On the incompatibility of relations $P=hk$ and $\Delta M=E/c^2$ with wave-particles dualism for the electromagnetic waves in a substance

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Main problem - if we have material with negative refraction, it can be described on the base of negative n and negative k values.
The consequences from appearance of negative n are presently more or less clear

Really, many famous formulas of electrodinamics and optics are not valid for negative n, because they are written in so called "nonmagnetic approach", namely for materials with *B*=*H*. If *B* differs with *H*, many formulas should be modified – see the next slide

Nonmagnetic approach – wrong and very dangerous

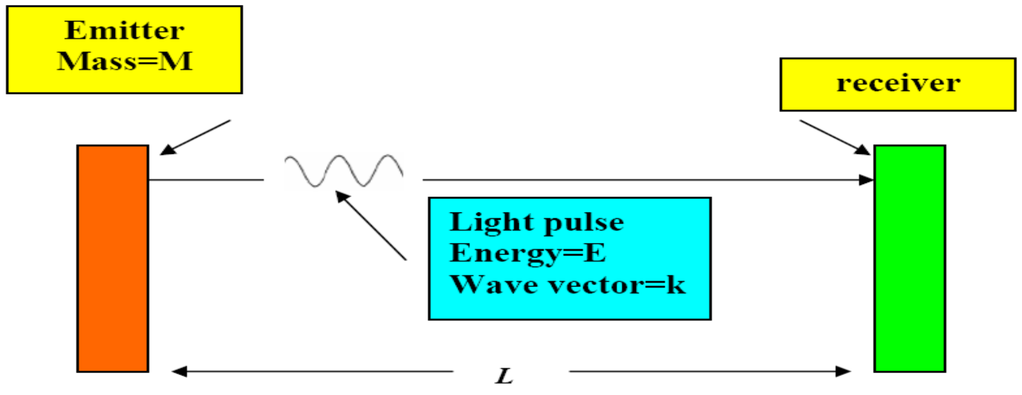
Physical law	Equation for nonmagnetic approach	Correct equation
Snellius, Doppler, Cherenkov $n=\sqrt{\varepsilon}\rightarrow n=\sqrt{\varepsilon\mu}$ if $\varepsilon,\mu<0$, than $n<0$	$\sin\varphi/\sin\psi=n_{21}=\sqrt{\varepsilon_2/\varepsilon_1}$	$\sin\varphi/\sin\psi=n_{21}=\sqrt{\varepsilon_2\mu_2/\varepsilon_1\mu_1}$
Fresnel $n=\sqrt{\varepsilon}\rightarrow 1/z=\sqrt{\varepsilon/\mu}$	$r_\perp=\dfrac{n_1\cos\varphi-n_2\cos\psi}{n_1\cos\varphi+n_2\cos\psi}$	$r_\perp=\dfrac{z_2\cos\varphi-z_1\cos\psi}{z_2\cos\varphi+z_1\cos\psi}$
Reflection coefficient for normal fall of light on the border between two media	$r=(n_1-n_2)/(n_1+n_2)$	$r=(z_2-z_1)/(z_2+z_1)$
Condition for full matching	$n_1=n_2$	$z_1=z_2$
Brewster angle	$tg\varphi=n$	$tg\varphi=\sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}\dfrac{\varepsilon_2\mu_1-\varepsilon_1\mu_2}{\varepsilon_2\mu_2-\varepsilon_1\mu_1}}$

The more complicated question is about negative k. Does it mean, that instead of light pressure, like in vacuum, we have in LHM, following relation P=hk, light attraction?

- This problem could not be resolved, if we do not know, what is a value of light pressure in more simple case, namely in materials with positive n>1 and $k=n\omega/c > k_0=\omega/c$
- Sorry, at present this problem has many approaches, but not a single convincing decision.

As a first step, let us consider the energy, linear momentum and mass transfer from emitter to receiver in vacuum, following Einstein's paper

Ann. Phys..20, 627 (1906)



Time t of passing light from emitter to receiver

$$t = L/c$$

Momentum p of light pulse gives recoil momentum for emitter

$$p = hk = E/c$$

Velocity of emitter

$$V = p/M$$

Movement of emitter

$$X = Vt = pL/Mc$$

Mass Δm, transferred from emitter to receiver, following center of mass conservation law MX= ΔmL

$$\Delta m = P/c = E/c^2$$

Return back to text on previous slide, namely to very famous **equation** $E=mc^2$.

$E=mc^2=m*c*c$

What does it mean two chars “c” ?

May be “c” means only constants? Or they have some definite physical meaning?

Answer – red equations on previous slide.

One “c” is “c_{gr}“- group velocity of light,

the second “c” is “c_{ph}“ - phase velocity.

So, two “c” in equation $E=mc^2$

have definite, but different physical meanings, namely phase and group velocities.

So, we have now not $E=mc^2$

but $E=mc_{ph}c_{gr}$

This result is very important !

There is a natural question - what happens if the space between the emitter and the receiver does not fill the vacuum, but a substance with the phase and group velocity V_{ph} and V_{gr}?

