Materials Science and Engineering 405
Prof. Mark C. Hersam, Fall 2006
PHYSICS OF SOLIDS

Homework #2

Due: Monday, October 9, 2006

1.) **(35 points)** Griffiths, Problem 2.27

2.) **(35 points)** Throughout this problem, consider the following rectangular potential barrier:

$$V(x) = \begin{cases} V_0, & \text{if } 0 \le x \le d \\ 0, & \text{otherwise} \end{cases}$$

(a) Assume that $V_0 > E > 0$. Under these conditions, show that the transmission coefficient is:

$$T = \frac{1}{1 + \dfrac{V_0^2}{4(V_0 - E)E}\sinh^2(\gamma d)}, \text{ where } \gamma = \frac{\sqrt{2m(V_0 - E)}}{\hbar} \text{ and } \sinh(\gamma d) = \frac{1}{2}\left(e^{\gamma d} - e^{-\gamma d}\right)$$

Note: $\cosh(\gamma d) = \frac{1}{2}\left(e^{\gamma d} + e^{-\gamma d}\right)$; $\cosh^2 x = 1 + \sinh^2 x$

(b) In the limit where $\gamma d >> 1$, show that: $T = \dfrac{16(V_0 - E)E}{V_0^2} e^{-2\gamma d}$ (i.e., tunneling through a rectangular barrier varies exponentially with barrier width).

3.) **(30 points)** Throughout this problem, consider the following spherically symmetric three dimensional potential: $V(r) = 0$ if $a < r < b$, $V(r) = \infty$ otherwise.

Note: $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$; $\int_0^\pi \sin^2\phi \, d\phi = \pi/2$

(a) Show that the energy of the ground state is: $E_1 = \dfrac{\hbar^2\pi^2}{2m(b-a)^2}$.

(b) Show that the normalized ground state wavefunction is: $\psi_{100}(\mathbf{r}) = \dfrac{1}{r\sqrt{2\pi(b-a)}} \sin\left(\pi \dfrac{r-a}{b-a}\right)$