NEEDS Thermoelectric Compact Model Documentation

Version 1.0.0

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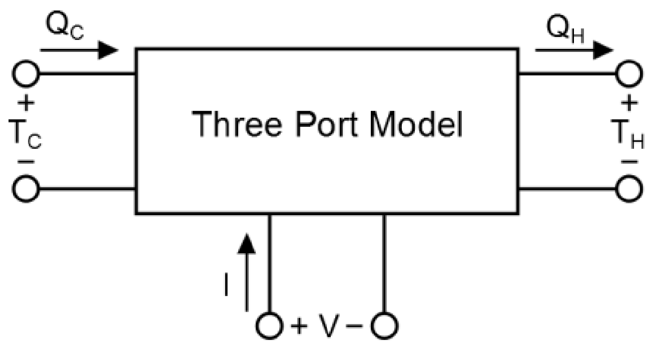
Introduction

The NEEDS thermoelectric compact model (*TEsegment.va*) describes a homogeneous segment of thermoelectric material and serves as a basic building block for complex electrothermal system. This work is derived from Kyle Conrad's thesis (Conrad 2015).

The compact model has three sets of terminal pairs (or ports). The compact model, at its heart, is a simple linear resistor between the electrical port. To calculate the thermal effects, we also need to know the temperature at each electrical contact. Thus, there are two additional ports for temperatures.

Build-in effects include the electrical and thermal resistances, Seebeck and Peltier effects, Joule heating, and heat capacitances. Modeling details are discussed in the *Model Equations* section.



Instance Parameters

Name	Default	Min.	Max.	Unit	Description
l	$5x10^{-3}$	0	Inf	m	Thermoelectric segment length
area	$1x10^{-6}$	0	inf	m^2	Thermoelectric segment cross-section area

Special Parameters

Name	Default	Min.	Max.	Unit	Description
version	1	0	Inf		
subversion	0	0	Inf		
revision	0	0	Inf		
pieces	50	2	inf		Number of discretization pieces the segment is divided into. While higher number will yield better accuracy, the number of circuit elements grows and slows down simulation time.

Model Parameters

Name	Default	Min.	Max.	Unit	Description

res	$2x10^{-5}$	0	inf	Ωm	Electrical resistivity
density	$7.75x10^{6}$	0	Inf	Kg/m^3	Material density
s	$2.45x10^{-4}$	-inf	Inf	V/K	Seeback coefficient
cp	0.158	0	Inf	j/gK	Heat capacity
kappa	1.4	0	Inf	W/mK	Thermal conductivity

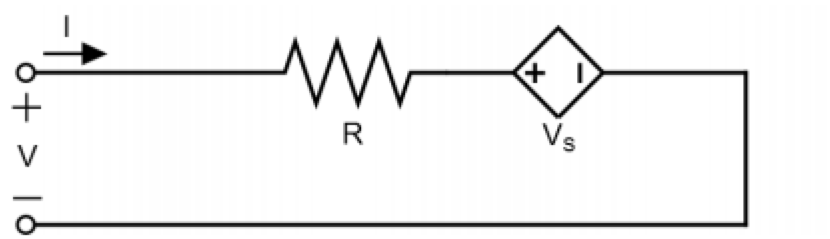
Model Equations

Here we examine the physical details within the thermoelectric compact model.

The entire compact model can be divided into two parts: the electrical and thermal circuits. Both the electrical and thermal circuits are simulated under the electrical nature, using thermal elements' electrical counterparts (shown in Table #1). This allows us flexibility and simplicity in designing the circuit. This also makes integrating the compact model into an electrical circuit easy.

Thermal Variable		Electrical Variable	
Heat Flow	W	Current	A
Temperature	V	Voltage	V
Thermal Resistance	K/W	Resistor	Ω
Thermal Mass	J/K	Capacitor	F

Electrical Circuit



The electrical circuit is composed of two components: the electrical resistance and the Seebeck voltage. The model equation for the electrical resistance is straightforward being

