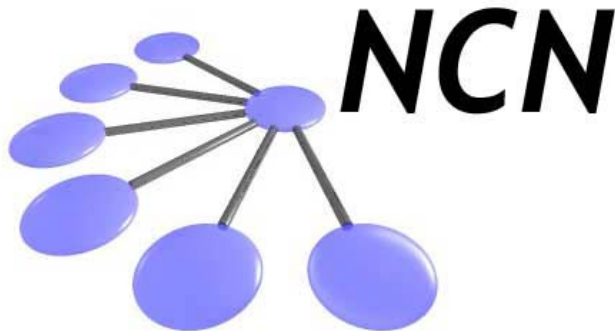


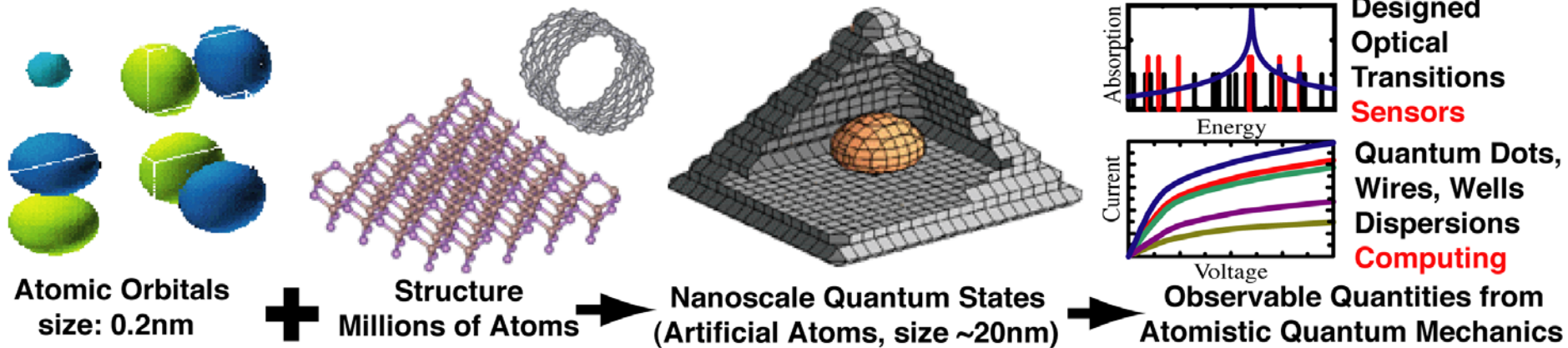
# Network for Computational Nanotechnology (NCN)

Berkeley, Univ. of Illinois, Norfolk State, Northwestern, Purdue, UTEP

## Introduction to the NEMO3D Tool

Gerhard Klimeck





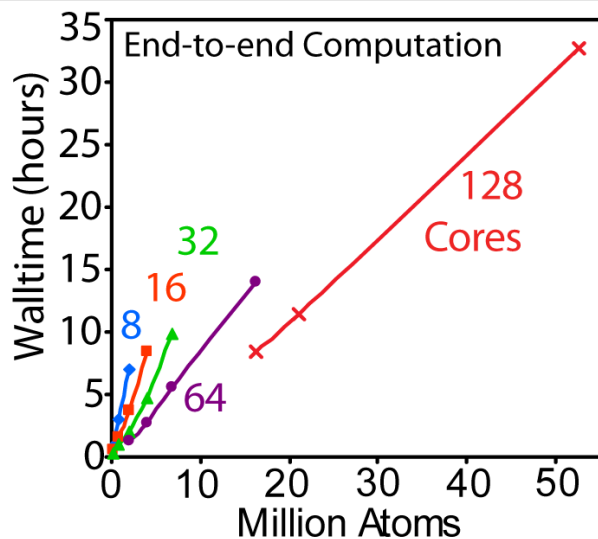
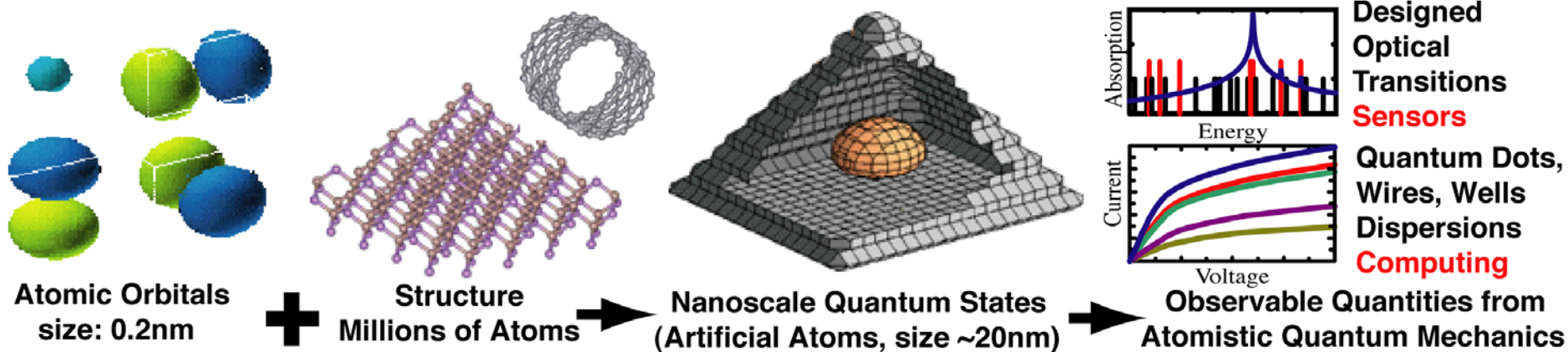
## Problem:

Nanoscale device simulation requirements:

- Cannot use bulk / jellium descriptions, need description atom by atom  
=> use pseudo-potential or local orbitals
- Consider finite extend, not periodic  
=> local orbital approach
- Bonds in semiconductors are stable!  
=> Do not need ab-initio methods!
- Need to include about one million atoms.  
=> need massively parallel computers
- The design space is huge:  
=> need a design tool

## Approach:

- Use local orbital description for individual atoms in arbitrary crystal / bonding configuration
  - Use s, p, and d orbitals.
  - Use genetic algorithm to determine material parameter fitting
- Compute mechanical strain in the system.
- Develop efficient parallel algorithms to generate eigenvalues/vectors of very large matrices



## Demonstration / Capability / Impact:

- 52 million atom electronic structure (101nm)<sup>3</sup>.
- Quantum dots, nanowires, quantum computing...

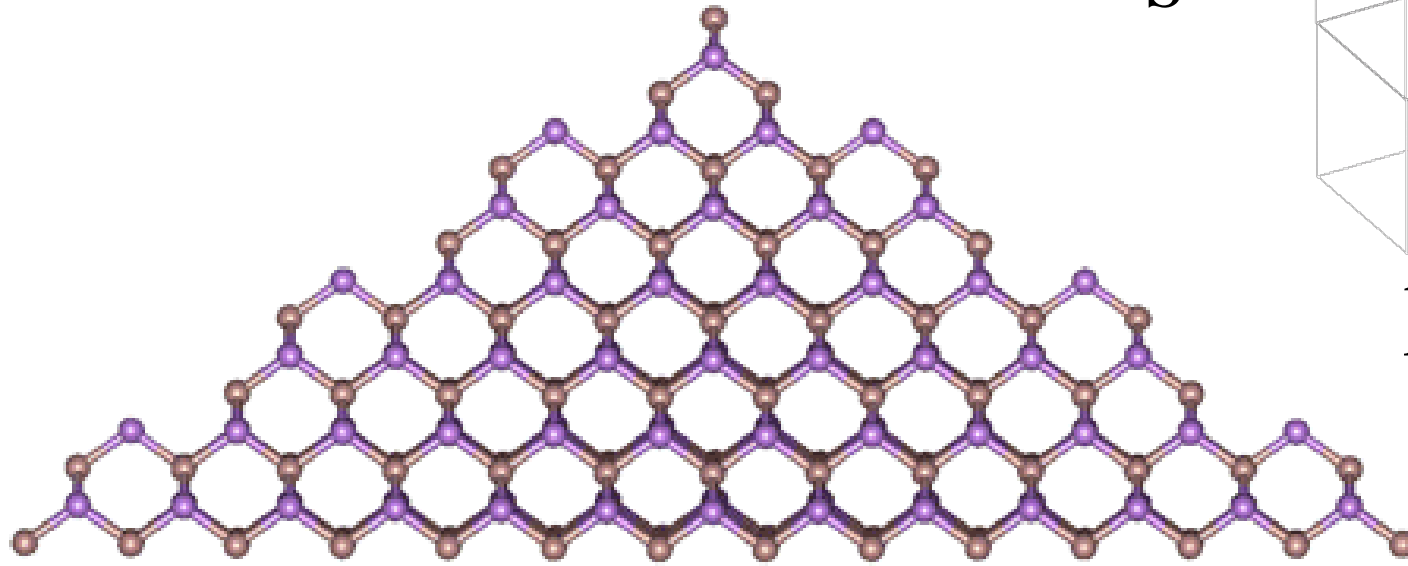
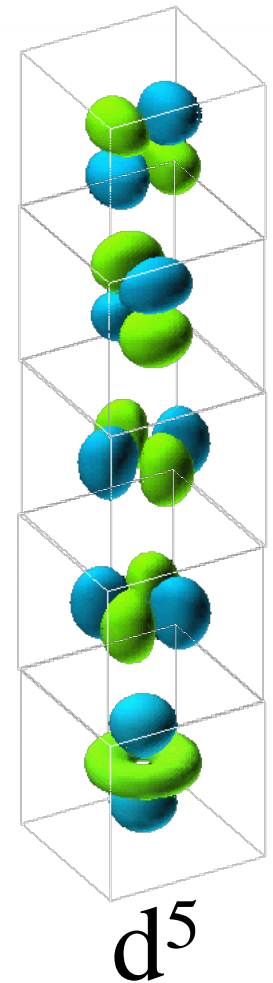
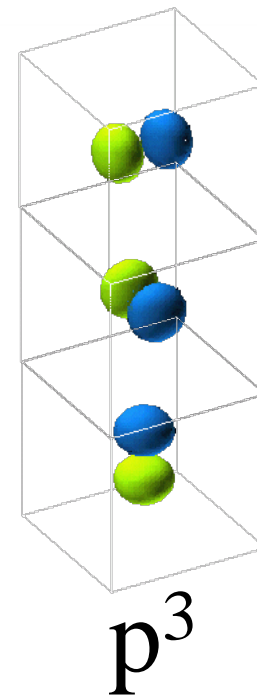
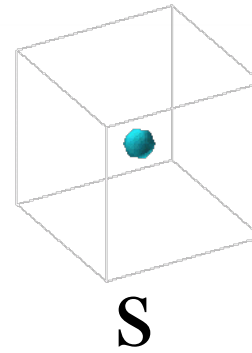
## Approach:

- Use local orbital description for individual atoms in arbitrary crystal / bonding configuration
  - Use s, p, and d orbitals.
  - Use genetic algorithm to determine material parameter fitting
- Compute mechanical strain in the system.
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$$\psi_{i,j,k} = \sum_1^{\text{orbitals}} C_{i,j,k}^l \phi_l$$

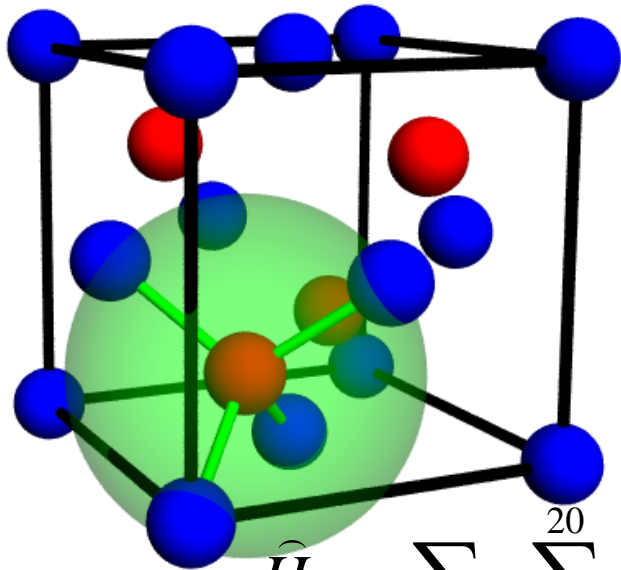
$$\phi_l = s, p^3, d^5$$

## Wavefunction Basis



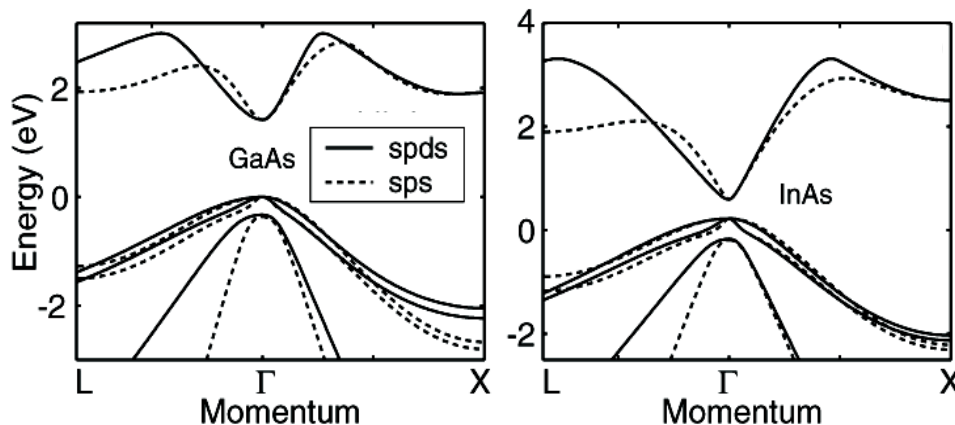
Crystalline Structure





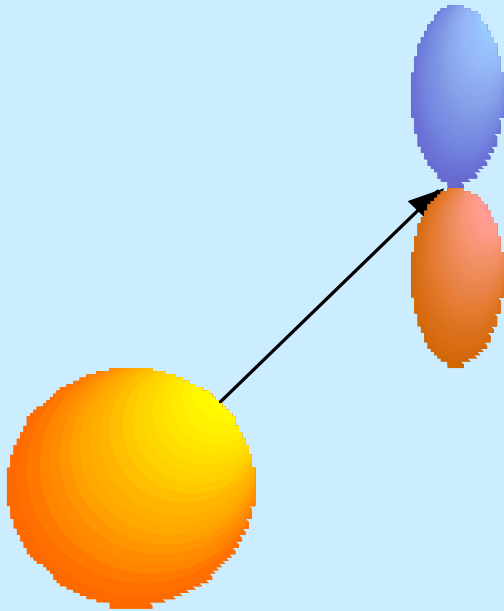
- Realistic zinc-blende lattice
- Each atom has 10 spin-degenerate orbitals ( $sp^3d^5s^*$ )
- Each atom has 4 nearest neighbors
- TB parameters found by fitting of bulk band structure to experiment

$$\hat{H} = \sum_{atoms\ R} \sum_{\alpha=1}^{20} \mathcal{E}_{R\alpha} C_{R\alpha}^+ C_{R\alpha} + \sum_{atoms\ R} \sum_{\substack{atoms\ R'=1 \\ bands}}^{4nn} \sum_{\alpha=1}^{20} \sum_{\alpha'=1}^{20} t_{R\alpha, R'\alpha'} C_{R\alpha}^+ C_{R'\alpha'}$$



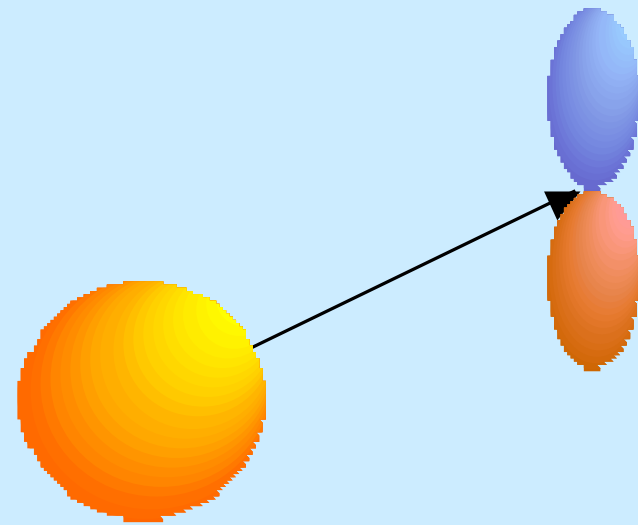
**Systems with up to  
52 million atoms**