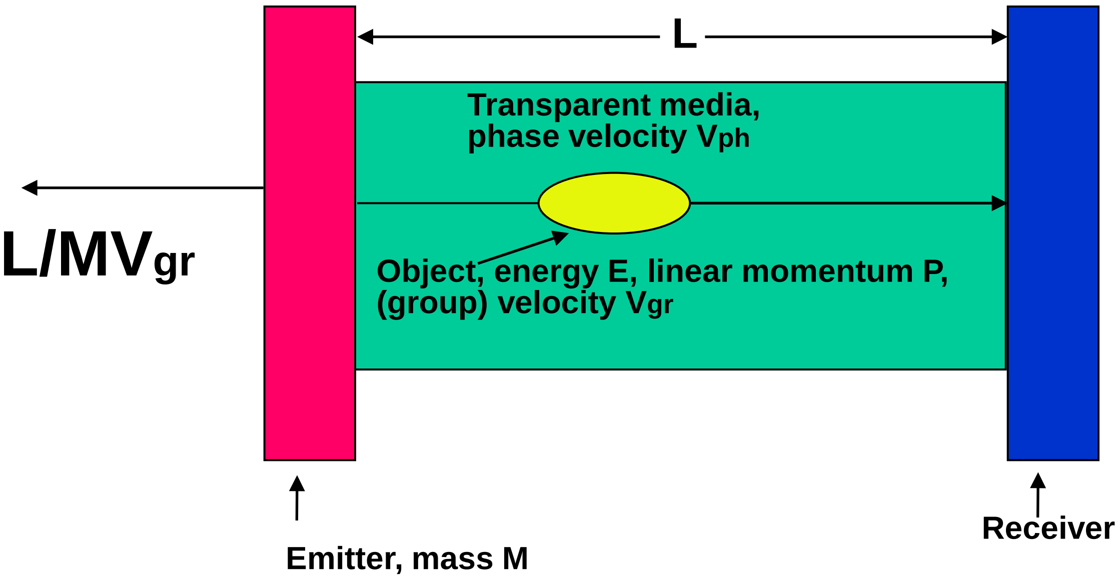
Should not we in this case replace the equation

$E = mc^2$ on $E = mV_{ph}V_{gr}$?

And what about sign of E if V_{ph} and V_{gr} has opposite directions?

Displacement X of emitter for the propagation time $t=L/V_{gr}$ of the object to the receiver.

$$X=PL/MV_{gr}$$



The movement of mass M to left on distance X should be compensated by the movement of mass $m=P/V_{gr}$ to right on distance L.

Let us consider the relations between energy E and linear momentum P of object.

Two different relations are possible:

1. **Object is a wave packet (photon, pulse of light).** $P=E/V_{ph}.$

   $m=E/V_{gr}V_{ph}$

2. **Object is material body (bullet, elementary particle).** $P=EV_{gr}/c^2$

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Light – wave or particle?

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See V.G.Veselago, Physics-Uspekhi **52**,(6), 649-654, (2009)

WHY TENSORS?
Tensors are the sources for calculations of ponderomotive forces

Both tensors have similar component, except value of linear momentum density g

$$f_i = \frac{\partial T_{ik}}{\partial x_k}$$

$$\mathrm{T}_{\iota\kappa} = \begin{bmatrix} \theta_{\alpha\beta} & gc \\ S/c & W \end{bmatrix}$$

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Minkowski

$$\theta_{\alpha\beta} = \frac{1}{4\pi}(E_\alpha D_\beta + H_\alpha B_\beta) - \frac{1}{8\pi}\delta_{\alpha\beta}(ED + HB)$$

$$S = \frac{c}{4\pi}[EH]$$

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Abrakham

$$\theta_{\alpha\beta} = \frac{1}{4\pi}(E_\alpha D_\beta + H_\alpha B_\beta + E_\beta D_\alpha + H_\beta B_\alpha) - \frac{1}{8\pi}\delta_{\alpha\beta}(ED + HB)$$

$$S = \frac{c}{4\pi}[EH]$$

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CONCLUSIONS

- 1. Relation between the mass m of a transferred light and its energy E in the general case is $m=E/V_{ph}V_{gr}$. The well-known relation $m=E/c^2$ is a special case.
- 2. Abraham tensor is not relativistically invariant and can not be used for calculation of the forces of light pressure. This fact completely resolves the century-old debate about “Minkowsky – Abrakham controversy”
- 3. In materials with negative phase velocity does not occur light pressure but presents light attraction. In this case the mass transferred not from the emitter to the receiver , but from receiver to emitter .

