UCSB Graphene-Nano-Ribbon (GNR) Interconnect model
(VERSION = 0.9.0)

by Junkai Jiang, Jiahao Kang, Wei Cao, Chuan Xu, Prof. Kaustav Banerjee

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Terminal and Distributed Circuit Definitions

This model includes two terminals, P and N, which represents the positive and negative terminal of a GNR interconnect. A distributed RLC equivalent circuit for GNRs is considered in the model. $R_Q$ is the quantum contact resistance, $r$ is the GNR resistance per unit length, $l_M$ is the GNR magnetic inductance per unit length (neglected in current version), $l_K$ is the GNR kinetic inductance per unit length, and $c_Q$ and $c_E$ are the GNR quantum capacitance and electrostatic capacitance (neglected in current version) per unit length.

Also to notice that terminal P and N are symmetric, i.e., completely interchangeable.

User-defined Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Default Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$</td>
<td>1e-6 m</td>
<td>Length of GNR</td>
</tr>
<tr>
<td>$w$</td>
<td>1e-7 m</td>
<td>Width of GNR</td>
</tr>
<tr>
<td>$N_L$</td>
<td>1</td>
<td>Number of layers of GNR</td>
</tr>
<tr>
<td>$dope$</td>
<td>0 m$^3$</td>
<td>Dopant concentration in GNR</td>
</tr>
<tr>
<td>$l_D$</td>
<td>1e-6 m</td>
<td>Carrier mean free path (MFP)</td>
</tr>
</tbody>
</table>
**Introduction**

UCSB GNR interconnect model is based on a distributed RLC circuit, in which carrier mean free path, graphene doping concentration (Fermi level) and number of layers are considered. The model was originally published by our group in *(Xu, Chuan et al. TED 2009)*. Using a simple tight-binding model and the linear response Landauer formula, the resistance per unit length of GNR is derived. In addition to the resistance, the quantum capacitance, kinetic inductance and quantum contact resistance are considered. This model is compatible with both DC and transient simulations. Current version of the model only supports the model of armchair GNR (ac-GNR), and only complete diffusive edge (edge specularity \( p = 0 \)). Zigzag GNR (zz-GNR) and non-diffusive edge scatterings will be considered in later versions.

**GNR Interconnect Parasitics**

The GNR interconnect parasitics in this model include contact resistance, resistance per unit length, inductance per unit length and capacitance per unit length. The electrostatic capacitance \( c_E \) and magnetic inductance \( l_M \) are neglected in current version. Generally, the kinetic inductance \( l_K \) in monolayer GNRs is much larger than the magnetic inductance \( l_M \), while that is not always the case in multilayer GNRs. On the other hand, the quantum capacitance \( c_Q \) in multilayer GNRs is much larger than the electrostatic capacitance \( c_E \), while that is not always the case in monolayer GNRs.

**I. Quantum contact capacitance**

The quantum contact resistance is defined as

\[
R_Q = \left( \frac{h}{2q^2} \right) / N_{ch} N_{layer}
\]

where \( N_{ch} \) is the number of conducting channels (modes) in one layer, \( N_{layer} \) is the number of GNR layers, \( h \) is the Planck’s constant.

**II. GNR resistance per unit length**

Based on the linear response (small voltage drop along the length) Laudauer formula, the conductance of the \( n \)th conduction mode in a single GNR layer \( G_n \) can be expressed as

\[
G_n = 2q^2 / h \int T_n(E)(-\partial f_0 / \partial E)dE
\]

\[
f_0(E) = \{1 + \exp[(E - E_F) / k_B T]\}^{-1}
\]
where $T_n(E)$ is the transmission coefficient, $f_0(E)$ is the Fermi-Dirac distribution function, $E_F$ is the Fermi level, $k_B$ is Boltzmann’s constant, and $T$ is the temperature. Assuming complete diffusive edge, by using the Matthiessen’s rule, $T_n(E)$ can be obtained by

$$\frac{1}{T_n(E)} = 1 + \frac{L}{l_D \cos \theta} + \frac{L}{w \cot \theta} \approx \frac{L}{l_D \cos \theta} + \frac{L}{w \cot \theta}$$  \hspace{1cm} (4)$$

where $l_D$ is the carrier mean free path, $w$ is the width of GNR, and $\cot \theta$ is the ratio of longitudinal (along the wire length) to transverse (across the wire width) velocities. In (4), the term “1” is due to quantum conductance, which can be ignored when $L \gg l_D$.