**Matrix order:  $1 \times 10^9$   
Lanczos Diagonalization**



$$\mathbf{d} = d(l\mathbf{e}_x + m\mathbf{e}_y + n\mathbf{e}_z)$$

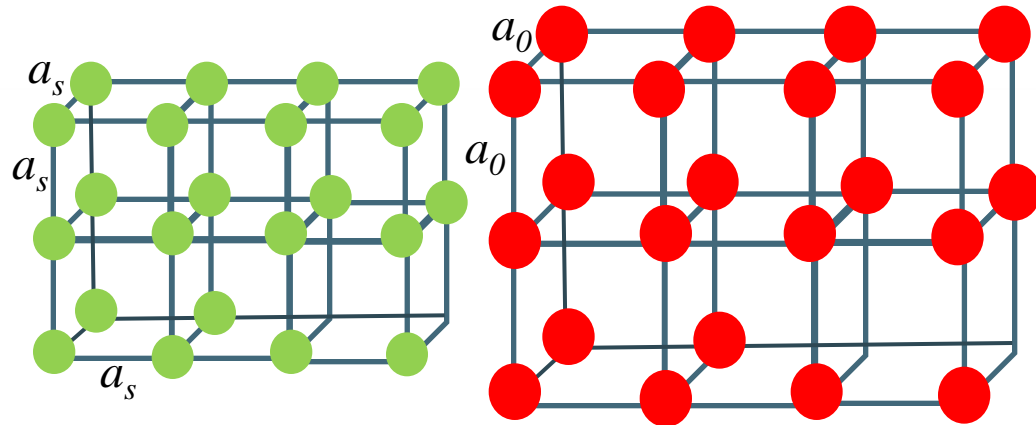
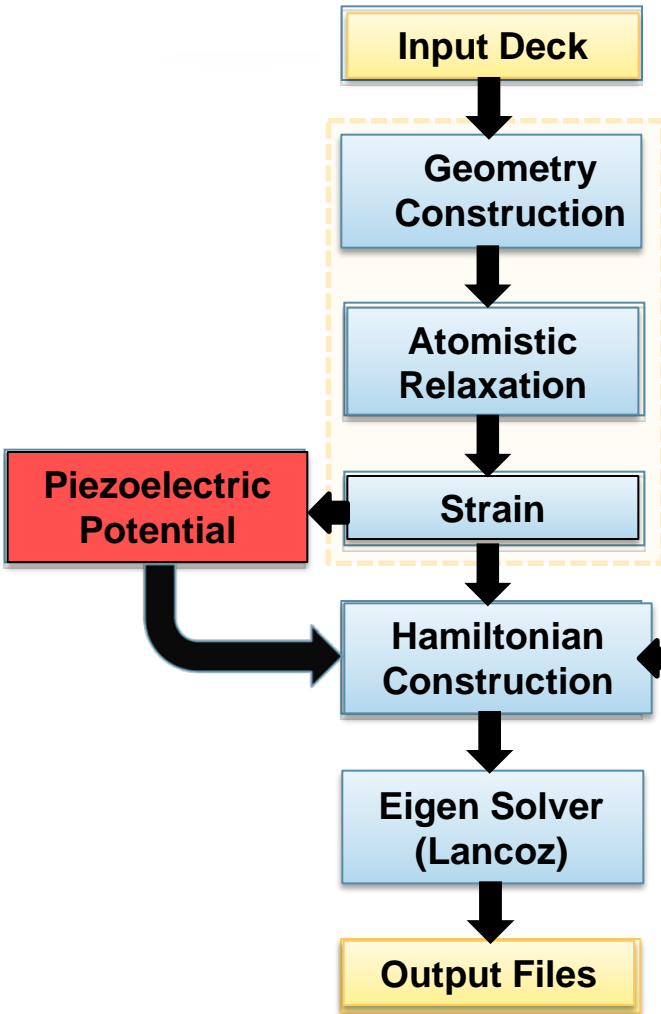
$$E_{sa, zc} = n V_{sa, pc\sigma}$$



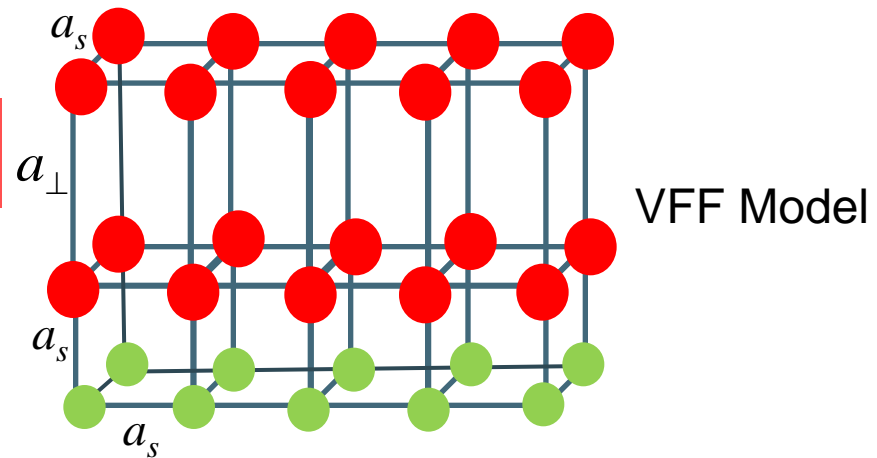
$$\mathbf{d}' = d'(l'\mathbf{e}_x + m'\mathbf{e}_y + n'\mathbf{e}_z)$$

$$E_{sa, zc} = n' V_{sa, pc\sigma} \left( \frac{d'}{d} \right)^\eta$$

# NEMO 3-D Calculation Flow - VFF Strain Model



An epitaxial layer is grown, on a substrate with different lattice constant, the epilayer deforms (strain) :



$$a_{\perp} = a_0 - 2 \frac{C_{12}}{C_{11}} [a_s - a_0] \iff \mathbf{R}' = (\mathbf{1} + \boldsymbol{\varepsilon}) \mathbf{R}$$

↑  
Strain tensor

## Strain Impact on Electronic Structure

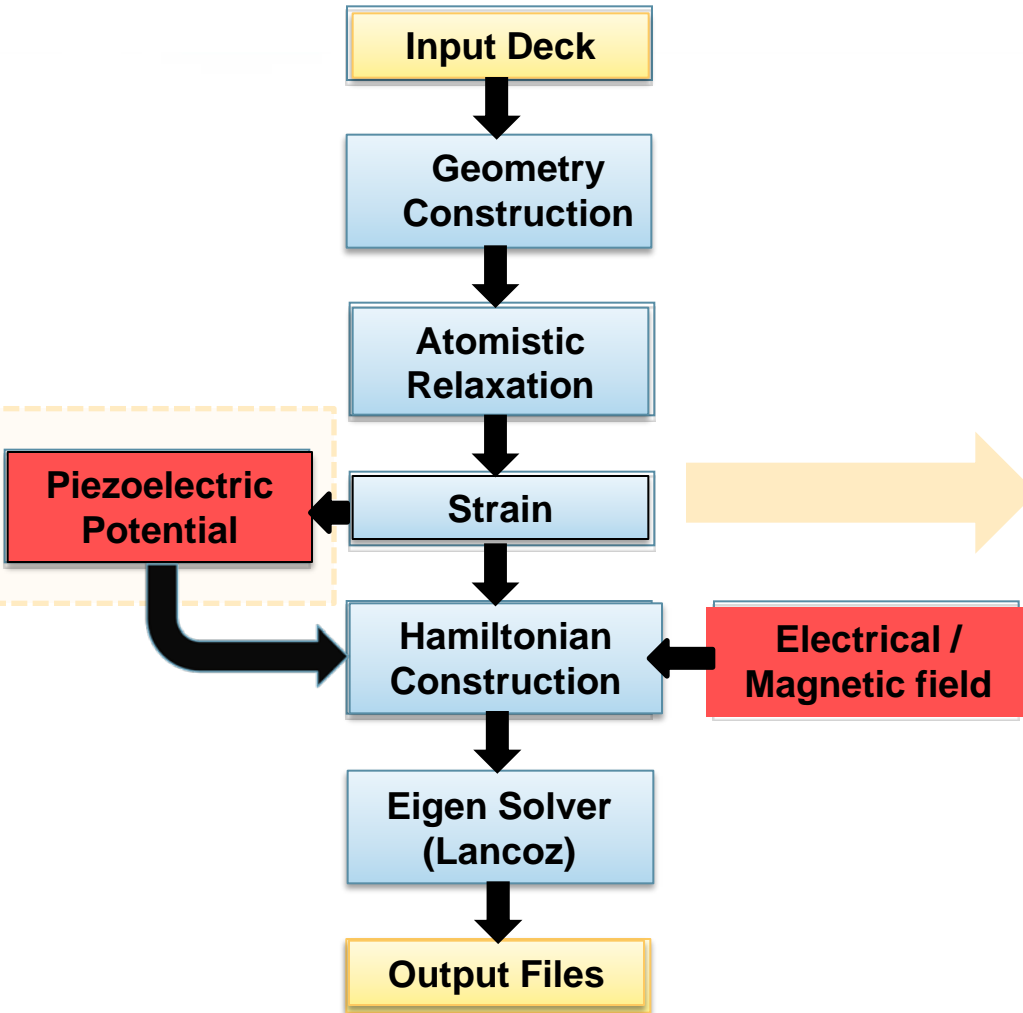
- Strain → Changes bond lengths/angles → deforms crystals
- Band edges are shifted and degeneracy of HH-LH is lifted
- Strain impact on electronic structure:

Conduction Band →  $E_c = E_{c0} + a_c(\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz})$

Valence Band →  $E_{HH/LH} = E_{HH0/LH0} + a_v(\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}) \pm 0.5b_v(\epsilon_{xx} + \epsilon_{yy} - 2\epsilon_{zz})$

where  $a_c = -5.08\text{eV}$ ,  $a_v = 1.0\text{eV}$  and  $b_v = -1.8\text{eV}$  for InAs

# NEMO 3-D Calculation Flow -Piezo-Electric Model



Strain tensor  $\rightarrow$  Polarization

$$P_1 = 2e_{14} \begin{pmatrix} \epsilon_{yz} \\ \epsilon_{xz} \\ \epsilon_{xy} \end{pmatrix}$$

$$P_2 = 2B_{114} \begin{pmatrix} \epsilon_{xx} \epsilon_{yz} \\ \epsilon_{yy} \epsilon_{xz} \\ \epsilon_{zz} \epsilon_{xy} \end{pmatrix} + 2B_{124} \begin{pmatrix} \epsilon_{yz} (\epsilon_{yy} + \epsilon_{zz}) \\ \epsilon_{xz} (\epsilon_{zz} + \epsilon_{xx}) \\ \epsilon_{xy} (\epsilon_{xx} + \epsilon_{yy}) \end{pmatrix} + 4B_{156} \begin{pmatrix} \epsilon_{xz} \epsilon_{xy} \\ \epsilon_{yz} \epsilon_{xy} \\ \epsilon_{yz} \epsilon_{xz} \end{pmatrix}$$

Polarization  $\rightarrow$  Charge density

$$\rho_{piezo}(r) = -\nabla \cdot P$$

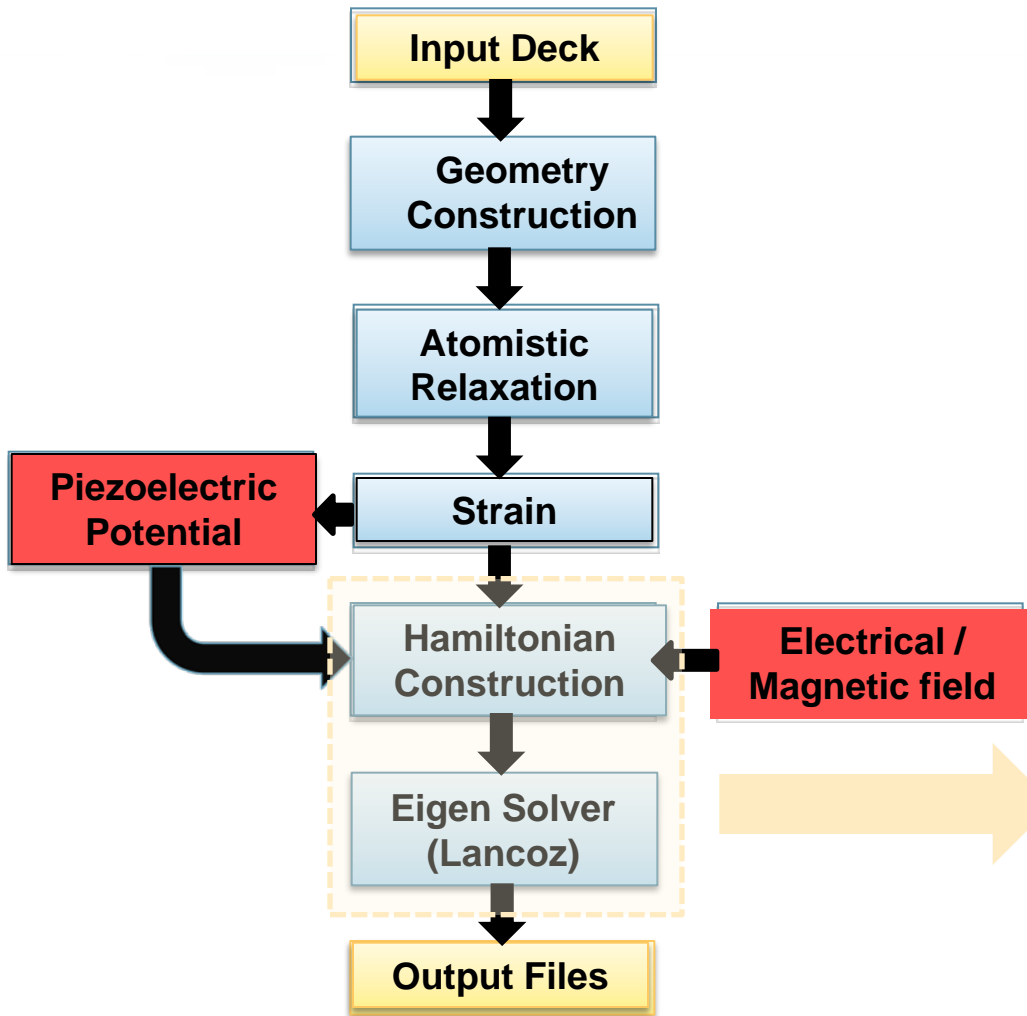
$$P = P_1 + P_2$$

Charge density  $\rightarrow$  Potential

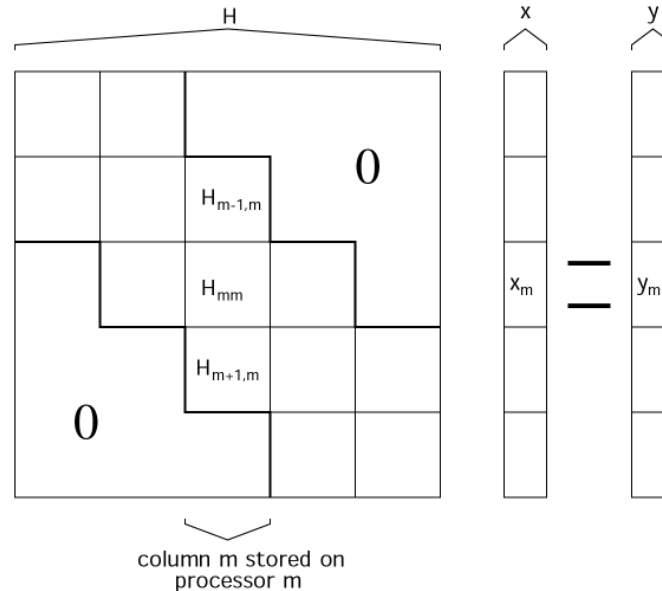
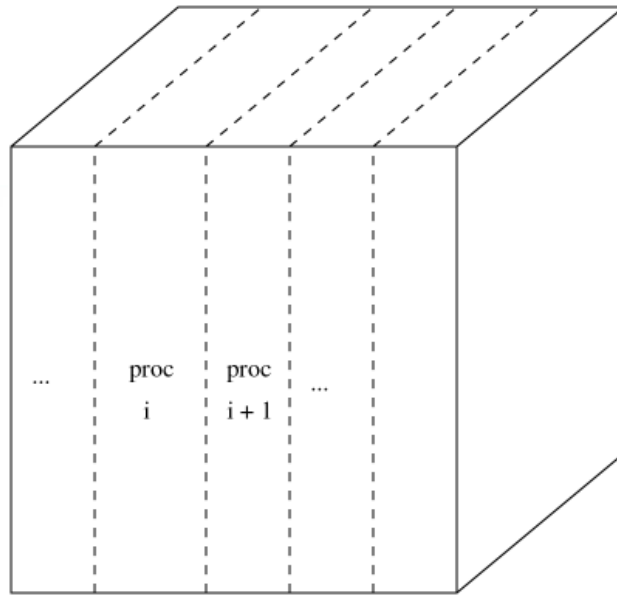
$$\rho_p(r) = \epsilon_0 \nabla \cdot [\epsilon_s(r) \nabla V_p(r)]$$



# NEMO 3-D Calculation Flow - Electronic Structure Model



- Real-size QD typically involves multi-million atoms.
- $sp^3d^5s^*$  TB requires 20 bases per each atom with consideration of spin-orbit couplings  $\rightarrow$   $(N_{\text{atom}} \times 20)$  by  $(N_{\text{atom}} \times 20)$  Hamiltonian.
- Need to obtain interior spectrum eigenstates
- NEMO 3-D: Designed from the bottom up to be parallel: MPI.
- NEMO 3-D code has been tested to run on (almost) any HPC cluster



- Divide Simulation domain into slices.
- Communication only from one slice to the next (nearest neighbor)
- Communication overhead across the surfaces of the slices.
- Limiting operation:  
complex sparse matrix-vector multiplication
- Enable Hamiltonian storage or re-computation on the fly.
- Electronic structure needs eigenvalues and eigenvectors. Matrix is Hermitian
- NEMO 3-D methods:
  - Standard 2-pass Lanczos
  - PARPACK about 10x slower
  - Folded Spectrum Method (Zunger), also typically slower than Lanczos

# From Beowulf Concept (1998 JPL, Tom Sterling) to Commodity Products in 4 Generations

Hyglac (1997)  
16 Pentium Pros 200MHz  
128 MB RAM per node  
*2 GB total*  
5GB Disc per node  
*80 GB total*  
100 Mb/s ethernet crossbar  
Linux, MPI  
3.2GFlops

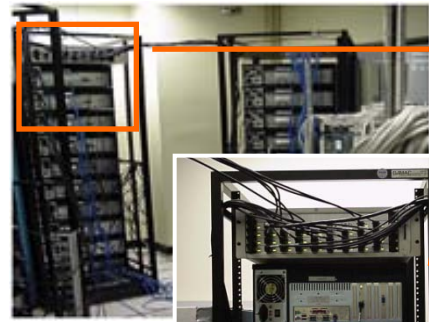
Gordon Bell Prize 1997



Nimrod (1999)  
32 Pentium IIIs 450MHz  
512 MB RAM per node  
*16 GB total*  
8GB Disc per node  
*256 GB total*  
100 Mb/s ethernet crossbar  
Linux, MPI  
14.4 GFlops



