Lecture 14
Introduction to dynamic AFM

Arvind Raman
Mechanical Engineering
Birck Nanotechnology Center

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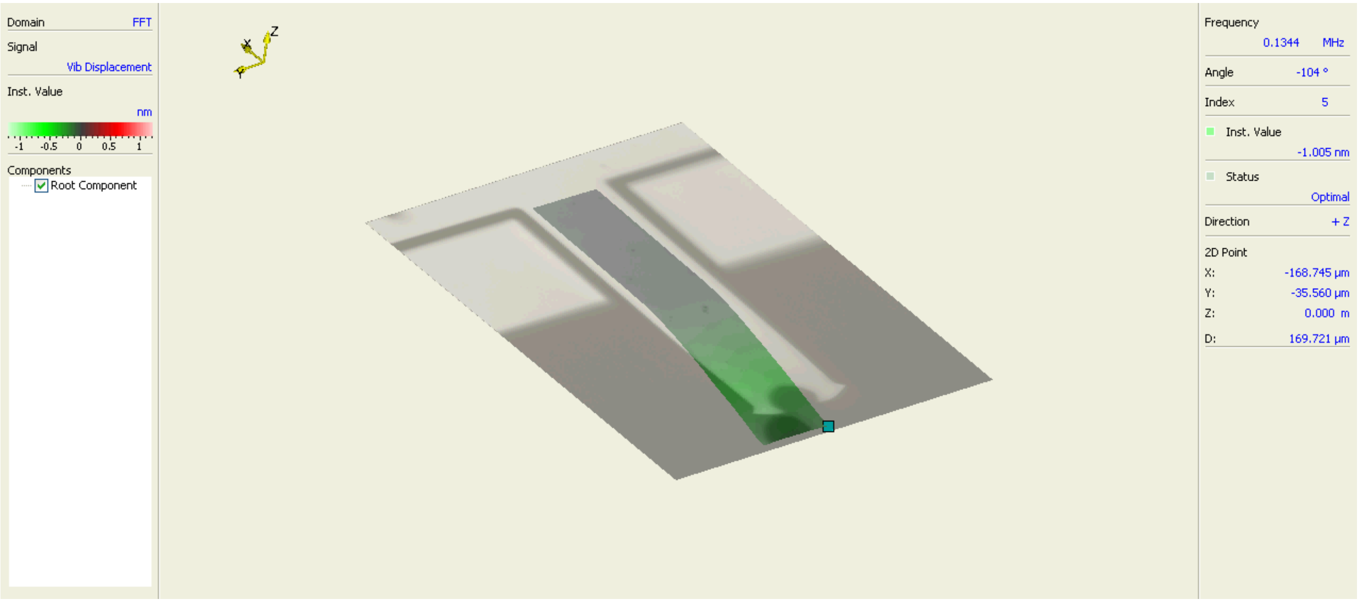
Dynamic AFM



- Cantilever driven near resonance
- The cantilever's resonant frequency, phase and amplitude are affected by short-scale force gradients
- In Amplitude Modulated AFM (AM-AFM) or tapping mode, driving frequency is fixed while cantilever approaches the sample
- In Frequency Modulated AFM (FM-AFM) the phase and amplitude are held constant while approaching the sample

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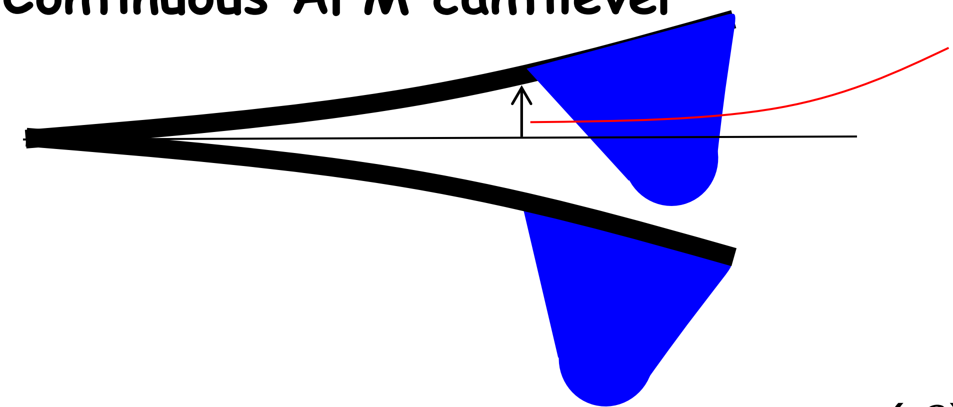
The point mass model

