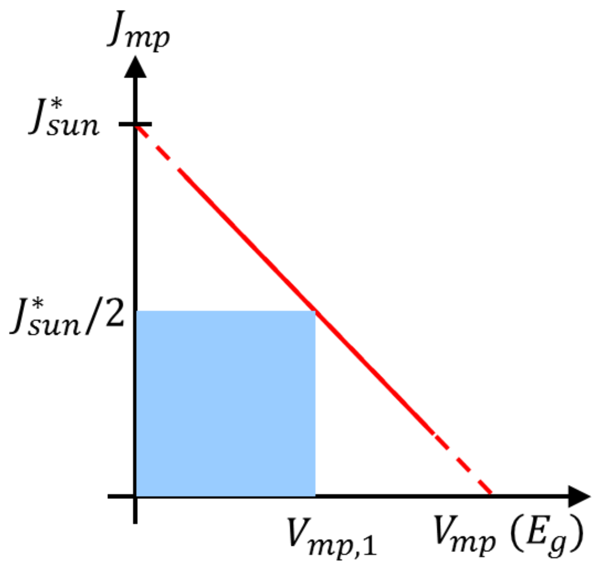
1) Motivation
2) Derivation of SQ-Triangle
3) Application of SQ-Triangles
4) Conclusions

# Defining the SQ-Triangle

1. Calculate $V_{mp}$ and $I_{mp}$

$$qV_{mp} = 0.95E_g - 0.3$$

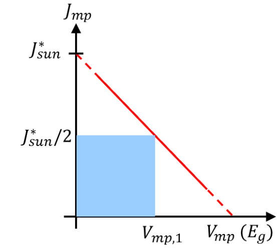
$$J_{mp} = 82(1 - 0.428E_g)$$

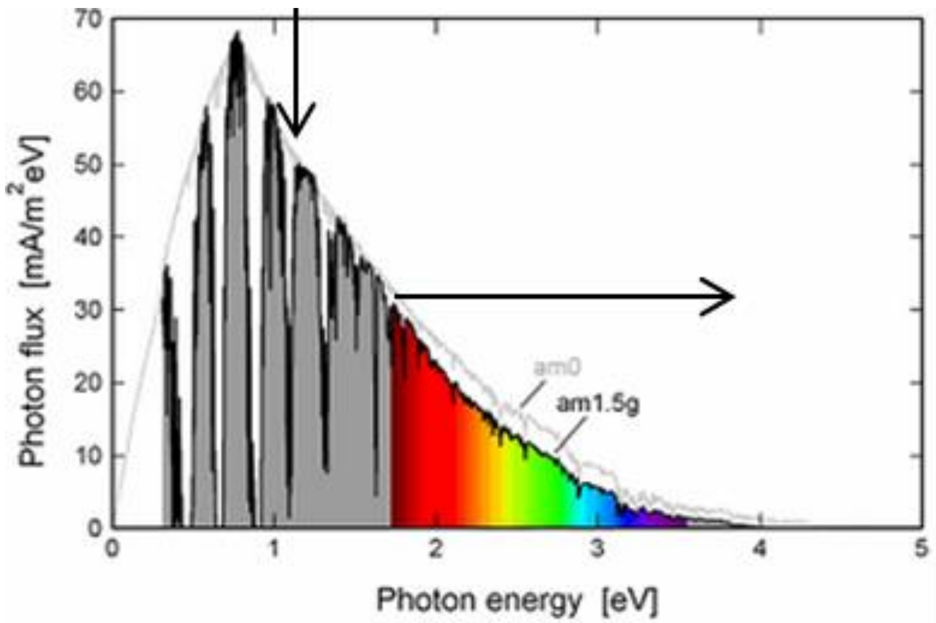


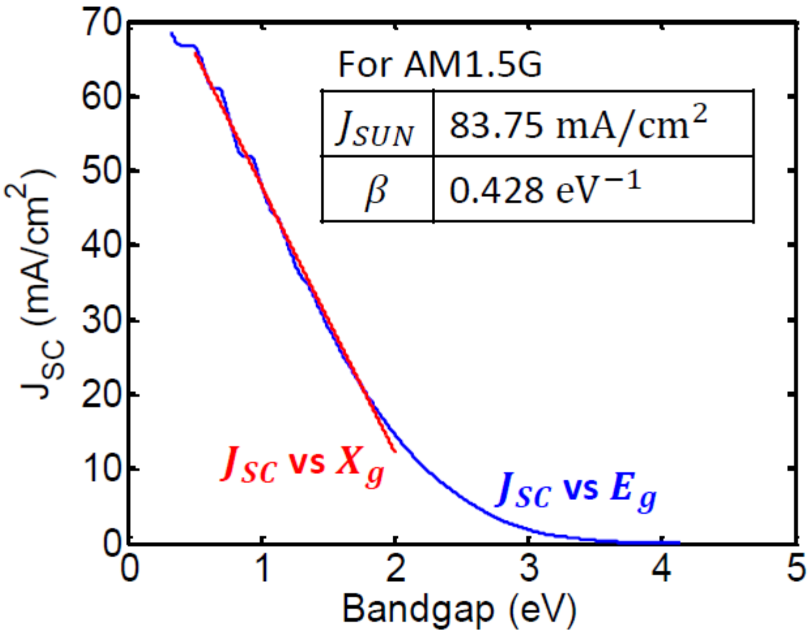

2. Calculate efficiency:
$\eta = V_{mp}\, I_{mp} / P_{in}$

3. Locate in the triangle $J_{mp} = 72(1 - 0.52\, V_{mp})$

# Step 1: Calculate $J_{mp}$

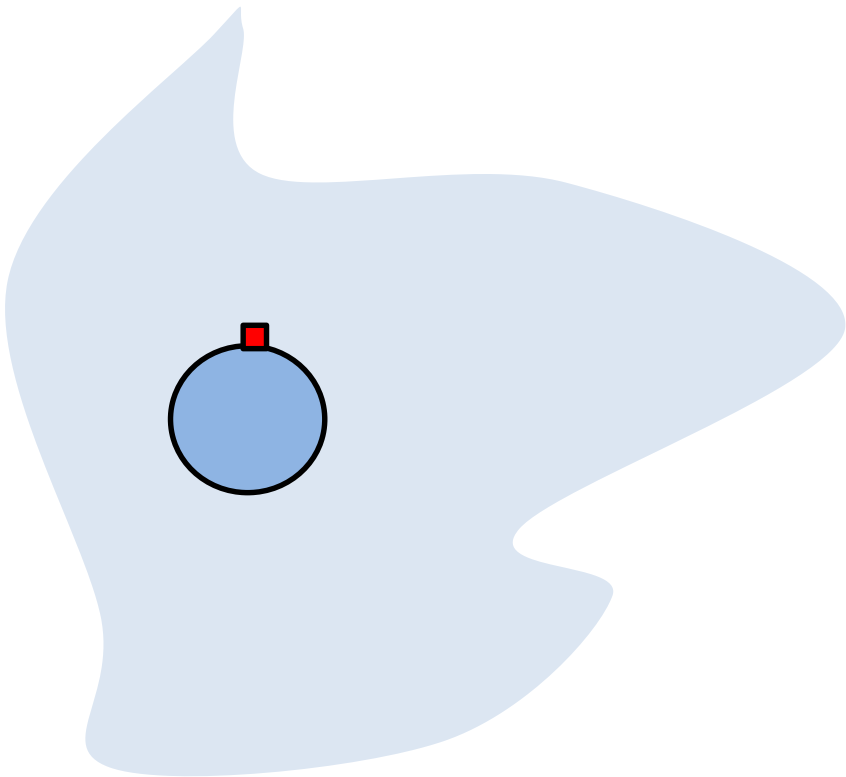


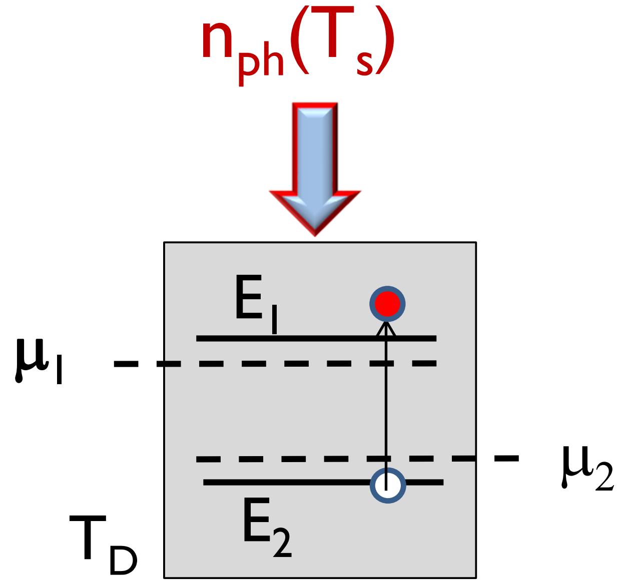




$$J_{mp} = 83.75(1 - 0.428E_g)$$

# Step 2: Derive $V_{mp}$ by a 2-Level Atom

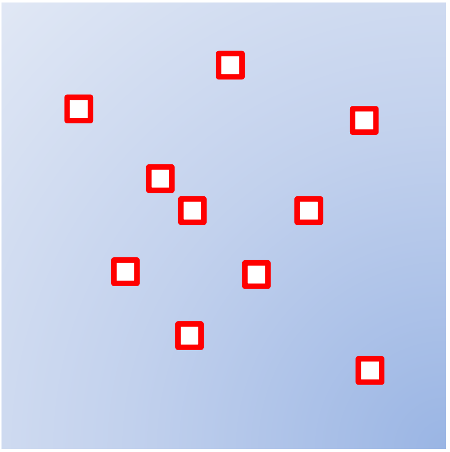


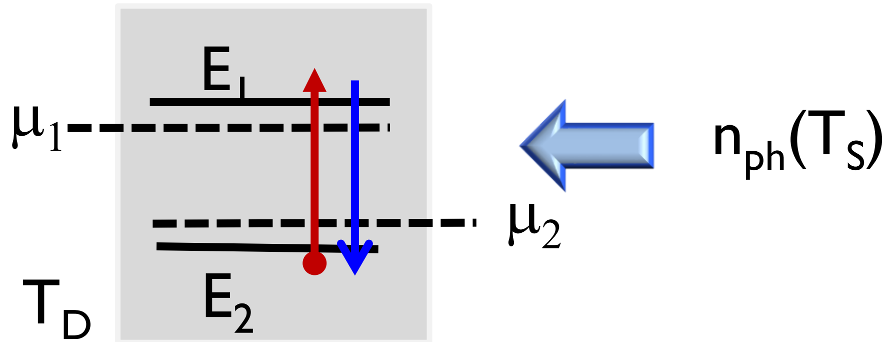
Single frequency filtered light
Isotropic illumination
Atom with only two levels




Some deep sea Plackton live
in a similar diffused light environment

# Detailed balance between Photons and Electrons






$$k_B\theta_1 = E_1 - \mu_1 \quad k_B\theta_2 = E_2 - \mu_2$$

Upward transition $= f_2(1 - f_2)n_{ph}$

Downward transition $= f_1(1 - f_2)(n_{ph} + 1)$

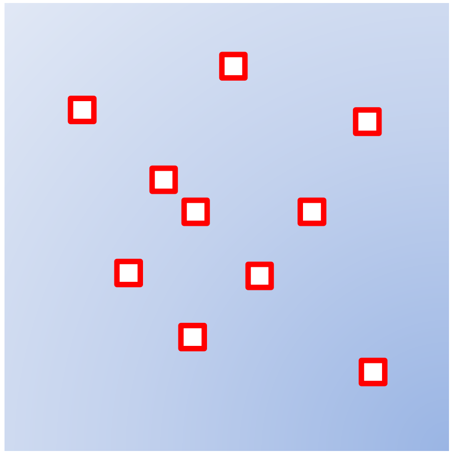
$$f_1 = \frac{1}{e^{\theta_1/T_D} + 1}$$