Pluto (2001)  
64 Pentium IIIs 800MHz  
*dual CPUs*  
2 GB RAM per node  
*64 GB total*  
10 GB Disc per node  
*320 GB total*  
2 Gb/s Myricom crossbar  
Linux, MPI  
51.2 GFlops



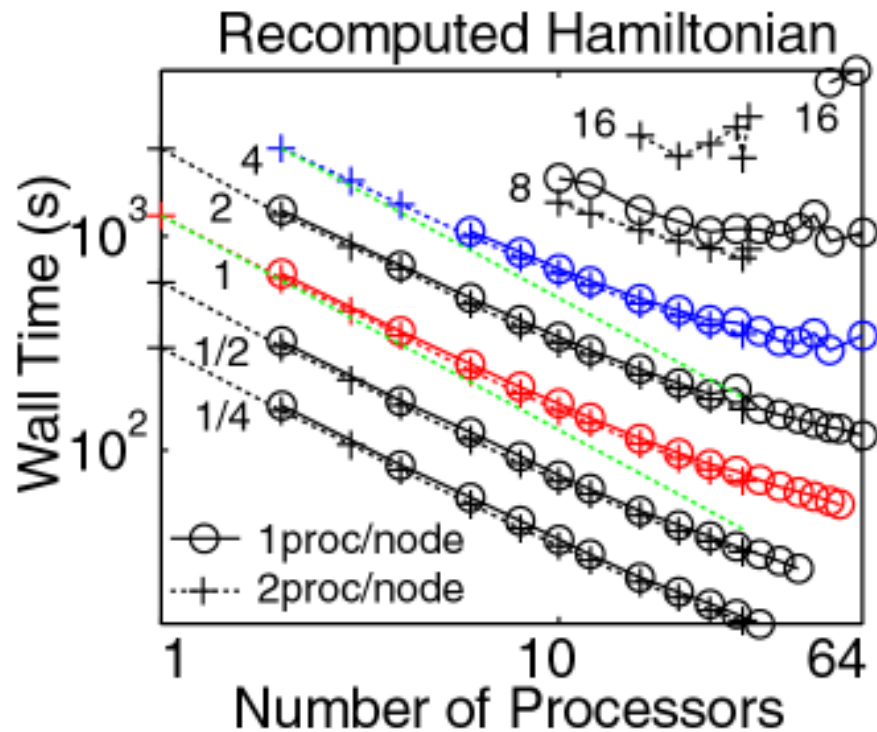
NewYork (2002)  
66 Xserve G4 1GHz  
1GB RAM per node  
*33 GB total*  
60 GB Disc per node  
*2 TB total*  
100 Mb/s ethernet crossbar  
MAC OS X, MPI  
495GFlops



Affordable Supercomputer  
for ~\$100k

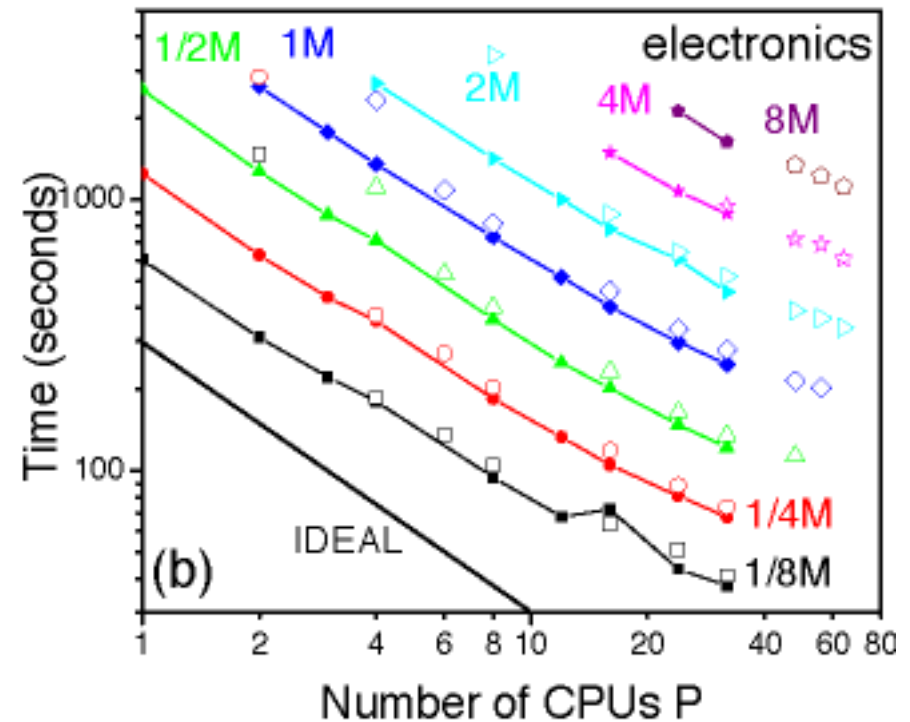
2000

32 nodes Dual Intel P3, 800MHz,  
1GB RAM, 100Mbs ethernet



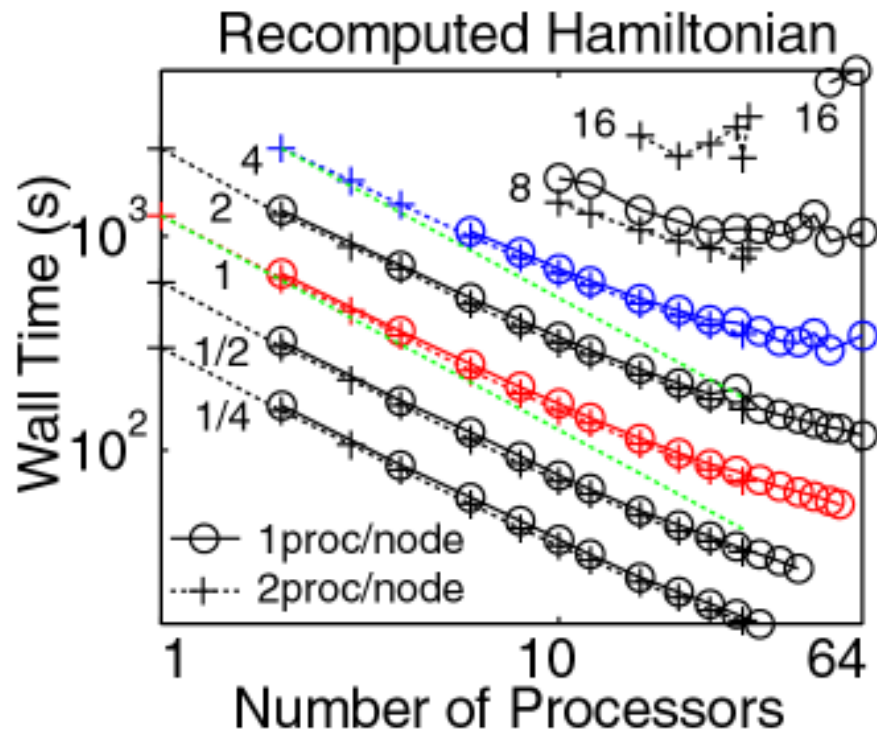
2003

32 node, G5 Apple cluster  
2GB RAM, Dual Gigabit Ethernet.



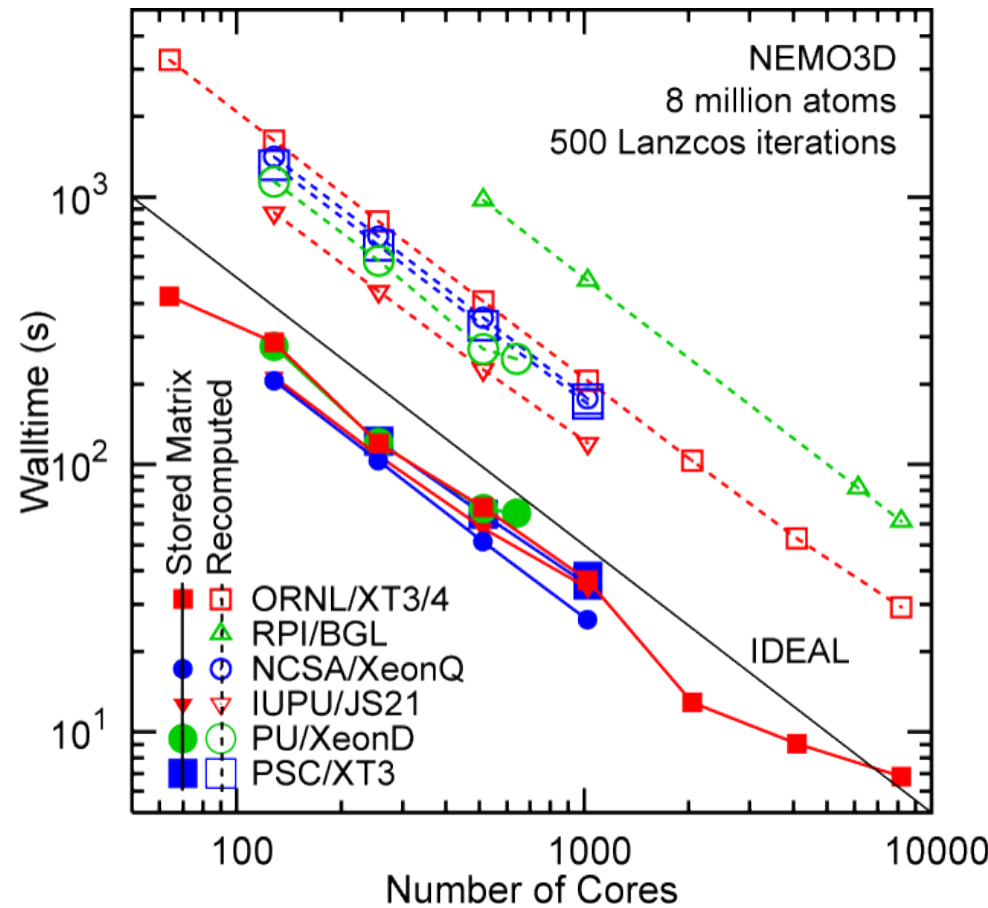
2000

32 nodes Dual Intel P3, 800MHz,  
1GB RAM, 100Mbs ethernet



2007

IBM BGL, Cray XT3,  
Intel Clusters, IBM JS21



**Demonstrated scaling  
to 8,192 cores**

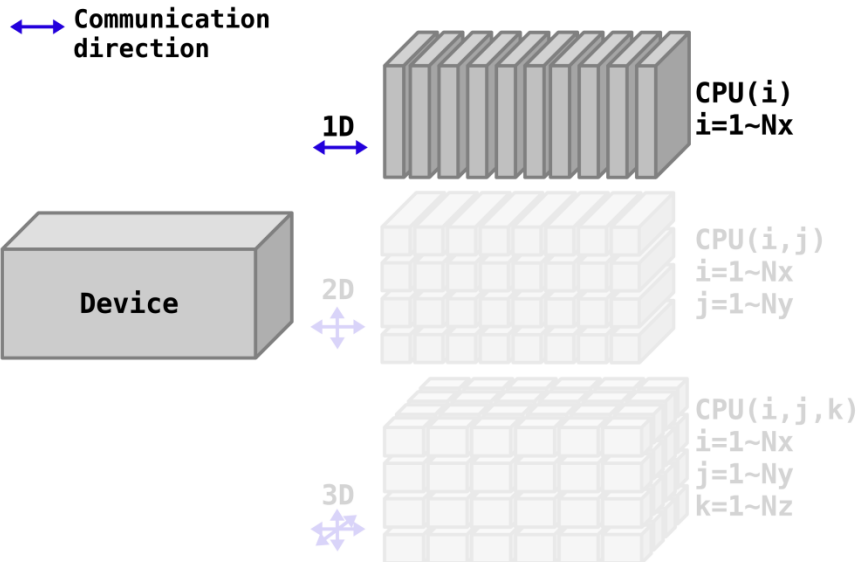


## Scaling to 8,192 processors in NEMO3D

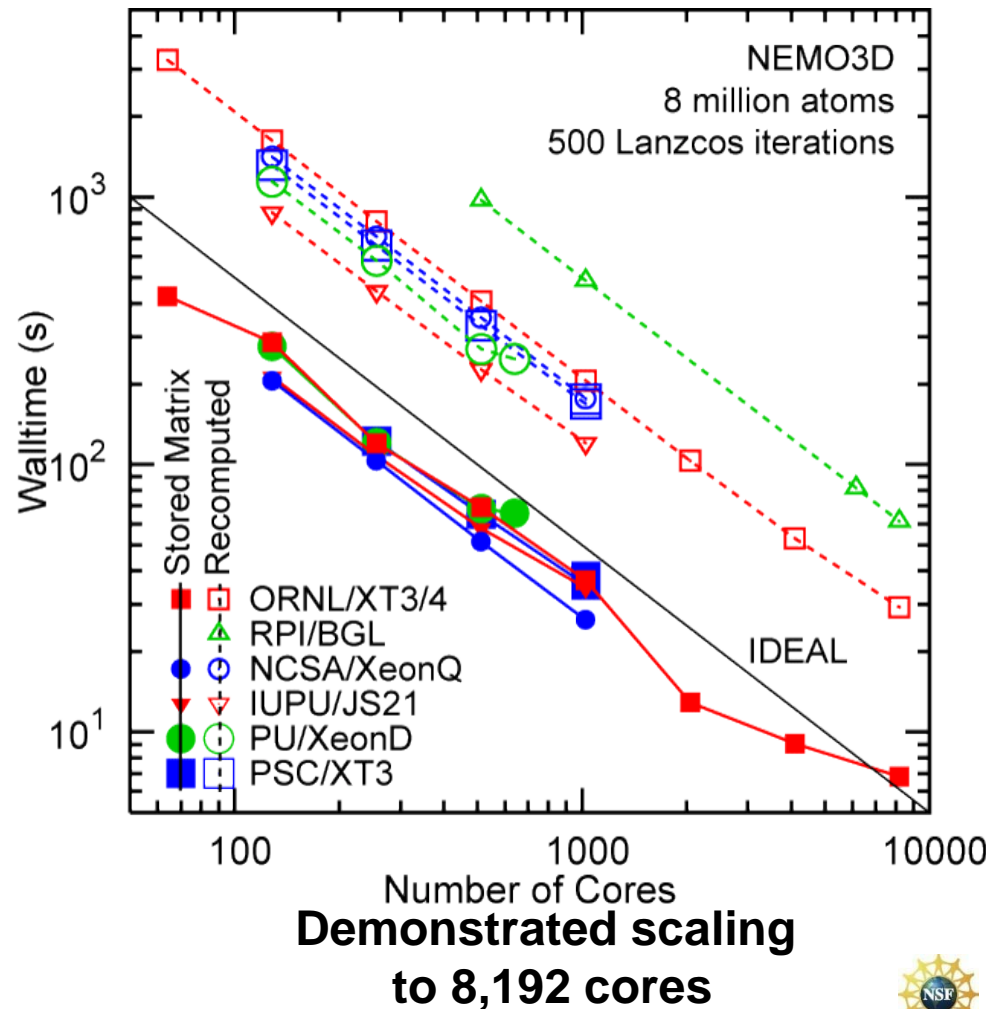
Is a bit of a cheat!  
NEMO3D does a 1D decomposition!!

The domain is  
a very thin and very long nanowire  
NOT a Quantum Dot!

Need a 2D and 3D Decompositions



2007  
IBM BGL, Cray XT3,  
Intel Clusters, IBM JS21



## Objective

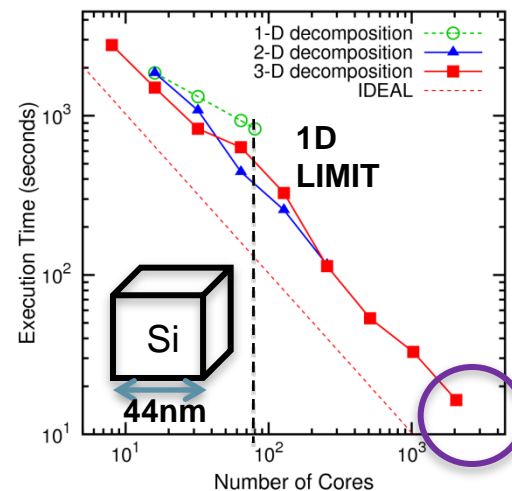
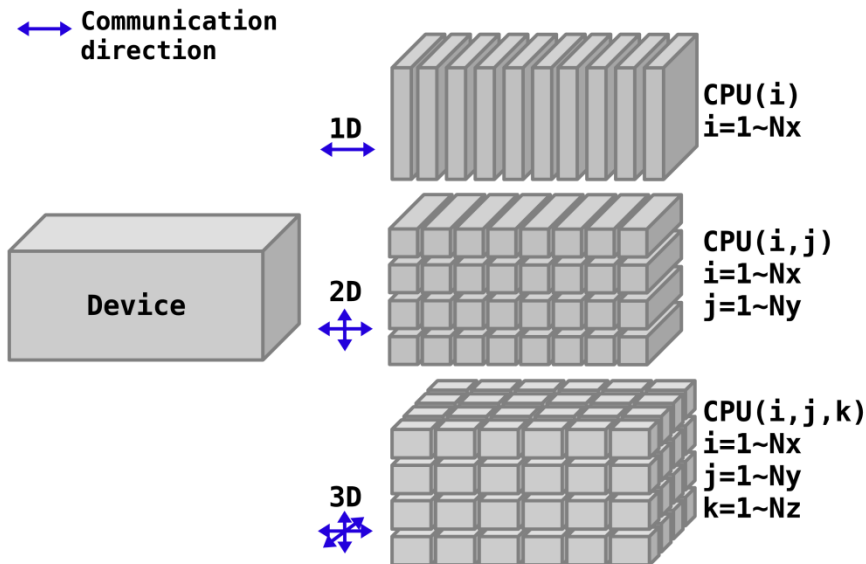
- Run large atomistic electronic structure simulations in minutes
- Represent realistically large structures atomistically
- Utilize available peta-scale computers

## Approach

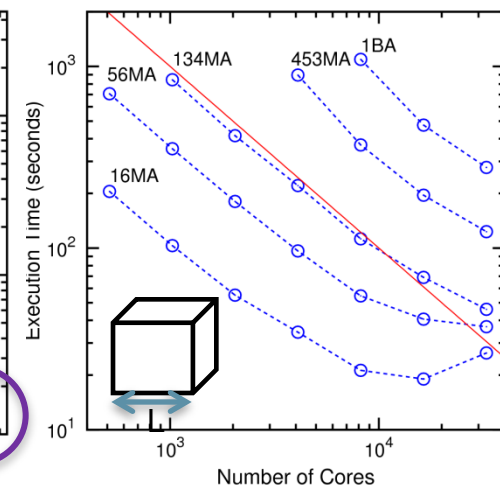
- Improve decomposition parallelism
- Include self-consistent calculations

## Performance / Impact

- Strong scaling of 3D decomposition => have overcome 1D limitation
- Utilized over 10,000 cores for realistic simulation domain size
- Expecting dramatic reduction of simulation time => 10hours to 20 minutes
- Published in SCIDAC proceedings, 2009, Haley, Lee et al.