The main stages in the development of new scientific ideas

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- All this is true, but long been known.
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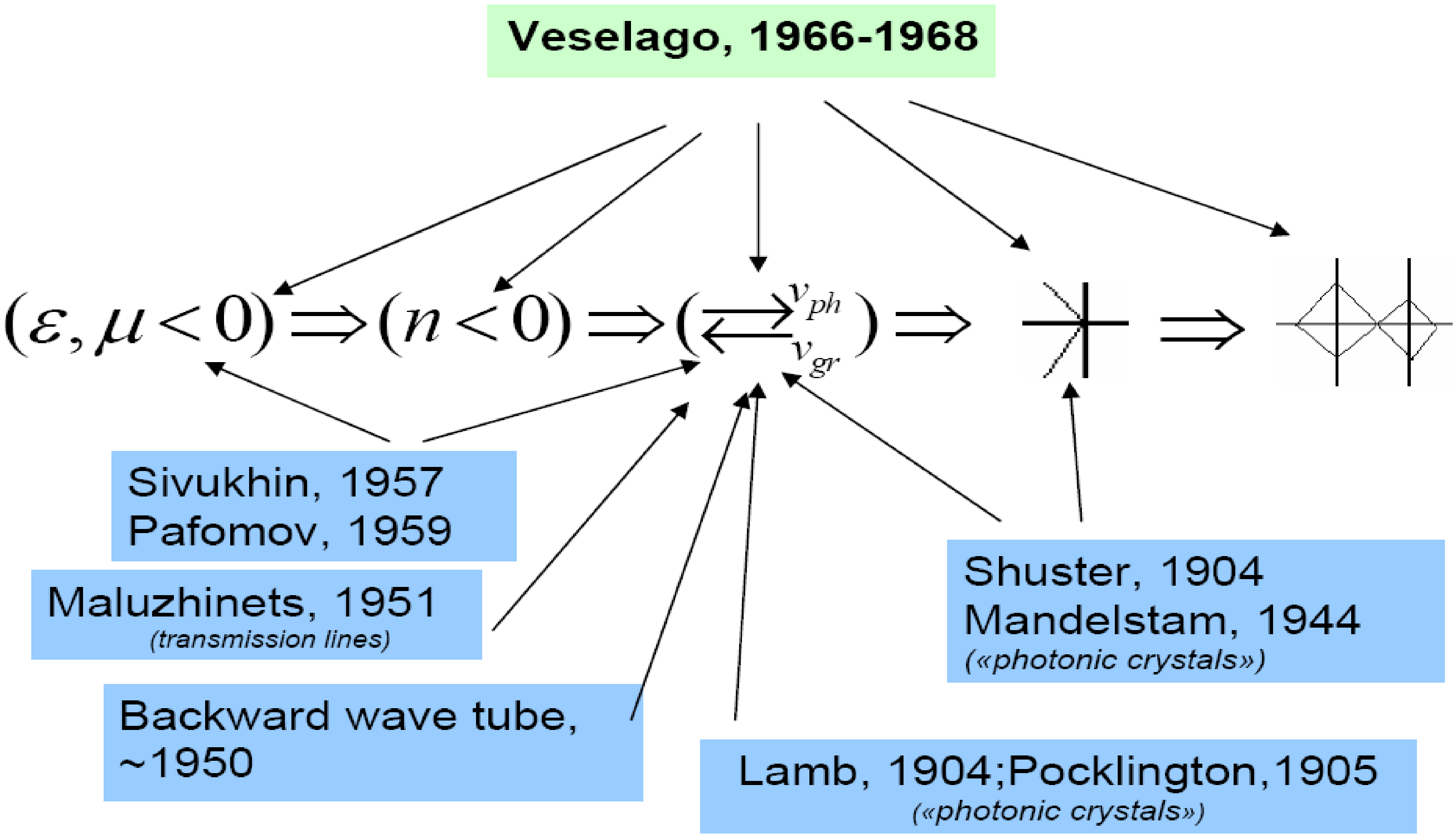
About history of negative refraction…



What is the difference?

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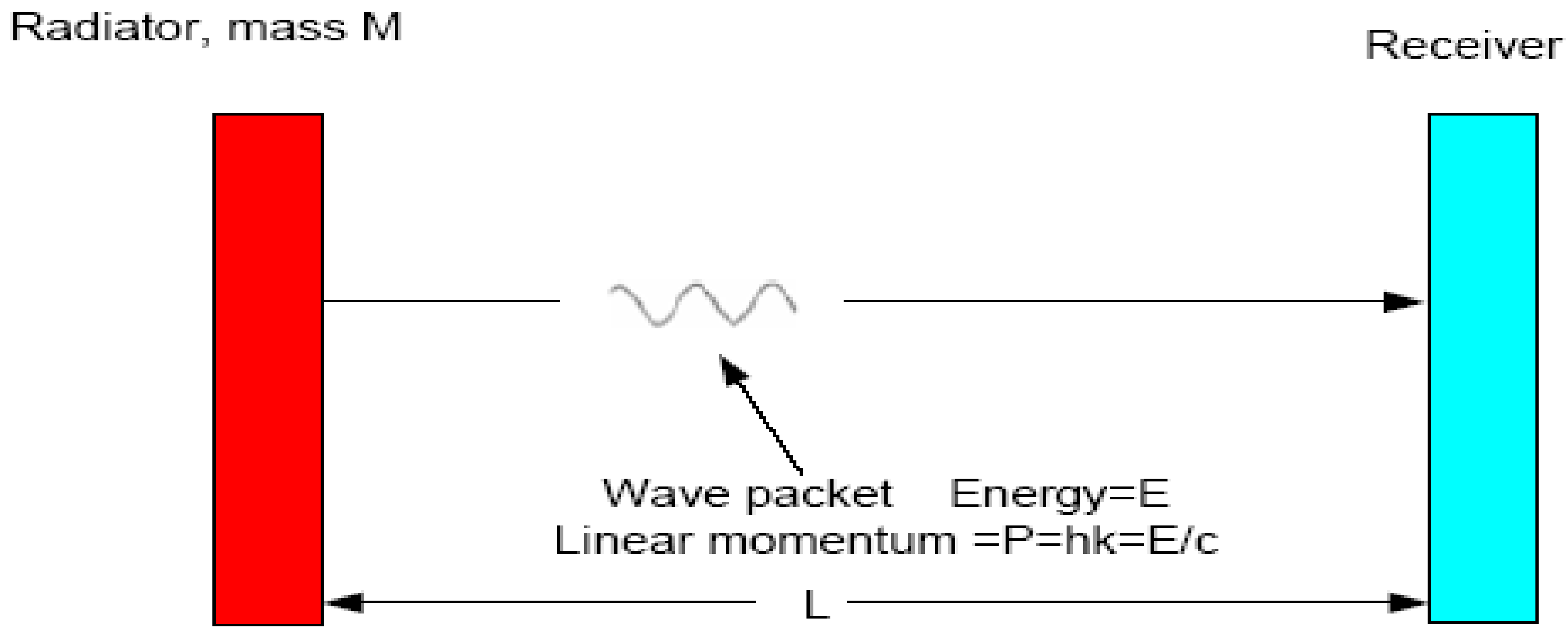
- Both tensors are not valid for materials with hegative ε and μ, because of negative sign before energy density W.



$$P = E/c$$

$$\Delta M = E/c^2$$

$$\mathrm{v} = P/M = E/Mc$$

$$t = L/c$$

$$\Delta x = t\mathrm{v} = LE/Mc^2$$

$$\Delta x M = LE / c^2$$

$$\Delta M = E / c_{ph} c_{gr}$$

$$\Delta M = E / \mathrm{v}_{ph} \mathrm{v}_{gr}$$

Waves or particles?