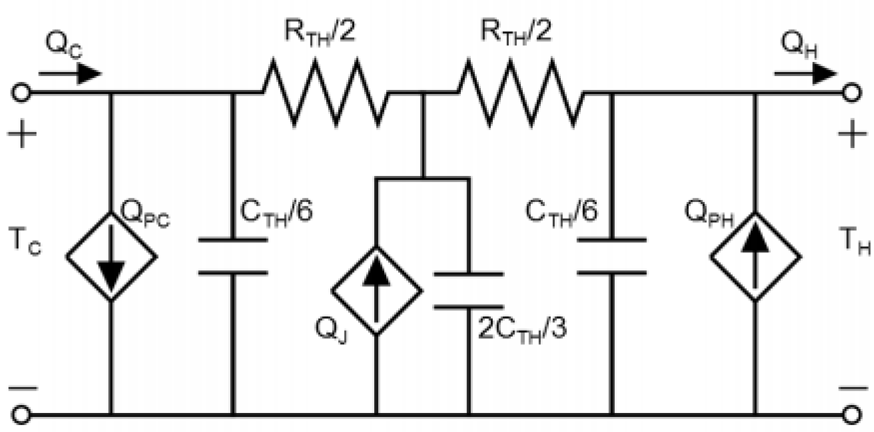
$$V = IR \, , \tag{1.1}$$

where V is the applied voltage, R is the electrical resistance, I is the resulting electrical current. The Seeback voltage, induced by the temperature gradient, is modeled with

$$V_s = s\Delta T = s(T_H - T_C) \, , \tag{1.2}$$

where V_s is the Seeback voltage, s is the Seeback coefficient, T_H is the hot-side temperature, and T_C is the cold-side temperature. Recall that the temperatures are manifested as voltages in the compact model.

Thermal Circuit



The thermal circuits are consisted of three distinct parts: 1) thermal resistance, 2) Joule heating, 3) Peltier heating/cooling, and 4) heat capacitance.

The thermal resistance is modeled as R_{th}. It is equally divided into two halves, so the Joule heating can be generated in the middle of the structure. The Joule heating is modeled as

$$Q_J = I^2 R. \tag{1.3}$$

The Peltier cooling/heating occur at the contacts and are defined as

$$Q_{PC} = sIT_C \tag{1.4}$$

$$Q_{PH} = sIT_H \ . \tag{1.5}$$

The thermal capacitances are partitioned into three portions: $C_{TH}/6$, $C_{TH}/6$, $2C_{TH}/3$. The reason behind this particular partition ratio is discussed in detail in (McAndrew and Bettinger 2012).

Benchmarks

Single Bi_2Te_3 thermoelectric leg

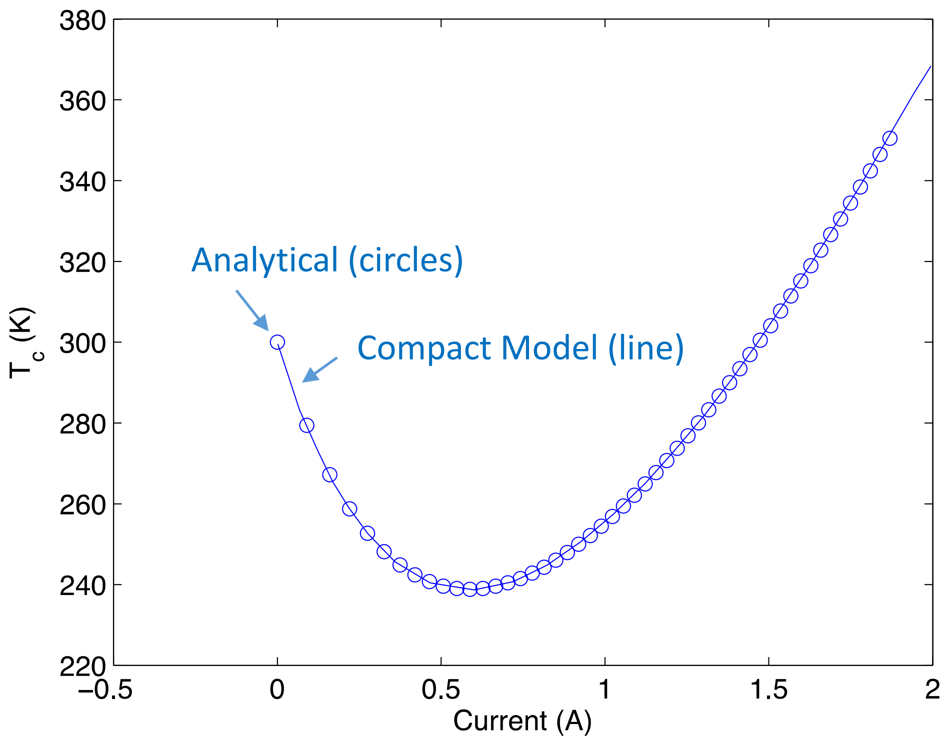
(See benchmark file: *benchmark_kyle_master.m*)

The performance of a single thermoelectric leg is an analytically solvable problem. In this section, we compare the analytical solution to the one found by using the NEEDS Thermoelectric Compact Model.