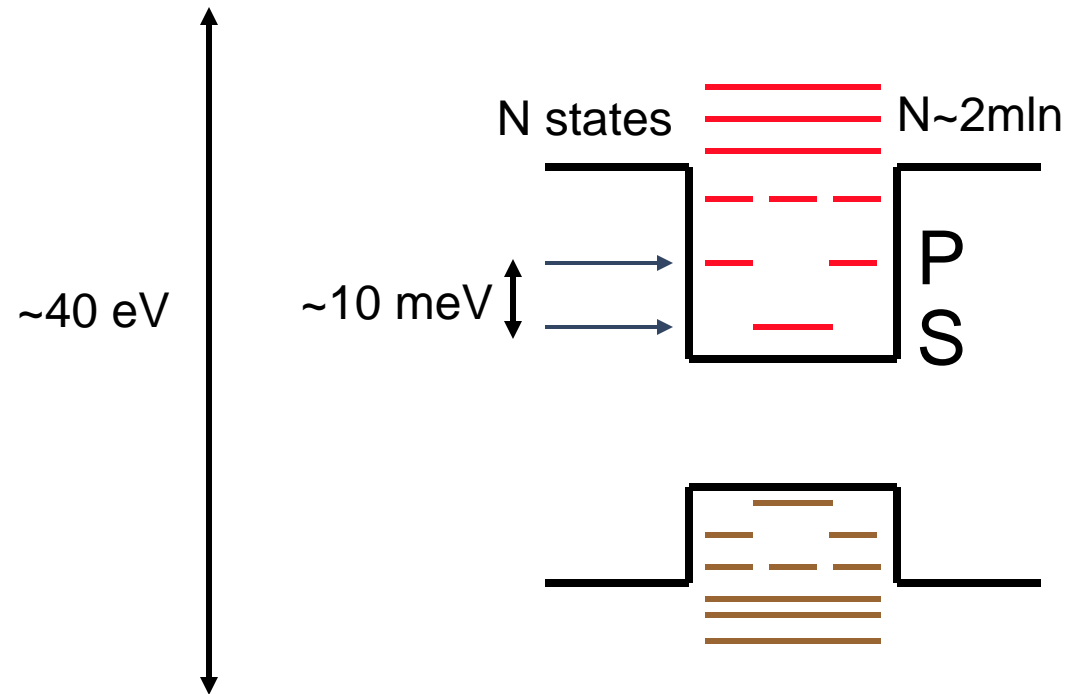
Tested on Ranger

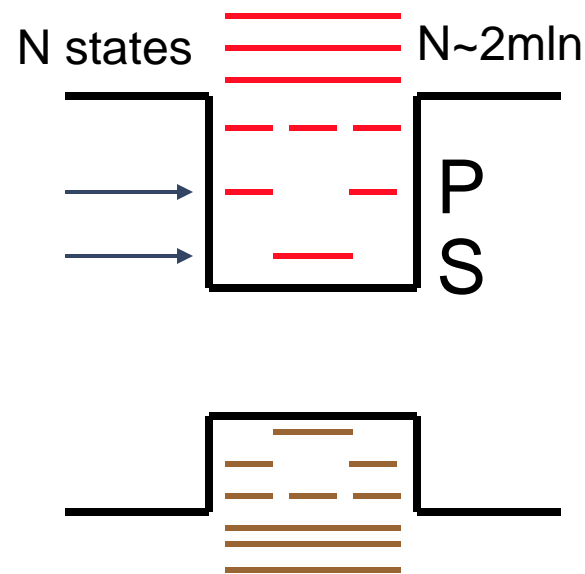
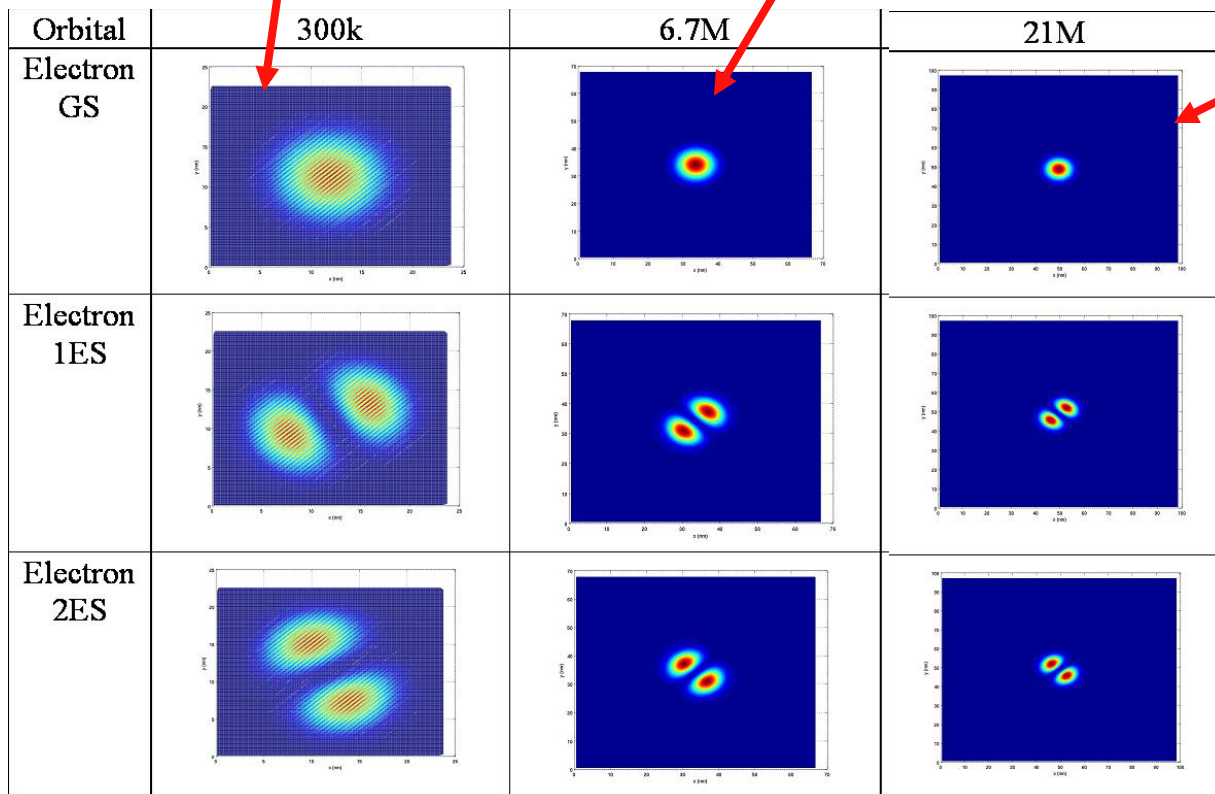
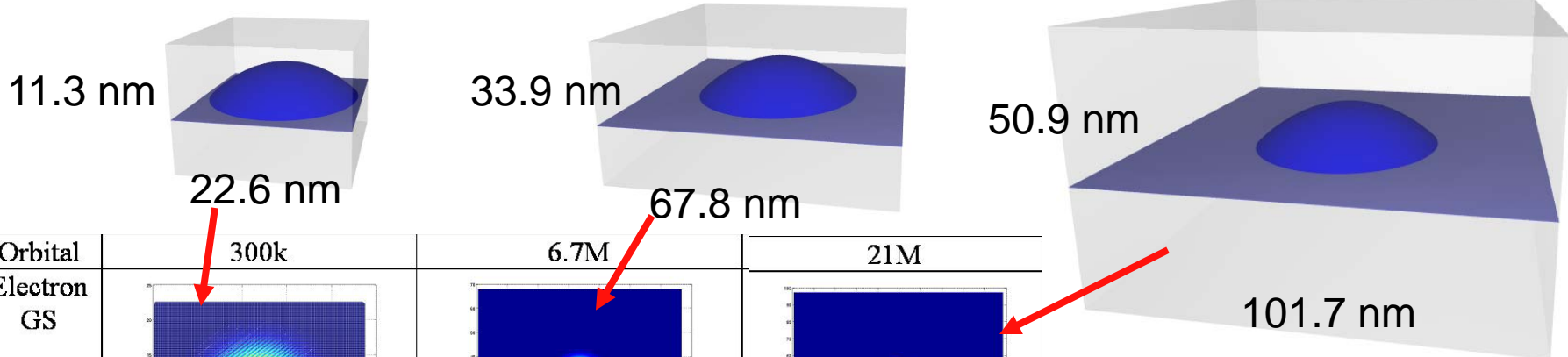


Tested on Kraken

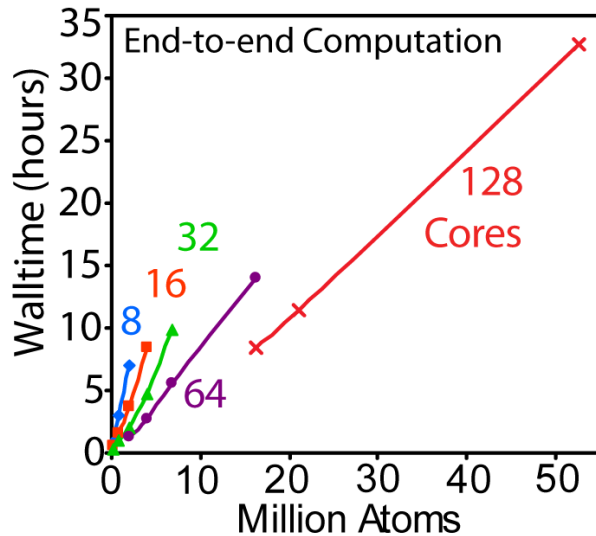
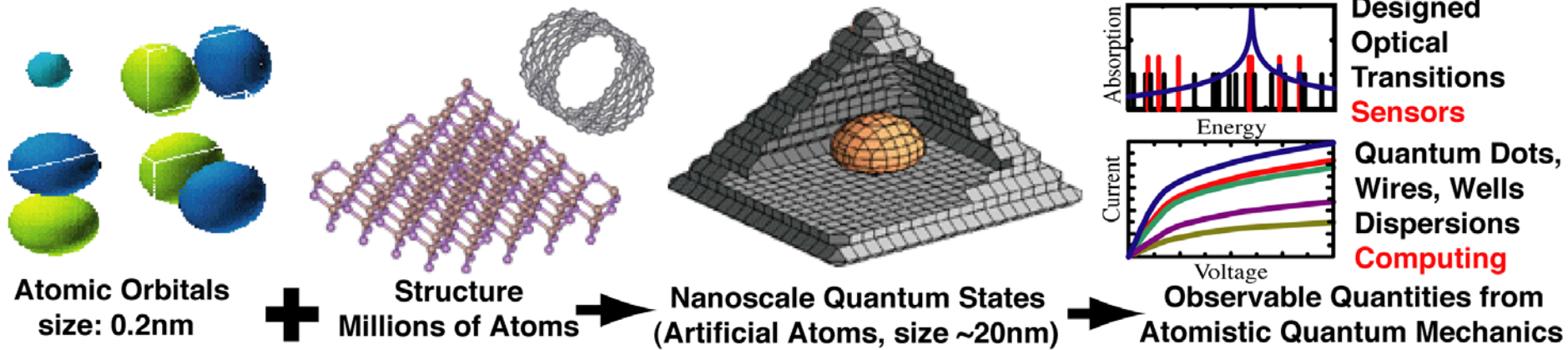
# Proof of Concept

## Extraction of Targeted Interior Eigenstates





Unique and targeted eigenstates of correct symmetry  
can be computed in all electronic computational domains



### Demonstration / Capability / Impact:

- 52 million atom electronic structure (101nm)<sup>3</sup>.
- Quantum dots, nanowires, quantum computing...

### Capabilities:

- Arbitrary 3D zincblende, wurzite crystals
- Strain in Valence Force Fields
- Piezo Electricity through potential perturbation
- Electronic Structure with empirical tight binding

### Applications:

- InAlGaAs/GaAs quantum dots
- Valley splitting in Silicon
- Alloy disorder in bulk, QD, nanowires
- Single impurities in Si<sub>19</sub>



# Multi-Million Atom Simulation of Quantum States and Transport in Realistically Extended Devices in Silicon for Quantum Computing

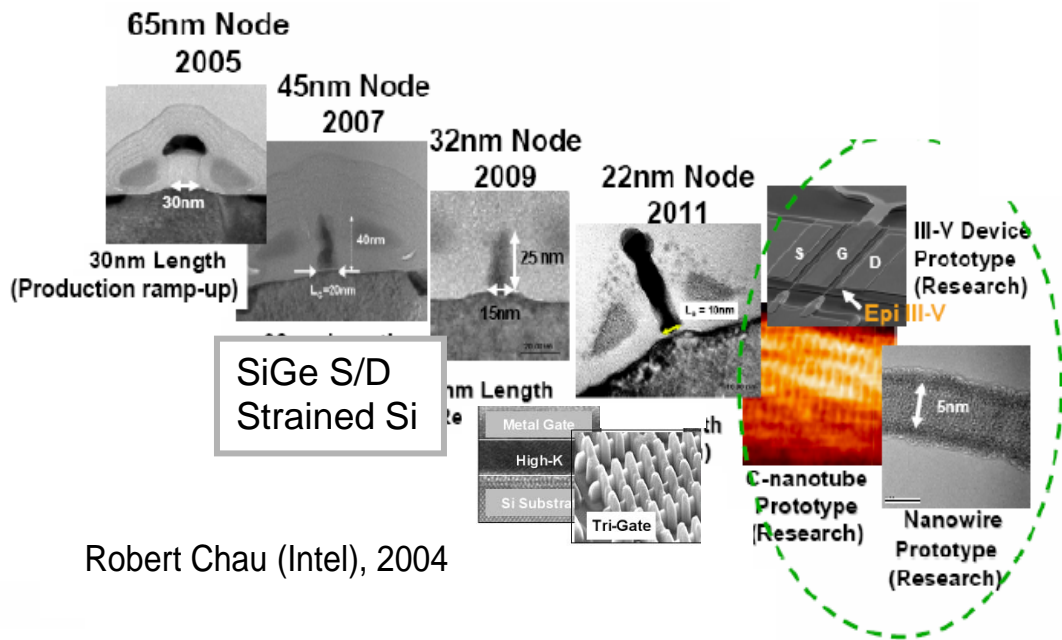
Gerhard Klimeck, Rajib Rahman, S-H Park, Hoon Ryu,  
Sunhee Lee, Muhammad Usman, Neerav Kharche, **Purdue**

Lloyd Hollenberg, **Melbourne**

Sven Rogge, G. Lansbergen, **Delft**

Michelle Simmons, Bent Weber, S. Mahapatra, **UNSW**

Rajib Rahman, Richard Muller, **Sandia**



Robert Chau (Intel), 2004

## Observations:

- 3D spatial variations on nm scale
- Potential variations on nm scale
- New channel materials (Ge, III-V)

## Questions / Challenges

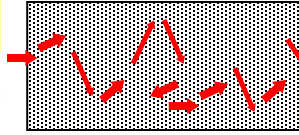
- Strain ?
- Quantization?
- Crystal orientation?
- Atoms are countable; does granularity matter? Disorder?
- New material or new device?