Continuous AFM cantilever

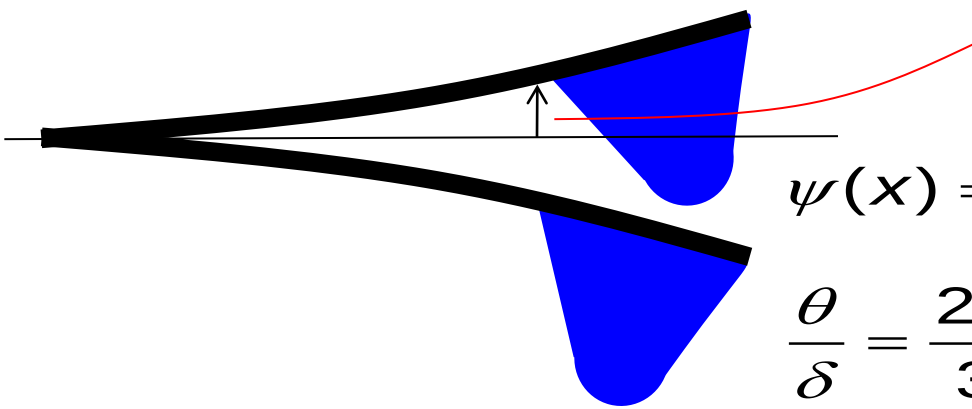


$w(x,t) = A \sin(\omega t)\, \psi(x)$

$\theta/\delta = ?$

$$\psi(x) = \cos(\beta \frac{x}{L}) - \cosh(\beta \frac{x}{L}) - \frac{\cos(\beta) + \cosh(\beta)}{\sin(\beta) + \sinh(\beta)} \left[\sin(\beta \frac{x}{L}) - \sinh(\beta \frac{x}{L}) \right]$$

Point mass model



$w(x,t) = A \sin(\omega t)\, \psi(x)$

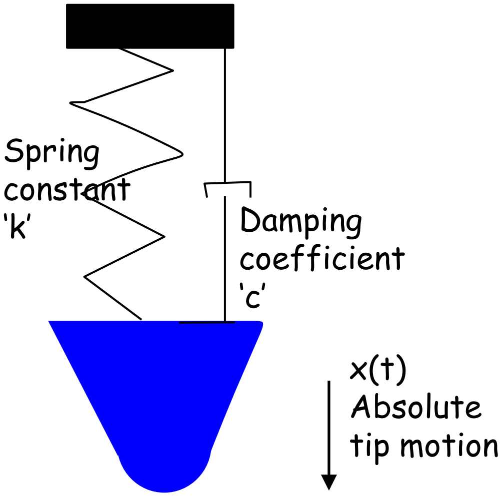
$$\psi(x) = -\frac{L^3}{6EI} \left(\frac{x}{L} \right)^3 x^3 + \frac{1}{2} \frac{L^3}{EI} \left(\frac{x}{L} \right)^2$$

$$\frac{\theta}{\delta} = \frac{2L}{3}$$

- Tip is massive, cantilever inertia negligible
- Replace cantilever by a spring of spring constant= static bending stiffness of lever
- Cantilever oscillates such that $\theta/\delta = 2L/3$

Point mass model – free oscillations

$$m\ddot{x} = -kx - c\dot{x} \quad \text{or} \quad \frac{\ddot{x}}{\omega_0^2} + x + \frac{1}{\omega_0 Q}\dot{x} = 0 \quad (1)$$

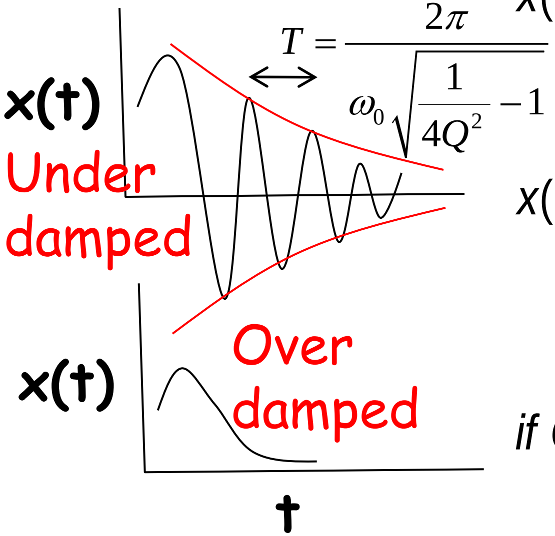


$$\text{where} \quad \omega_0 = \sqrt{\frac{k}{m}}, Q = \frac{m\omega_0}{c} = \frac{\sqrt{mk}}{c}$$

General solution of type

$$x(t) = e^{\lambda t} \Rightarrow \frac{\lambda^2}{\omega_0^2} + 1 + \frac{\lambda}{\omega_0 Q} = 0 \Rightarrow \lambda_{1,2} = -\frac{\omega_0}{2Q} \pm \omega_0\sqrt{\frac{1}{4Q^2} - 1} \quad (2)$$

$$x(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}, \quad \text{integration constants to be determined from } x(0), \dot{x}(0)$$



$$T = \frac{2\pi}{\omega_0\sqrt{\frac{1}{4Q^2} - 1}}$$

$$x(t) = e^{-\frac{\omega_0}{2Q}t}\left(x(0)\cos\left(\sqrt{1-\frac{1}{4Q^2}}\,\omega_0 t\right) + \frac{\dot{x}(0) + \omega_0\frac{x(0)}{2Q}}{\left(\sqrt{1-\frac{1}{4Q^2}}\right)\omega_0}\sin\left(\sqrt{1-\frac{1}{4Q^2}}\,\omega_0 t\right)\right) \quad (3)$$

$$\text{if } Q > \frac{1}{2} \text{ Underdamped oscillation}$$

$$x(t) = c_1 e^{\left(-\frac{\omega_0}{2Q} + \omega_0\sqrt{\frac{1}{4Q^2} - 1}\right)t} + c_2 e^{\left(-\frac{\omega_0}{2Q} - \omega_0\sqrt{\frac{1}{4Q^2} - 1}\right)t} \quad \text{if } Q < \frac{1}{2} \text{ Overdamped oscillation (4)}$$