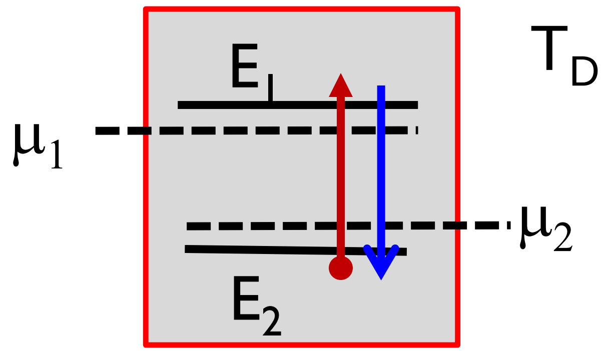
$$f_2 = \frac{1}{e^{\theta_2/T_D} + 1}$$

*At steady state up and down transitions are equal …*

$$f_1(1 - f_2)(n_{ph} + 1) = f_2(1 - f_2)n_{ph}$$

# Detailed balance of photons and electrons





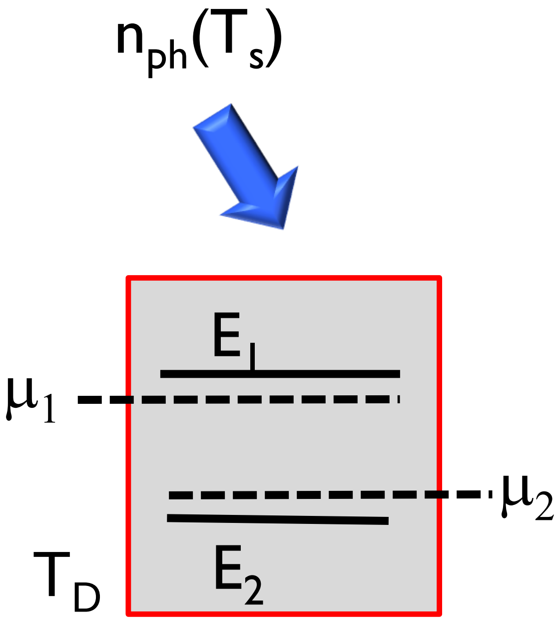

$$f_1(1-f_2)(n_{ph}+1) = f_2(1-f_1)n_{ph}$$

$$\frac{1}{e^{\theta_1/T_D}+1}\,\frac{e^{\theta_2/T_D}}{e^{\theta_2/T_D}+1}(n_{ph}+1) = \frac{1}{e^{\theta_2/T_D}+1}\,\frac{e^{\theta_1/T_D}}{e^{\theta_1/T_D}+1}\times(n_{ph})$$

Bose Einstein distribution

$$n_{ph} = \frac{1}{e^{\hbar\omega/k_B T_S}-1} \qquad n_{ph}+1 = \frac{e^{\hbar\omega/k_B T_S}}{e^{\hbar\omega/k_B T_S}-1}$$

# PV Efficiency of 2-level System




$$\frac{E_2-\mu_2}{T_D}+\frac{E_2-E_1}{T_S}=\frac{E_1-\mu_1}{T_D}$$

$$(\mu_1-\mu_2)=(E_1-E_2)\left[1-\frac{T_D}{T_S}\right]$$

$$(\mu_1-\mu_2)=0$$
$$\text{if } T_D=T_S$$

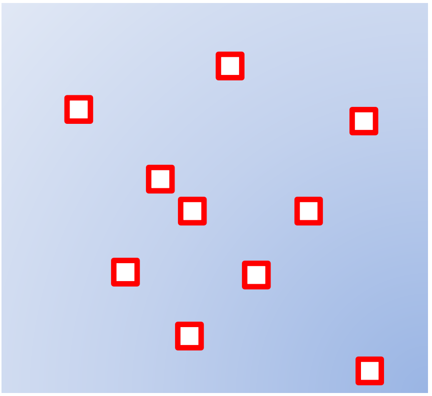
$$\eta=\frac{(\mu_1-\mu_2)R}{(E_1-E_2)R}=\left[1-\frac{T_D}{T_S}\right]$$

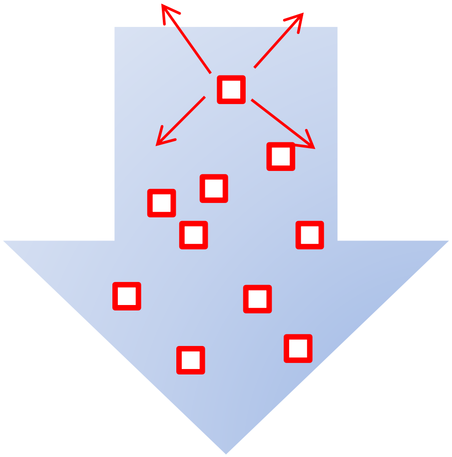
Can not extract energy at same temperature

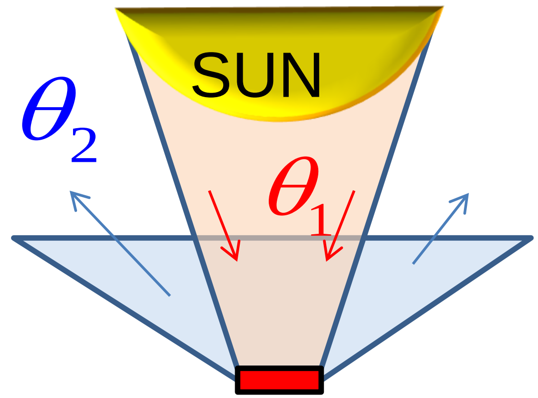
$$\eta=\left[1-\frac{300}{6000}\right]=0.95$$

Single molecule-single frequency gives Carnot efficiency!