## Assertions of importance

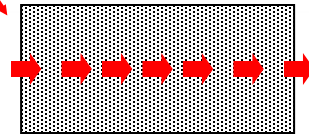
- High bias / non-equilibrium
- Quantum mechanics
- Atomistic representation
  - » Band coupling, non-parabolicity, valley splitting
  - » Local (dis)order, strain and orientation

Macroscopic dimensions

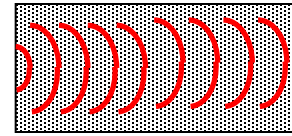
Diffusive



Ballistic



Quantum

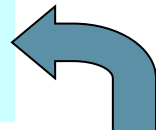


Non-Equilibrium Quantum Statistical Mechanics

Drift / Diffusion



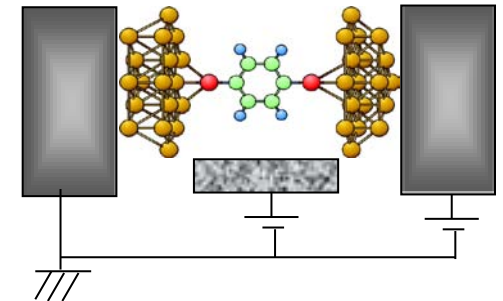
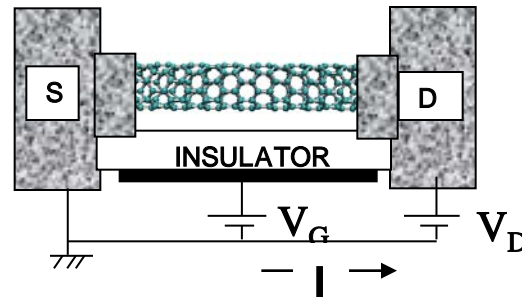
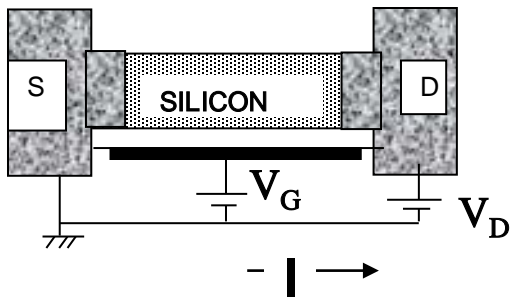
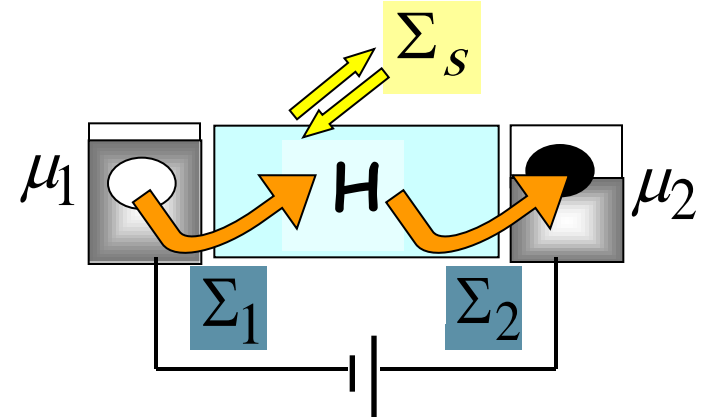
Boltzmann Transport

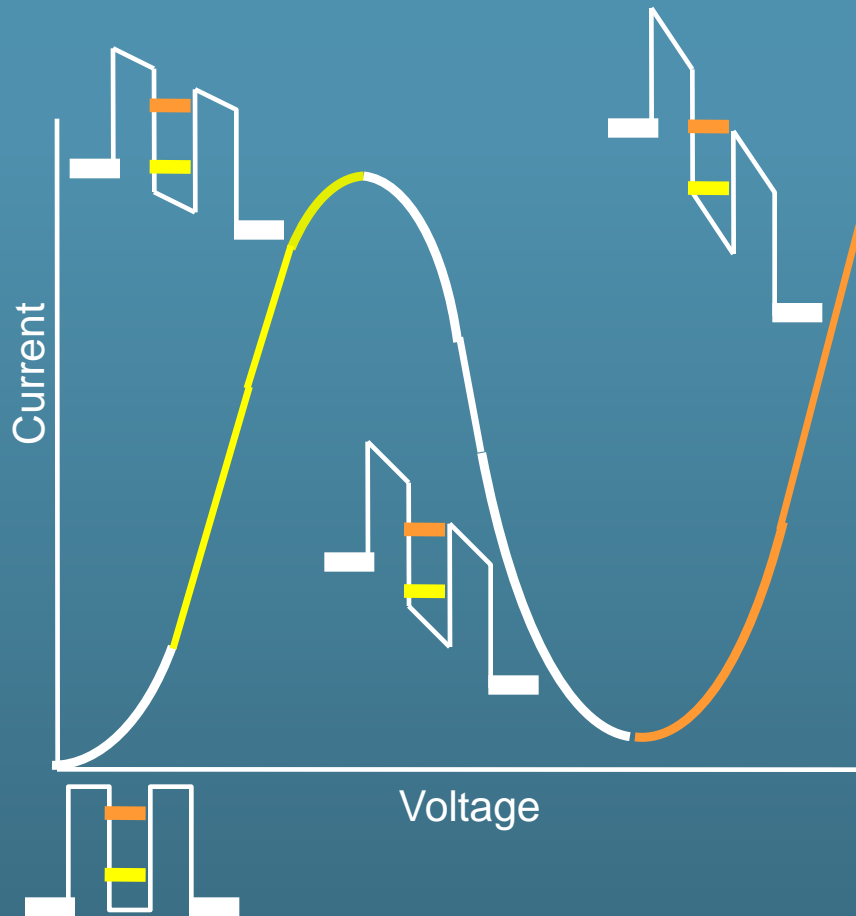


Non-Equilibrium Green Functions

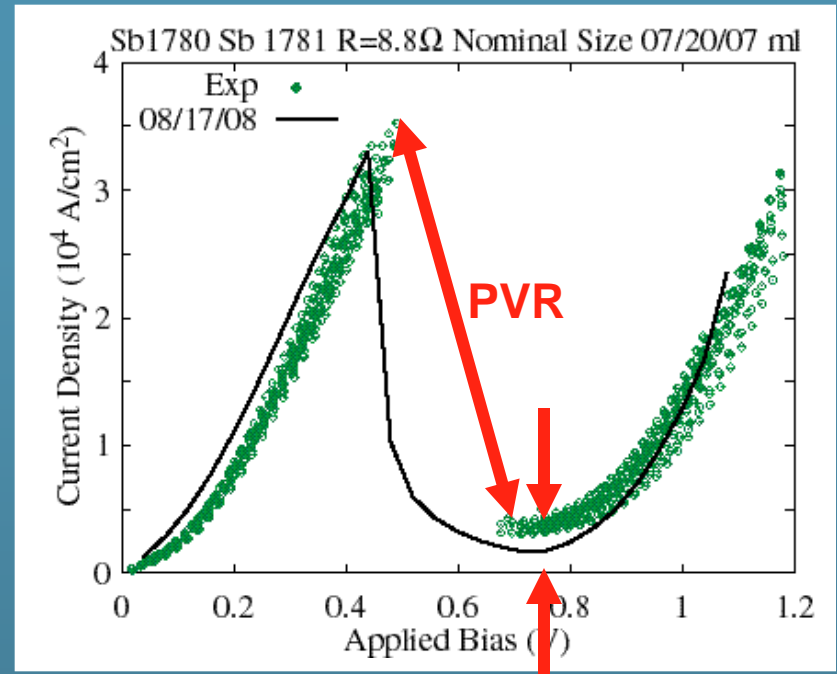
Unified model

Atomic dimensions





Conduction band diagrams for different voltages and the resulting current flow



12 different I-V curves: 2 wafers, 3 mesa sizes

**PVR – Peak-to-Valley-Ratio**

1994: Best experiment PVR=80

=> On-Off-Ratio should to be >1,000

1994: What is the valley current physics?

1997: Can overlay experiment and theory.

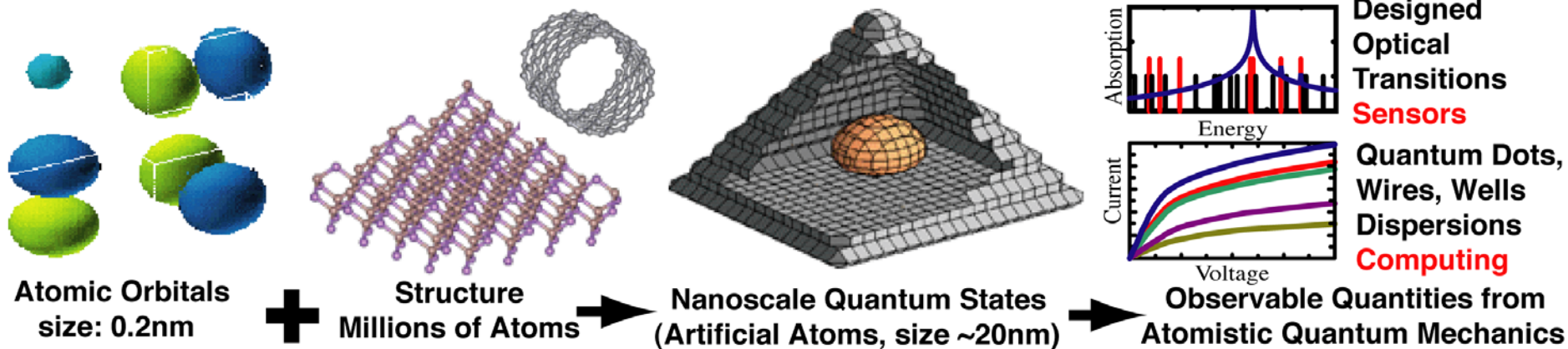
What are the key insights?

Atomistic device resolution (2-5nm dim.)

Extended contacts with scattering

Extended electrostatics

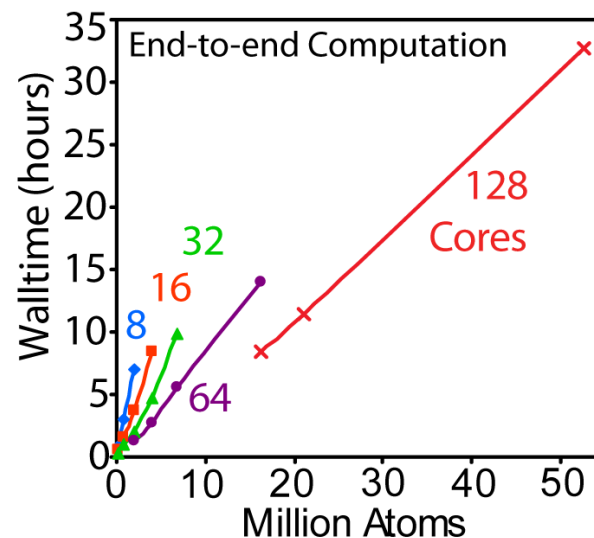
First industrial NEGF device simulator



## Problem:

Nanoscale device simulation requirements:

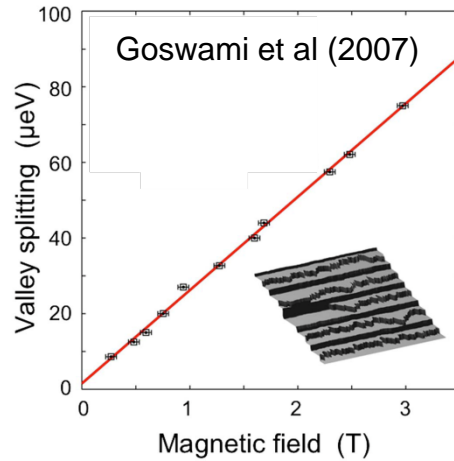
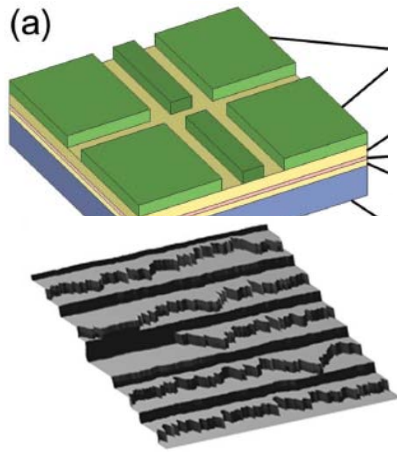
- Cannot use bulk / jellium descriptions, need description atom by atom  
=> use pseudo-potential or local orbitals
- Consider finite extend, not periodic  
=> local orbital approach
- Bonds in semiconductors are stable!  
=> Do not need ab-initio methods!
- Need to include about one million atoms.  
=> need massively parallel computers
- Tight binding in a design tool: NEMO3D



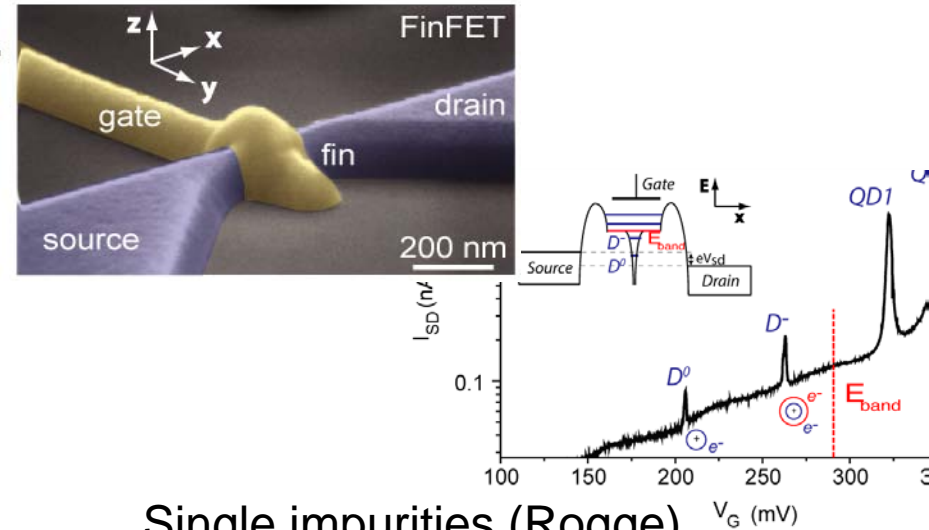
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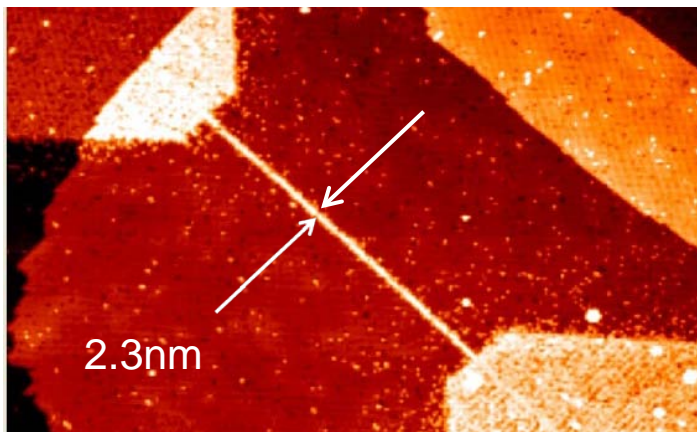




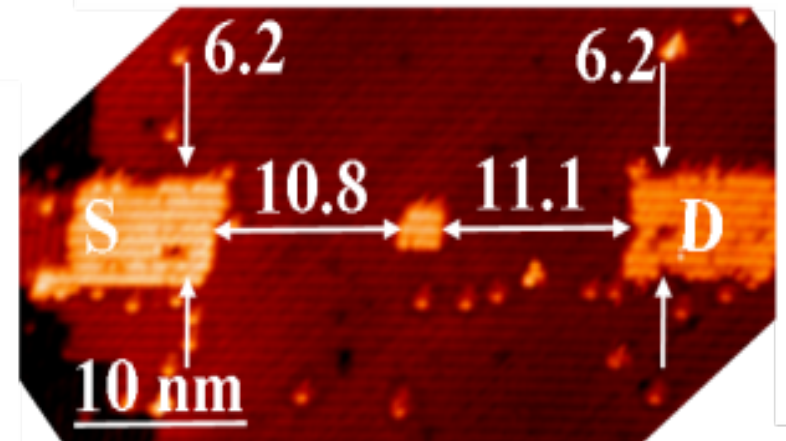
Si QD on SiGe (Eriksson)



Single impurities (Rogge)



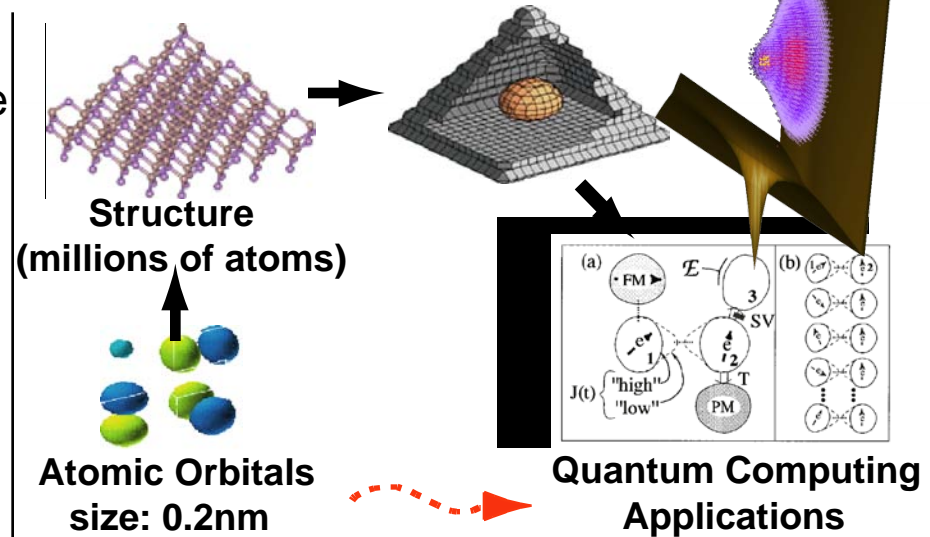
Si:P nanowires (Simmons)



Si:P quantum dots (Simmons)

## Objective

- Narrow experimental qubit parameter space
- Development of a comprehensive suite of modeling tools for solid state qubit devices.
- Atomic level material description.
- Model both spin and charge coherent devices.
- Magnetic field effect on electronic structure and state coherence.
- Compute interactions.