- Damped natural frequency is different from natural frequency
- Q can be regarded as number of oscillation cycles before transients become small

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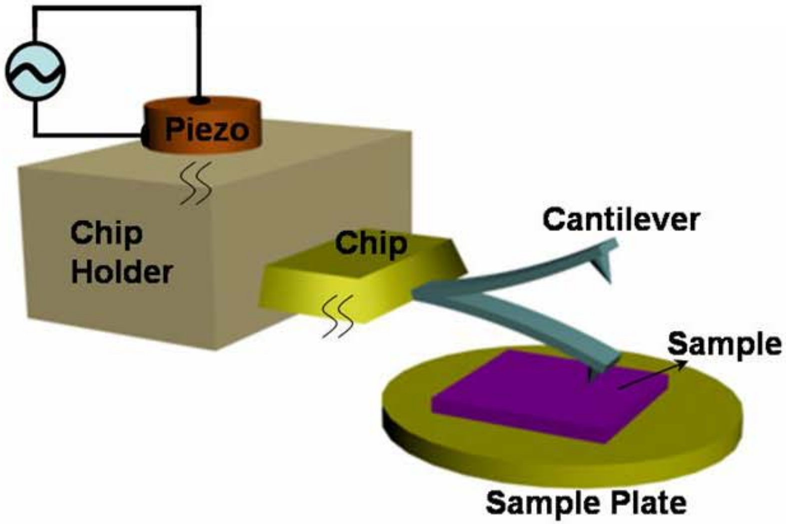
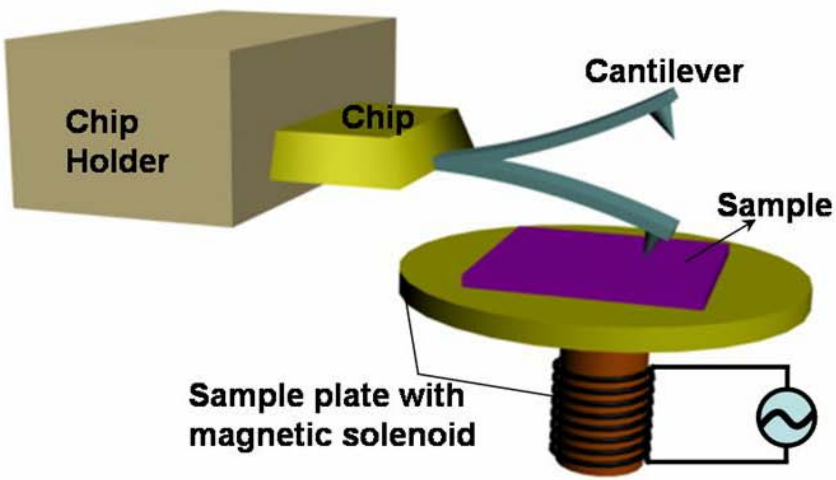
Forced vibrations



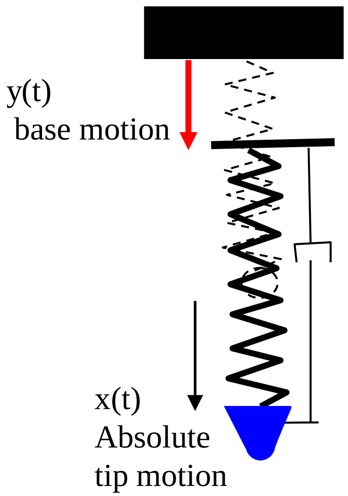
a. Acoustic excitation



b. Magnetic excitation

- Mechanical (acoustic or piezo excitation)
- Magnetic excitation
- Magnetostrictive excitation
- Photothermal excitation
- Lorentz force excitation
- Ultrasound excitation
- Direct piezoelectric excitation

Response of acoustically excited levers



Acoustic (inertial or piezo)

$$m\ddot{x} = -k(x-y) - c\dot{x}$$

$$\frac{\ddot{x}}{\omega_0^2} + x + \frac{1}{\omega_0 Q}\dot{x} = y(t);\ \text{with} \quad \omega_0 = \sqrt{\frac{k}{m}},\ Q = \frac{m\omega_0}{c}$$

$$\text{Measured motion } z(t) = x(t) - y(t)$$

$$\frac{\ddot{z}}{\omega_0^2} + z + \frac{1}{\omega_0 Q}\dot{z} = -\frac{\ddot{y}}{\omega_0^2} - \frac{1}{\omega_0 Q}\dot{y}$$

$$y(t) = Y_0 \sin(\omega t)$$

$$z^p(t) = A\sin(\omega t + \phi_{acoustic})$$

$$|H_{acoustic}(\omega)| = \frac{A}{Y_0} = \left(\frac{r^4 + (r/Q)^2}{(1-r^2)^2 + (r/Q)^2}\right)^{1/2}$$

$$\phi_{acoustic}(\omega) = \tan^{-1}\left(\frac{Q}{r(1+Q^2 r^2 - Q^2)}\right)$$

$$\text{where } r = \frac{\omega}{\omega_0}$$

- ω_0 is the natural freq, ω is the drive freq
- Maximum amplitude occurs when $\omega > \omega_0$!
- Base motion amplitude $Y_0(r=1) = A/Q$ when Q is large!