# Problem of Angular Anisotropy








$$\theta_2 f_1(1-f_2)(n_{ph}+1) = \theta_1 f_2(1-f_2)n_{ph}$$

$$\text{or, } -\ln\left(\frac{\theta_1}{\theta_2}\right) + \left(\frac{E_2-\mu_2}{k_B T_D}\right) + \left(\frac{E_1-E_2}{k_B T_S}\right) = \left(\frac{E_1-\mu_1}{k_B T_D}\right)$$

$$\therefore \mu_1 - \mu_2 = V_{mp} = E_g\left(1-\frac{T_D}{T_S}\right) + k_B T_D \ln\left(\frac{\theta_1}{\theta_2}\right) \longleftarrow \text{negative number}$$

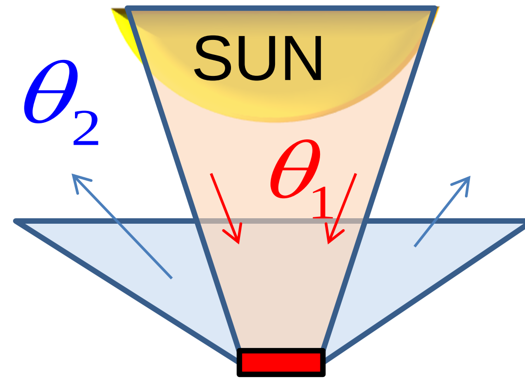
The angle mismatch leads to Vmp loss

# Step 1: Derivation of Vmp complete

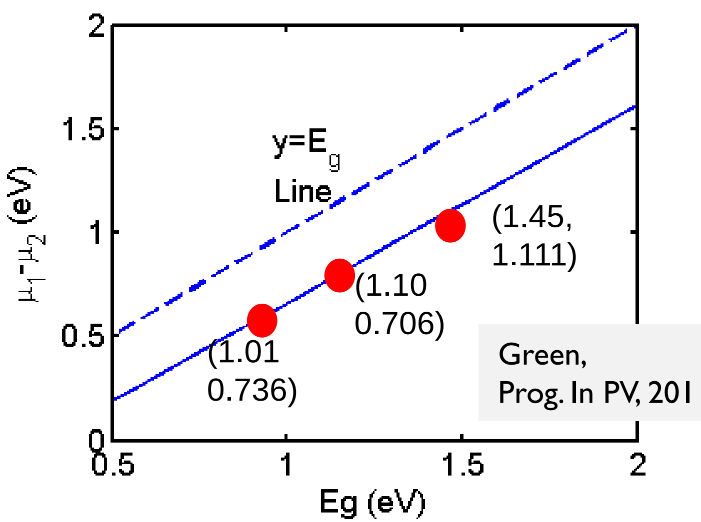
$$\theta_1 = 6\times10^{-5}$$

$\theta_2 = 4\pi$ Without mirror

$\theta_2 = 2\pi$ With mirror




$$\eta = \frac{\mu_1-\mu_2}{E_1-E_2} = \left(1-\frac{T_D}{T_S}\right) + \frac{k_B T_D}{E_1-E_2}\ln\left(\frac{\theta_1}{\theta_2}\right)$$



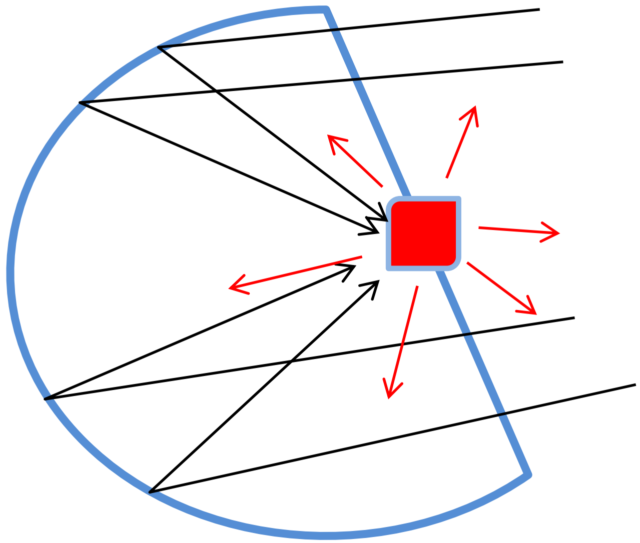

$$\text{Set}, T_S = 6000K, T_D = 300K,$$

$$E_g = E_1 - E_2, \theta_2 = 2\pi :$$

$$qV = \Delta\mu = \left(1-\frac{T_D}{T_S}\right)E_g + k_B T_D \ln\left(\frac{\theta_1}{\theta_2}\right) = 0.95\times E_g - 0.31$$

Consistent with all experimental data !

# Concentrator cells fool the cells



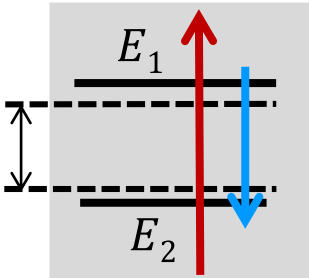
$$\theta_{S,C} = \theta_D$$

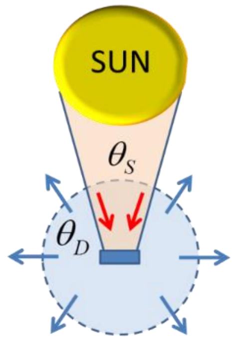
Number of suns …

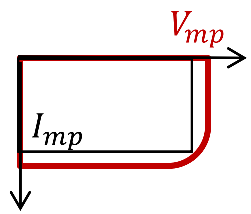
$$N = 2\pi / \theta_S \approx 104720$$

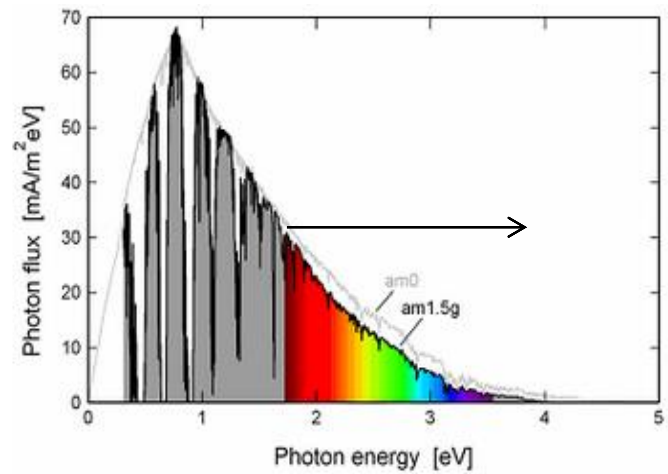
$$V_{mp} = \left(1 - \frac{T_D}{T_S}\right) E_g + k_B T_D \ln\left(\frac{\theta_S}{\theta_D}\right) \rightarrow \left(1 - \frac{T_D}{T_S}\right) E_g$$