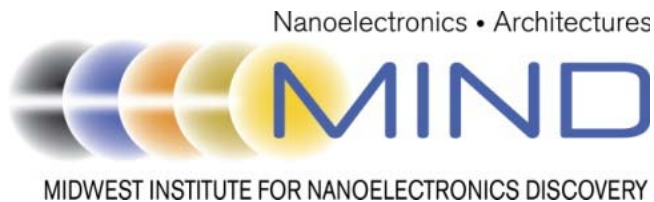
## Objective Approach

- Leverage about 50,000 hours NEMO 1-D and 20,000 hours NEMO 3-D development.
- Model semicond. Quant. Comput. Syst.
  - Si on SiGe (Wisconsin / Eriksson)
  - P in Si (Australia / Dzurak)
  - Si QD in SiGe (Purdue / Rokhinson)
- Atomistic of treatment semiconductors.
- Incorporate many-body effects through Hartree & configuration interaction.

## Impact / Studies

- VS in flat Si QW - (2004)
- VS in slanted Si/SiGe QW - (2007) - exp
- VS in Si/SiGe QD - (2008)
- VS in Si [111] QW - (2009) – exp
- Single impurities in Si transport (2008) - exp
- Single impurities in Si Stark Eff (2008) - exp
- Single impurities g-factors (2009)
- Single impurities CTAP design (2009)
- Single impurities Wavefunction Map (2009)
- Dense impurities in Si QW – ongoing - exp
- Dense impurities in Si Wire – ongoing - exp

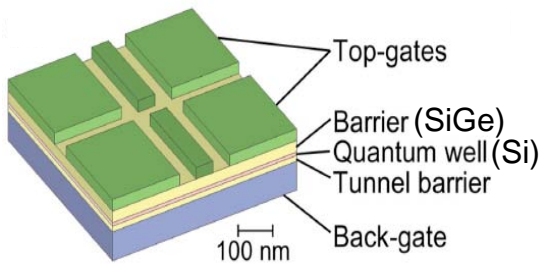




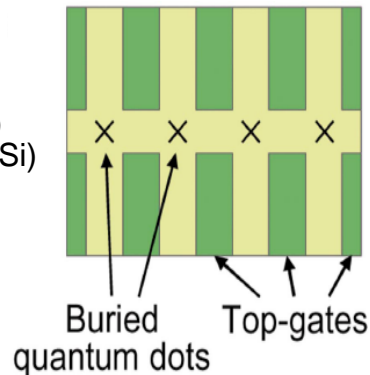
- Motivation
- NEMO Numerical Engine
- Valley splitting
  - » Si quantum wells
  - » Si Quantum Dots
- Single impurities in Si
  - » Metrology – impurity identification
  - » Wavefunction mapping
  - » g-factor engineering
  - » CTAP
- Dense impurities in Si – Si:P
  - » Infinite sheets
  - » Infinite wires
  - » Gating, transport through wires
  - » Impurity-based quantum dots

Authors: **Neerav Kharche**, Marta Prada, Timothy Boykin, and Gerhard Klimeck

## Spin based quantum computing in Si/SiGe heterostructures

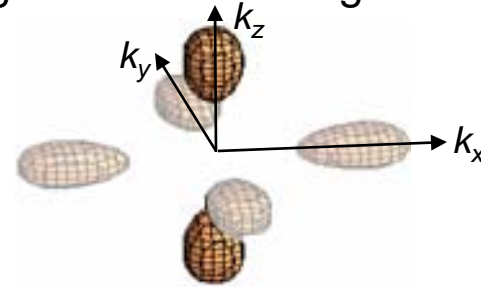


Friesen et al. Phys. Rev. B, **67**, 121301(R) (2003)

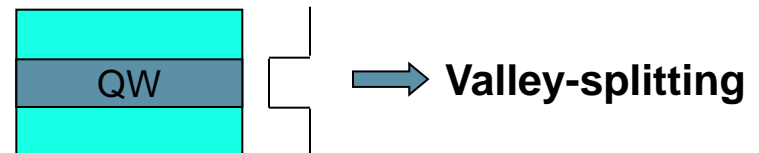


Biaxial strain splits **six-fold** valley-degeneracy into

- Lower **two-fold** degeneracy
- Higher **four-fold** degeneracy



- Lowest two-fold degenerate **valleys split** in the presence of sharp confinement potentials in QWs.



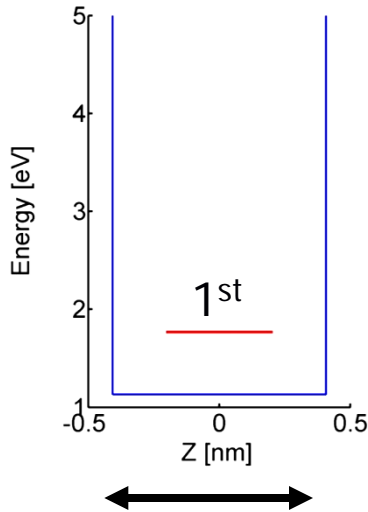
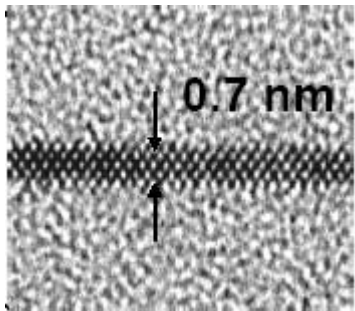
- Valley-splitting is a critical design parameter for QC devices.

## Potential problem: Valley degeneracy

- Six-fold degenerate valleys lead to spin decoherence



Five Atomic Layer  
Si (001)  
IEDM, Uchida



Most basic  
quantum mechanical problem:  
Particle in a box!

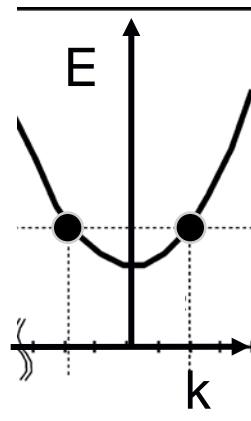
**Expect / Remember:**

- State quantization
- 2 counter-propagating states
- 1 bound state
- $E \sim 1/L^2$

Schrödinger Equation

$$\left( E - \frac{\hbar^2}{2m^*} \frac{d^2}{dz^2} - V(z) \right) \Psi(z) = 0$$

$$E = \frac{\hbar^2 k^2}{2m^*} \quad \text{Parabolic Dispersion}$$



$$k_1 = \frac{\pi}{L}$$

quantize

$$\Psi(z) = e^{-ik_1 z} \quad \Psi(z) = e^{+ik_1 z} \quad \text{2 propagating states}$$

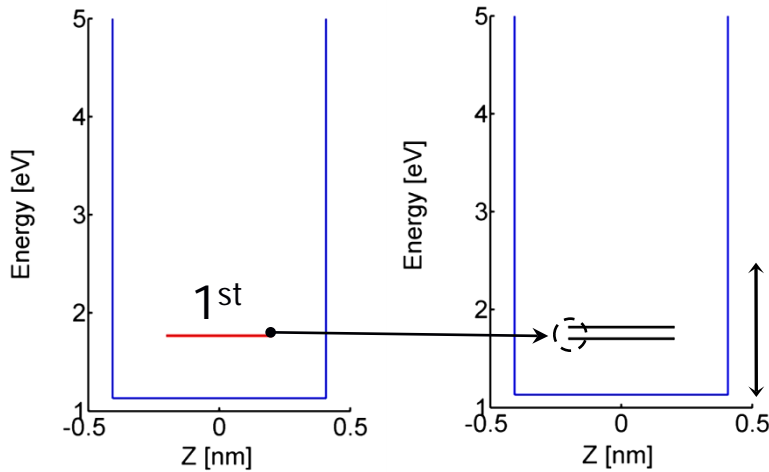
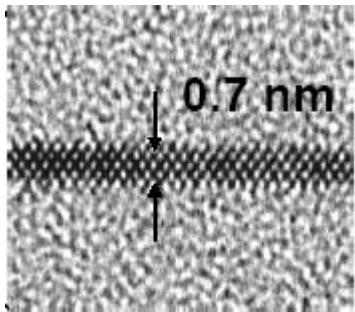
$$\Psi(z) = \sin(k_1 z) \quad \text{1 bound state}$$

$$E = \frac{\hbar^2}{2m^*} \frac{\pi^2}{L^2}$$

$$E \sim \frac{1}{L^2}$$

## Special Considerations in Si

Five Atomic Layer  
Si (001)  
IEDM, Uchida



Envelope:

$$k_{1/2} \sim a/na$$

a: lattice constant

n: QW layers ( $L=na$ )

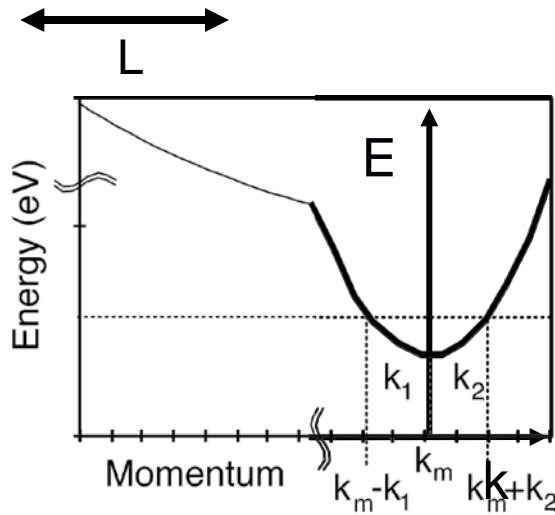
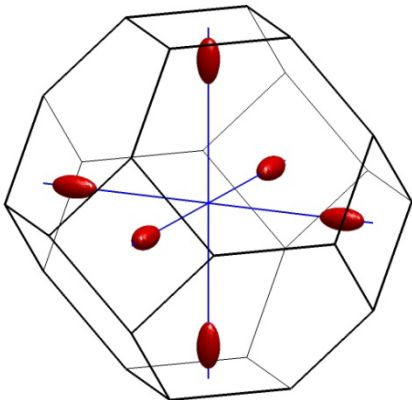
Phase control:

$$k_m \sim 0.85 a/a$$

Depends on

atomic overlaps

$sp^3s^*d^5$  (Klimeck *et al*) CB,  $\Delta E_c = +100$  meV

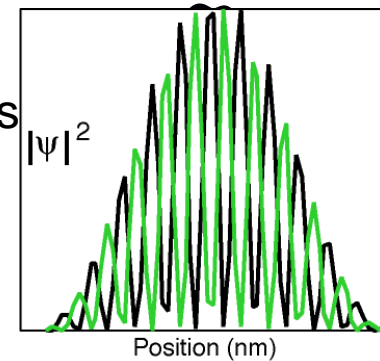


2 valleys  
4 propagating states

2 bound states

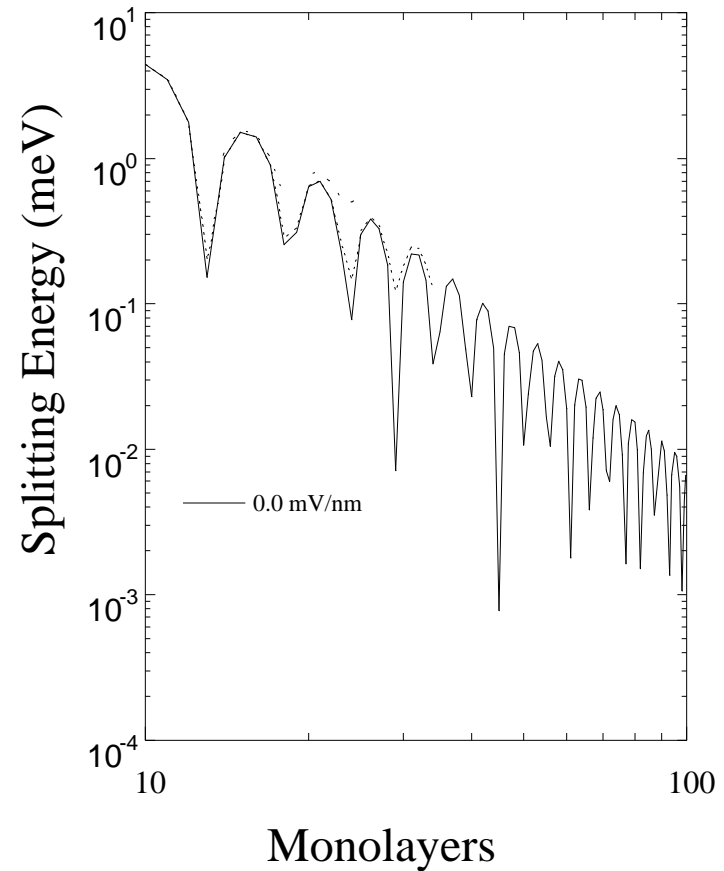
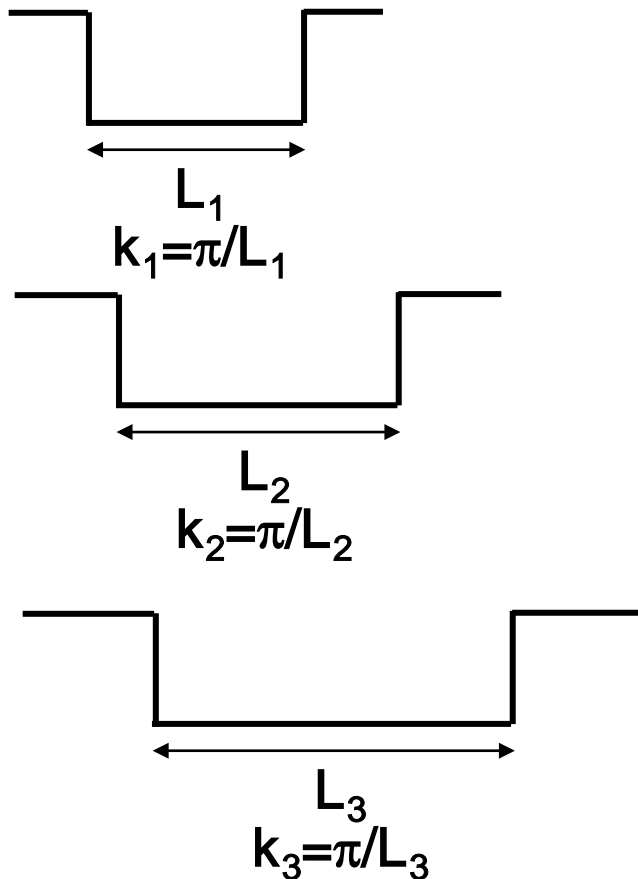
$k_{1,2}$  envelope

$k_m$  fast oscillations



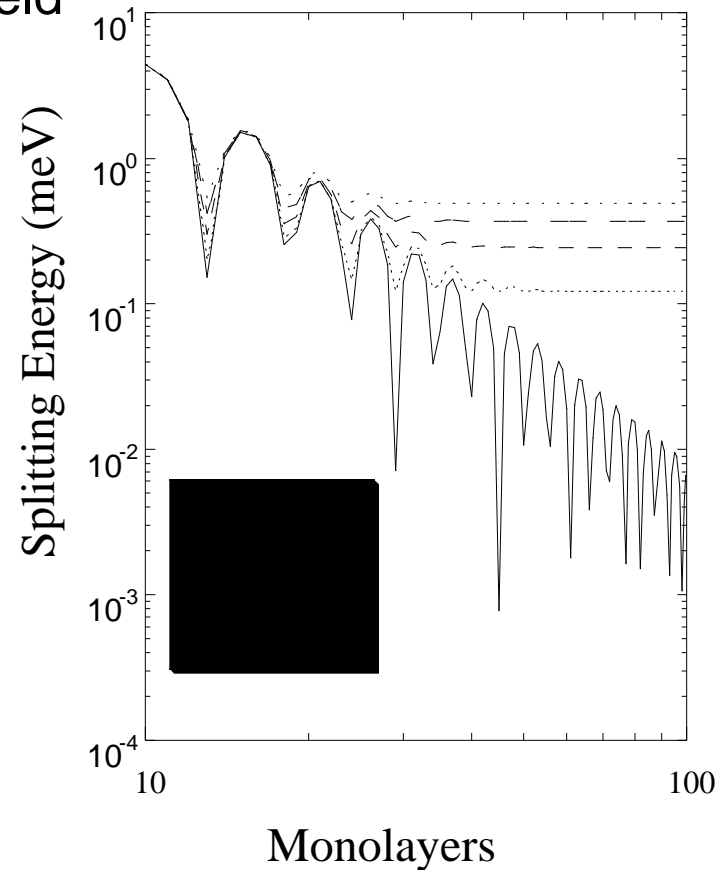
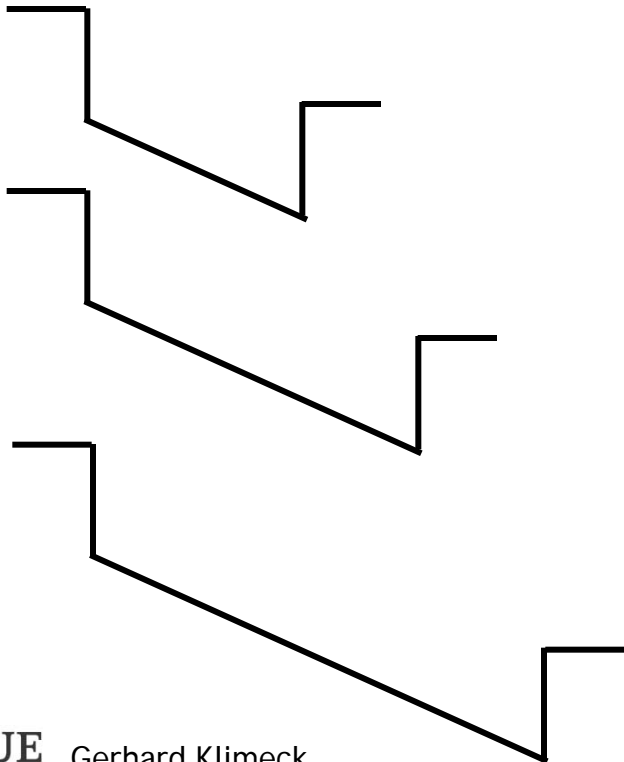
# Splitting Behavior With Quantum Well Width

