ME597/PHYS57000
Fall Semester 2010
Lecture 21

Frequency Modulated AFM

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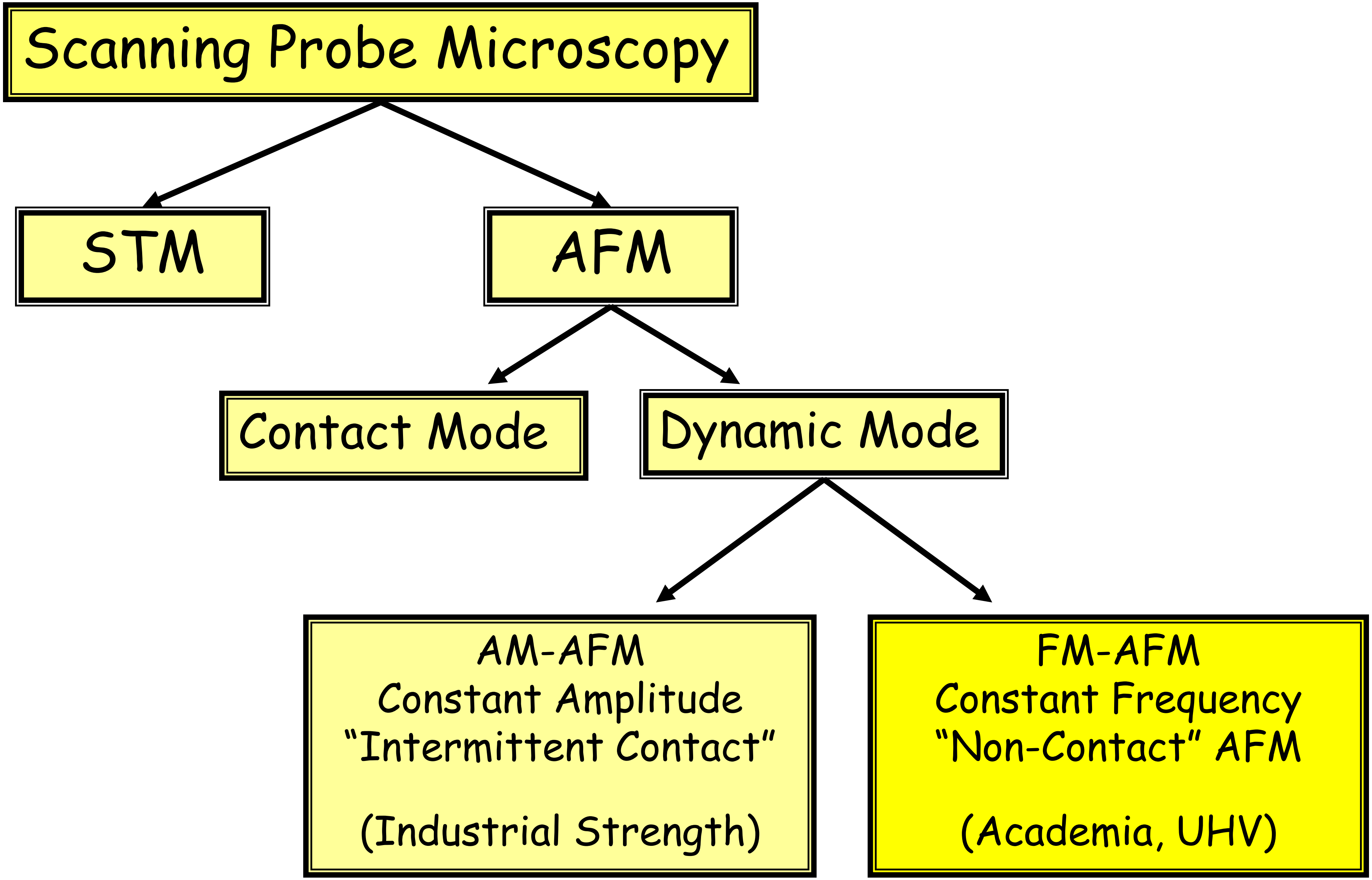


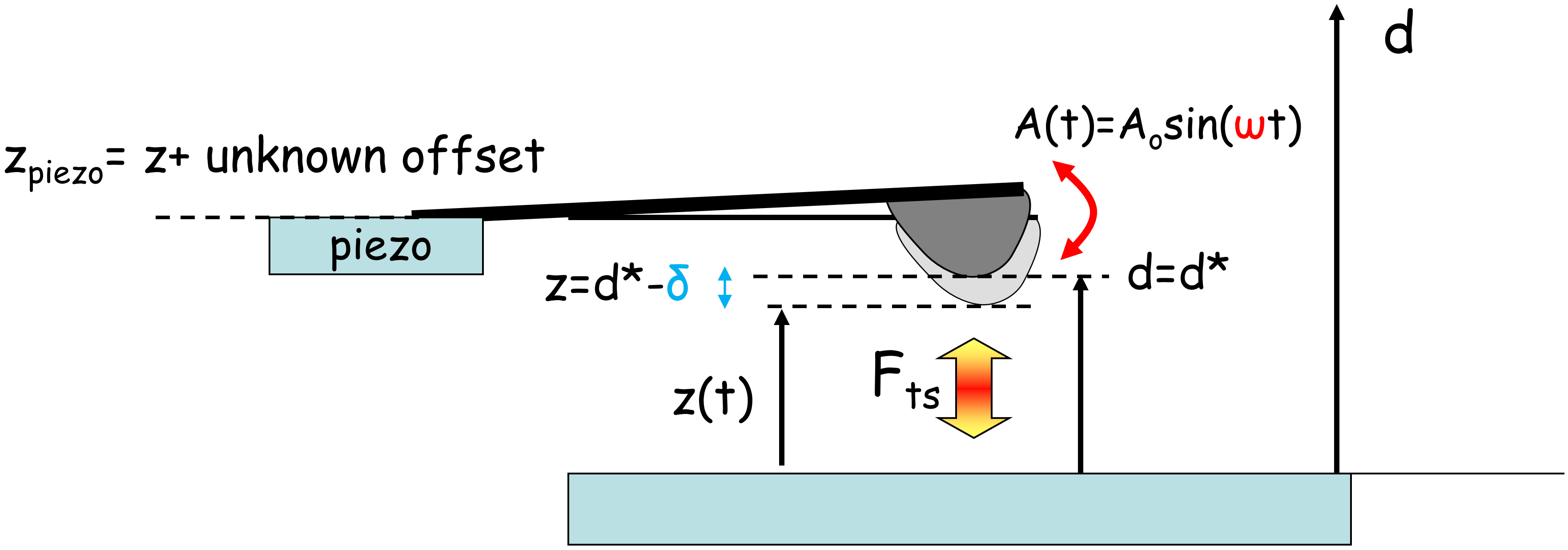
Note: FM means Frequency Modulation, NOT Force Modulation

Questions

How to finely control $F_{ts}(z)$?

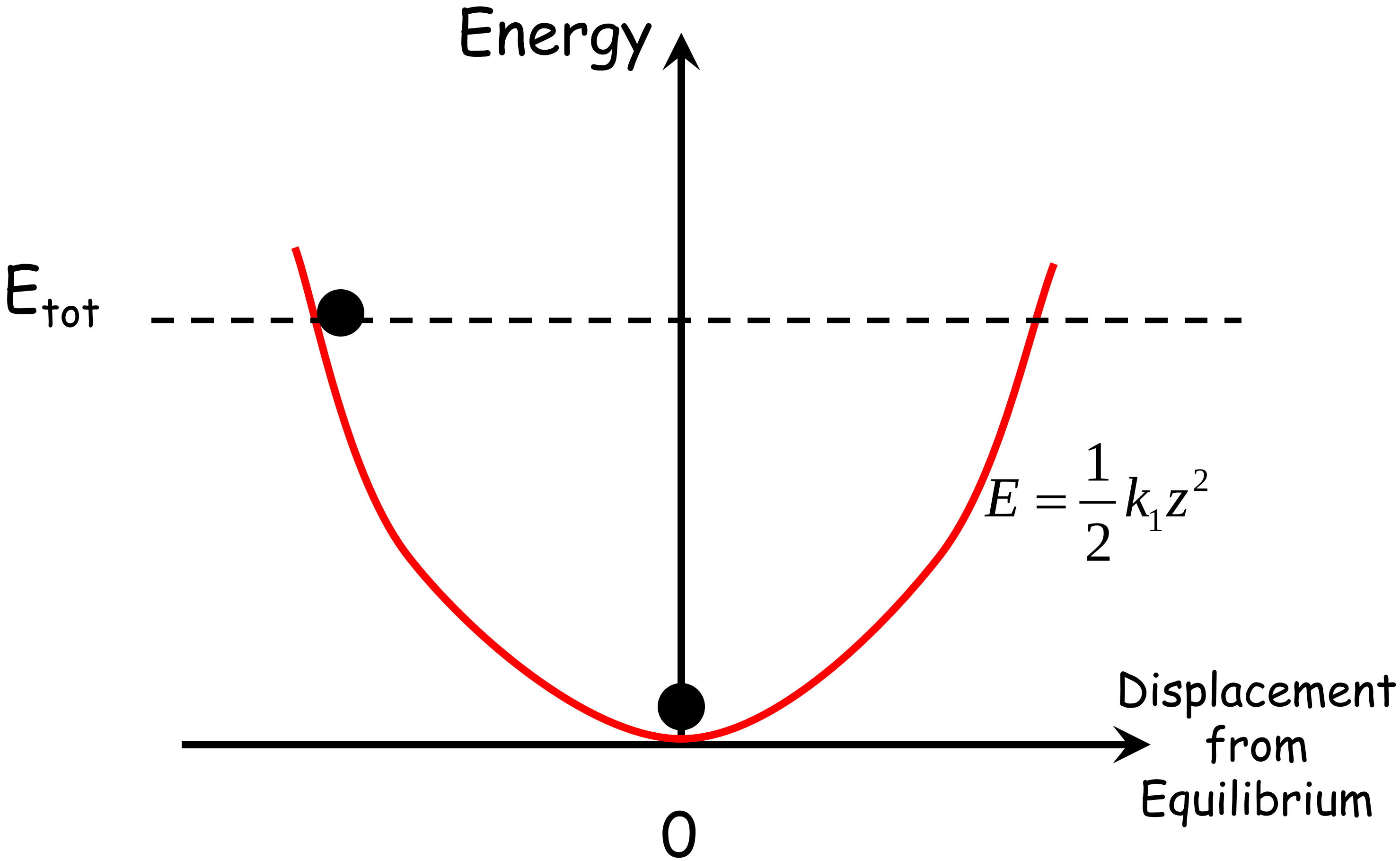
How to measure $F_{ts}(z)$?

What benefits are gained?