See V.G.Veselago, Physics-Uspekhi **52**,(6), 649-654, (2009)

$$P = M\mathrm{v}$$

$$P = hk = h\omega / \mathrm{v}$$

$$M = E / c^2$$

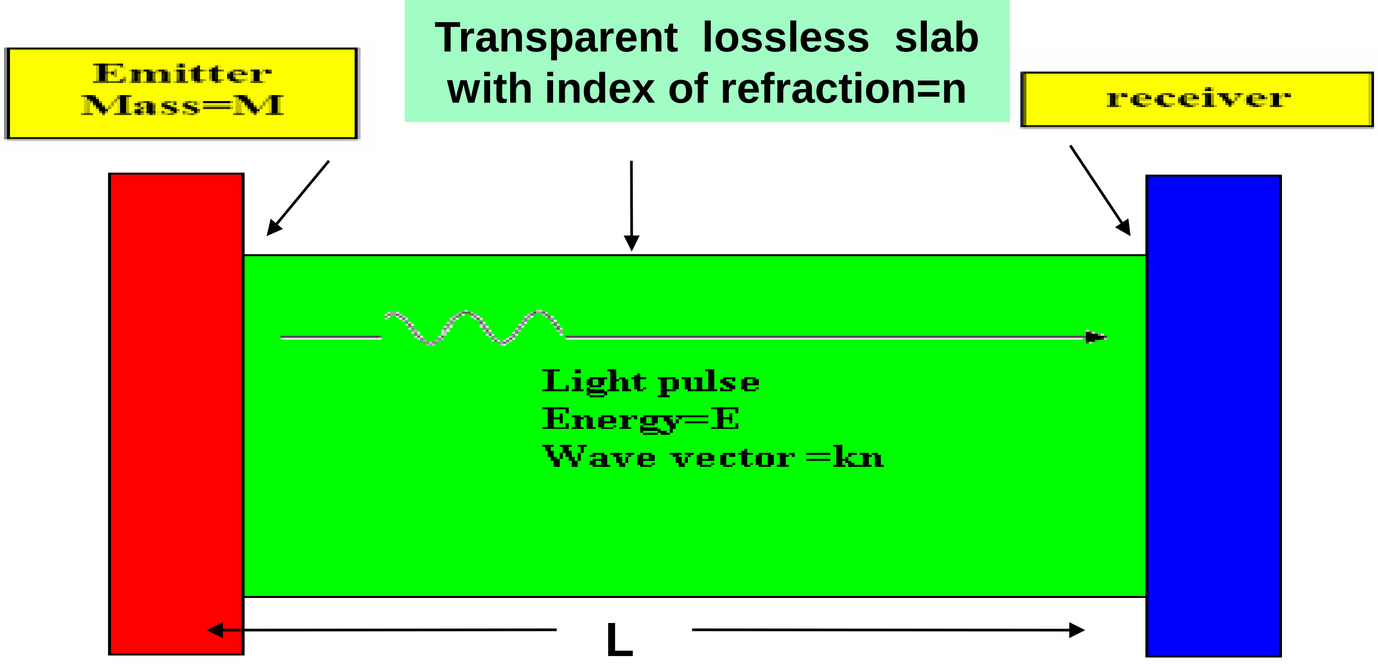
$$E = h\omega$$

$$v = c / n$$

$$v = c / n$$

$$P = E / cn$$

$$P = E \square n / c$$

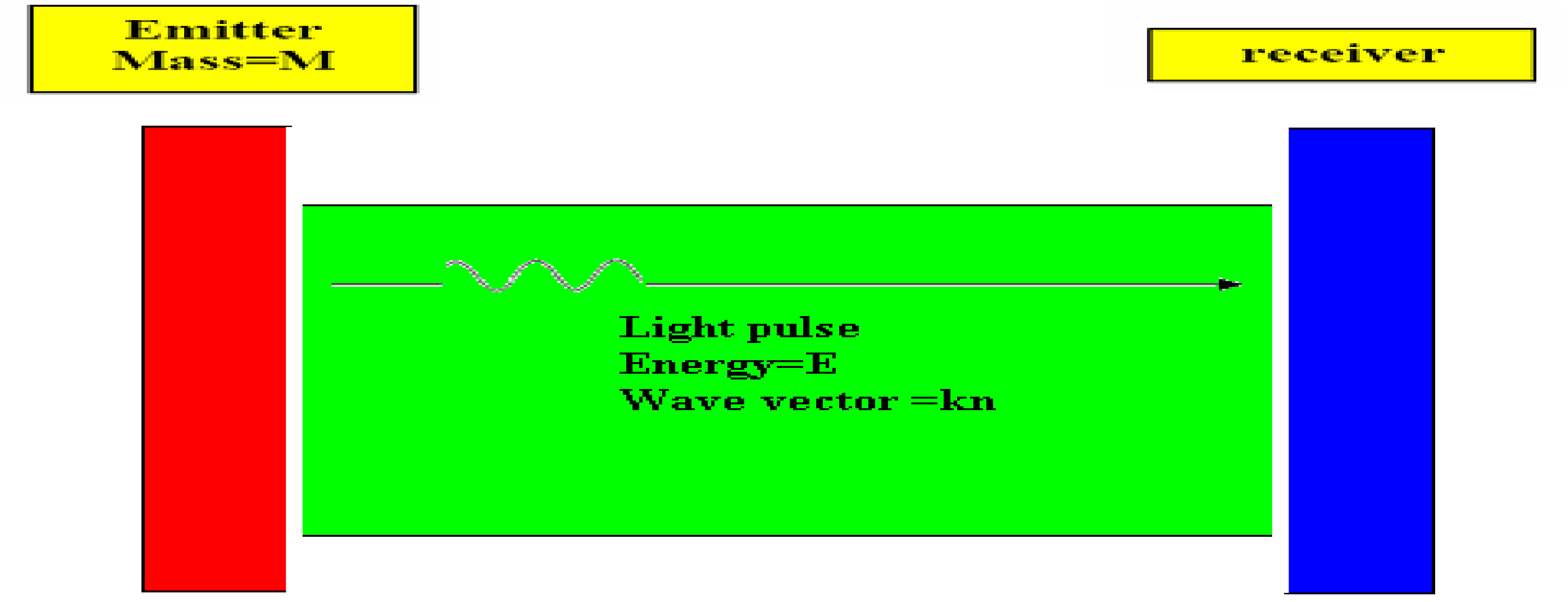


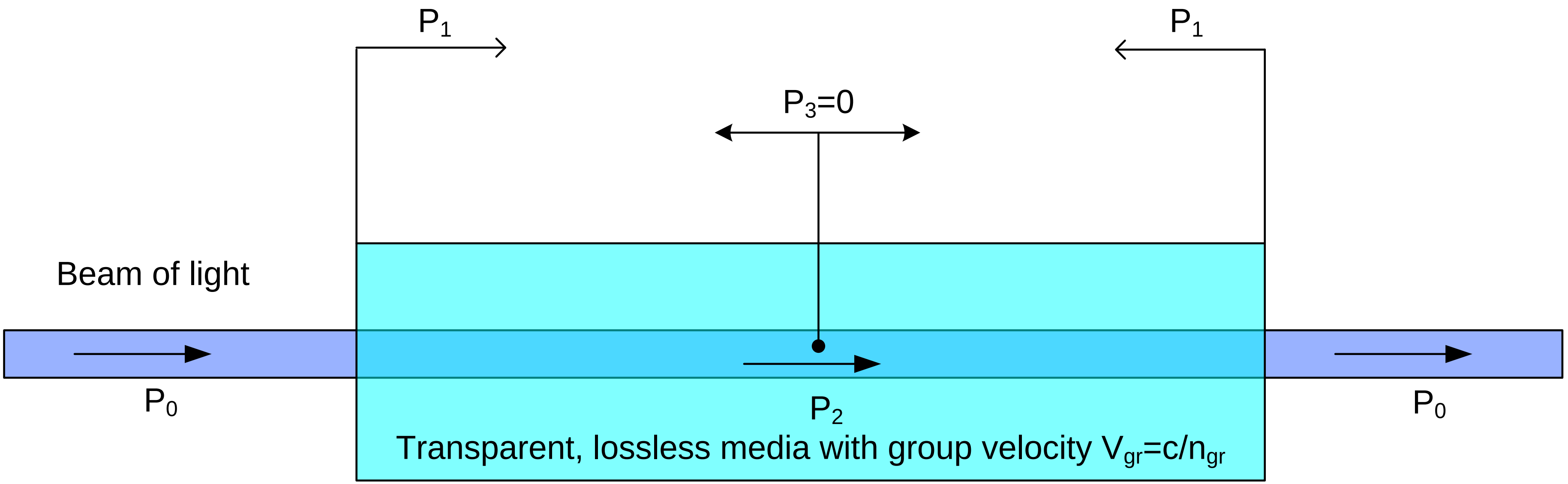
Main questions:

1. What is a pressure on the interface between slab and emitter (E) and receiver (R)?

2.What is a mass, transferred from (E) to (R) , if wave packet with energy E transferred from (E) to (R) inside the slab of media?

Let us consider a little modified picture from previous slide. Let us include some small vacuum gaps between slab and emitter and receiver. By this choice of geometry we could calculate separately the forces on the interfaces of slab, emitter and receiver. As to forses on the interfaces of emitter and receiver, inserted in vacuum, one could estimate it, following our first slide.





When a light beam entered into the transparent slab, it pays a portion of its linear moment to the field inside the slab and part of moment transferred to the slab, causing his movement.

The light pressuse (P_1) could presents on forward and backward interfaces only. The light pressure inside the media (P_3) is equal zero, because the light drag does not exist.

The main question is – how momentum P_0 splits on two part, that are the real values P_1 and P_2?

There are two possible approaches for calculating distribution of P_0 between P_1 and P_2.