- Strained Si QW, Hardwall BCs
- Oscillations with Monolayers (decay as  $ML^{-3}$ ) - published analytical results



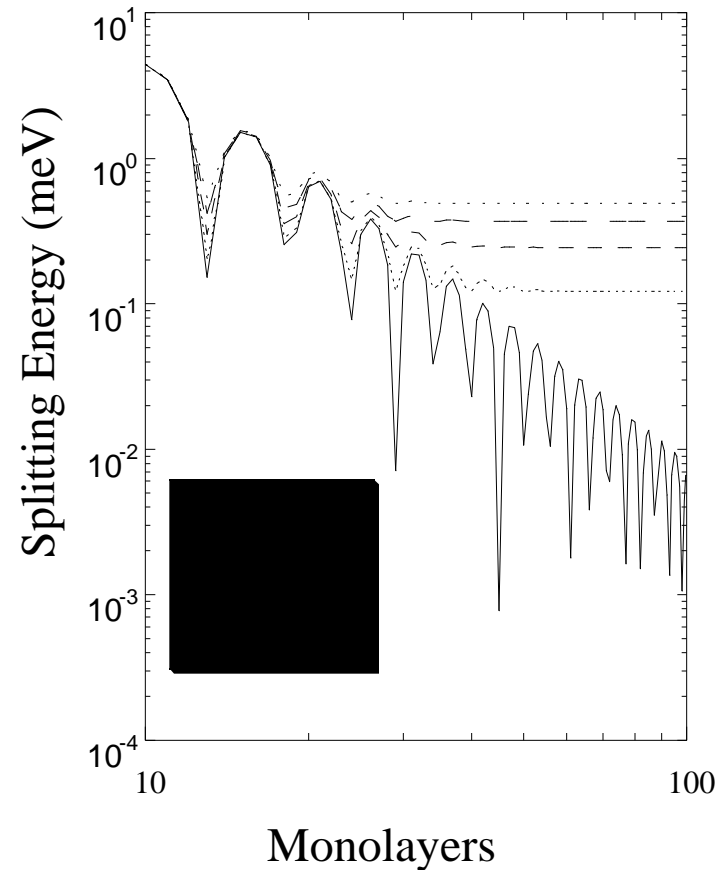
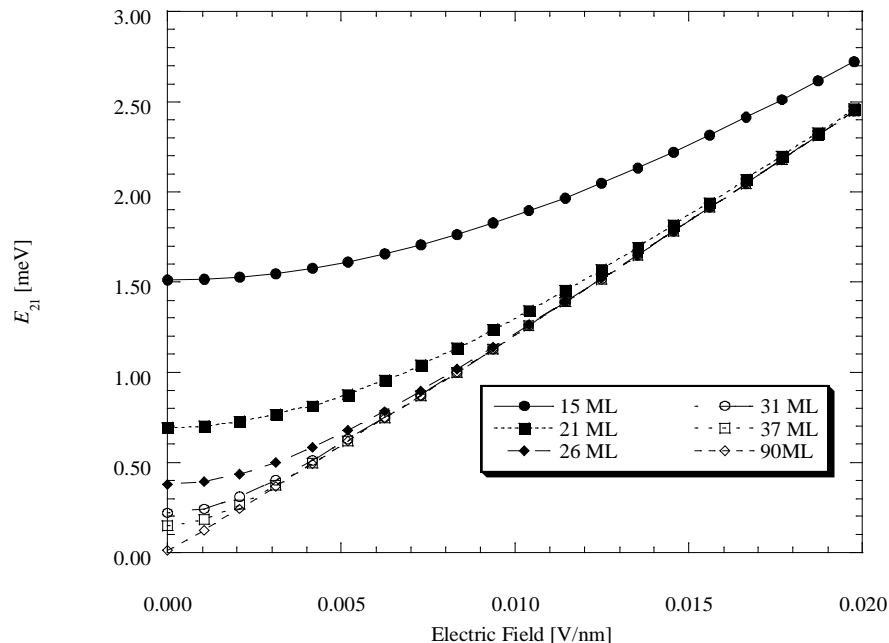
# Splitting Behavior With Quantum Well Width and Electric Field

- Strained Si QW, Hardwall BCs
- Oscillations with Monolayers
- Oscillations decay with increasing Field
- Decay significant once drop/ML is of order zero-field splitting



# Splitting Behavior With Quantum Well Width and Electric Field

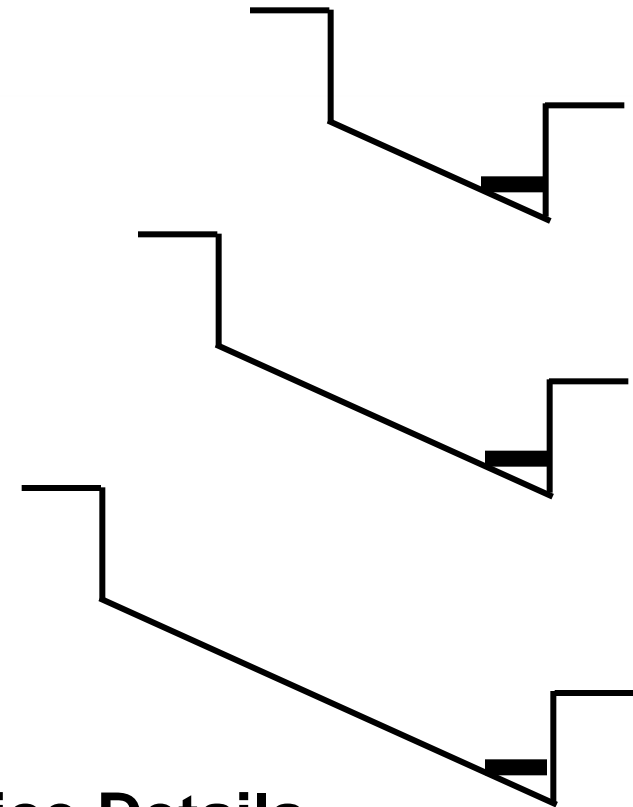
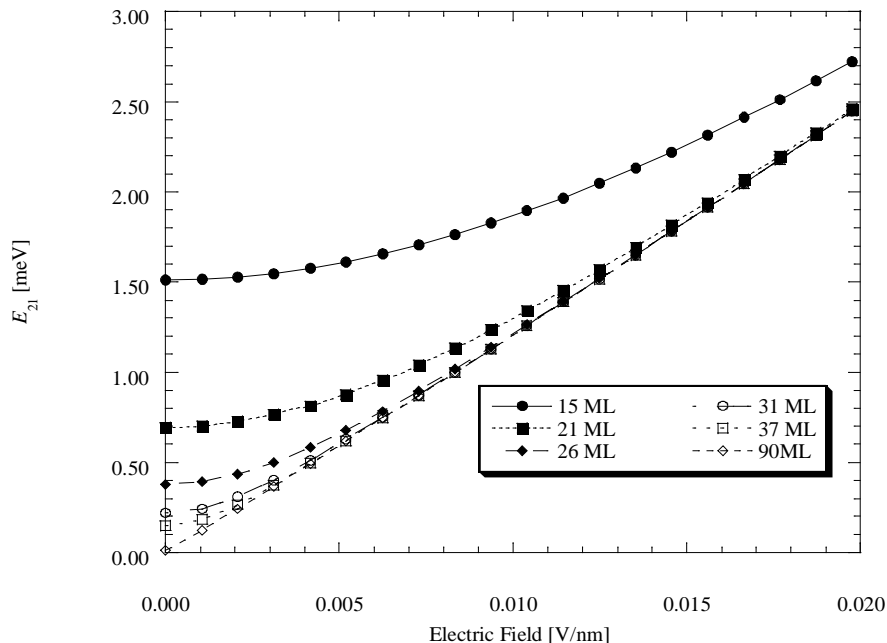
- Strained Si QW, Hardwall BCs
- Oscillations with Monolayers
- Oscillations decay with increasing Field
- Decay significant once drop/ML is of order zero-field splitting
  - For fixed  $L$ , splitting linear at high field, nonlinear at low field



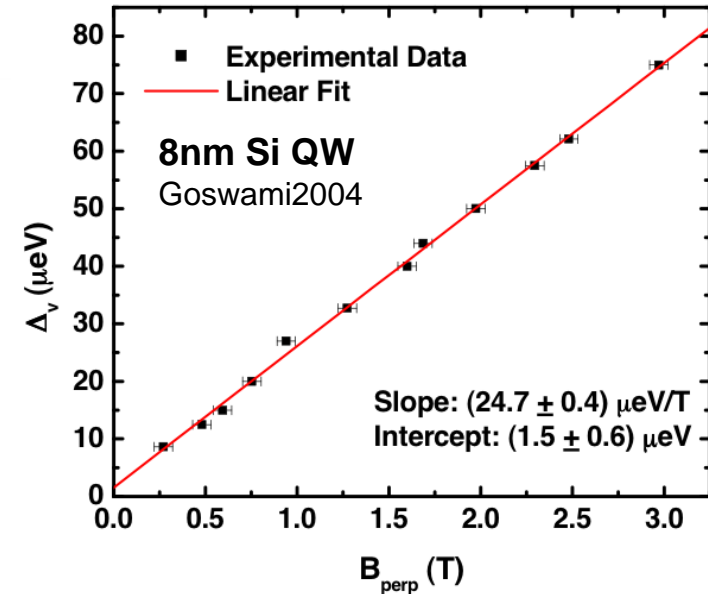
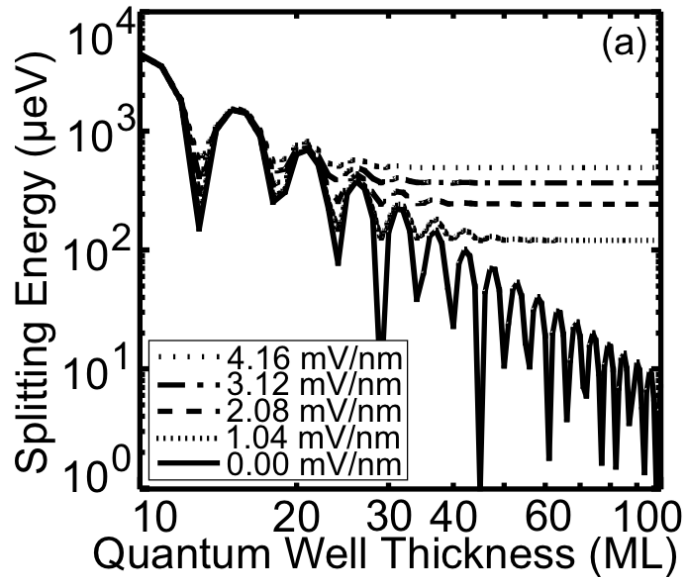
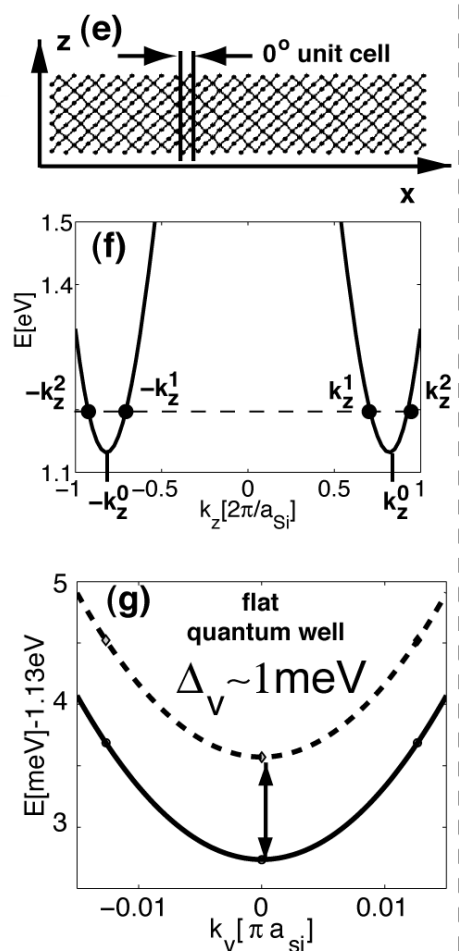


# Splitting Behavior With Quantum Well Width and Electric Field

- Strained Si QW, Hardwall BCs
- Oscillations with Monolayers
- Oscillations decay with increasing Field
- Decay significant once drop/ML is of order zero-field splitting
- For fixed  $L$ , splitting linear at high field, nonlinear at low field



**Device Details  
(structure, materials,  
local fields, )  
determine energy splitting**



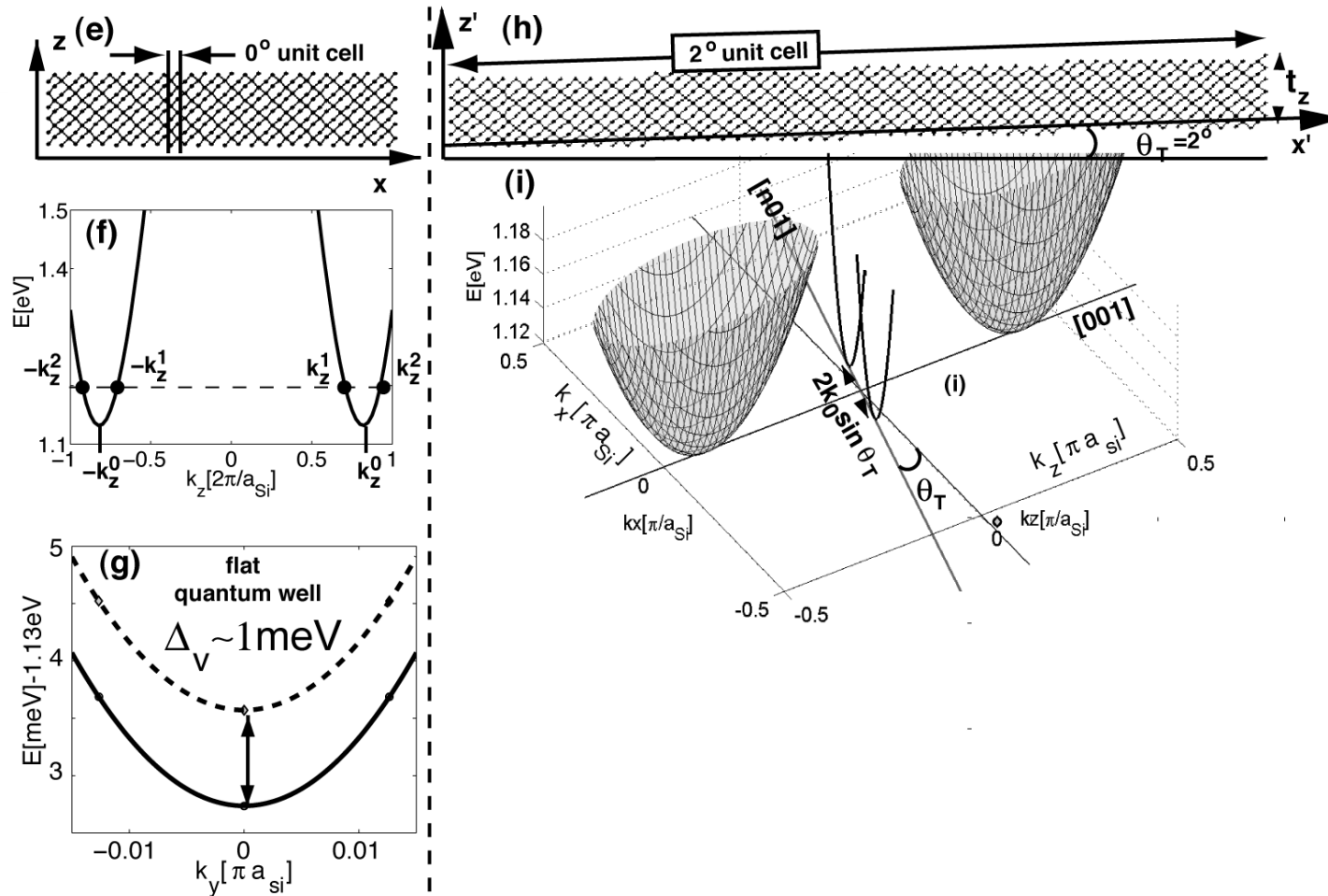
Why is the experimental valley splitting so much smaller?



