Compact Model for Double-Clamped Silicon NEMS Resonators

User Manual

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Introduction

Micro/Nanoelectromechanical systems (M/NEMS) are gaining great momentum and interest in a variety of applications, such as high-sensitivity mass sensing, tunable signal filtering and precision timing. They possess inherently high quality factors and can provide narrow bandwidth operation. This model is built for a silicon-based, double-clamped (source and drain), double-gate (back and side gate) beam (Figure 1). The model takes into account capacitive modulation with the two gates, piezoresistive modulation through the beam and electrical parasitic elements.
Mechanical Equation of Motion

The beam’s mechanical equation of motion, based on classical Bernoulli-Euler beam model and mid-plane stretching, is given by

$$\rho w h \frac{\partial^2 y(x, t)}{\partial t^2} + c \frac{\partial y(x, t)}{\partial t} + E I \frac{\partial^4 y}{\partial x^4} - T \frac{\partial^2 y(x, t)}{\partial x^2} = F(x, t)$$

Here, \( T \) is the axial force given by

$$T = S_r w h + \frac{E w h}{2L} \int_0^L \left[ \frac{\partial y(x, t)}{\partial x} \right]^2 dx$$

where \( L, w \) and \( h \) are the length, width, and thickness of the beam, \( \rho \) is the mass density, \( E \) is the modulus of elasticity for the material, \( y(x, t) \) is the deflection of the beam at time \( t \) and a distance along the beam, \( x \), \( c \) is the specific viscous damping coefficient, \( I = \frac{1}{12} w h^3 \) is the moment of inertia for out-of-plane motion and \( S_r \) is the average, uniform residual stress in the beam.
Forcing Model

To account for the beam’s mode shape for small deflections, the forcing model presented in [3] is dimensionalized and expanded in a Taylor series around \( z(t) = 0 \), keeping terms up to \( z^3 \) to allow the forcing term to be properly integrated. The final forcing equation is

\[
F(x, t) = \frac{\varepsilon_0 w V^2(t)}{9 g^5} \left[ 45 g^3 + [90 \phi(x) - 2 g^2 [15 \phi''(x) + g^2 \phi'''(x)]] g^2 z(t) + \{135 g \phi^2(x) + g^3 [-15 \phi'^2(x) + 3 g^2 \phi''^2(x) + 4 g^2 \phi'''(x) \phi'(x)]
+ 2 g^3 [-15 \phi''(x) \phi(x) + g^2 \phi'''(x) \phi(x)] \} \phi(x) \right] z^2(t)
+ \{180 g \phi^3(x) - 30 g^2 \phi'^2(x) \phi(x) - 30 g^2 \phi''(x) \phi^2(x) + 48 g^4 \phi''(x) \phi'^2(x) \} \phi(x) z^3(t)
\]

where \( \phi(x) \) is the mode shape normalized such that the deflection of the beam midpoint is unity.

Capacitance Modulation

When operating, the beam and gate act as a variable capacitor, allowing AC current to flow through them. The capacitance can be obtained by integrating along the length of the beam and the back gate so that the beam’s curvature is taken into consideration. The final capacitance between the beam and gate is given by

\[
C(t) = \frac{\varepsilon_0 L}{g} \left[ L + \kappa_1 z(t) + \kappa_2 z^2(t) + \kappa_3 z^3(t) \right]
\]

where \( \varepsilon_0 \) is vacuum permittivity and the constants \( \kappa_1, \kappa_2, \kappa_3 \) are defined in Table 1. \( g \) and \( L \) are the nominal gap size and beam length, respectively. The charge at time \( t \) is

\[
Q(t) = C(t) V_{\text{gap}}(t)
\]

The current is given by

\[
i(t) = \frac{dQ}{dt}
\]

\[
i_{\text{cap}}(t) = \dot{C}(t) V_{\text{gap}}(t) + C(t) \dot{V}_{\text{gap}}(t)
\]

This is the current contribution from the variable capacitance with the gate.