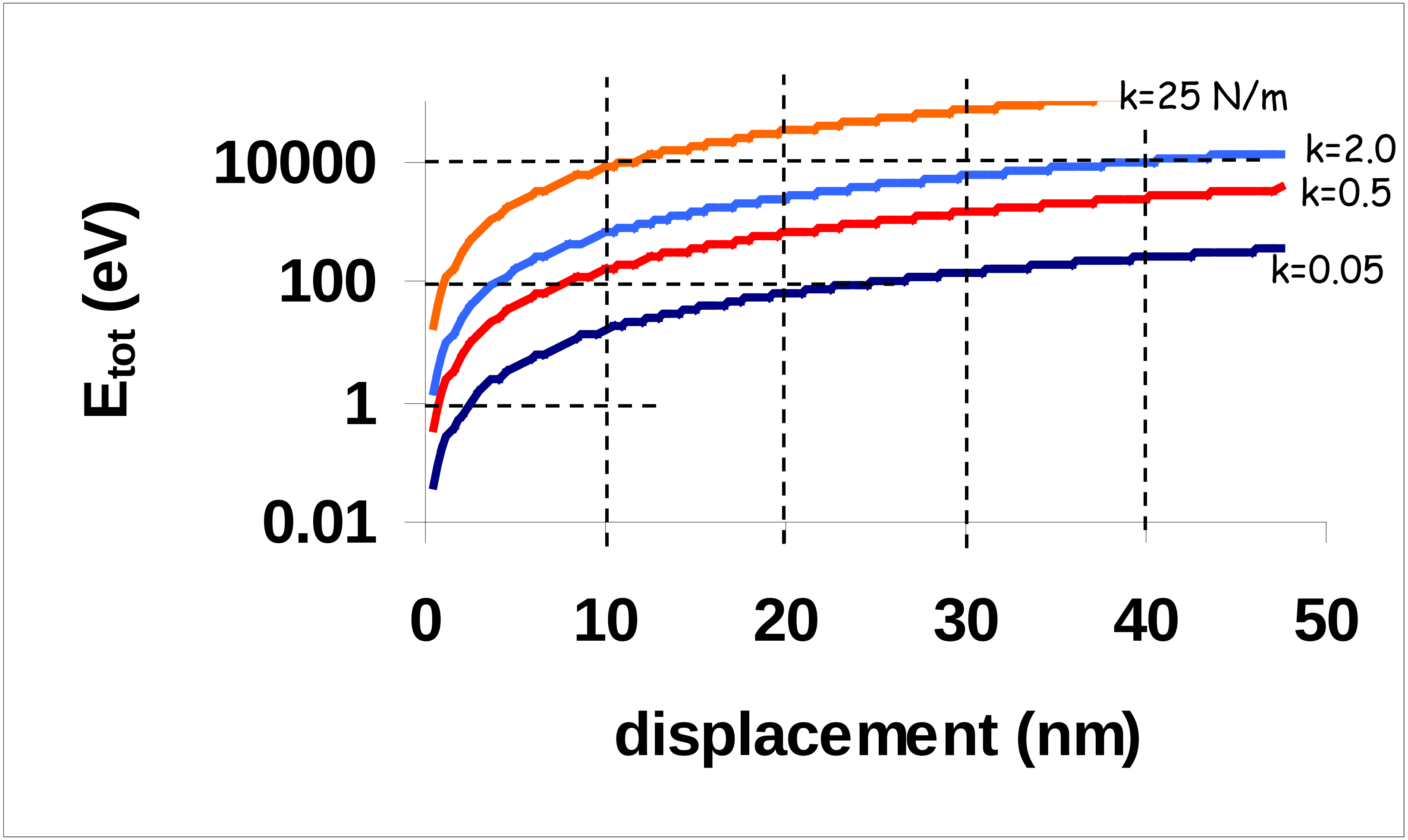
Answer: Focus on the frequency, not the amplitude!

Oscillations in quadratic potential well
(no energy loss)



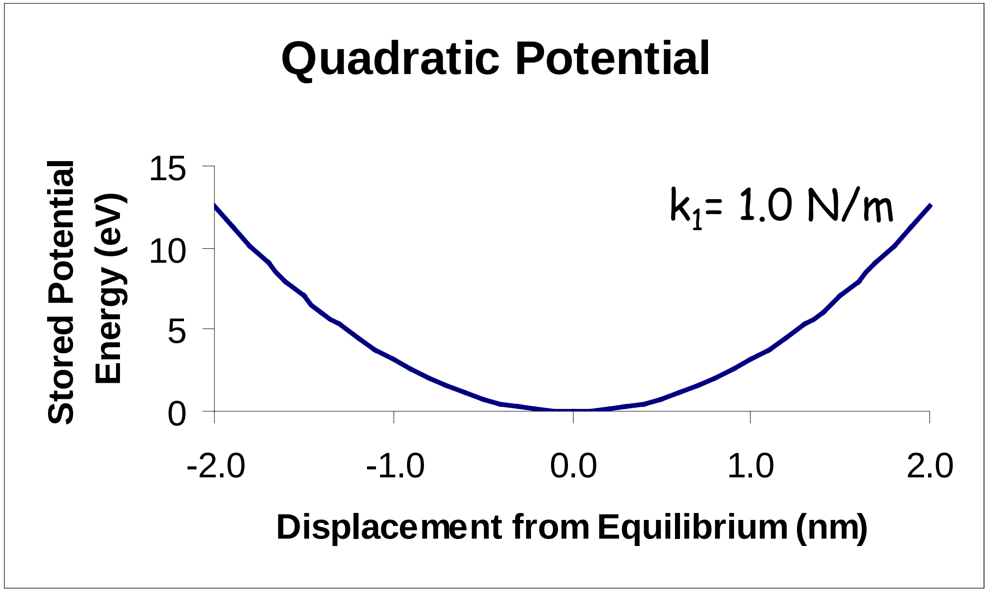
2 clicks

How much energy is stored?



1 eV/atom = 23 kcal/mole=96 kJ/mole

The Simple Harmonic Oscillator (SHO) is really simple!



Equation of Motion

$$\text{Let } V(z) = \frac{1}{2} k_1 z^2$$

$$F \equiv -\partial V(z) / \partial z = ma$$

$$\Rightarrow \; m \frac{d^2}{dt^2} z + k_1 z = 0$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k_1}{m}}$$

$$T \equiv \frac{1}{f} = 2\pi \sqrt{\frac{m}{k_1}}$$

Conservation of Energy

$$E_{tot} = \frac{1}{2} m \left(\frac{dz}{dt} \right)^2 + V(z)$$

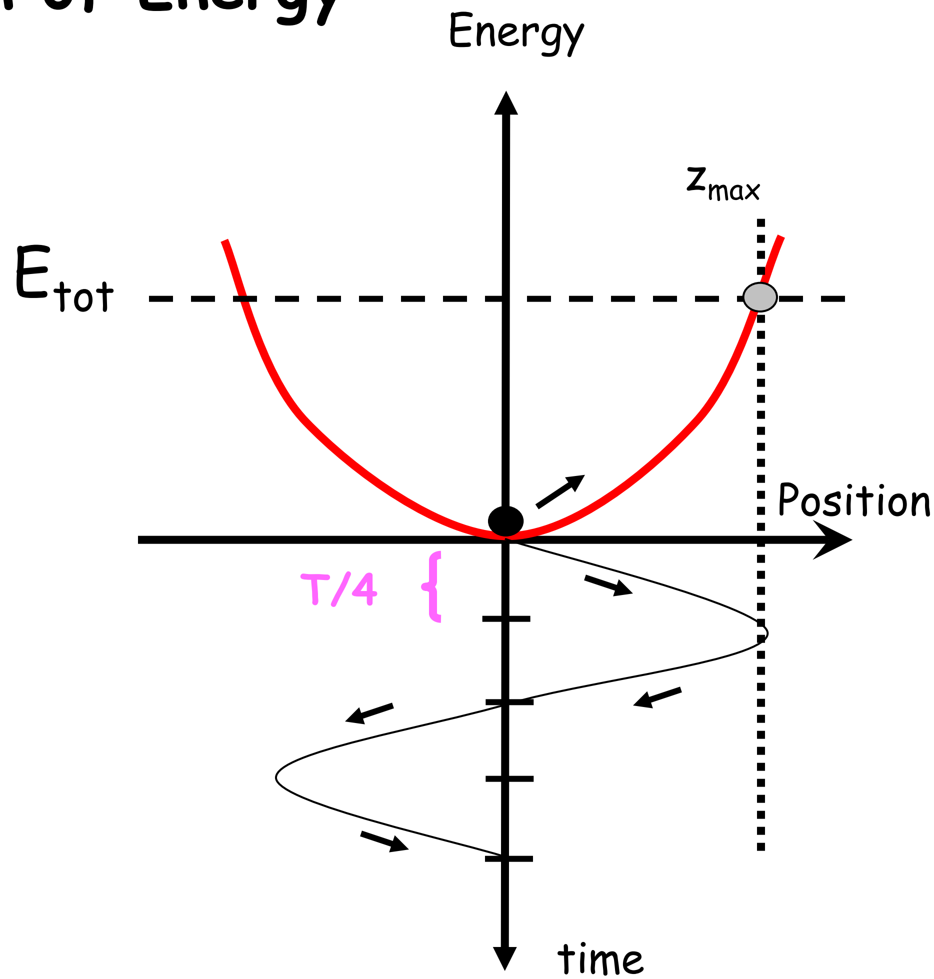
$$\frac{dz}{dt} = \sqrt{\frac{2E_{tot}}{m} - V(z)}$$

$$\int dt = \int \frac{dz}{\sqrt{\frac{2E_{tot}}{m} - V(z)}} \qquad (1)$$

$$\int_0^{T/4} dt = \int_0^{z_{max}} \frac{dz}{\sqrt{\frac{2E_{tot}}{m} - \frac{1}{2} k_1 z^2}}$$

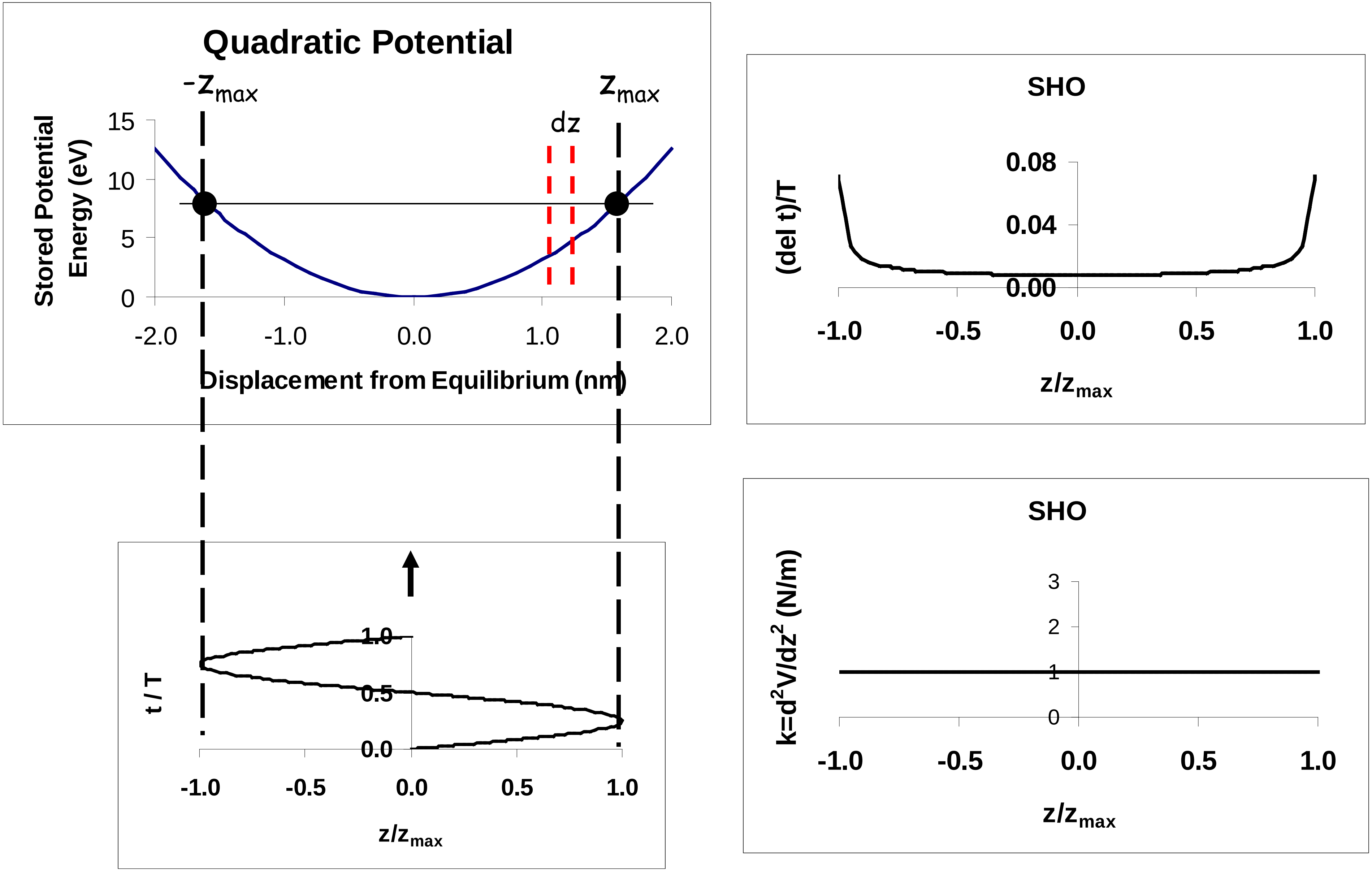
$$\frac{T}{4} = \sqrt{\frac{m}{k_1}} \sin^{-1} \left(z \sqrt{\frac{k_1}{2E_{tot}}} \right)_0^{z_{max}} = \sqrt{\frac{m}{k_1}} \frac{\pi}{2}$$

$$T = 2\pi \sqrt{\frac{m}{k_1}} \quad \Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{k_1}{m}}$$