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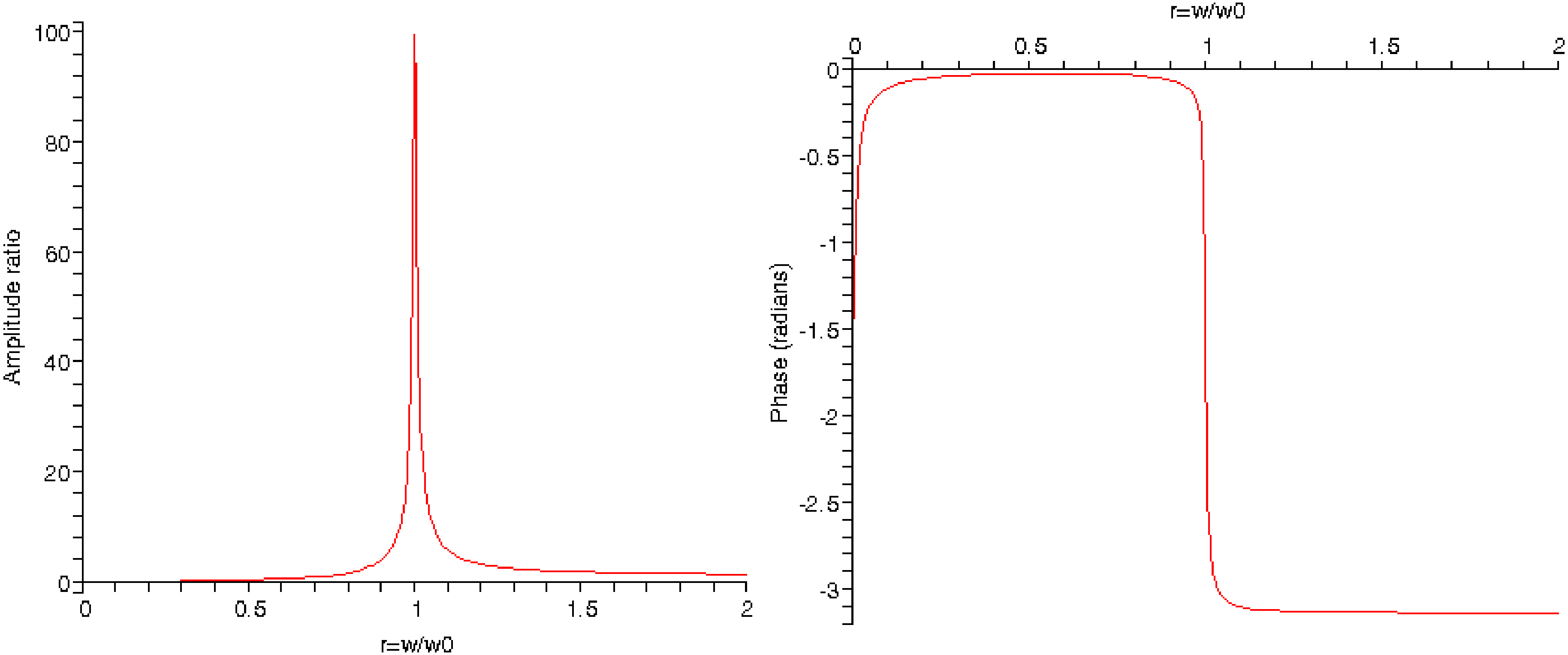
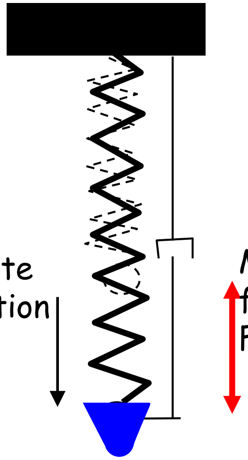
Response of acoustically excited levers



- For Q=100, see response above
- Asymmetric peak, amplitude greater when $\omega > \omega_0$

Response of directly excited AFM levers



$$m\ddot{x} = -kx - c\dot{x} + F_{mag}(t)$$

$$\frac{\ddot{x}}{\omega_0^2} + x + \frac{1}{\omega_0 Q}\dot{x} = \frac{1}{k}F_{mag}(t); \text{ with } \quad \omega_0 = \sqrt{\frac{k}{m}}, Q = \frac{m\omega_0}{c}$$