Piezoresistive Effect

The resistance of the beam will change when the beam deflects, and will contribute to the total current as well. The axial strain from mid-plane stretching is

\[
\varepsilon(t) = \frac{1}{2L} \int_0^L \left[ \frac{\partial y(x, t)}{\partial x} \right]^2 dx = 2.44 \left[ \frac{z(t)}{L} \right]^2
\]
With the transverse and shear stresses neglected, the beam resistance can be described as

\[ R_{\text{beam}}(t) = R_0 [1 + \varepsilon(t) G_R] \]

where \( R_0 = \frac{\rho_r L}{w h} \) is the nominal beam resistance, \( G_R \) is the resistance gauge factor given by

\[ G_R = 1 + 2\nu + E\pi_L, \]

which accounts for both geometric effects, \( 1 + 2\nu \) (\( \nu \) is Poisson’s ratio) and piezoresistive effects, represented by \( E\pi_L \). Here, \( \pi_L \) is the effective longitudinal piezoresistive coefficient, which has a dependence on crystal orientation and other parameters, such as doping and temperature. With all these effects accounted for, the final beam resistance equation can be described as,

\[ R_{\text{beam}}(t) = \frac{\rho_r L}{w h} \left[ 1 + 2.44 \left( \frac{z(t)}{L} \right)^2 (1 + 2\nu + E\pi_L) \right] \]

which depends on the beam deflection.

**Equivalent Circuit Representation**

![Equivalent Circuit Representation](image)

*Figure 3. Equivalent circuit representation of the beam, which combines beam mechanical and electrical behavior [2]*

Figure 3 is the equivalent circuit representation of the beam. There are variable capacitances between the gates and the beam and piezoresistors for each half of the beam. The equations that describe the variable capacitances and beam piezoresistors can be found in the above section, which is the point between the two subdomains, electrical and mechanical. When the device operates, there will be two AC currents flowing into the source, one from the drain, the other from the gates. These currents are modulated by how the beam deflects and both contribute to the output current. Figure 4 shows the block diagram of the interplay of electrical and mechanical behavior of the beam [2].
### Table 1. Coefficients – Dimensional Form [2]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_0$</td>
<td>$\rho A \int_0^L \phi^2 , dx$</td>
</tr>
<tr>
<td>$B_1$</td>
<td>$c \int_0^L \phi^2 , dx$</td>
</tr>
<tr>
<td>$B_2$</td>
<td>$EI_b \int_0^L \phi''' \phi , dx - S_r w h \int_0^L \phi'' \phi , dx$</td>
</tr>
<tr>
<td>$B_3$</td>
<td>$\frac{EA}{2L} \int_0^L \phi^2 , dx \int_0^L \phi'' \phi , dx$</td>
</tr>
<tr>
<td>$f_0$</td>
<td>$45g^3 \int_0^L \phi , dx$</td>
</tr>
<tr>
<td>$f_1$</td>
<td>$g^2 \left[ 90 \int_0^L \phi^2 , dx - 2g^2 \left( 15 \int_0^L \phi'' \phi , dx + g^2 \int_0^L \phi''' \phi , dx \right) \right]$</td>
</tr>
<tr>
<td>$f_2$</td>
<td>$135g \int_0^L \phi^3 , dx + g^3 \left( -15 \int_0^L \phi^2 \phi , dx + 3g^2 \int_0^L \phi'' \phi , dx + 4g^2 \int_0^L \phi''' \phi , dx \right) + 2g^3 \left( -15 \int_0^L \phi'' \phi^2 , dx + g^2 \int_0^L \phi''' \phi^2 , dx \right)$</td>
</tr>
<tr>
<td>$f_3$</td>
<td>$180 \int_0^L \phi^4 , dx - 30g^2 \int_0^L \phi^2 \phi^2 , dx - 30g^2 \int_0^L \phi'' \phi^3 , dx + 48g^4 \int_0^L \phi'''' \phi' \phi , dx$</td>
</tr>
<tr>
<td>$f_c$</td>
<td>$\frac{\varepsilon_0 w V^2(t)}{90g^2}$</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>$\frac{1}{g} \int_0^L \phi , dx$</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>$\frac{1}{g^2} \int_0^L \phi^2 , dx + \frac{1}{3} \int_0^L \phi^2 , dx + \frac{g^2}{45} \left( \int_0^L \phi'' \phi , dx + 2 \int_0^L \phi''' \phi' , dx \right)$</td>
</tr>
<tr>
<td>$\kappa_3$</td>
<td>$\frac{1}{45g^3} \left( 45 \int_0^L \phi^3 , dx + 6g^4 \int_0^L \phi'' \phi^2 , dx + 15g^2 \int_0^L \phi''' \phi , dx \right.$ \hspace{1cm}$-g^4 \int_0^L \phi'''' \phi , dx - 2g^4 \int_0^L \phi'''' \phi' , dx \right)$</td>
</tr>
</tbody>
</table>
Simulation