# Step 3: Creating the SQ Triangle



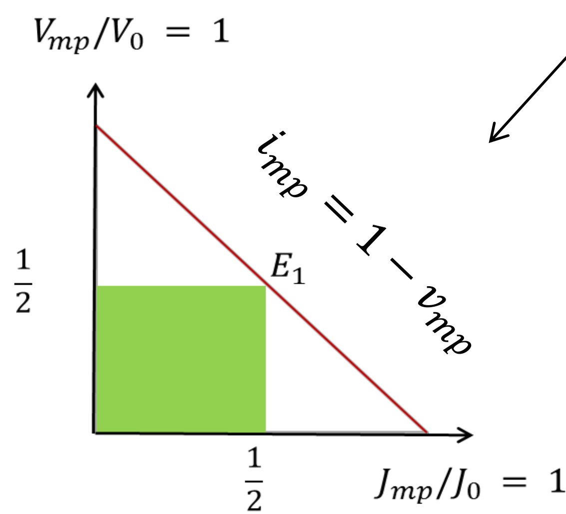




$$qV_{mp} = E_g\left(1 - \frac{T_D}{T_S}\right) - k_B\, T_D \ln\left(\frac{\theta_D}{c\,\theta_S}\right)$$

$$= c_f E_g - \Delta$$

$$J_{mp}(E_g) = cI_{sun}(1 - \beta' E_g)$$
$$= J_0(1 - \beta\, V_{mp})$$

$$\beta \equiv \beta' / c_f$$




$$V_0 \equiv (1 - \beta\Delta)/\beta$$

$$J_0 \equiv cI_{sun}(1 - \beta\Delta)$$

Hirst, PIP, 2011
Khan & Alam, APL, 2015,
Also, see AJP, 2012

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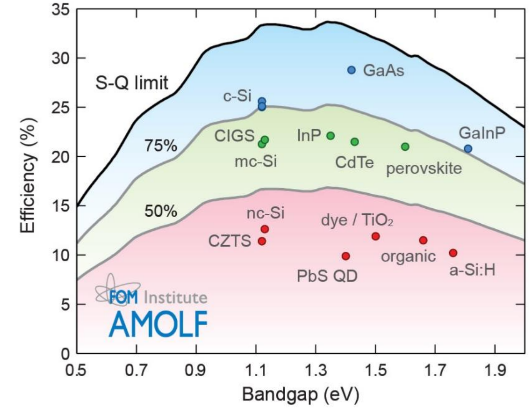
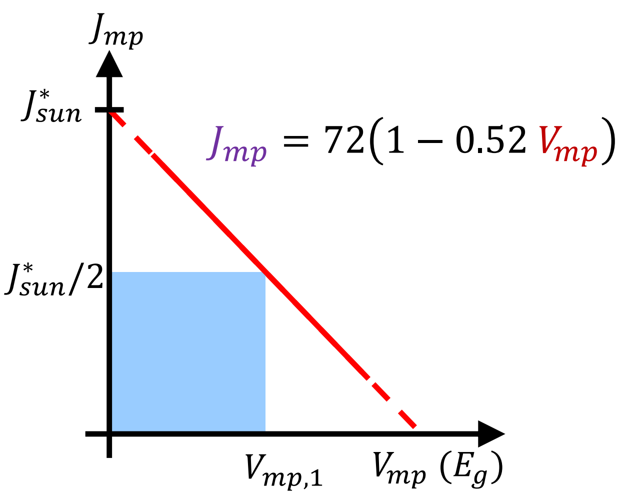
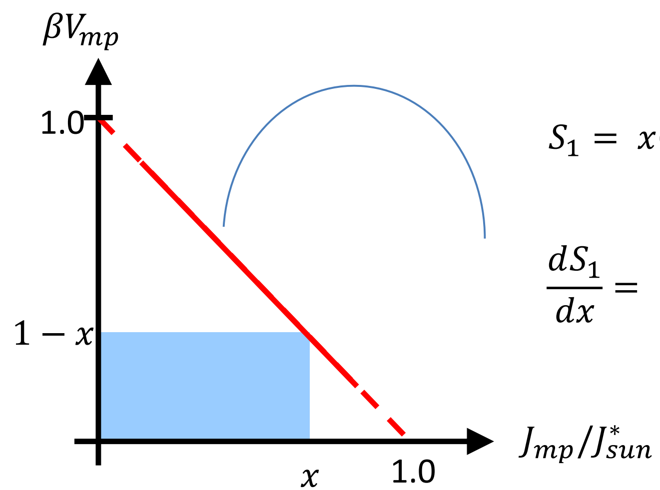
# Maximum Single Junction Efficiency



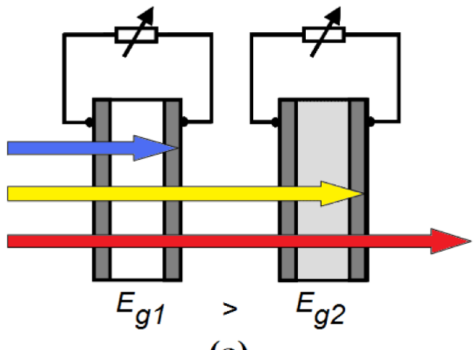



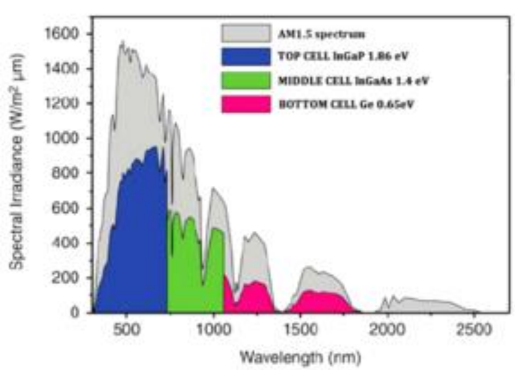

$$S_1 = x(1 - x)$$

$$\frac{dS_1}{dx} = 1 - 2x = 0 \rightarrow x = \frac{1}{2}$$

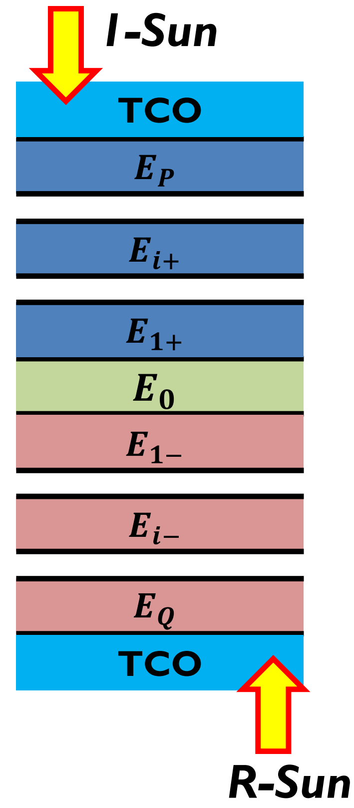
$$0.52 * V_{mp} = \frac{1}{2}, \qquad V_{mp} = 0.96, \qquad 0.95 \times E_g - 0.3 = V_{mp} = 0.96, \qquad E_g = 1.33V$$

# Tandem Solar Cell



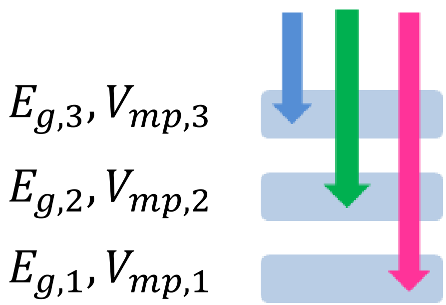



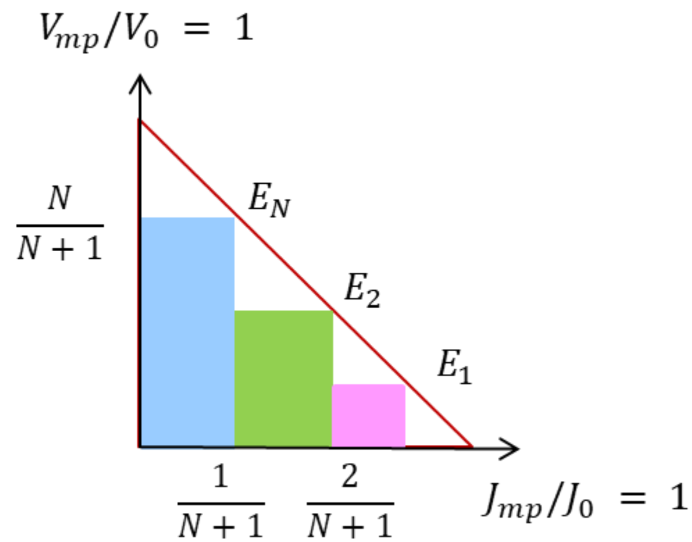

(b)



# N-Junction Tandem Cell

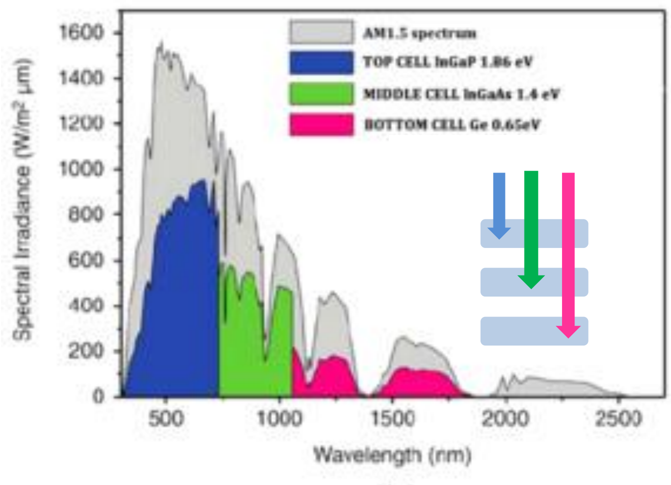
$$V_{mp} = c_f\,E_g - 0.31 - k_B\,T_D \ln c$$