1. Following the law of conservation of center mass of full system (mechanical approach).
2. Following the validity of the relation P=hk for electromagnetic momentum inside the media P_2 (quantum approach)

When beam of light with momentum P_0 enter into slab on the front interface, it divides on two parts :

a) electromagnetic momentum inside slab P_e

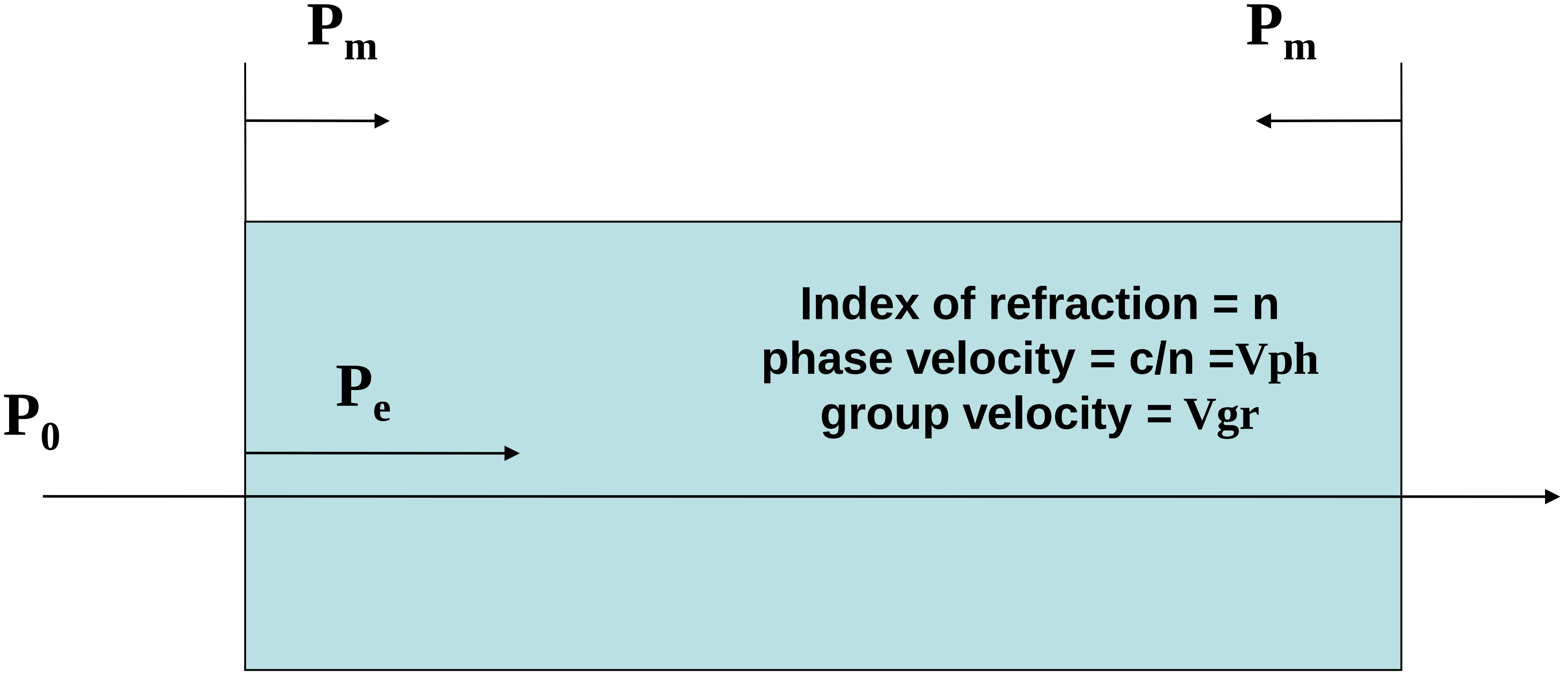
b) mechanical momentum on interface P_m

Momentum on back interface is $-P_m$

$$P_0 = P_e + P_m$$

The mechanical forces on interfaces are proprtional to P_m

Main problem- how to divide P_0 between P_e and P_m ?



Mechanical approach to the problem, following Abragham

Full momentum in vacuum $P_0=E/c$

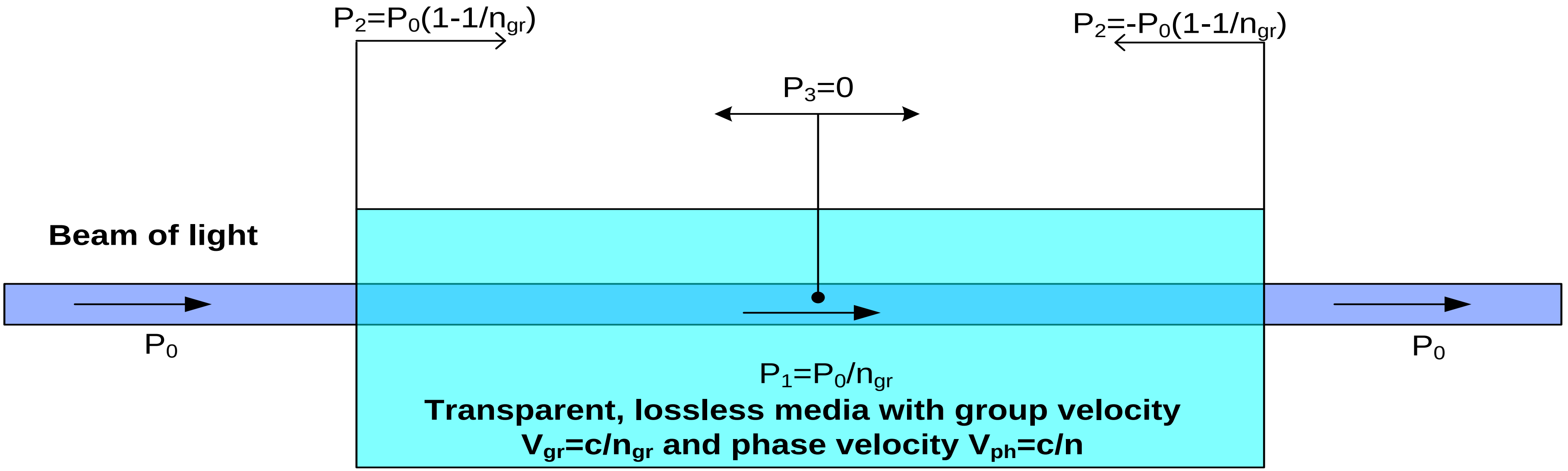
Electromagnetic momentum inside the body $P_1=P_0/n_{gr}$

Mechanical momentum of the body $P_2=P_0(1-1/n_{gr})$

$$P_0=P_1+P_2$$

No optical drag effect, no forces inside the slab

This result does not depends on phase velocity!