The total conductance of a single GNR layer is the summation of conductance of electrons and holes, as in (5).

$$G_{total} = \sum_n G_n (\text{electrons}) + \sum_n G_n (\text{holes})$$ \hspace{1cm} (5)$$

And the summation in (5) can be transformed to an integration form as follows:

$$G_{total} = \frac{2}{\Delta E_n} \left[ \int_0^\infty G_n (\text{electrons}) dE_n + \int_{-\infty}^0 G_n (\text{holes}) dE_n \right]$$ \hspace{1cm} (6)$$

which can further be derived as

$$G_{total} = \frac{4q^2}{Lh^2v_f} \cdot 2k_BT \ln \left[ 2 \cosh \left( \frac{E_F}{2k_BT} \right) \right] \cdot func(w, l_D)$$ \hspace{1cm} (7)$$

where $func(w, l_D)$ is defined as

$$func(w, l_D) = \begin{cases} \frac{\pi w - 2l_D}{l_D} + \frac{4\sqrt{l_D^2 - w^2}}{l_D} \cdot \text{arctanh} \left( \frac{l_D - w}{l_D + w} \right), & l_D \geq w \\ \frac{\pi w - 2l_D}{l_D} - \frac{4\sqrt{l_D^2 - w^2}}{l_D} \cdot \text{arctan} \left( \frac{l_D - w}{l_D + w} \right), & l_D < w \end{cases}$$ \hspace{1cm} (8)$$

**III. GNR capacitance per unit length**

By neglecting the electrostatic capacitance in the current version of GNR interconnect model, the capacitance per unit length is calculated as

$$c_{total} = c_q = N_{layer}N_{ch}4q^2/\hbar v_f$$ \hspace{1cm} (9)$$

where $v_f$ is the Fermi velocity, which is estimated as $10^6$ m/s in the model.
VI. GNR inductance per unit length

By neglecting the magnetic inductance in the current version of GNR interconnect model, the inductance per unit length is calculated as

\[ l_{\text{total}} = l_k = \frac{(h/4q^2v_f)}{N_{\text{layer}}N_{\text{ch}}} \]  
\[ (10) \]

The number of conducting channels (modes) \( N_{\text{ch}} \) can be calculated as

\[ N_{\text{ch}} = N_{\text{ch,electron}} + N_{\text{ch,hole}} = \sum_n \left[ 1 + \exp \left( \frac{E_{n,\text{electron}} - E_F}{k_B T} \right) \right]^{-1} + \sum_n \left[ 1 + \exp \left( \frac{E_F - E_{n,\text{hole}}}{k_B T} \right) \right]^{-1} \]  
\[ (11) \]

where \( E_{n,\text{electron}} (E_{n,\text{hole}}) \) is the minimum (maximum) energy of the \( n \)th conduction (valence) subband.
**DC Circuit Simulation**

For both DC and transient simulations, the following circuit is applied.

![Circuit Diagram](image)

*Figure 2 Circuit for DC and transient simulations*

![Graph](image)

*Figure 3 GNR resistance per unit length (Ohm/um) with zero doping (EF=0) from DC simulation*
Figure 4 GNR resistance per unit length (Ohm/um) with doping concentration of 2.02e35 m$^{-3}$ ($E_F=0.21eV$) from DC simulation
Transient Circuit Simulation

Figure 5 Transient simulation results