$$\text{Measured motion} = x(t)$$

$$F_{mag}(t) = F_0 \sin(\omega t)$$

$$x^p(t) = A\sin(\omega t + \phi_{magnetic})$$

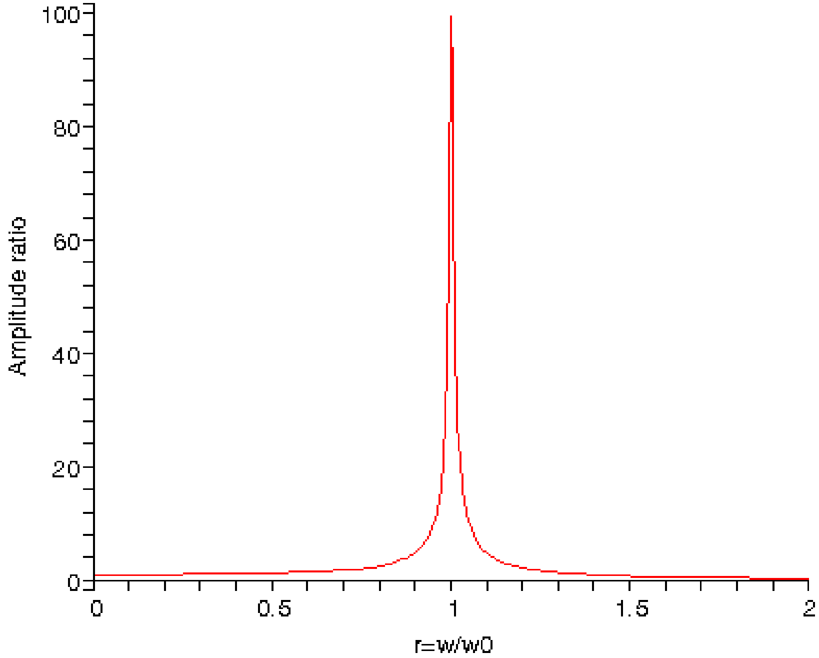
$$|H_{mag}(\omega)| = \frac{A}{F_0 / k} = \left(\frac{1}{(1-r^2)^2 + (r/Q)^2}\right)^{1/2}$$

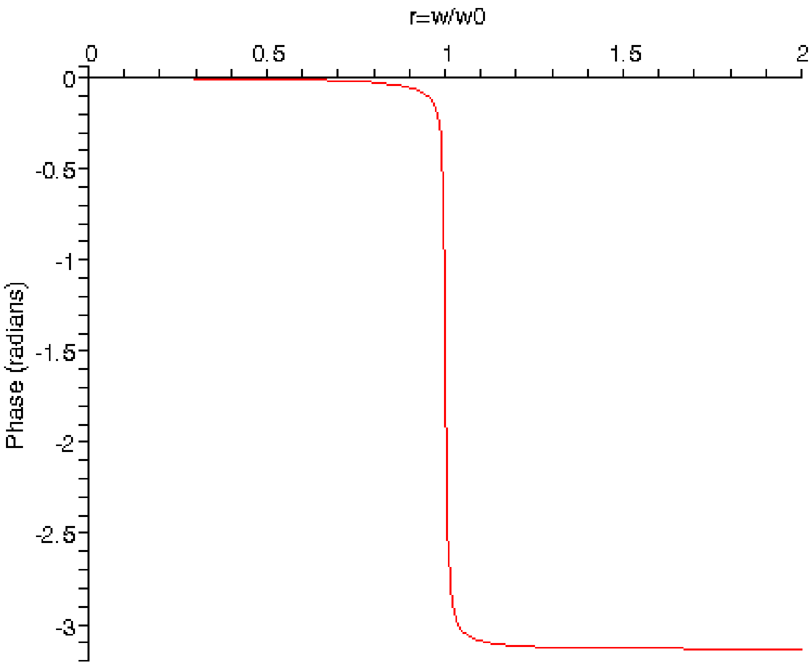
$$\phi_{mag}(\omega) = \tan^{-1}\left(\frac{r}{Q(r^2-1)}\right)$$

$$\text{where } r = \frac{\omega}{\omega_0}$$

- ω_0 is the natural freq, ω is the drive freq
- Maximum amplitude occurs when $\omega<\omega_0$!
- For $\omega<<\omega_0$ $A=F_{mag}/k$!

Response of directly excited AFM levers





- Asymmetric response with greater amplitude when $\omega<\omega_0$!
- Classical phase response

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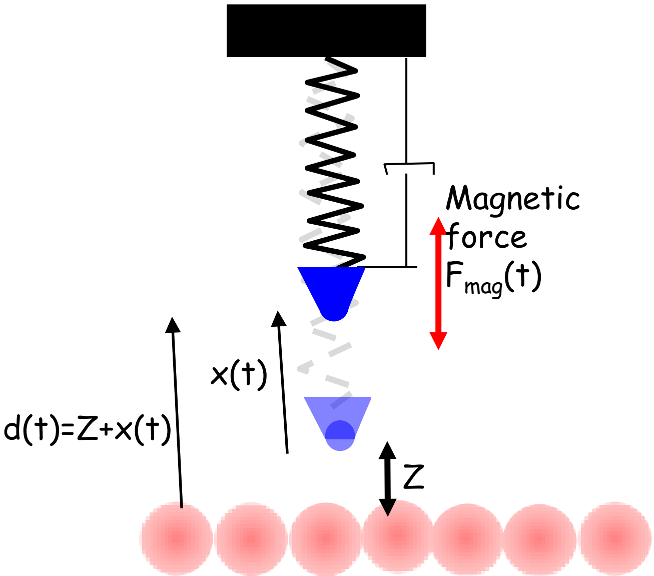
Driven point mass model with tip-sample interaction