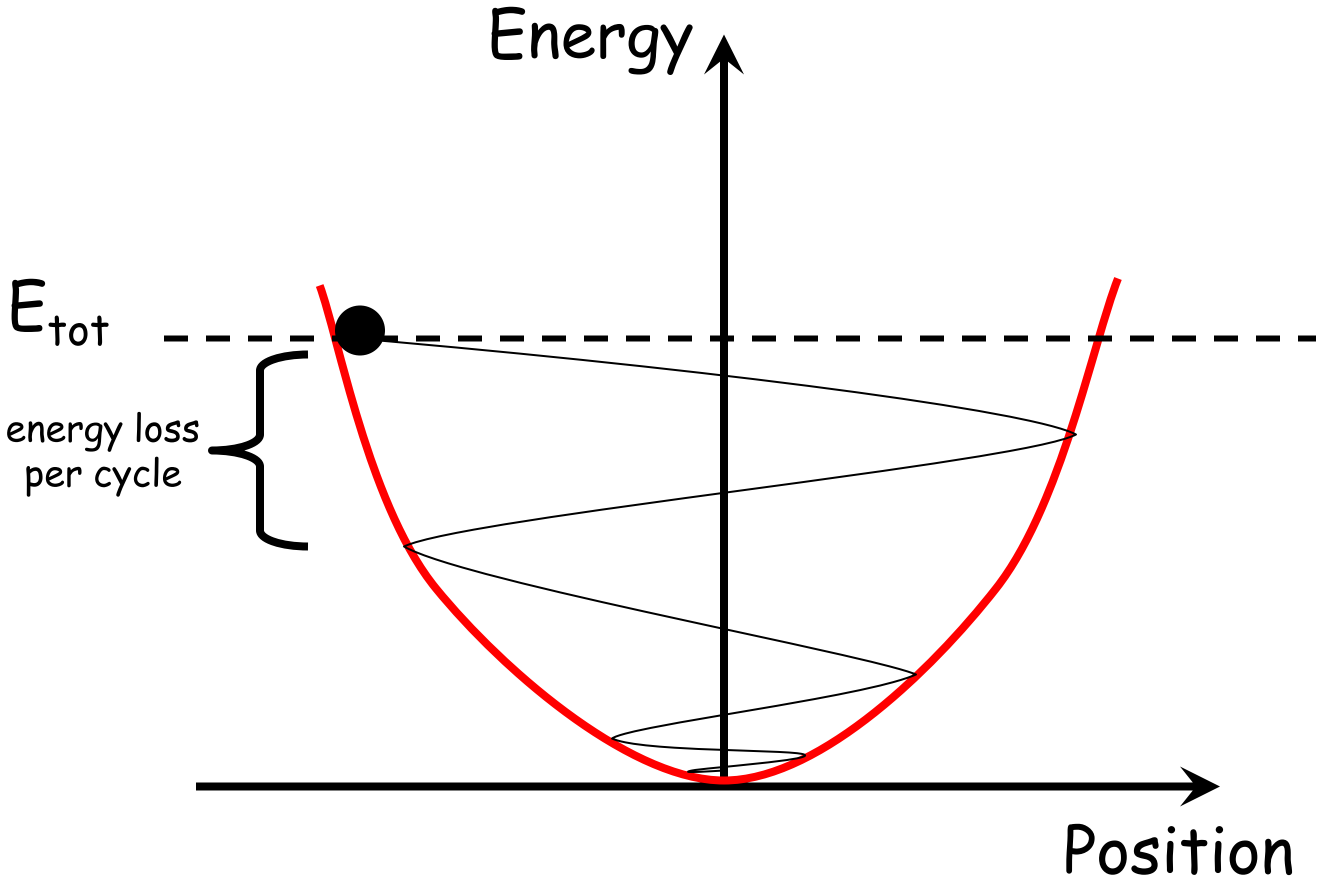


Isochronous!

Numerical Example



I. Oscillations in quadratic potential well with damping



1 click

loss mechanism?

Equation of Motion for Damped Oscillator

$$V(z) = \frac{1}{2} k_1 z^2$$

$$F \equiv -\partial V(z) / \partial z = ma \quad \Rightarrow$$

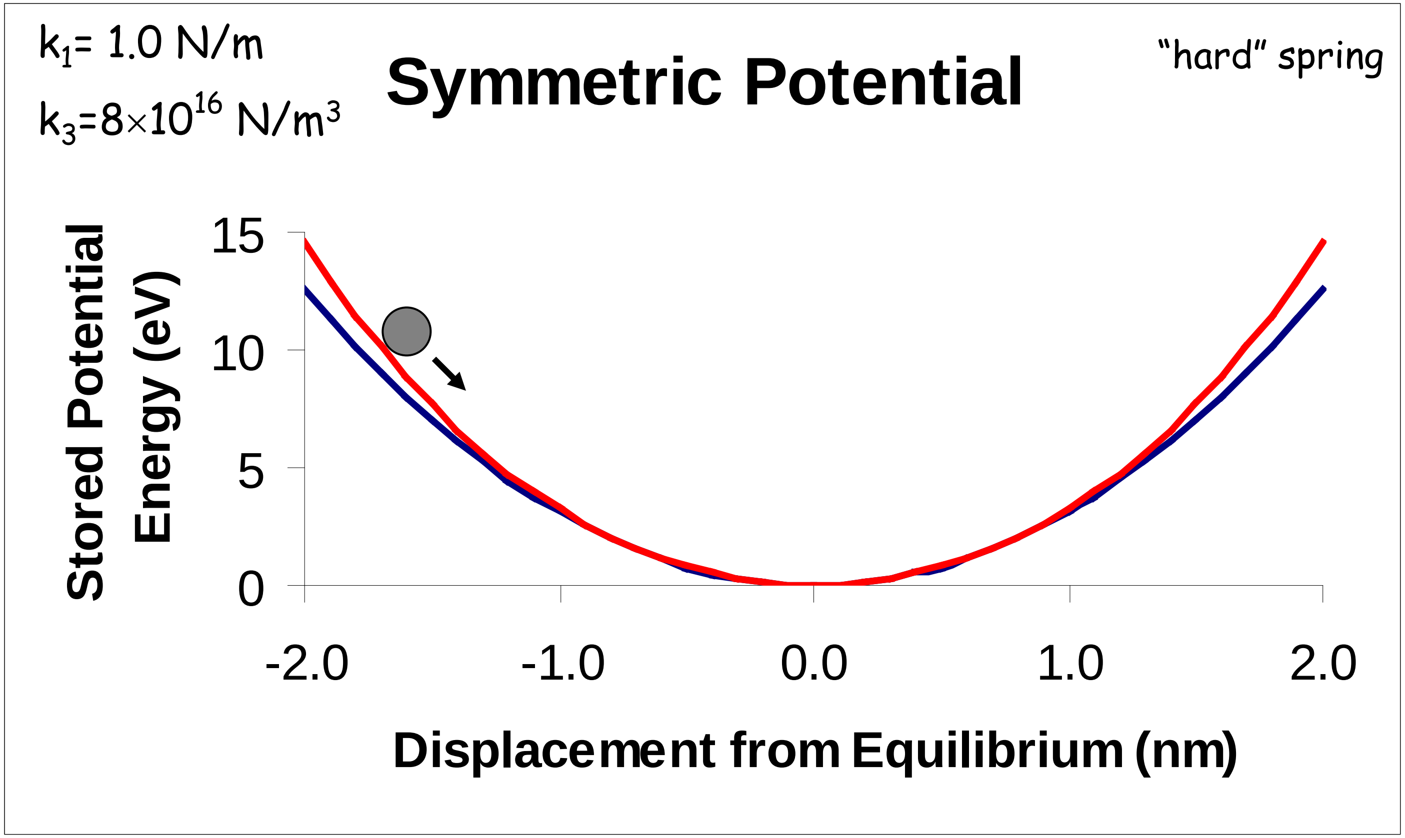
$$m \frac{d^2}{dt^2} z + c\dot{z} + k_1 z = 0$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k_1}{m}} \sqrt{1 - \frac{c^2}{4 k_1 m}}$$

II. Non-linear symmetric oscillator; no damping
[V(z)=V(-z)]

$$V(z) = \frac{1}{2}k_1 z^2 + \frac{1}{4}k_3 z^4$$

$$F \equiv -\partial V / \partial z = -k_1 z - k_3 z^3$$



Non-linear Symmetric Oscillator

$$V(z) = \frac{1}{2} k_1 z^2 + \frac{1}{4} k_3 z^4 + \ldots.$$

$$m \frac{d^2}{dt^2} z + k_1 z + k_3 z^3 + \ldots. = 0$$

$$z(t) = a_o + \sum_{n=1}^{\infty} \left(a_n \sin n\omega t + b_n \cos n\omega t \right)$$