Quantum approach , following Minkowsky.

Full momentum in vacuum $P_0=hk=h\omega/c$

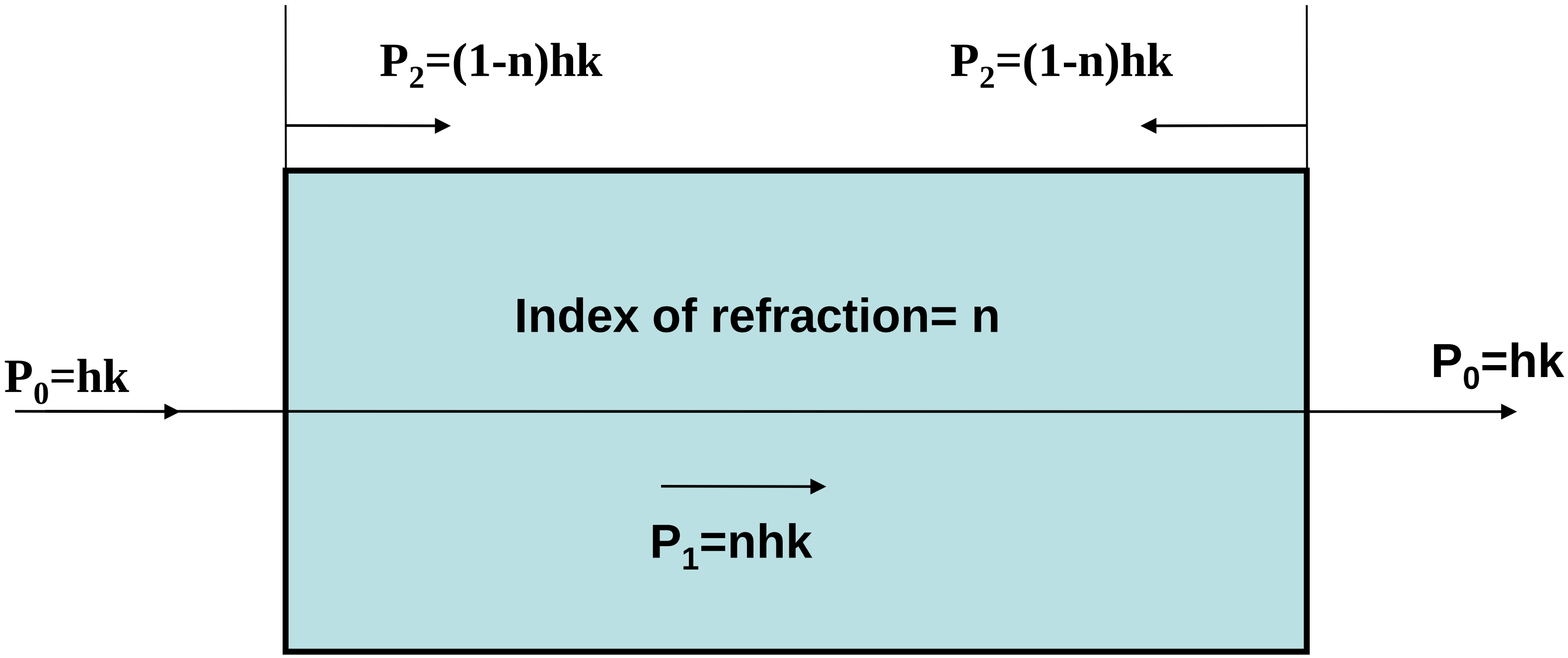
Electromagnetic momentum inside the body $P_1=nhk=nP_0$

Mechanical momentum of the body $P_2=(1-n)hk=(1-n)P_0$

$$P_0=P_1+P_2$$

No optical drag effect, NO FORCES INSIDE THE SLAB !

If n>1 we have optical tension, if n<1 we have optical compression



Main contradiction – main result

If the wish to save center of mass conservation and equation $E=mc^2$, we need to write P=hw/nc, not P=hnw/c, and it is a «mechanical approach»

If momentum of light is proportional to nhw/c, the center of mass conservation and Einstein relation $E=mc^2$ are not consistent. This statement based on «quantum approach».

Now the have very important contradiction.

If we prefer to have momentum of light inside the media as P=En/c, mass, transferred from emitter to receiver is not E/c^2, but E/VphVgr

This value depends on direction of vector K, and is negative in LHM!

Now the have very important contradiction.