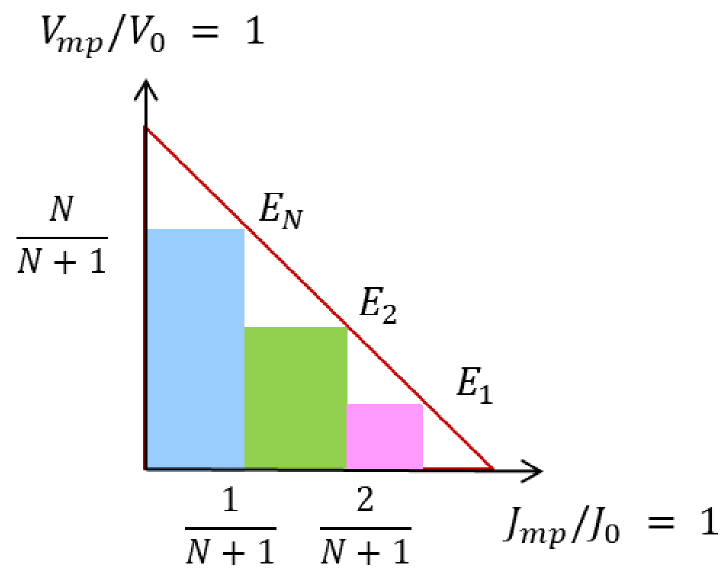


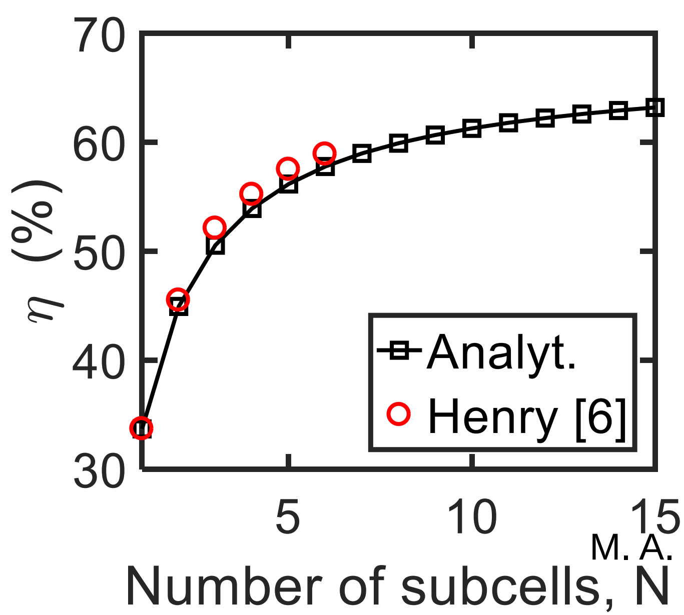




$$V_{mp}^{\{i\}} = iV_0/(N+1)$$
$$I_{mp}^{\{i\}} = I_0/(N+1)$$

# Efficiency of N-junction Tandem




$$V_0 \equiv (1 - \beta\Delta)/\beta$$

$$\beta = \beta'/c_f$$

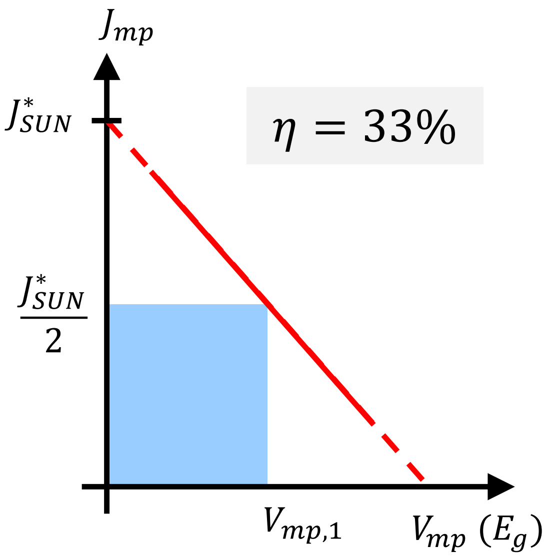
$$I_0 \equiv cI_{sun}(1 - \beta\Delta)$$

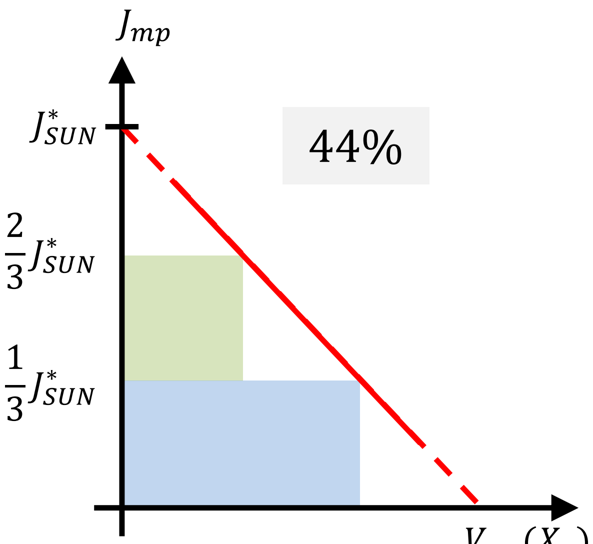
$$\eta_N(c) = (I_0 V_0/4c) \times 2N/(N+1)$$