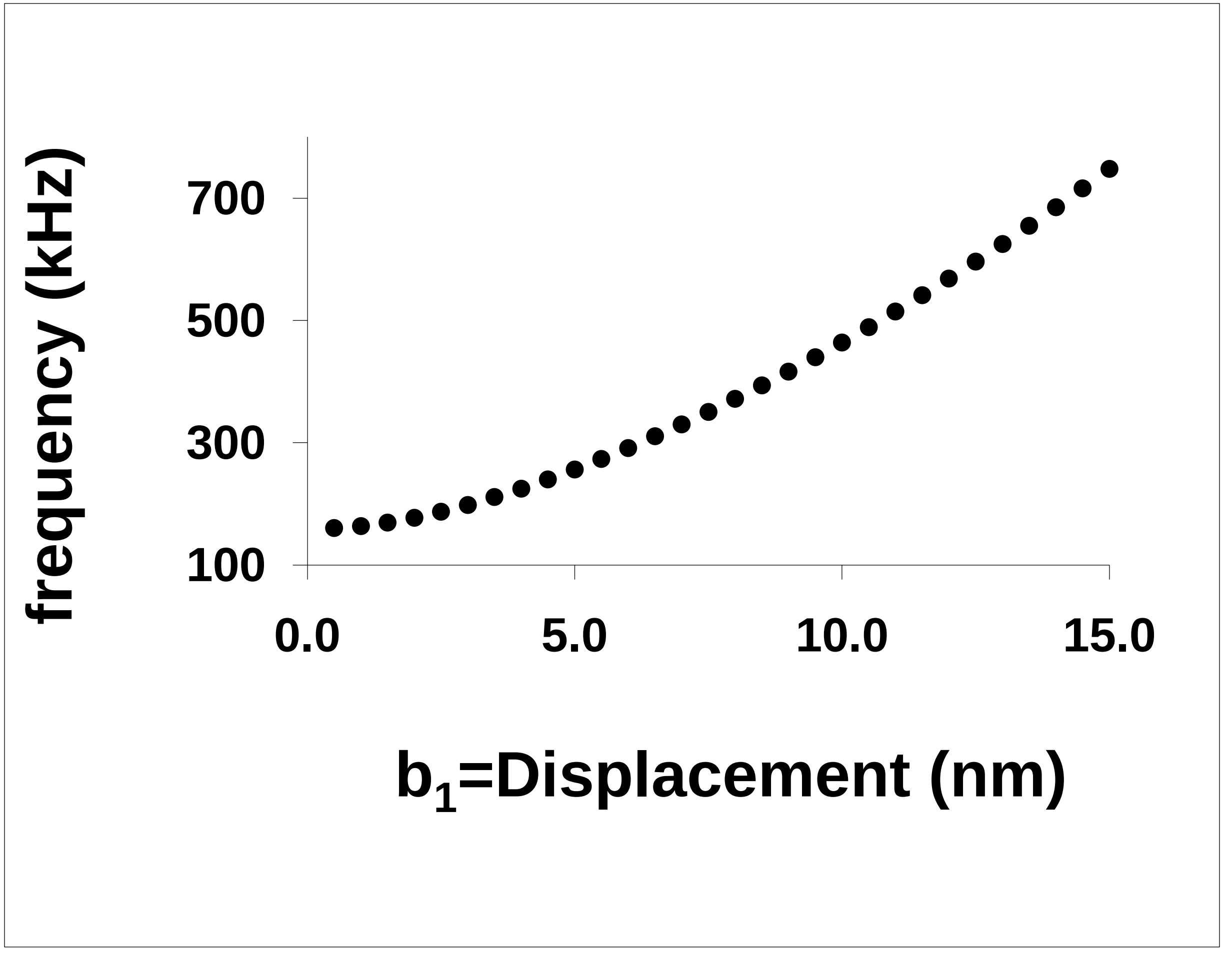
$$a_o = 0; \quad a_n = 0; \quad b_2 = b_4 = b_6 = \ldots. = 0 \;\; \textit{from symmtery}$$

$$\textit{Leading non-zero term is } b_1$$

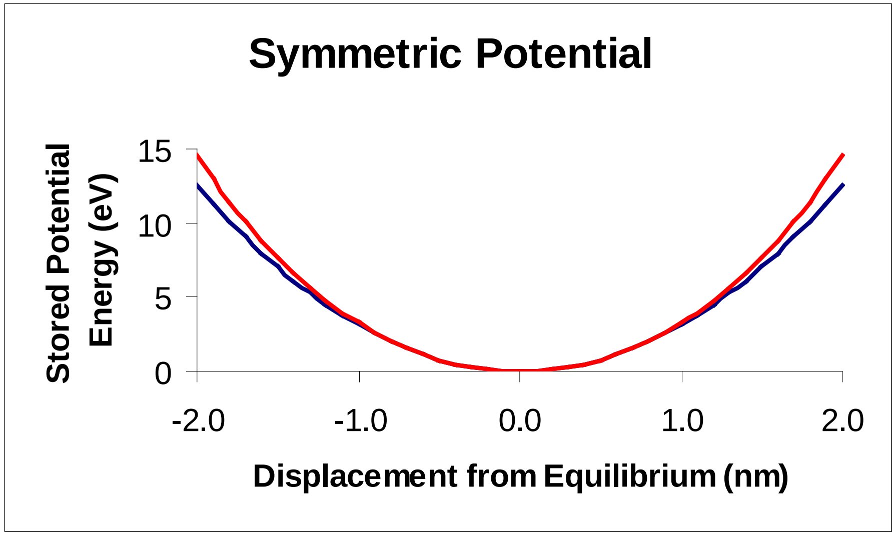
$$f \approx \frac{1}{2\pi} \sqrt{\frac{k_1}{m} + \frac{3}{4} \frac{k_3}{m} b_1^2 + \frac{3}{128} \frac{k_3}{m} \frac{k_3}{k_1} b_1^4 + \ldots} = f_o + \Delta f \left(b_1 \right)$$

- New feature: frequency depends on amplitude

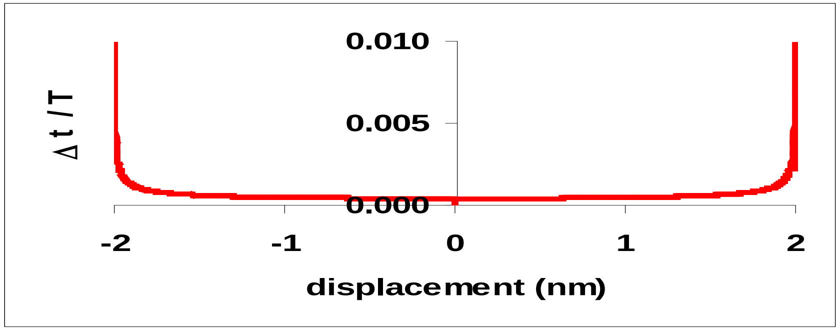
Non-linear Symmetric Oscillator no Damping

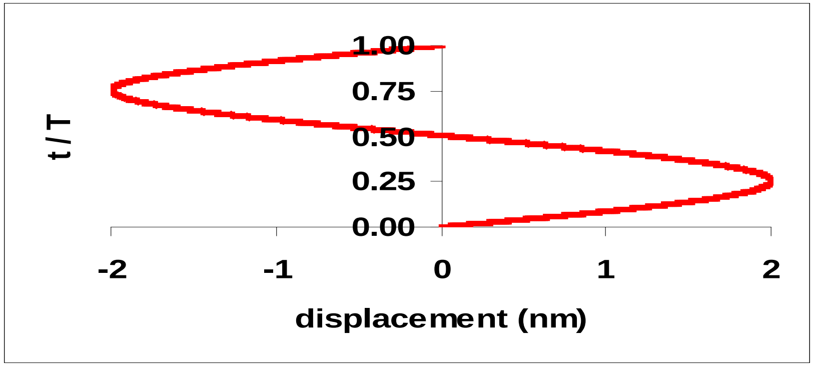


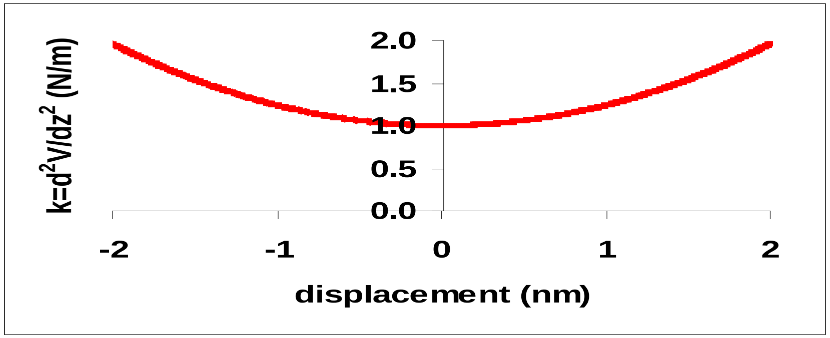
Non-linear Symmetric Oscillator no Damping



$$V(z) = \frac{1}{2}k_1 z^2 + \frac{1}{4}k_3 z^4$$







What is the effective spring constant?

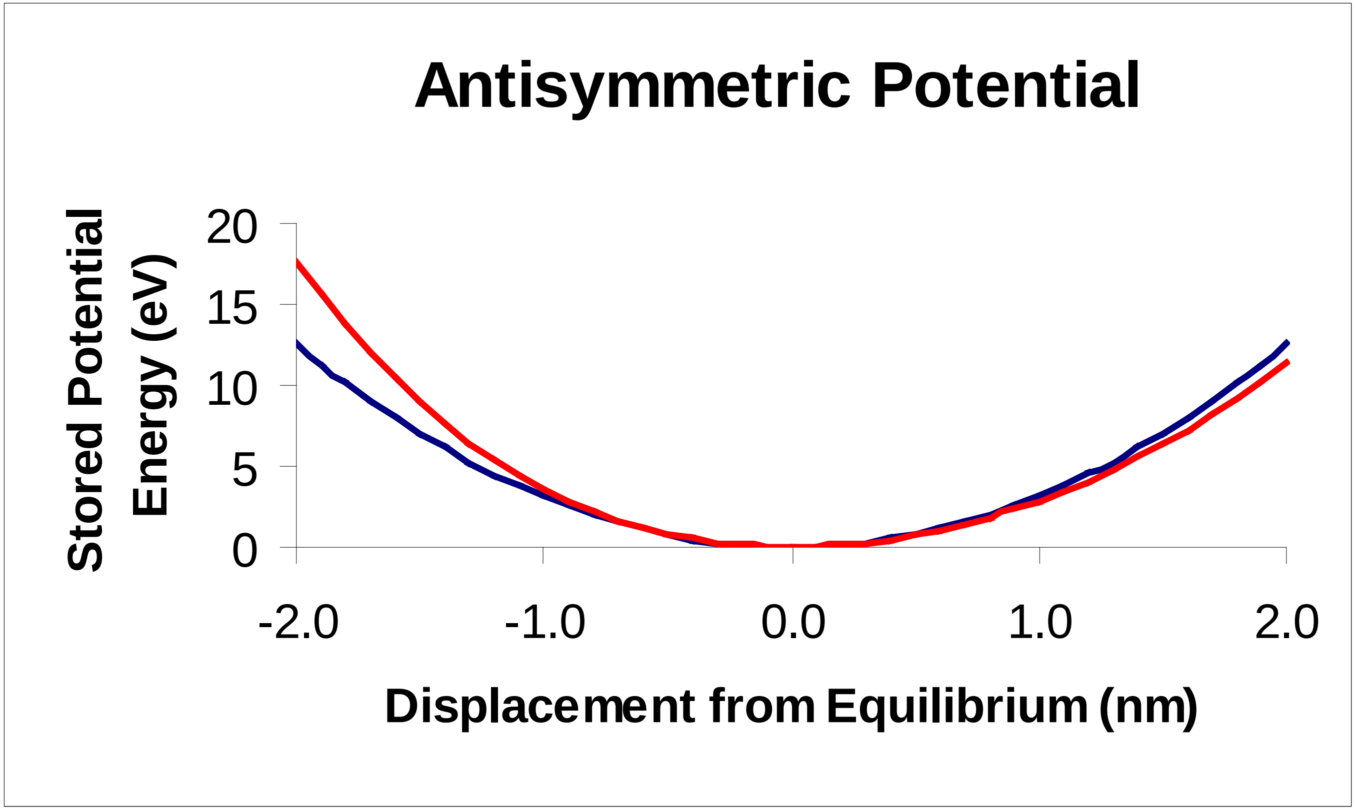
III. Non-linear antisymmetric oscillator; no damping

$$V(z) = \frac{1}{2}k_1 z^2 - \frac{1}{3}k_2 z^3 + \frac{1}{4}k_3 z^4$$

k_1= 1.0 N/m

k_2=1.9E8 N/m^2

k_3=8E16 N/m^3



Non-linear Antisymmetric Oscillator

$$V(z) = \frac{1}{2}k_1 z^2 - \frac{1}{3}k_2 z^3 + \frac{1}{4}k_3 z^4 + \ldots.$$