Magnetic

$$m\ddot{x} = -kx - c\dot{x} + F_{mag}(t) + F_{ts}(Z + x(t))$$

$$\frac{\ddot{x}}{\omega_0^2} + x + \frac{1}{\omega_0 Q}\dot{x} = \frac{1}{k}\left(F_{mag}(t) + F_{ts}(Z + x(t))\right);$$

$$\text{with} \quad \omega_0 = \sqrt{\frac{k}{m}}, \; Q = \frac{m\omega_0}{c}$$

$$\text{Measured motion} = x(t)$$

$$F_{mag}(t) = F_0 \sin(\omega t)$$

Acoustic excitation

$$\frac{\ddot{z}}{\omega_0^2} + z + \frac{1}{\omega_0 Q}\dot{z} = -\frac{\ddot{y}}{\omega_0^2} - \frac{1}{\omega_0 Q}\dot{y} + \frac{F_{ts}(Z + y(t) + x(t))}{\omega_0^2}$$

- Highly nonlinear ordinary differential equation

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Linearized analysis

Magnetic excitation

$$\frac{\ddot{x}}{\omega_0^2} + x + \frac{1}{\omega_0 Q}\dot{x} = \frac{1}{k}\left(F_0 \sin(\omega t) + F_{ts}(Z + x(t))\right); \qquad d(t) = Z + x(t) \qquad (1)$$

At a given Z the equilibrium deflection is

$$x^* = \frac{1}{k}F_{ts}(Z + x^*) \quad where \quad d^* = Z + x^* \qquad (2)$$

$$Let \quad x(t) = x^* + \bar{x}(t) \qquad (3)$$

Include time – dependent terms

$$\frac{\left(\ddot{\bar{x}} + \ddot{x}^*\right)}{\omega_0^2} + \left(\bar{x} + x^*\right) + \frac{1}{\omega_0 Q}(\dot{\bar{x}} + \dot{x}^*) = \frac{1}{k}\left(F_0 \sin(\omega t) + F_{ts}(Z + x^* + \bar{x})\right) \qquad (4)$$

$$\frac{\ddot{\bar{x}}}{\omega_0^2} + \left(\bar{x} + x^*\right) + \frac{1}{\omega_0 Q}\dot{\bar{x}} = \frac{1}{k}\left(F_0 \sin(\omega t) + F_{ts}(Z + x^* + \bar{x})\right) \qquad (5)$$

If $\bar{x} << Z + x^*$ *or when* $\bar{x} << d^*$ *then*

$$\frac{\ddot{\bar{x}}}{\omega_0^2} + \left(\bar{x} + x^*\right) + \frac{1}{\omega_0 Q}\dot{\bar{x}} = \frac{1}{k}\left(F_0 \sin(\omega t) + F_{ts}(Z + x^*) + \left.\frac{\partial F_{ts}(d)}{\partial d}\right|_{d=d^*}\bar{x}\right) \qquad (6)$$

$\Rightarrow$

$$\frac{\ddot{\bar{x}}}{\omega_0^2} + \left(1 - \frac{1}{k}\left.\frac{\partial F_{ts}(d)}{\partial d}\right|_{d=x^*}\right)\bar{x} + \frac{1}{\omega_0 Q}\dot{\bar{x}} = \frac{1}{k}\left(F_0 \sin(\omega t)\right) \qquad (7)$$

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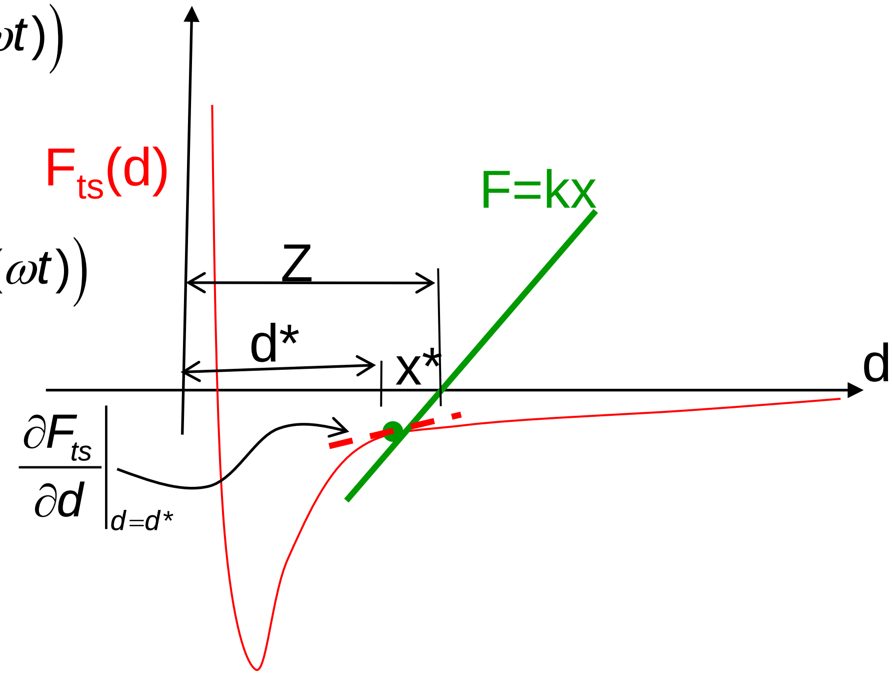
Linearized analysis

$$\frac{\ddot{\bar{x}}}{\omega_0^2} + \left(1 - \frac{1}{k}\frac{\partial F_{ts}(d)}{\partial d}\bigg|_{d=d^*}\right)\bar{x} + \frac{1}{\omega_0 Q}\dot{\bar{x}} = \frac{1}{k}\left(F_0 \sin(\omega t)\right)$$