The setup includes a single p-type thermoelectric leg. The hot side is fixed at room temperature at 300 K, and the cold side temperature is left “floating”. The material and device parameters are listed below:

s = 245 $\mu V/K$	κ = 1.4 W/mK	$1/\rho$ = 20 $\mu\Omega/m$	ρ = 7.75x10^6 kg/m^2
C_p = 0.158 J/gK	L = 5 mm	A = 1 mm^2	

It is predicted that, for this device, the minimal cold side temperature is 238.85 K, which is found at a current of 0.585 A. The simulation result is plotted below along with the analytical expression. The two show good agreement.



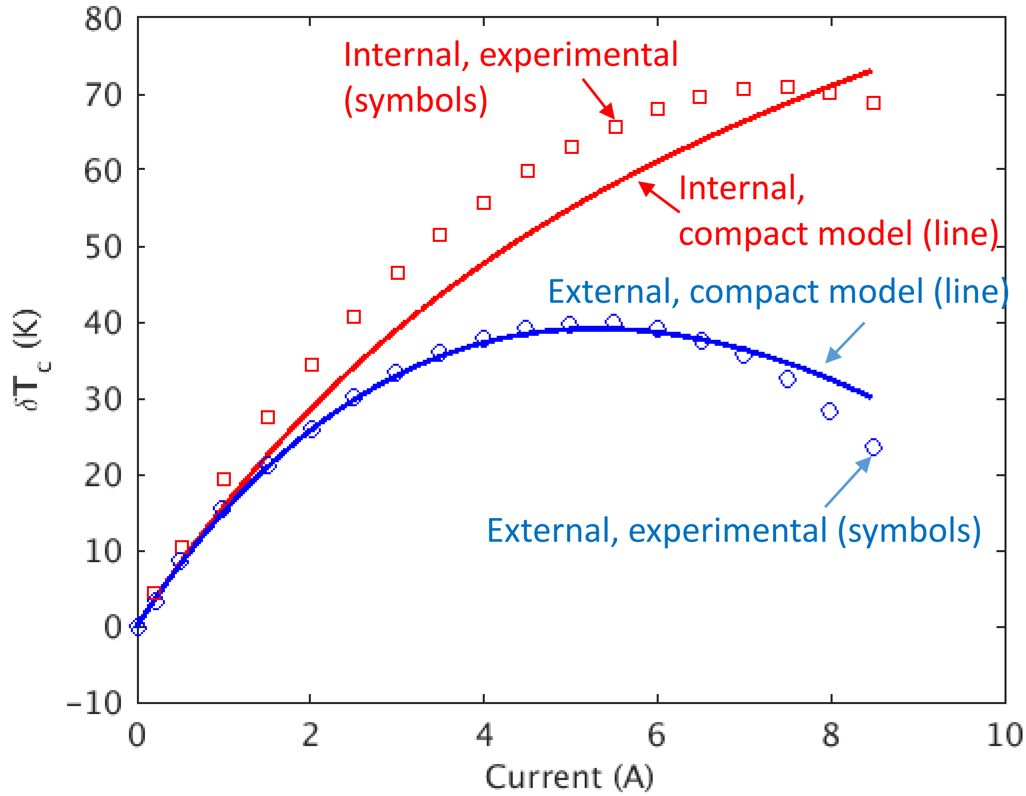
Initially from zero current, as the current increases, the Peltier cooling decreases the temperature at the cold side. As the current further increases, the Joule heating becomes significant enough to overcome the cold-side Peltier cooling effect, and the cold-side temperature begin to increase after reaching a minimum value at 238.85 K.

Experimental data by (Bulman, Siivola et al. 2006)

(See benchmark file *benchmark_bulman.m*)

We also benchmark our compact model with the experimental work in (Bulman, Siivola et al. 2006).

In this work, a functioning thermoelectric cooler is fabricated for hotspot cooling. We use the compact model to simulate the cooling performance. The results are shown below.



The *internal* ΔT denotes the temperature difference between the internal hot and cold sides of the thermoelectric leg. It is measured by placing a thermal couple directly to the contact layer. The *external* ΔT denotes the temperature difference reading from the contacts. The difference between internal and external temperature differences arises from the interface resistance between the contacting metal to the thermoelectric leg. Details of this discussion can be found in (Bulman, Siivola et al. 2006).

References

Bulman, G. E., et al. (2006). "Large external ΔT and cooling power densities in thin-film Bi[sub 2]Te[sub 3]-superlattice thermoelectric cooling devices." Applied Physics Letters **89**(12): 122117.

Conrad, K. (2015). A physics-based compact model for thermoelectric devices. Electrical and Computer Engineering, Purdue University. **Master of Science:** 128.

McAndrew, C. C. and T. Bettinger (2012). "Robust Parameter Extraction for the R3 Nonlinear Resistor Model for Diffused and Poly Resistors." IEEE Transactions on Semiconductor Manufacturing **25**(4): 555-563.