$$m\frac{d^2}{dt^2}z + k_1 z - k_2 z^2 + k_3 z^3 + \ldots. = 0$$

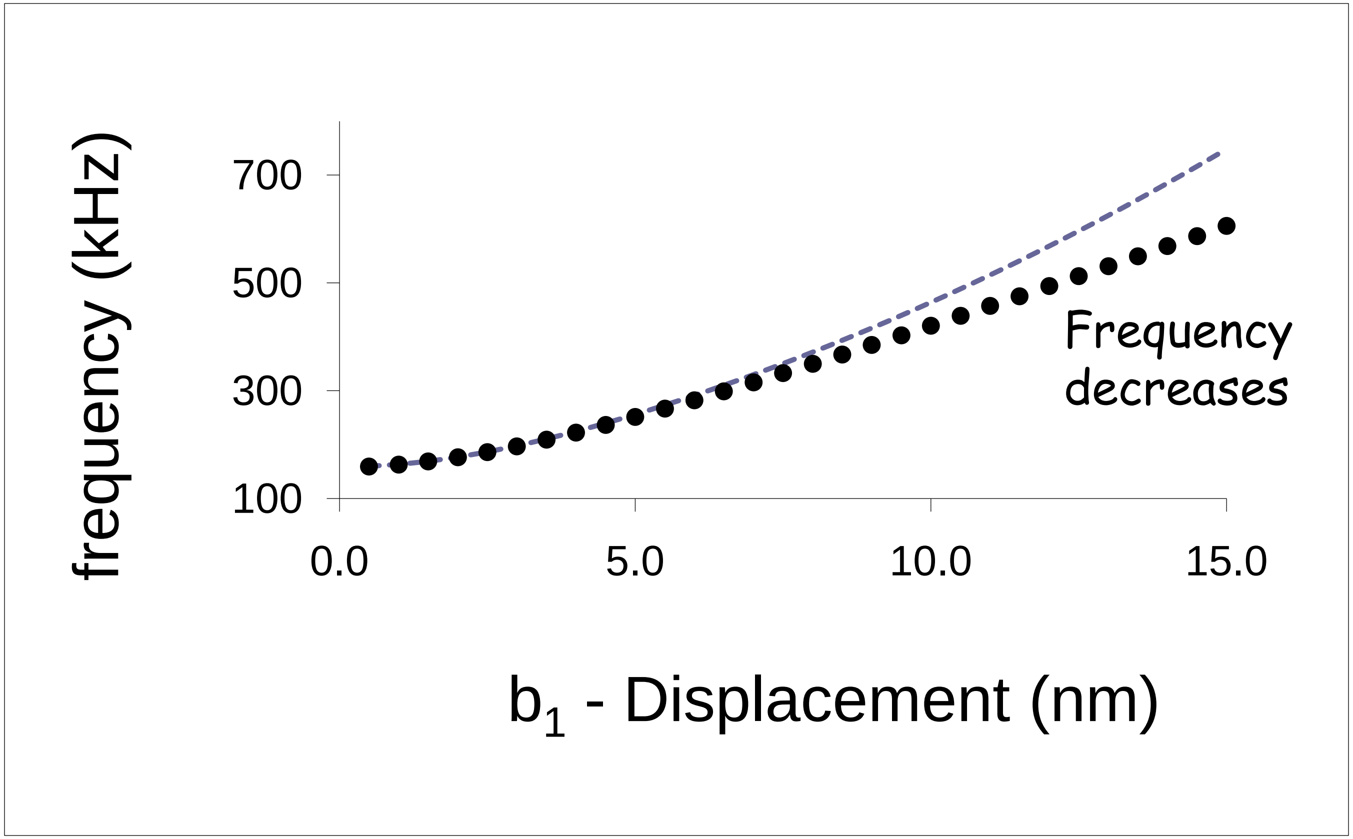
$$z(t) = a_o + \sum_{n=1}^{\infty}\left(a_n \sin n\omega t + b_n \cos n\omega t\right)$$

$a_n = 0;$ solution now contains BOTH even and odd harmonics[†]

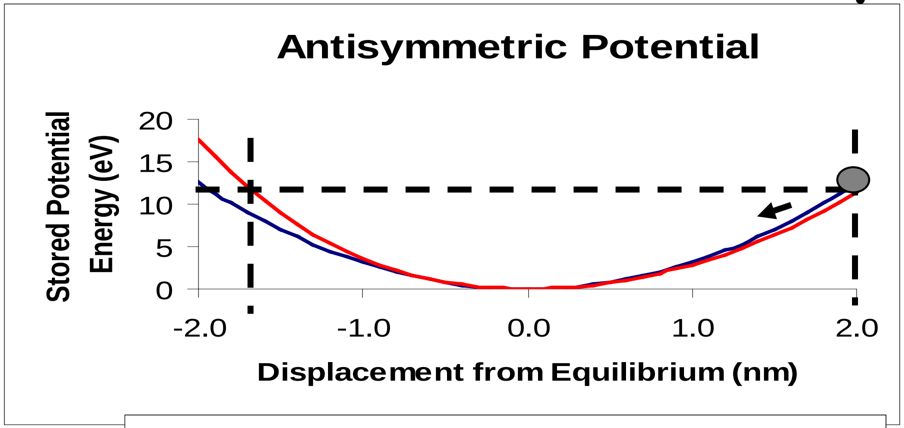
$$f \approx \frac{1}{2\pi}\sqrt{\frac{k_1}{m} + \frac{3}{4}\frac{k_3}{m}b_1^2 - \frac{5}{6}\frac{k_2}{m}\frac{k_2}{k_1}b_1^2 + \ldots}$$