Figure 5 shows the experimental testing circuit as well as the simulation testbench circuit. An AC signal (modulated or not) is applied to the drain of the beam, while the source is connected into a lock-in amplifier. A DC voltage is applied to the gate. Various parasitic elements are considered here, including the wire capacitance, wire resistance, contact resistance, leakages, etc. The output current/voltage at the lock-in amplifier is measured.

Figure 4. Block diagram of the interplay of electrical and mechanical behavior of the beam [2]

Figure 5. Experimental testing circuit schematic and testing circuit simulation schematic [2]
For the circuit simulation, the spectreRF harmonic balance solver is used. For this highly sensitive simulation, a conservative accuracy is set. Also, in the simulator option, the relative tolerance (reltol), voltage absolute tolerance (vabstol) and current absolute tolerance (iabstol) are set to be 1e-8. To help the convergence, the transient-aided HB (tstab) is set to be 1e-4 s.

**Single-tone Simulation**

This is a test of the system driven by a single-tone AC excitation. The beam is 4 μm in length, 110 nm in thickness and 180 nm in width, while the gap between the beam and gate is 144 nm. The rest of the parameters are listed in the file `parameters.txt`. Figure 6 depicts the frequency response under a 40 mV rms AC excitation and 6 V DC back gate bias. This simulation is to measure the output voltage across resistor of the lock-in amplifier under different sweeping frequencies. A clear resonant peak is observed at 64.4 MHz.

![Figure 6. Frequency response of the beam driven by a single-tone excitation. The red line represents the magnitude. The blue line represents the phase.](image)

**Multi-tone Simulation**

This is a test of the system driven by an amplitude modulated (AM) AC signal. The beam is 5.9 μm in length, 110 nm in thickness and 180 nm in width, while the gap between the beam and gate is 144 nm. The rest of the parameters are listed in the file `parameters.txt`. Figure 7 is the corresponding frequency response. The modulation factor (M) is 0.5 and the modulation frequency is 1 kHz. The amplitude of the carrier signal is 40 mV rms. 6 V DC is applied to the back gate. The simulation is to monitor the output current under the modulated frequency, 1 kHz,
while the carrier frequency is swept around the natural frequency. From Figure 7, under this excitation condition and bias condition, strong nonlinearity and hysteresis are observed.

![Figure 7. Frequency response of the beam under an amplitude modulated excitation. The left figure is the magnitude of the output current at modulated frequency 1 kHz as a function of carrier frequency. The right figure is the phase. Red curves represent the response when the carrier frequency is swept up while the black curves correspond to downward sweeps.](image)

**Comments**

While this model is built for a double-gate silicon beam, currently, only behaviors associated with out-of-plane vibrations induced by a back gate excitation are valid. The side gate capacitance is set to be static in the model. For a more detailed discussion about the model, please refer to the reference [2]