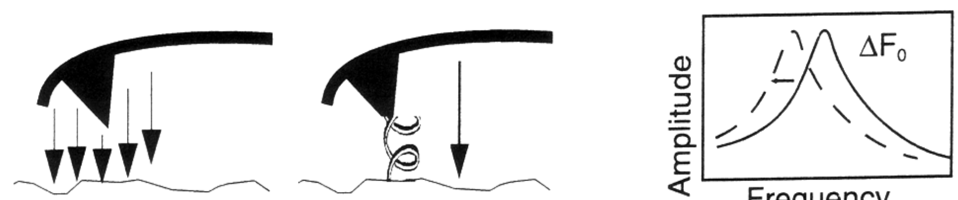
Or

$$\ddot{\bar{x}} + \omega_0^2\left(1 - \frac{1}{k}\frac{\partial F_{ts}(d)}{\partial d}\bigg|_{d=d^*}\right)\bar{x} + \frac{\omega_0}{Q}\dot{\bar{x}} = \frac{\omega_0^2}{k}\left(F_0 \sin(\omega t)\right)$$

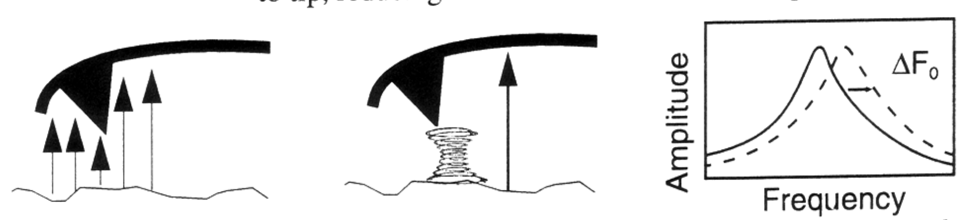
$$\hat{\omega}_0^2 = \omega_0^2\left(1 - \frac{1}{k}\frac{\partial F_{ts}(d)}{\partial d}\bigg|_{d=d^*}\right)$$



- When $\frac{\partial F_{ts}}{\partial d}\bigg|_{d=d^*} > 0$ attractive force and natural frequency decreases
- When $\frac{\partial F_{ts}}{\partial d}\bigg|_{d=d^*} < 0$ rep. regime and natural frequency increases

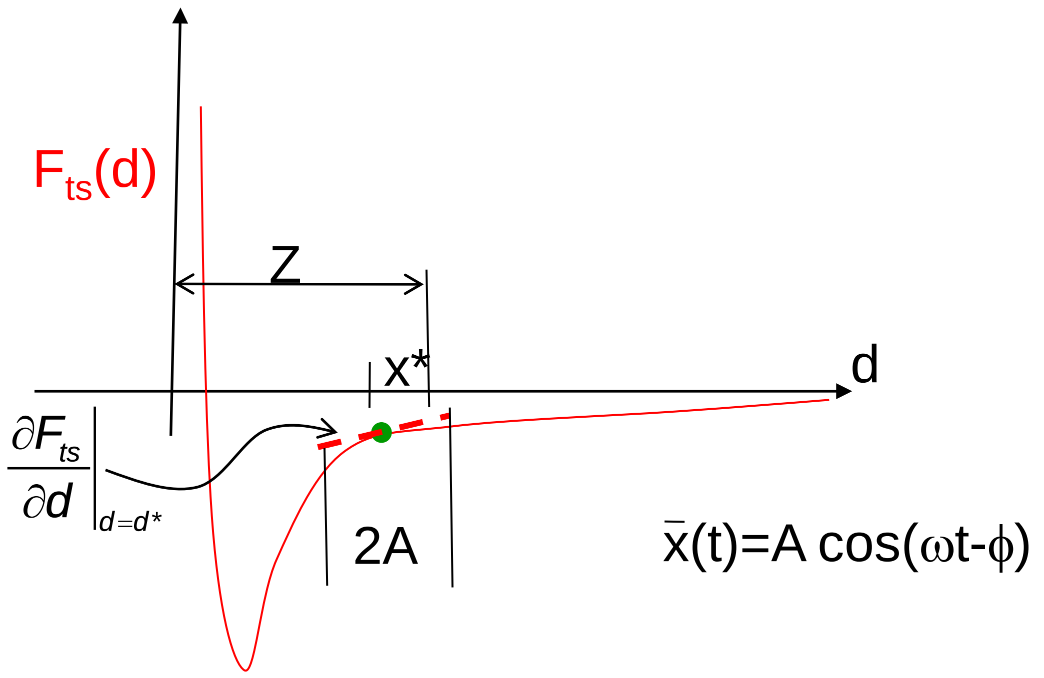


Attractive gradient equivalent to additional spring in tension attached to tip, reducing the cantilever resonance frequency.



Repulsive gradient equivalent to additional spring in compression attached to tip, increasing the cantilever resonance frequency.