- frequency depends on amplitude
- integer harmonics appear

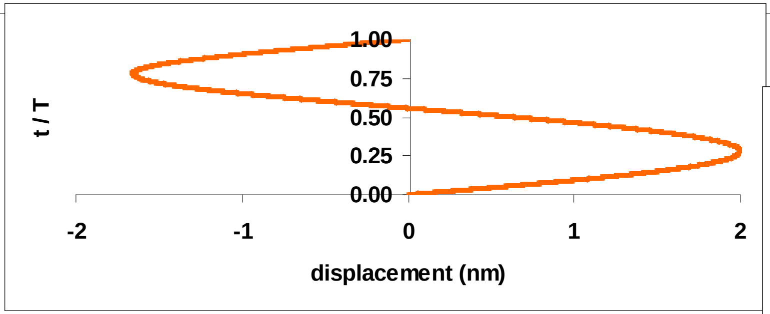
Non-linear Antisymmetric Oscillator

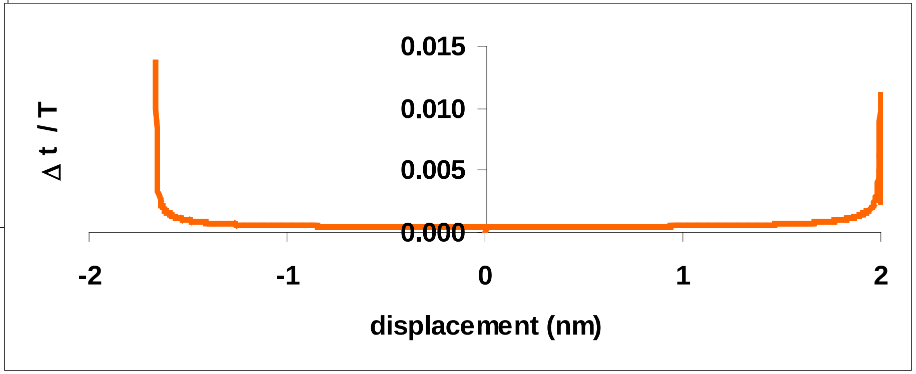


Non-linear Antisymmetric Oscillator

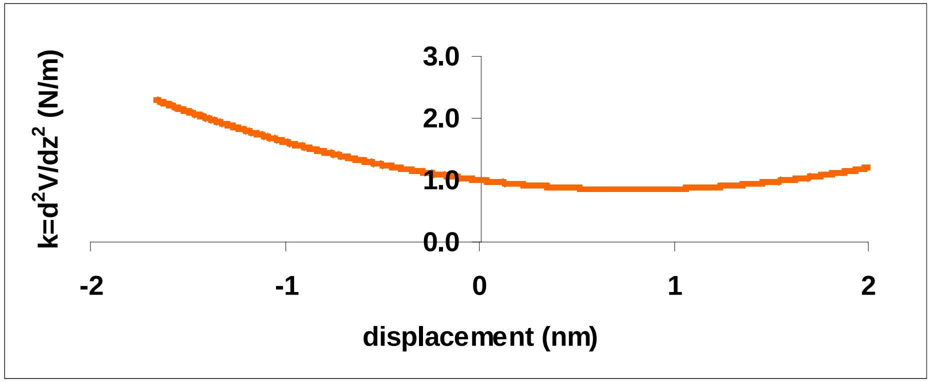


$$V(z) = \frac{1}{2}k_1 z^2 - \frac{1}{3}k_2 z^3 + \frac{1}{4}k_3 z^4$$

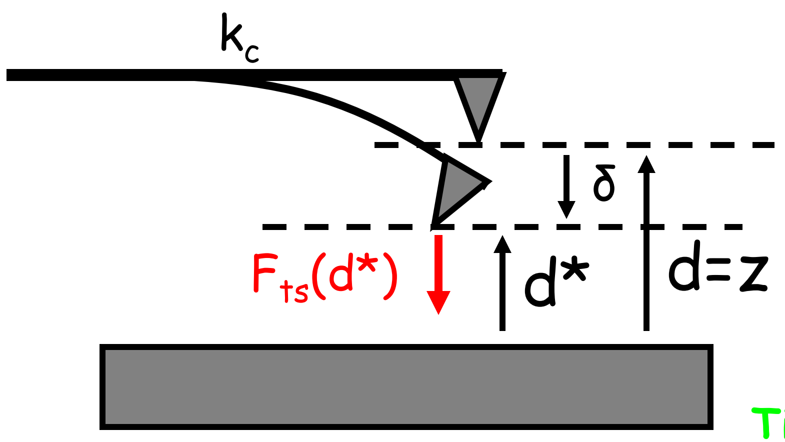


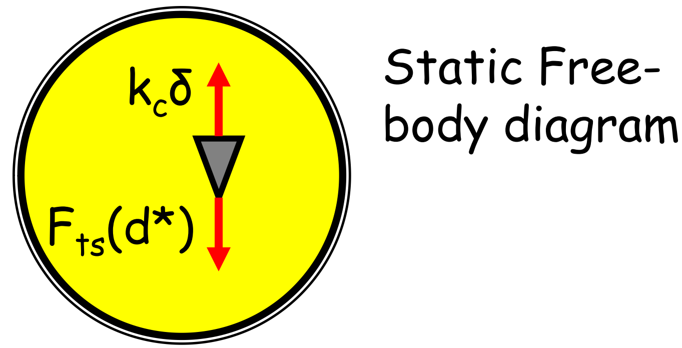


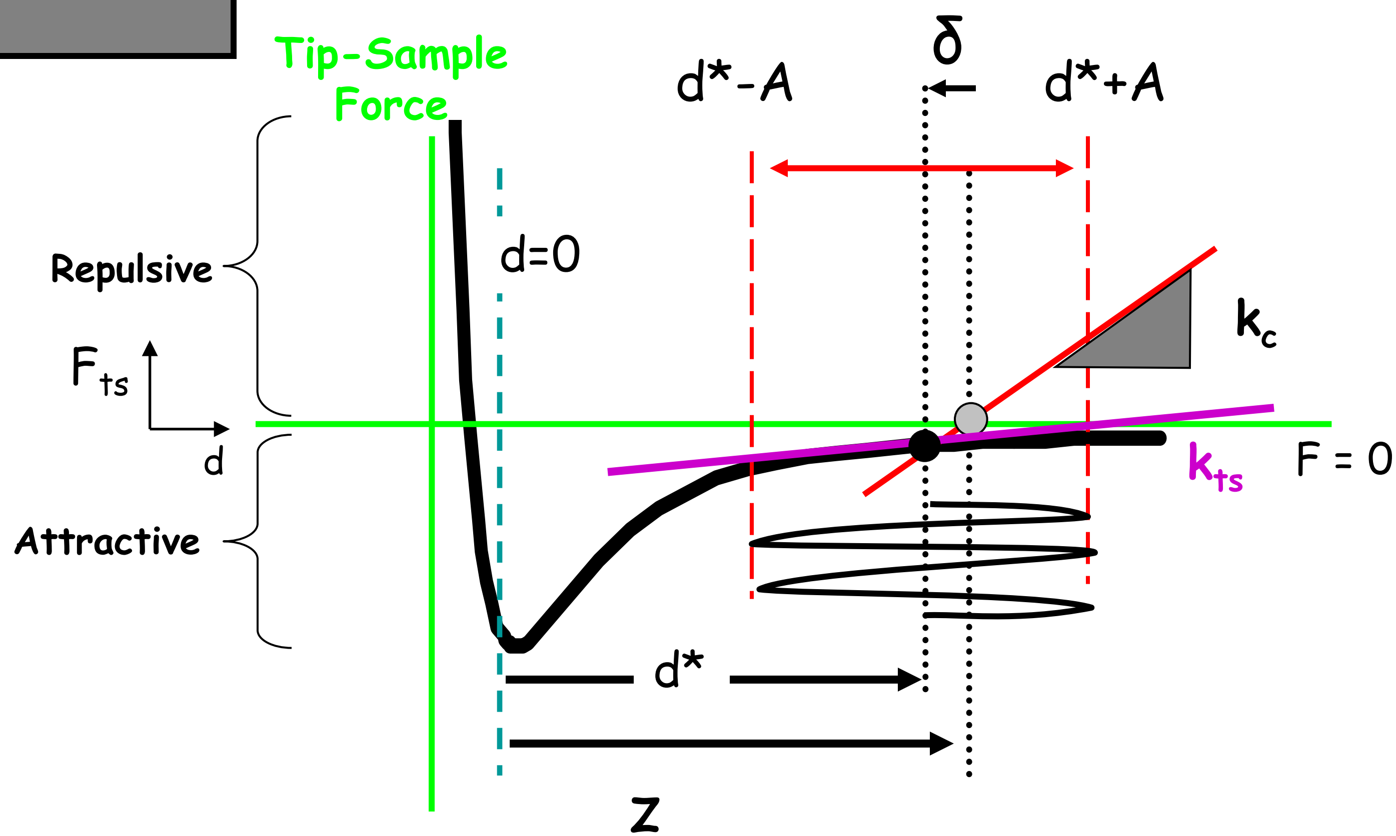
effective spring constant:
weighted average



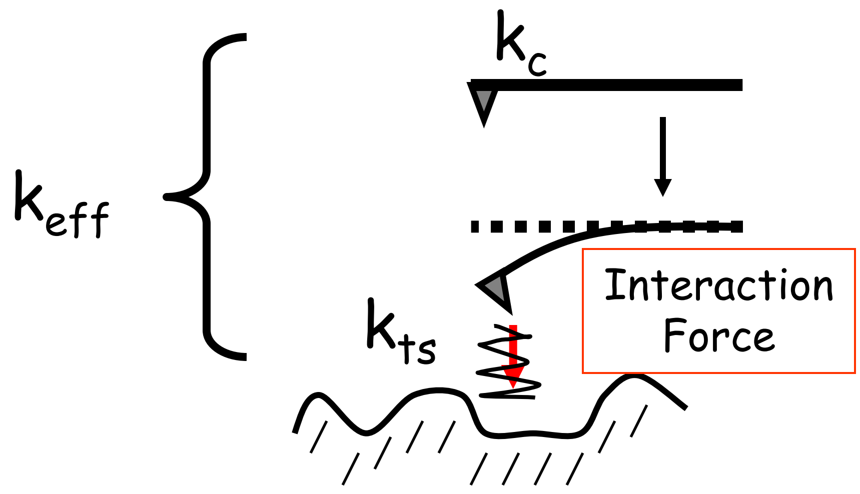
IV: The Oscillating AFM Tip





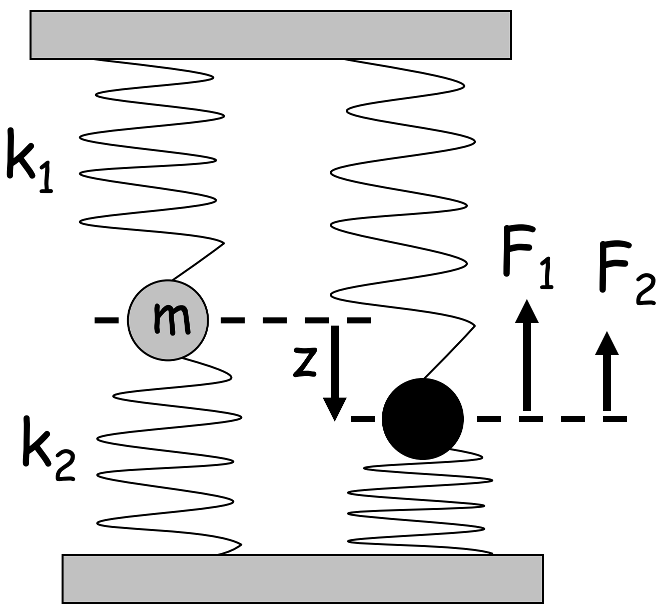


How does frequency change as d* decreases?



The tip-substrate spring is a different type of spring!

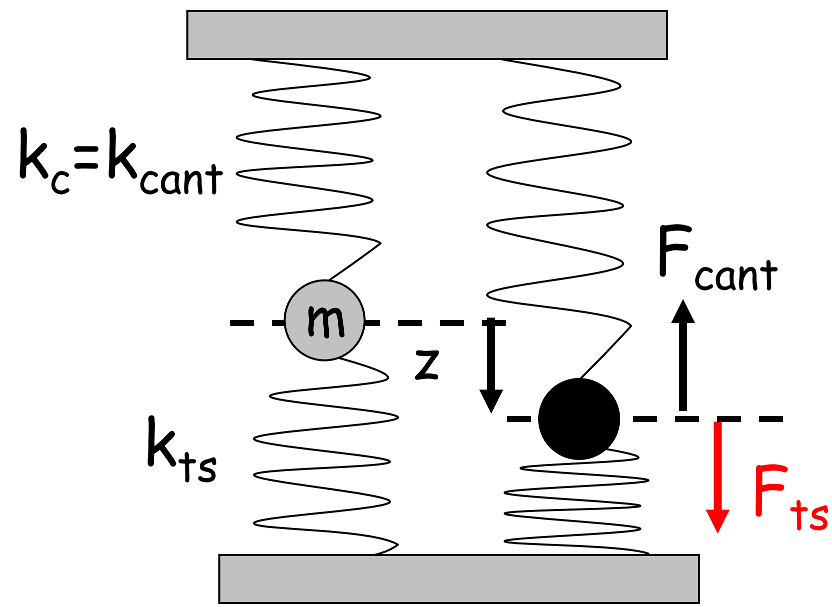
"Conventional" Spring



$$m\ddot{z} = -k_1 z - k_2 z$$

$$\omega = \sqrt{\frac{k_1 + k_2}{m}}$$

Tip-Substrate "Effective" Spring



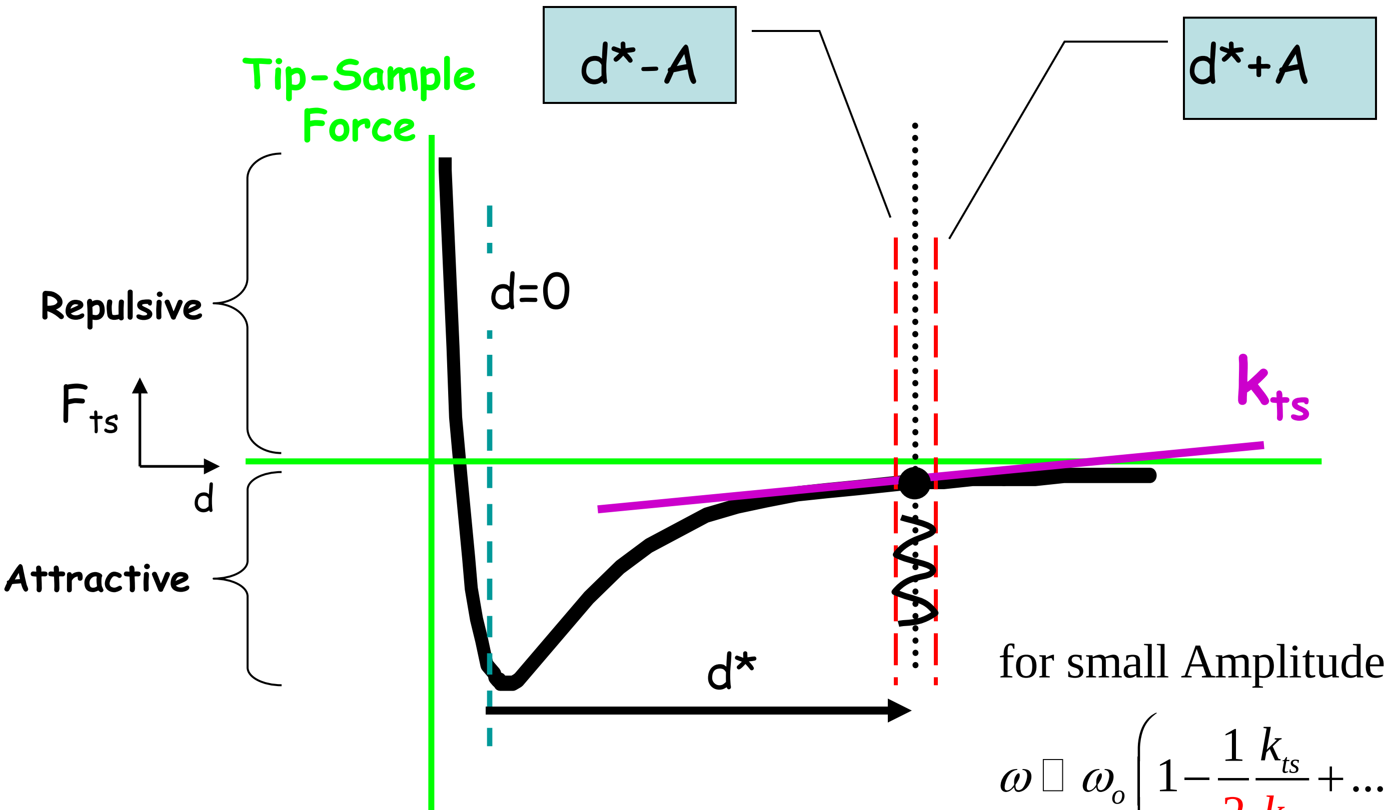
$$m\ddot{z} = -k_c z + k_{ts} z \quad or$$

$$m\ddot{z} = -(k_c - k_{ts})z$$

$$\omega = \sqrt{\frac{k_c - k_{ts}}{m}} = \sqrt{\frac{k_c}{m}\left(1 - \frac{k_{ts}}{k_c}\right)}$$

$$\approx \omega_o \left(1 - \frac{1}{2}\frac{k_{ts}}{k_c} + ...\right); \quad \omega_o \equiv \sqrt{\frac{k_c}{m}}$$

What's the frequency shift?