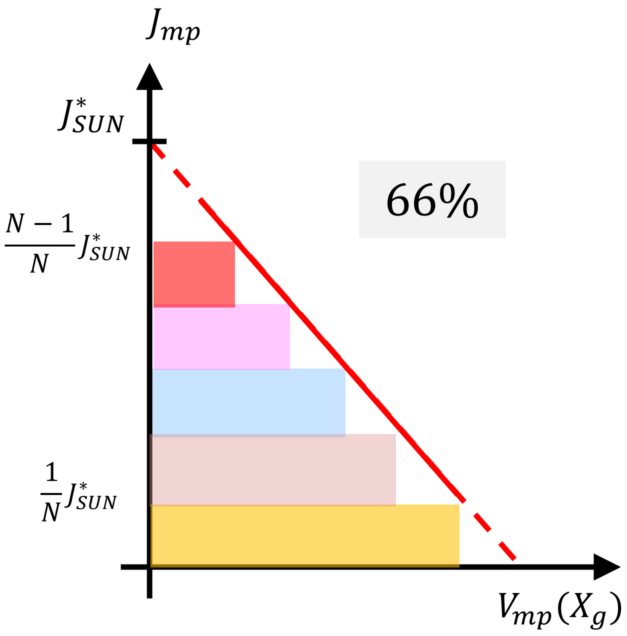
# Tandem Efficiency Limits

$$J_{mp} = 70\left(1 - 0.52\, V_{mp}\right)$$

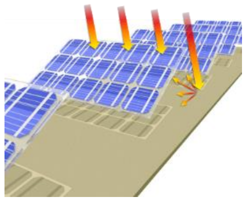
$$P_{out} = J_{mp} \times V_{mp}$$



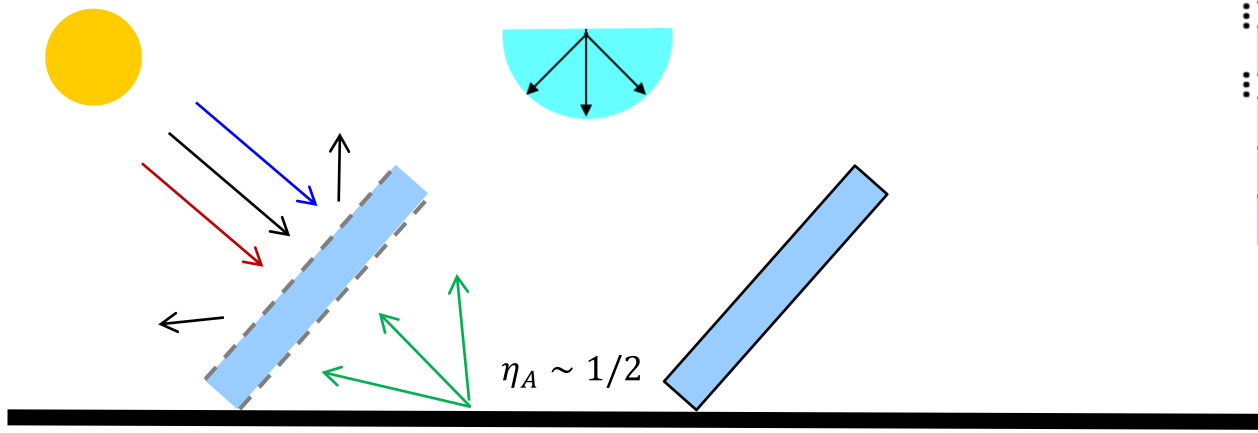



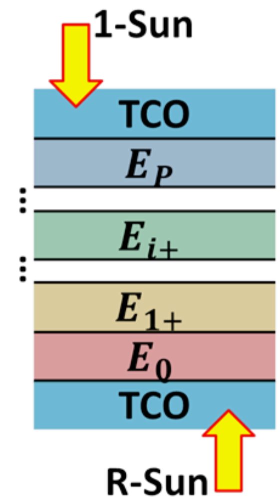



# Bifacial Solar Cell



$$\eta = \eta_N \times \eta_{SQ} \times \eta_M \times \eta_A = \frac{2}{\pi} \times \frac{1}{3} \times \frac{5}{6} \times \frac{1}{2} \sim \frac{1}{10}$$



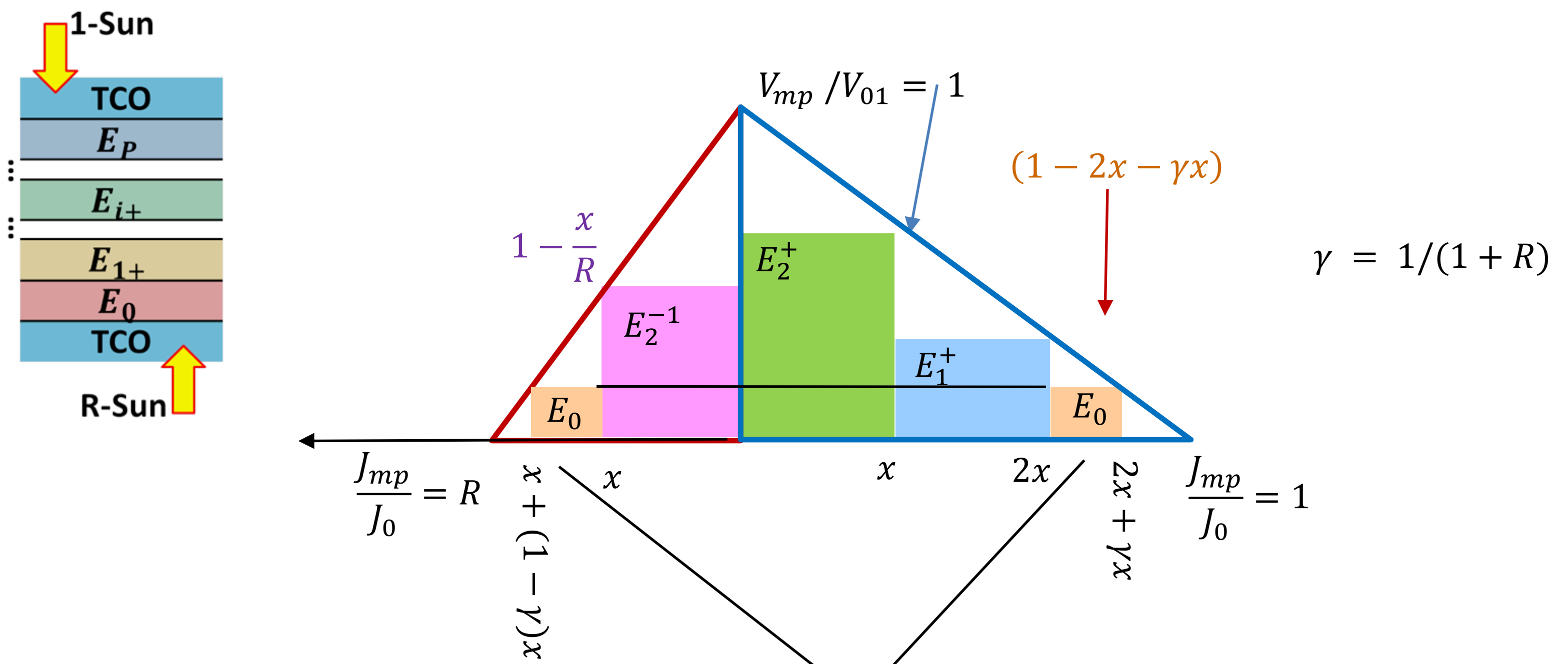






2015 bifacial cell productions

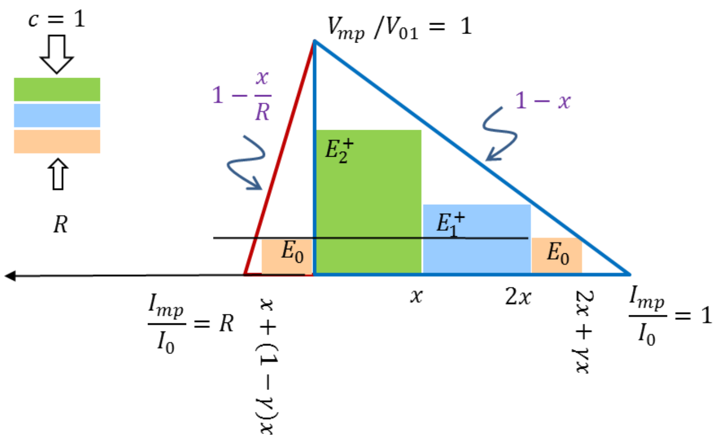
PVG Solutions
bSolar The Bifacial PV Cell Company
nexolon
MegaCell
LG Electronics

# For inverted bifacial design

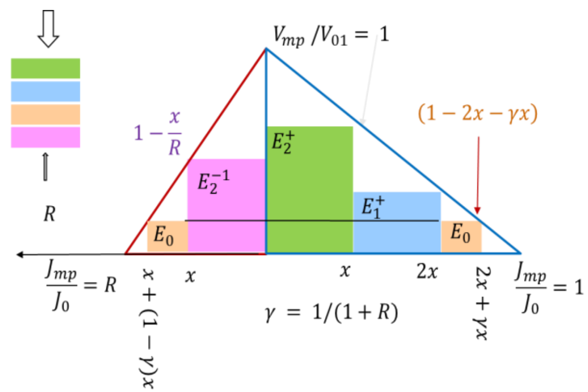



$$S_4 = x(1-x) + x(1-2x) + x(1-2x-\gamma x) + x\left(1-\frac{x}{R}\right)$$

# SQ Triangle for Bifacial PV




$$N_{crit} \leq 1 + R^{-1}$$




$$\frac{S_N}{S_1} = \frac{2(1+R)N^2}{N(N+1)(R+1) - 2R}$$

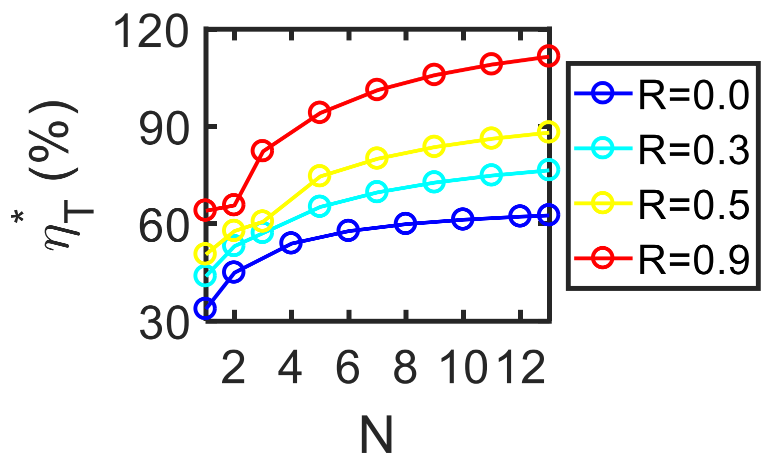
$$\frac{S_N}{S_1} = \frac{8R(1+R)N^2}{2R(2N^2+4N-1) - R^2 - 1}$$