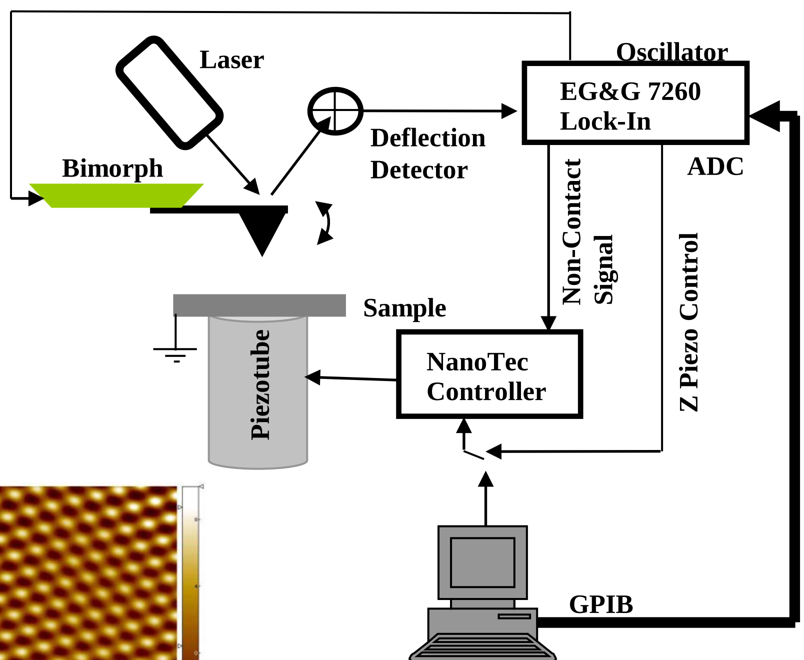
Limitations of the linearized analysis

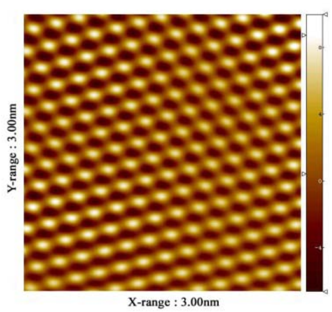


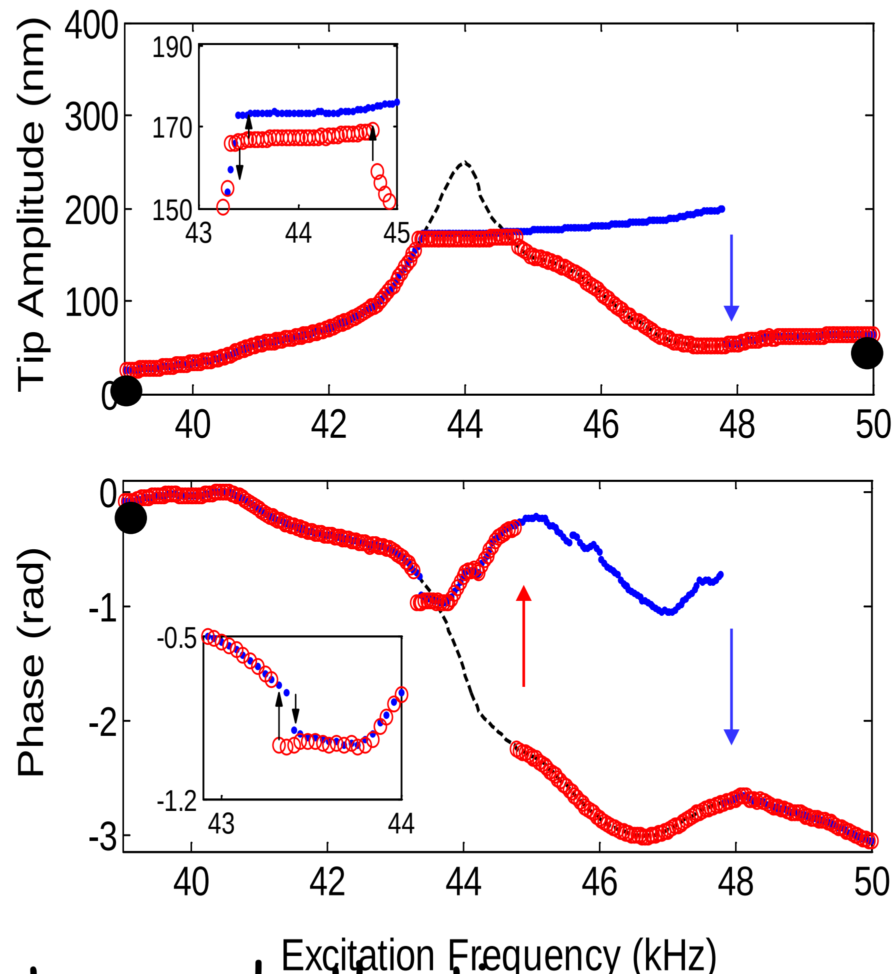
- For what driven oscillation amplitude is this approximation valid?

Experiments with conventional tips

- **Si tip / HOPG sample z=90 nm, frequency sweep**







- When brought closer to sample the tip sometimes sticks to the sample

Lee et al, Phys Rev B (2002)

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Next class

- Nonlinear response
- Dynamic Approach curves