for small Amplitude of oscillation

$$\omega \cong \omega_o \left(1 - \frac{1}{2} \frac{k_{ts}}{k_c} + ... \right)$$

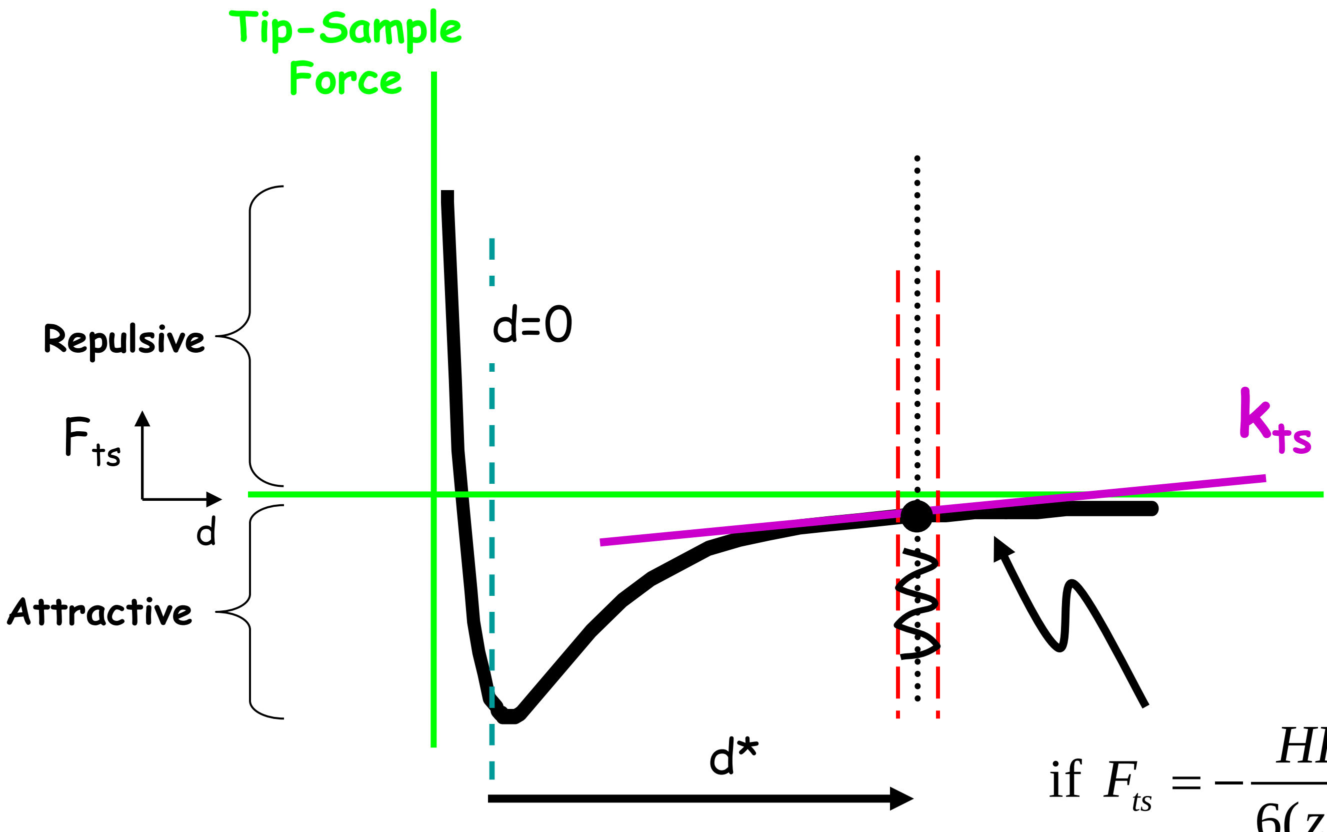
Frequency shift is negative

$$\omega - \omega_o \equiv \Delta\omega(d^*) \cong -\frac{\omega_o}{2k_c} k_{ts} |_{d*} = -\frac{\omega_o}{2k_c} \frac{dF_{ts}(d*)}{dz}$$

$$d* = z - \delta$$

$$\Rightarrow \quad F_{ts}(d^*) = -2k_c \int_{\infty}^{d^*} \frac{\Delta\omega}{\omega_o} dz$$

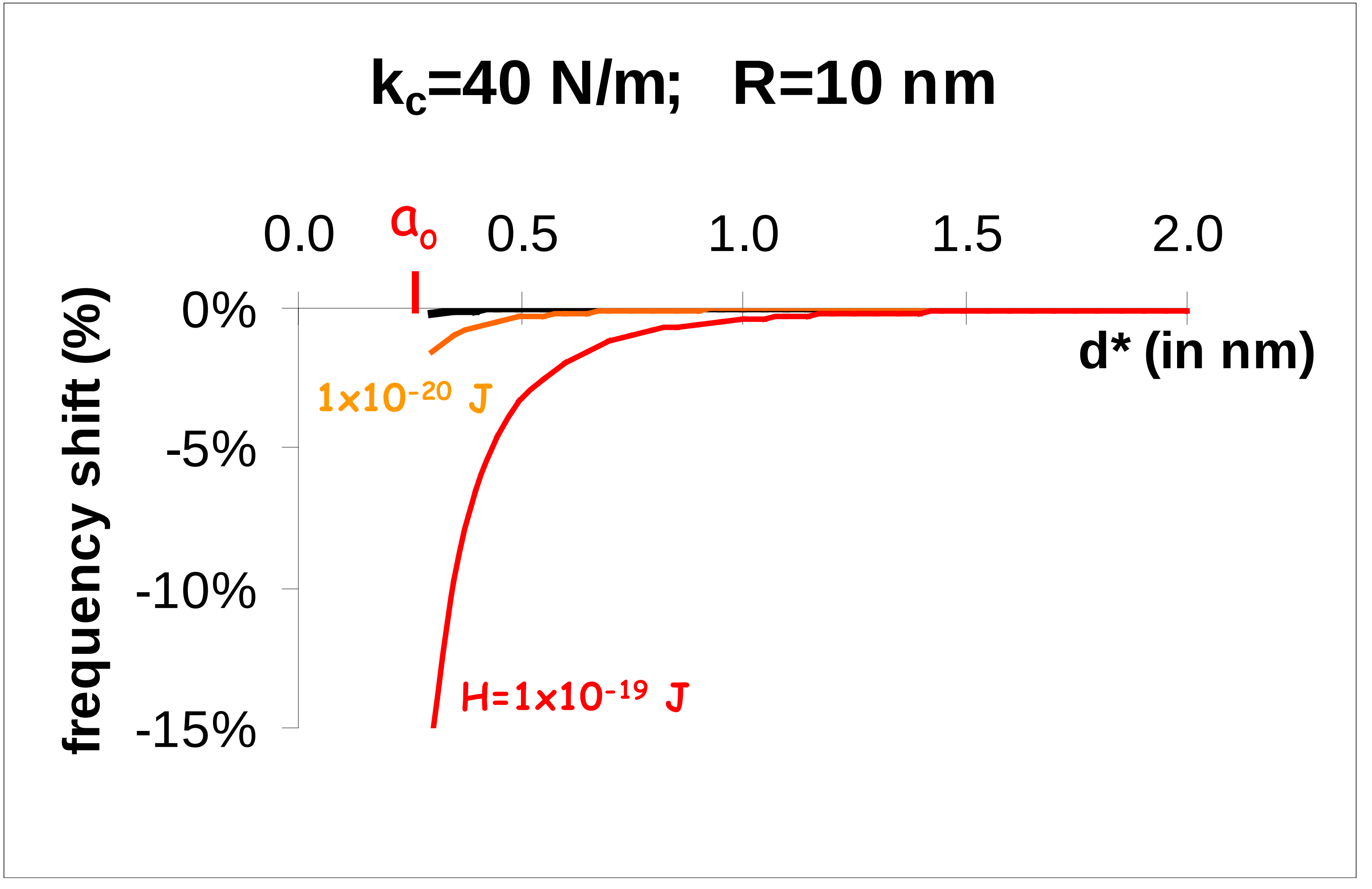
Putting in some numbers



$$\text{if } F_{ts} = -\frac{HR}{6(z)^2}$$

$$\text{and } k_{ts} = \left.\frac{dF_{ts}}{dz}\right|_{z=d^*} = \frac{HR}{3(d^*)^3}$$

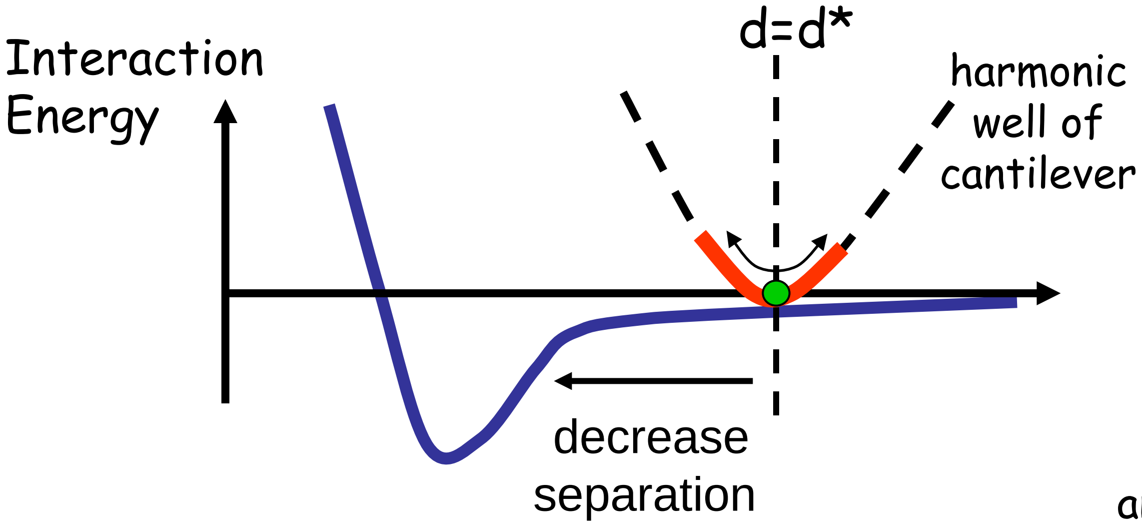
$$\therefore \frac{\Delta\omega}{\omega_o} = -\frac{1}{2k_c}\frac{dF_{ts}}{dz} = -\frac{1}{2k_c}\frac{HR}{3(d^*)^3}$$



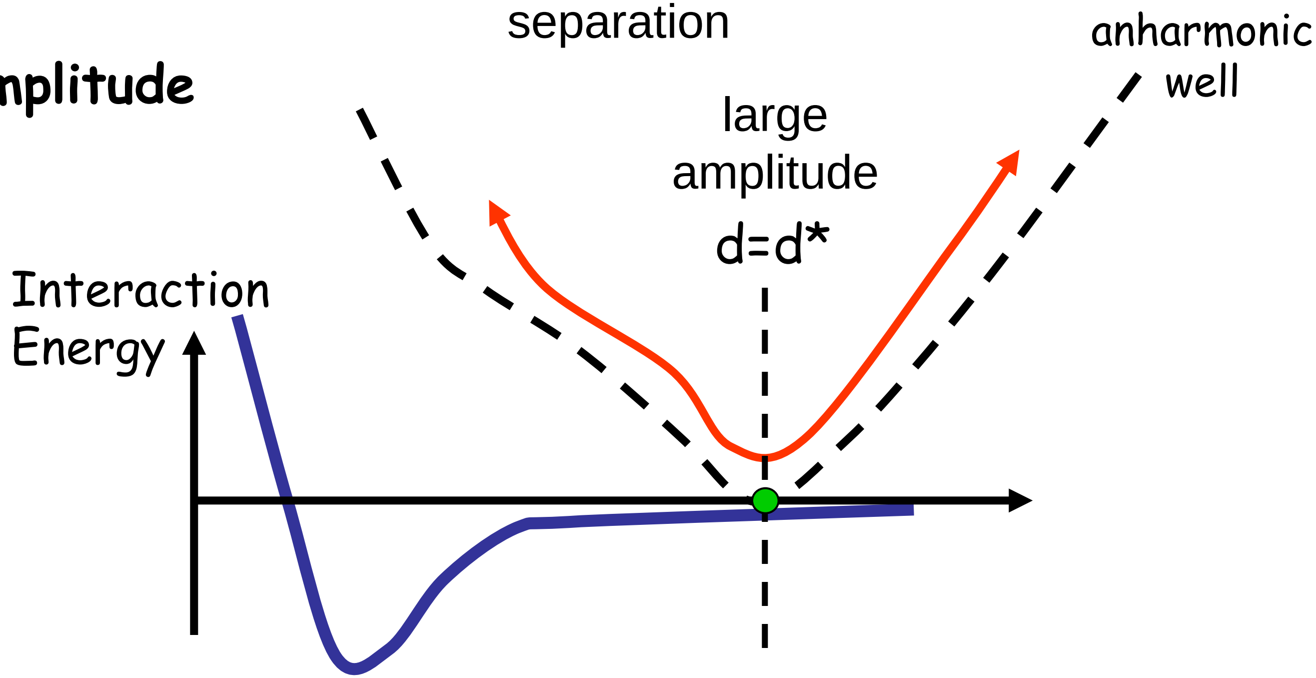
k_c=40 N/m;   R=10 nm
0.0
a_o
0.5
1.0
1.5
2.0
0%
-5%
-10%
-15%
frequency shift (%)
d* (in nm)
1x10^-20 J
H=1x10^-19 J