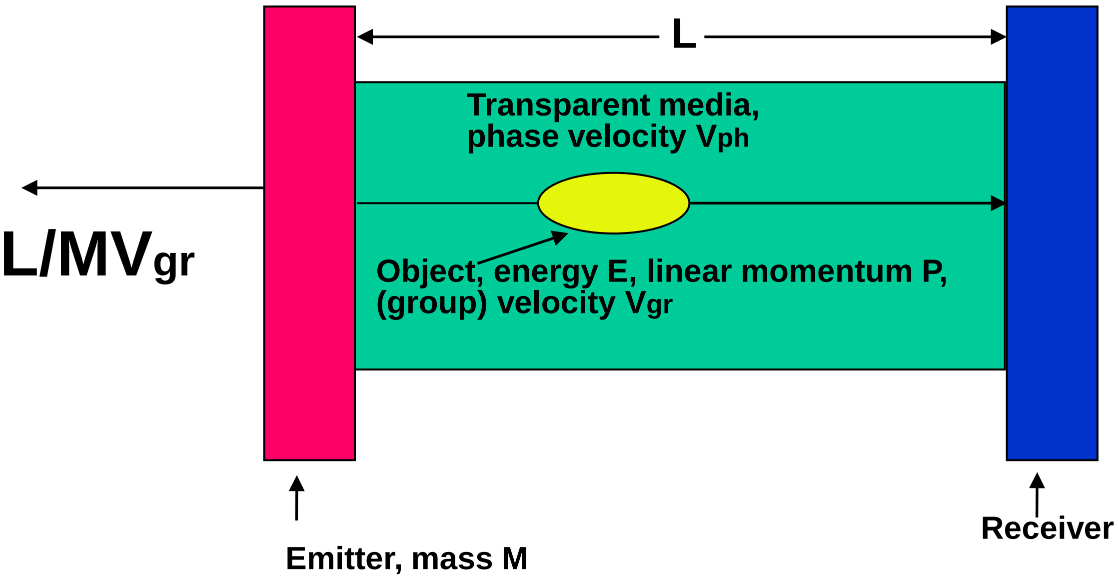
If we prefer to have mass, transferred from emitter to receiver as E/c^2, we need consider momentum of light P inside media as $P=E/cn_{gr}$

This value does not depend on direction of vector K, and is equal for LHM and RHM

Displacement X of emitter for the propagation time

t=L/Vgr of the object to the receiver.

$X = PL/MV_{gr}$



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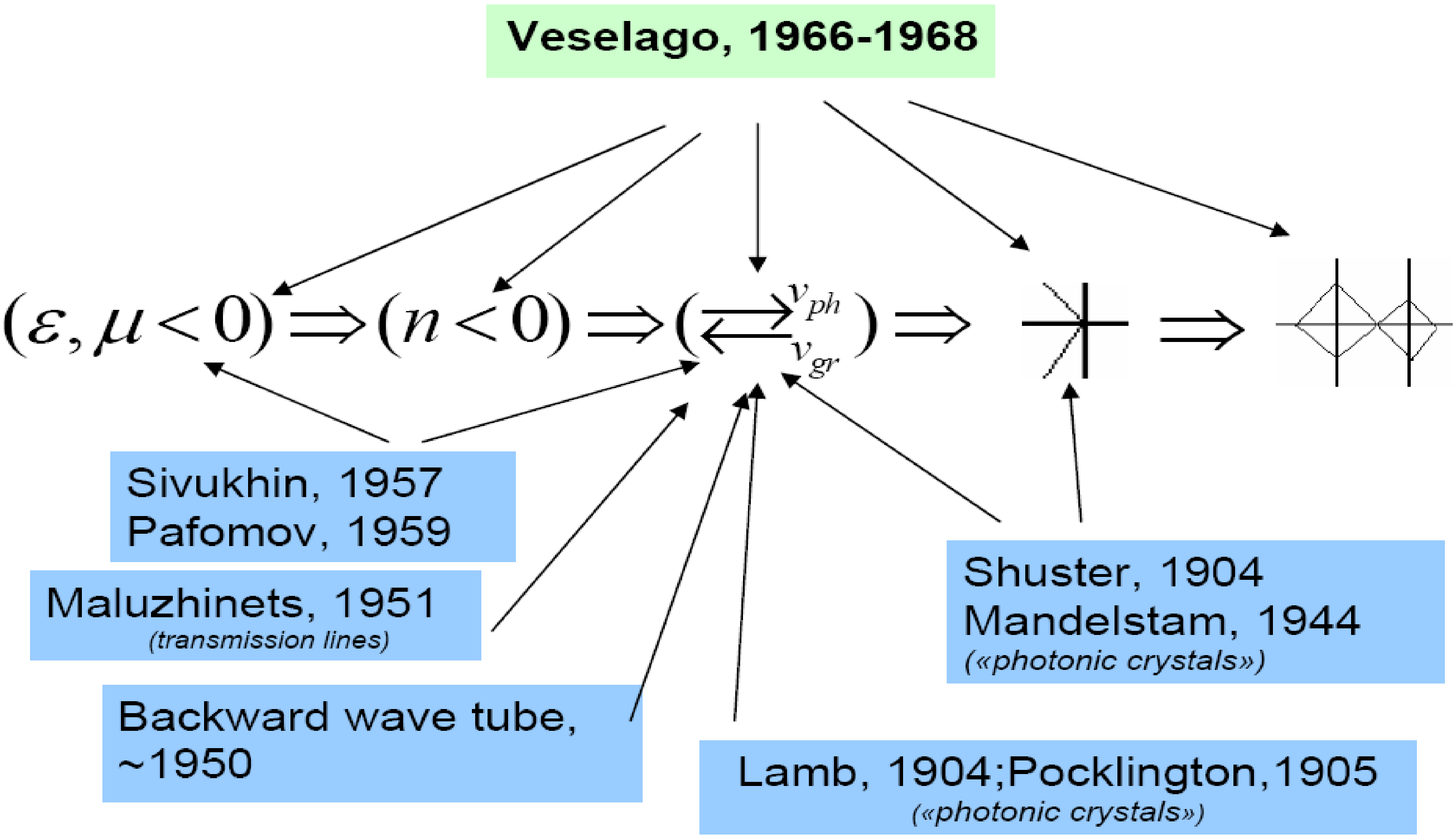

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