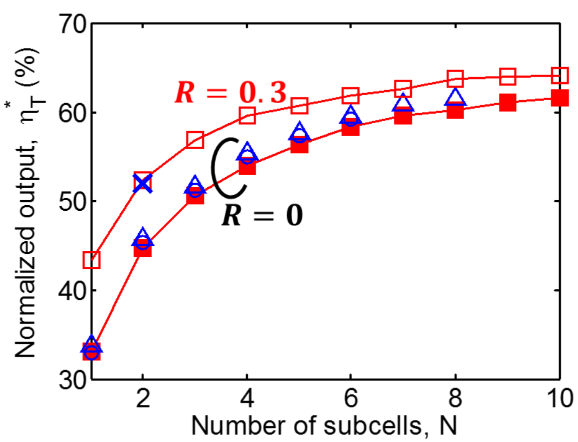
# Advantages of Bifacial PV

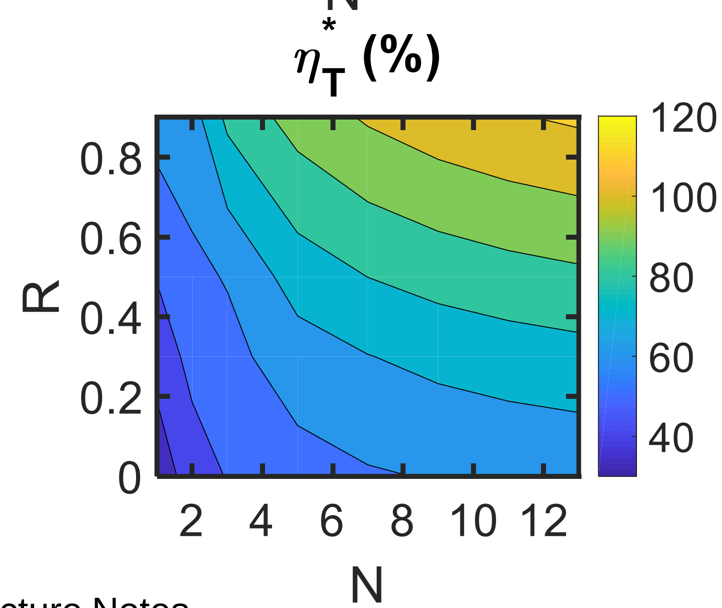
$N_{crit} \leq \; 1 + \; R^{-1}$

$$\frac{S_N}{S_1} = \frac{2(1+R)N^2}{N(N+1)(R+1) - 2R}$$

$$\frac{S_N}{S_1} = \frac{8R(1+R)N^2}{2R(2N^2+4N-1) - \; R^2 - 1}$$

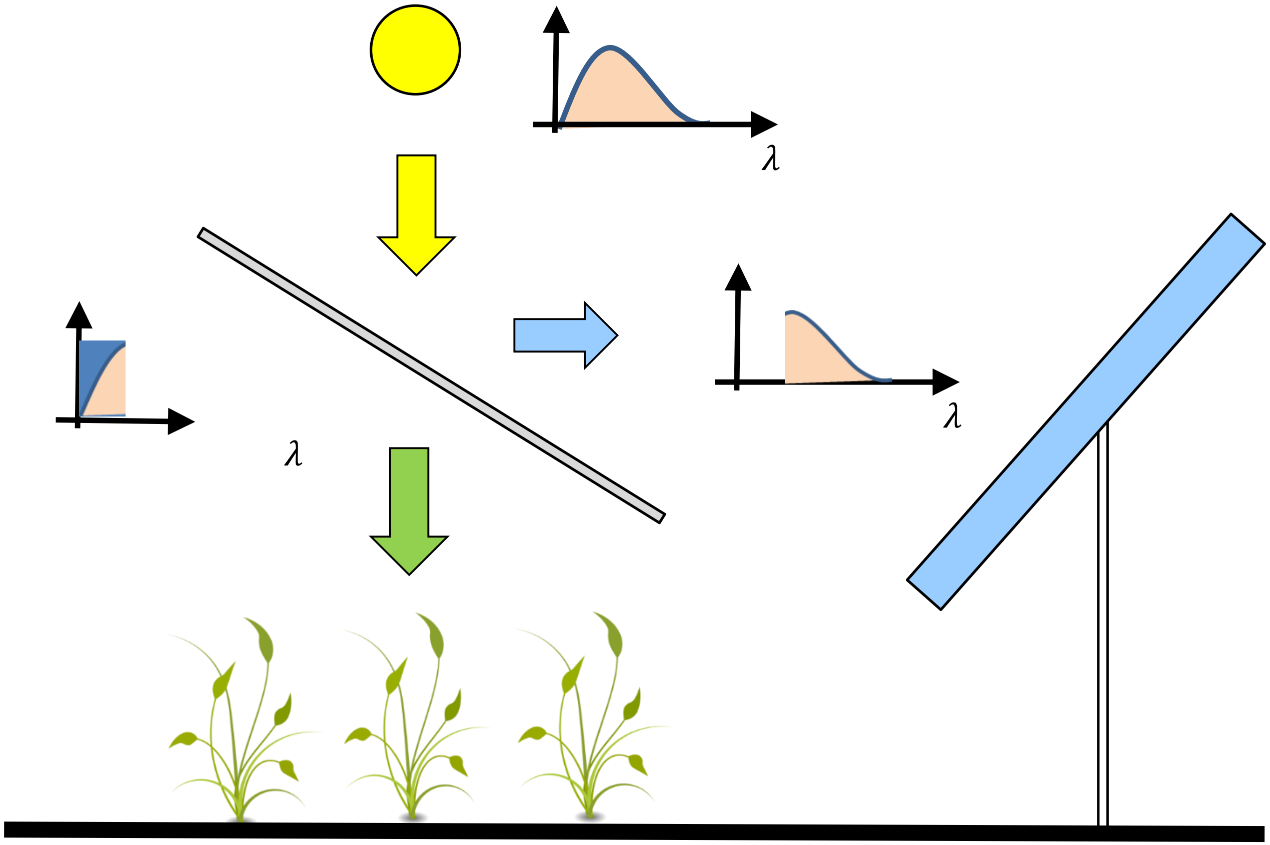


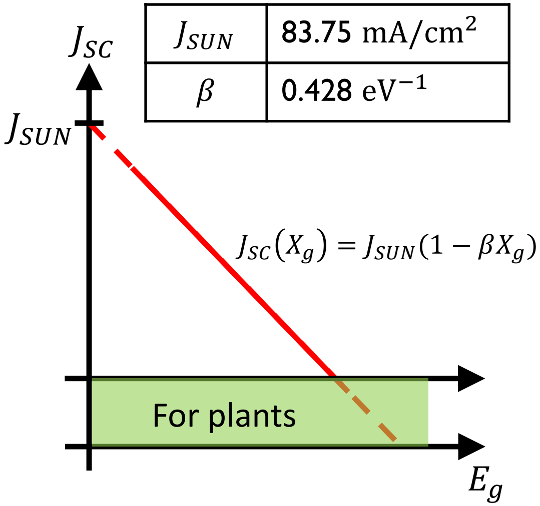

M. A. Alam, PV Lecture Notes

# Plant-PV Tandem cells

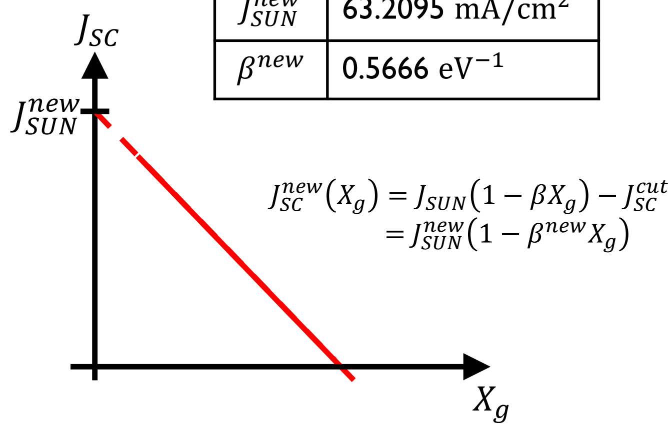


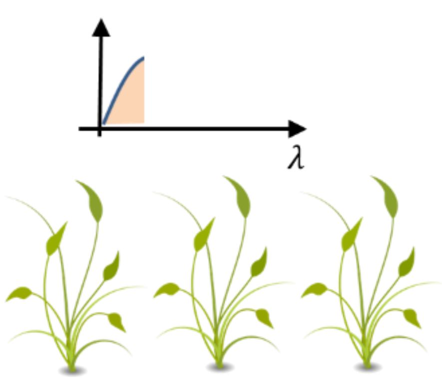
# Rescaled Tandem System

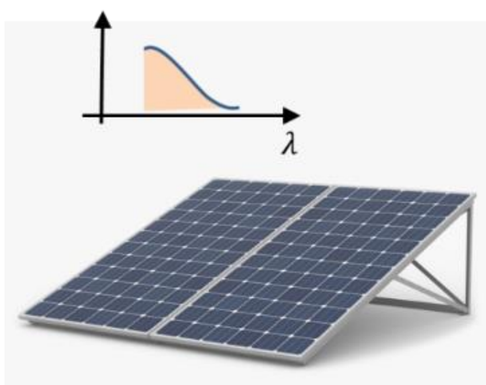
| | |
|---|---|
| $J_{SUN}$ | $83.75\ \mathrm{mA/cm^2}$ |
| $\beta$ | $0.428\ \mathrm{eV^{-1}}$ |




| | |
|---|---|
| $J_{SUN}^{new}$ | $63.2095\ \mathrm{mA/cm^2}$ |
| $\beta^{new}$ | $0.5666\ \mathrm{eV^{-1}}$ |