Probing the Interaction Potential (schematic)

A. Small amplitude



B. Large amplitude



vdW + DMT for d=d*=0.8 nm

Input Parameters:

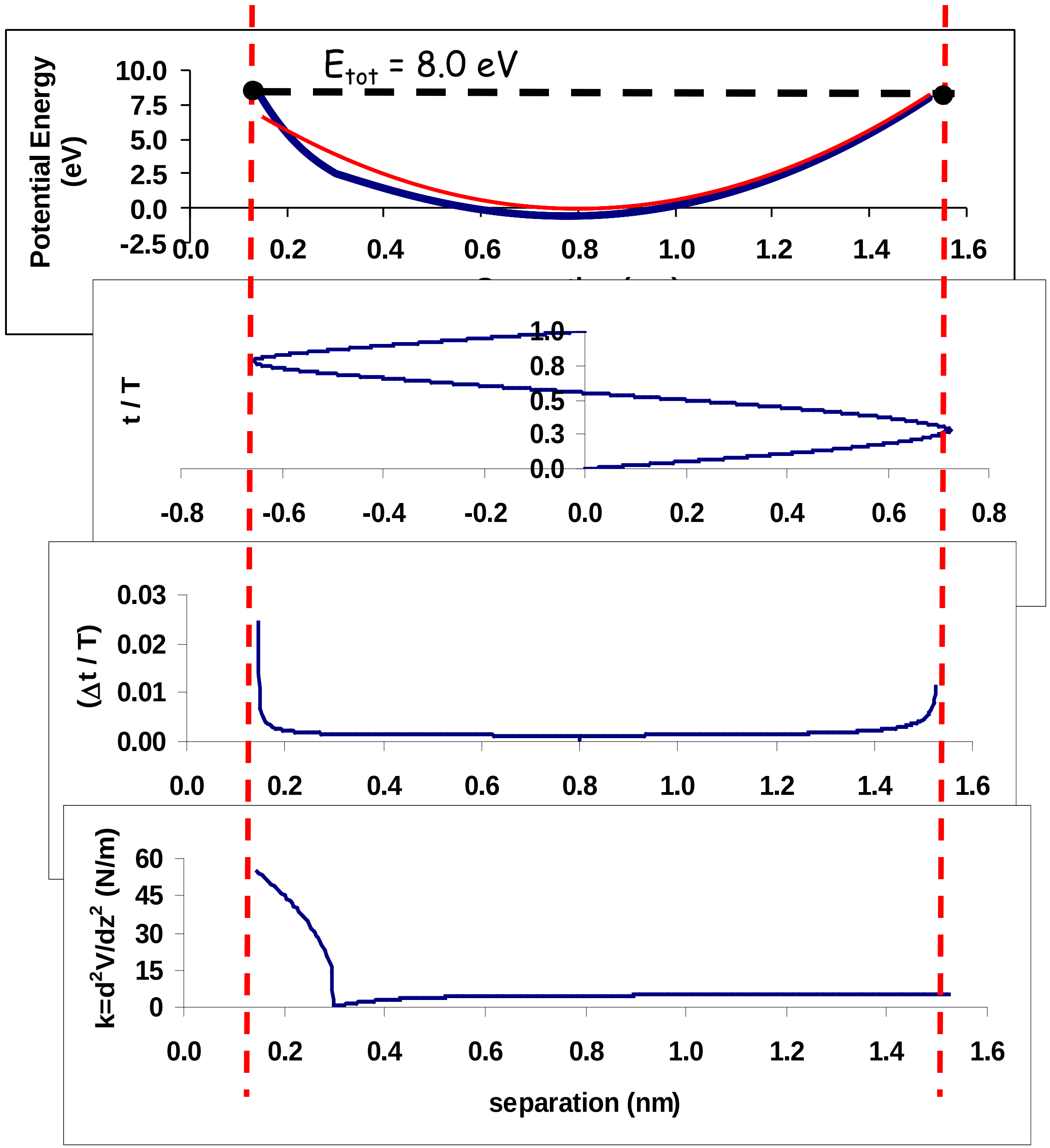
k= 5 N/m

a_o= 0.3 nm

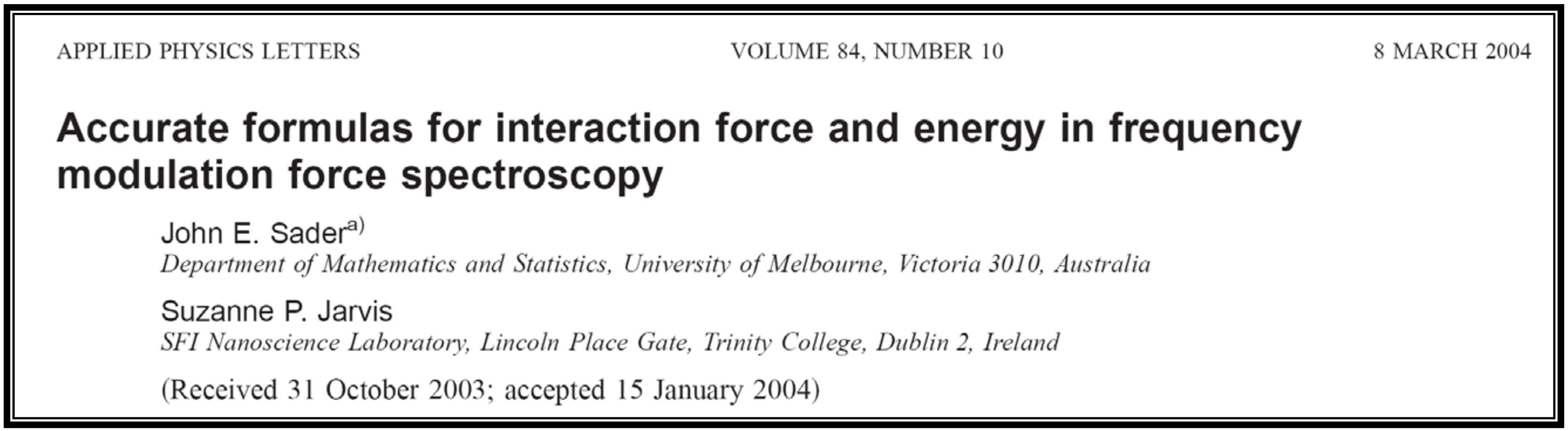
H= 4 $\times 10^{-20}$ J

R= 10 nm

E*=20 GPa



How to determine $F_{ts}(z)$ from $\Delta f(z)$ for arbitrary amplitude of oscillation?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Accurate formulas for interaction force and energy in frequency modulation force spectroscopy

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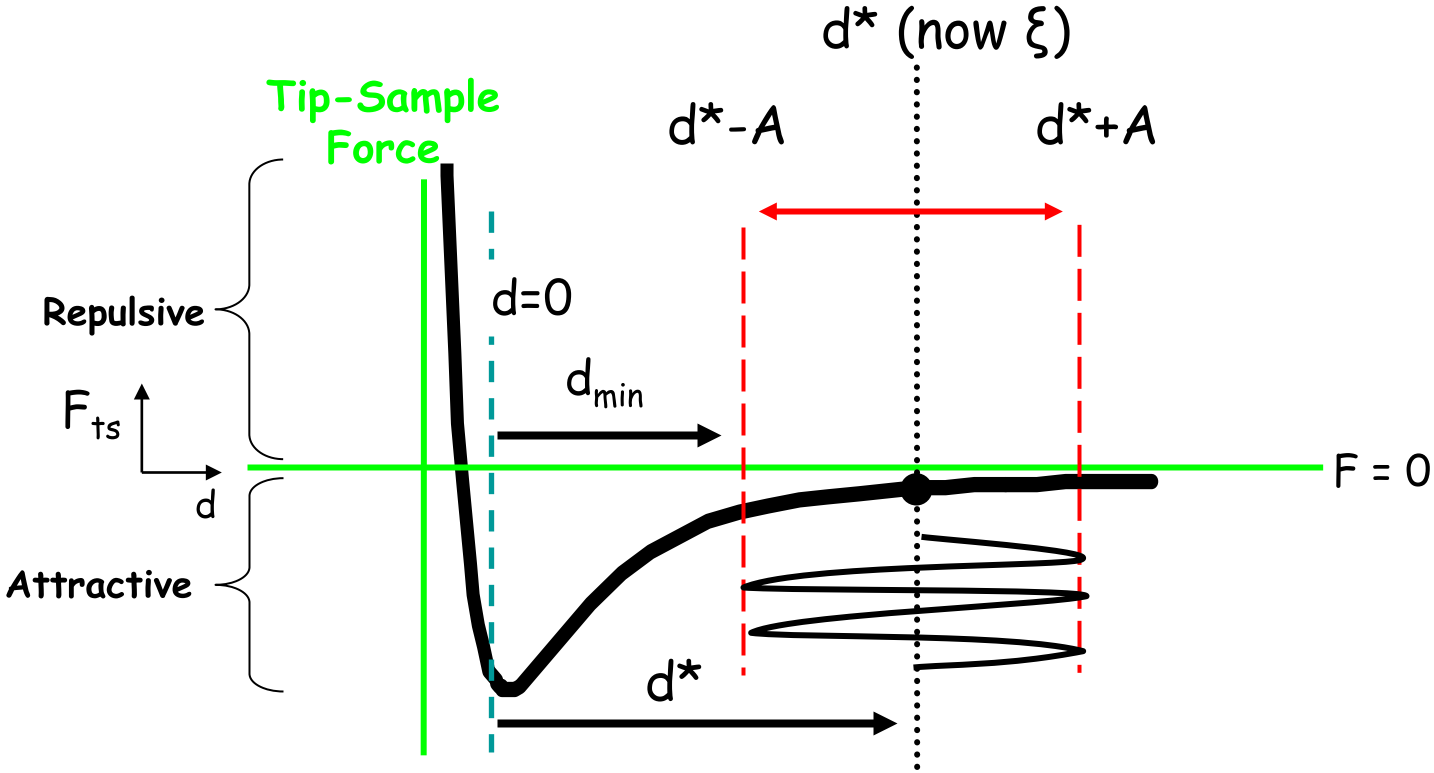
J. E. Sader and S. P. Jarvis, Appl. Phys. Lett. **84**, 1801 (2004).



small amplitude

"interpolation"

large amplitude

$$F_{ts}(d_{min}) = 2k_c \int_{d_{min}}^{\infty} \left\{ \left[1 + \frac{\sqrt{A}}{8\sqrt{\pi(\xi - d_{min})}}\right]\Omega(\xi) - \frac{A^{3/2}}{\sqrt{2(\xi - d_{min})}} \frac{d\Omega(\xi)}{d\xi} \right\} d\xi$$

where $\Omega(\xi) \equiv \dfrac{\Delta f(\xi)}{f_o}$ $\qquad \xi \Leftrightarrow d^*$

$\Rightarrow$ valid if $A(\xi)$ varies with ξ