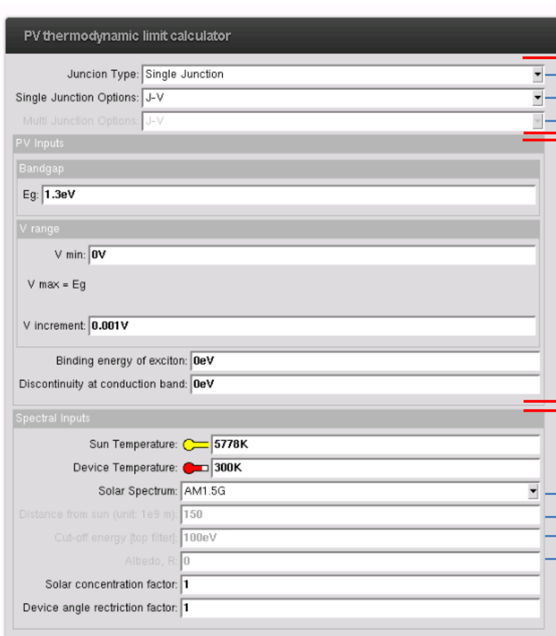

With the new parameters we can now redesign any N-J tandem

# Conclusions

- Solar cells are inefficient machines. A thermodynamic analysis shows how we can improve the performance.
- A simple SQ Triangle anticipates thermodynamic limits for tandem PV, bifacial PV, and concentrator PV cells.
- Tandem efficiency scales as: $\eta_N(c) = I_0\, V_0\, N/[2(N+1)c]$
- Bifacial PV has phase transition at $N_{crit} \leq 1 + R^{-1}$
- Thermodynamic limit serves as a beacon and a guardrail for next generation PV.

# nanohub.org/resources/pvlimits

http://arxiv.org/abs/1606.01176




***Simulation setup***

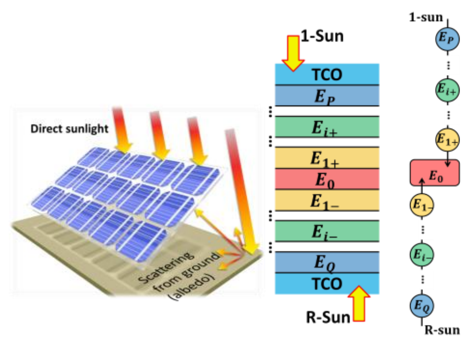
- Single (SJ) or multi-junction (MJ)
- SJ: J-V or Eg-sweep
- MJ: J-V or N-sweep

***Simulation specific input***

These set of inputs change based on the choice of simulation setup

***Spectral input***

- Input spectrum: AM1.5G or ideal Blackbody
- Distance from sun (for Blackbody input)
- Spectral low-pass filter (for MJ calculations)
- Reflectance of the ground (for MJ calc.)




(a) (b) (c)

# Self-assessment

1. How does the average bandgap of tandem cell compare to that of a single junction solar cell?

2. How does the efficiency of a triple junction solar cell compare to that of a single junction solar cell? Hint. Efficiency ratio is given by 2N/(N+1).

3. What is the maximum short-circuit current achievable from AM1.5 illumination? Ans. 70 mA/cm^2.

4. What is the maximum short circuit current for a 4-junction solar cell? Ans. 70/(N+1) = 14 mA/cm^2

5. For R=0.3, what is efficiency of a 4-junction bifacial solar cell?

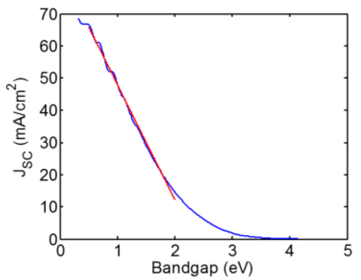
6. If R_s increases by a factor of 5, how much does the critical concentration for a CPV decrease by?

# Wait, Wait don't tell me …

Maximum Jsc (in mA/cm2) from AM1.5 spectrum is

a) 7000  b) 700  c) 70  d) 7

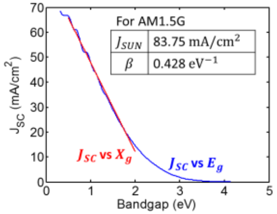
**70 mA/cm$^2$**



# Wait, Wait don't tell me …

Maximum Jsc for Eg=1.7 eV perovskite under AM1.5 illumination is

a) 100  b) 50  c) 20  d) 5



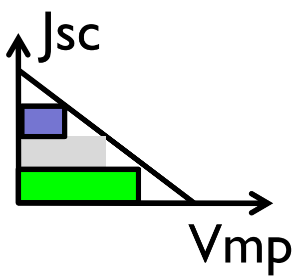

**Jsc=83.75(1-0.43Eg) =23 mA/cm$^2$**

# Wait, Wait don't tell me …

For a 3-junction tandem, maximum Jsc in mA/cm2 is

a) 100 b) 70 c) 17 d) 4

$$J_{sc} = J_{max}/(N+1)$$



# Wait, Wait don't tell me …

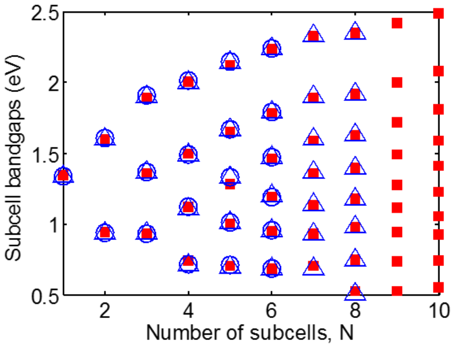
If Eg=1eV, maximum $V_{mp}$ is

a) 1.0  b) 0.65  c) 0.45  d) 0.25

**Voc=0.95Eg-0.31**

# Wait, Wait don't tell me …

In a N=3 tandem,
E1=1.9eV, E3=0.97,
what is E2?




a) 2.5 b) 1.6 c) 1.3 d) 1.1

$$E_{g,ave}=E_{g,SJ}=1.33$$

## Wait, Wait don't tell me …

In a N=3 tandem, What is Vmp?

a) 4    b) 3    c) 2    d) 1

$$V_{mp}/N=0.95E_{avg}-0.31$$

# A Little Formula Sheet

**SJ cell**

$J_{sc,SJ} = J_0\left(1 - \beta E_g\right)$ (AM1.5, $J_0 = 83.75, \beta = 0.428$).

$qV_{oc,SJ} = 0.95 \times E_g - 0.232$

$qV_{mp,SJ} = 0.95 \times E_g - 0.31$

$E_{g,SJ}^{opt} \cong 2.55\, kT_s$

$FF \sim (v_{oc}/v_{oc} + 4.7)$

$\eta_{T.SJ} = -26.45E_g^2 + 70.77E_g - 14.42$ (AM1.5, empirical)

**Tandem**

$J_{sc}(N) = \frac{2}{N+1} J_{sc}(\langle E_g \rangle)$

$qV_{mp}/N = \langle E_g \rangle \left(1 - \frac{T_D}{\langle E_g \rangle} \frac{E_{g,max}}{T_s}\right) - k_B\, T_D\, ln \frac{\theta_D}{\theta_S}$

$$E_{g,\max} = \frac{N-1}{\beta\, N} + \frac{\beta(1+R)E_0 - R}{\beta \times N}$$

**Module**

$(T - T_a) = P/h = 1000(1 - \eta - R)/h$