Well Understood

APL, Vol. 84, pg. 115-117, 2004

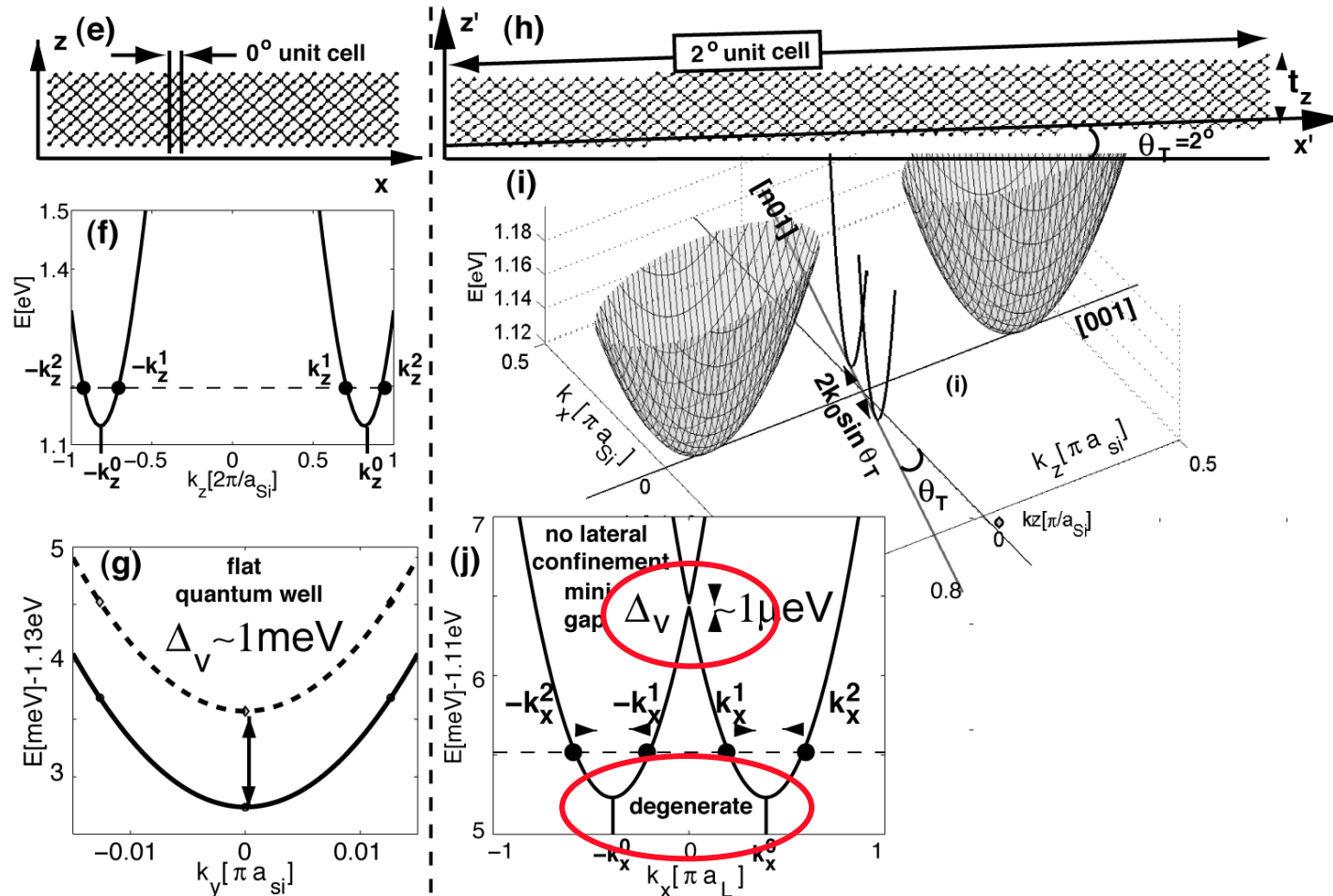
Phys. Rev. B. Vol. 70, 165325 (2004),



Well Understood

What happens with a tilted substrate?  
“Parabolas” do not overlap perfectly on the plane!

## ideal tilted quantum well - zero valley splitting



Well Understood

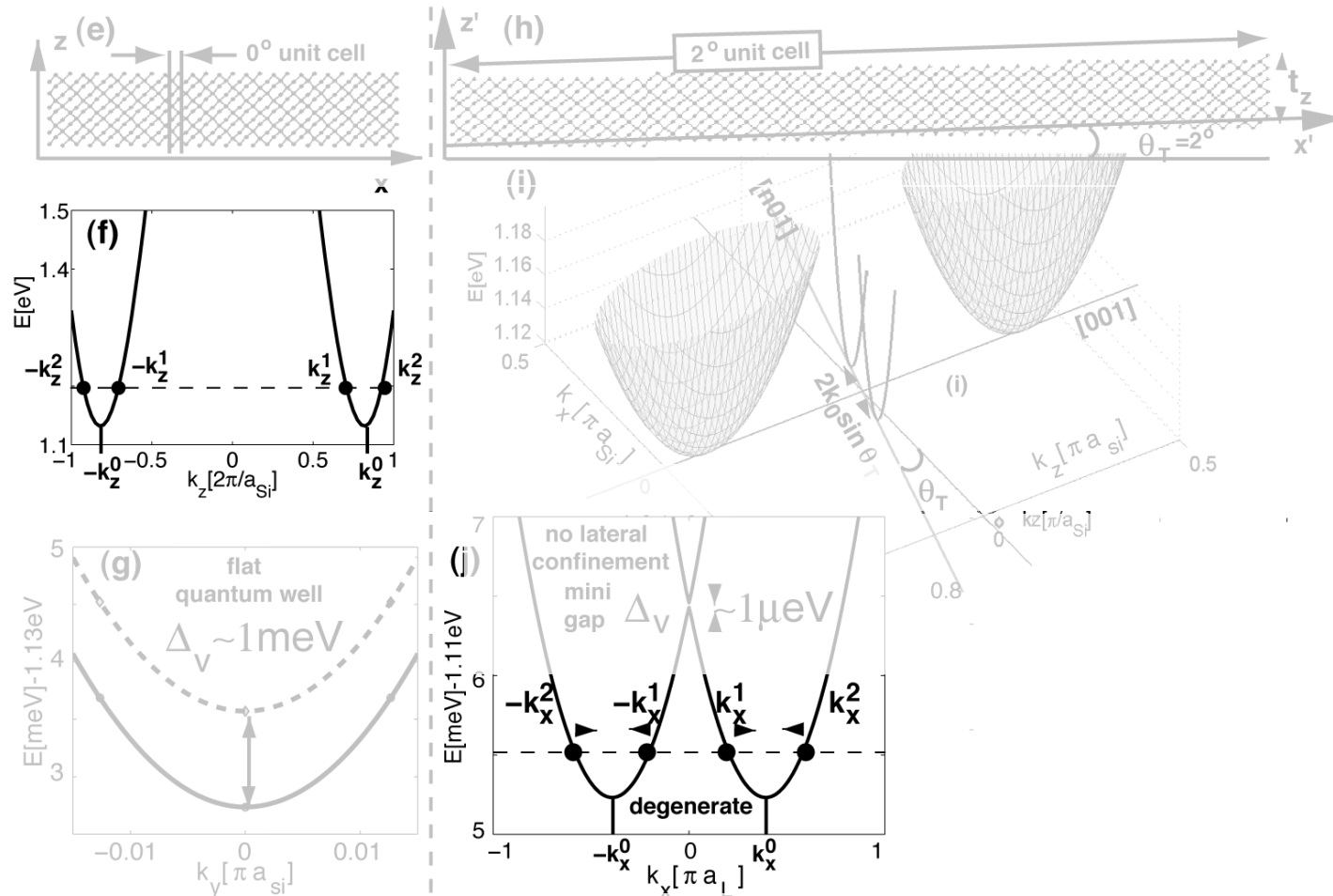
What happens with a tilted substrate?

“Parabolas” do not overlap perfectly on the plane!

Coherent overlap only at  $k=0$  leads to **minigap!**

**Valley Splitting is ZERO!**

need symmetry breaking to induce valley splitting



Well Understood

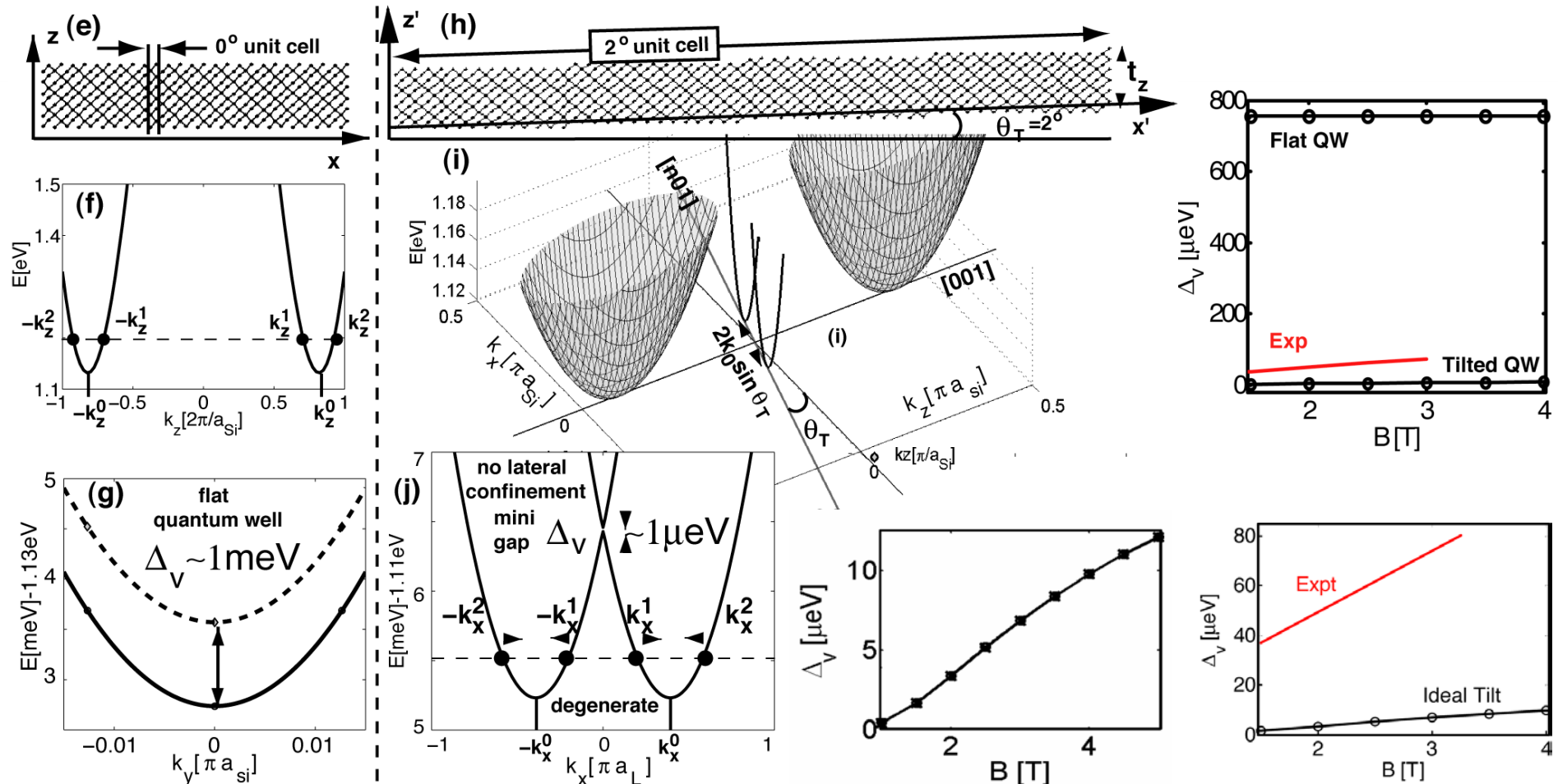
What happens with a tilted substrate?

“Parabolas” do not overlap perfectly on the plane!

Need symmetry breaking in x-direction to get valley splitting!

2 pairs of counter propagating states will create valley splitting!





Well Understood

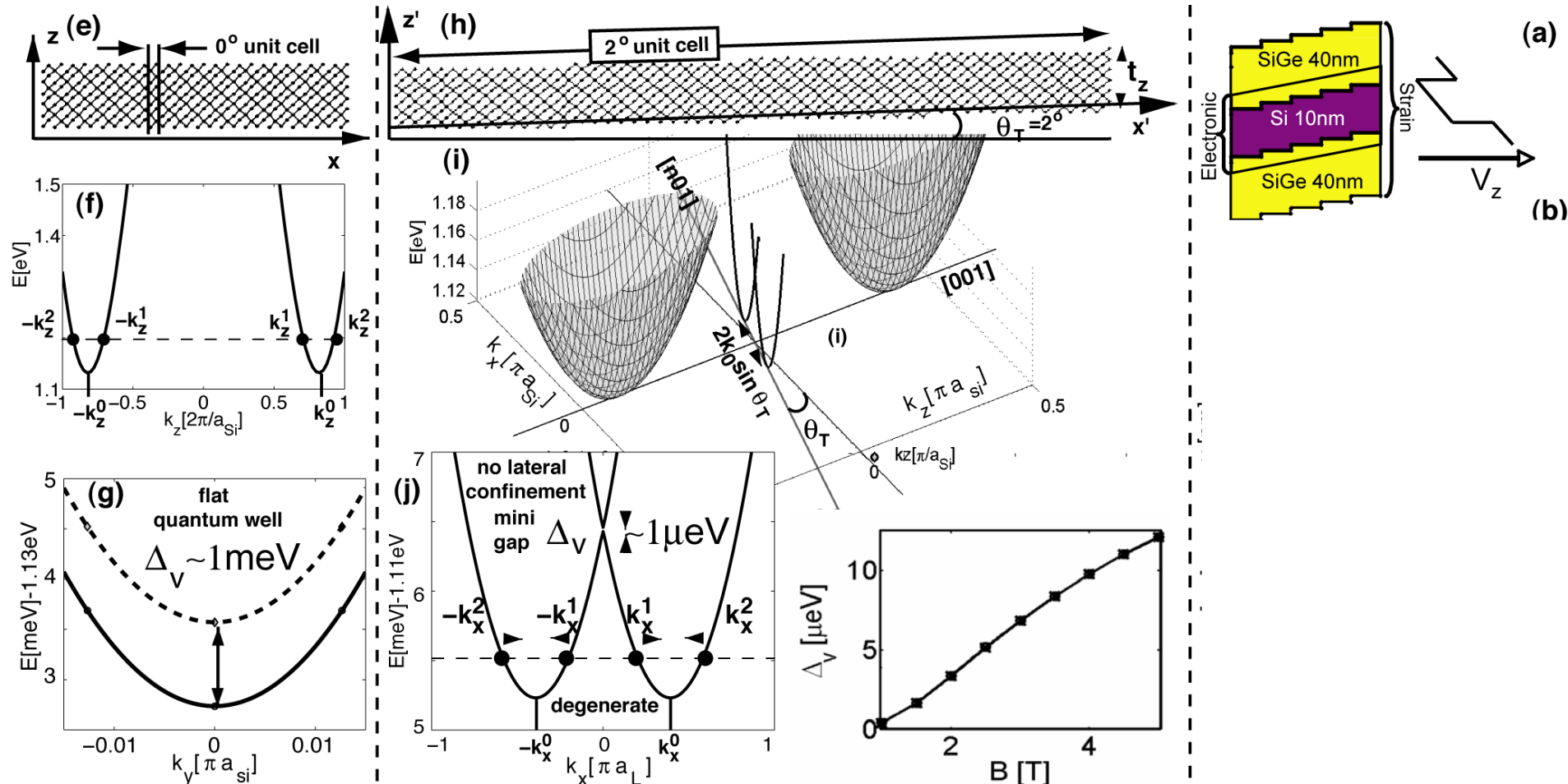
What happens with a tilted substrate?

Need symmetry breaking in x-direction – confinement / mag.field

Ideal tilted QW underpredicts,

Ideal flat QW overpredicts VS!

## Other symmetry breaking mechanisms



Well Understood

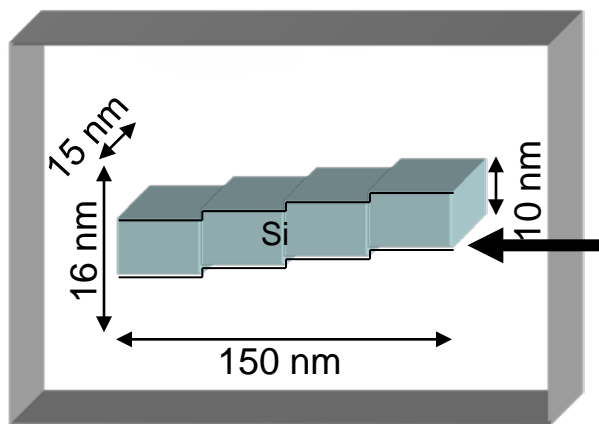
What happens with a tilted substrate?

Are there other symmetry breaking mechanisms?

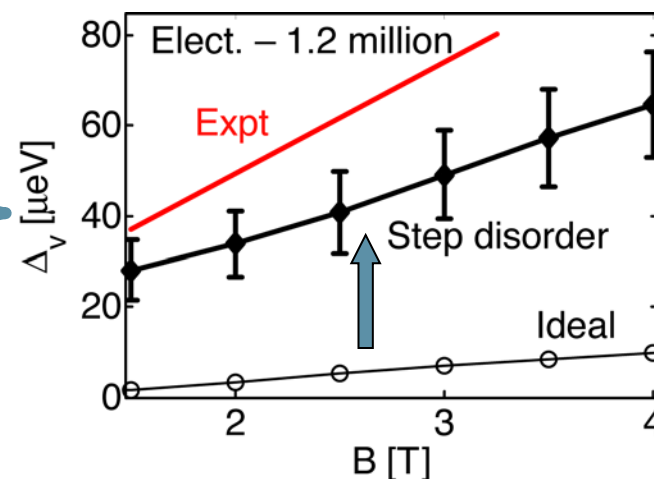
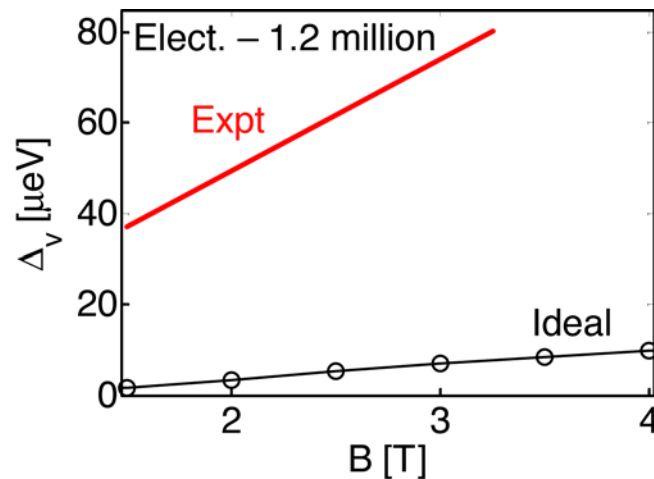
Step disorder (happens on an **atomic** scale)

Alloy disorder (happens on an **atomic** scale)

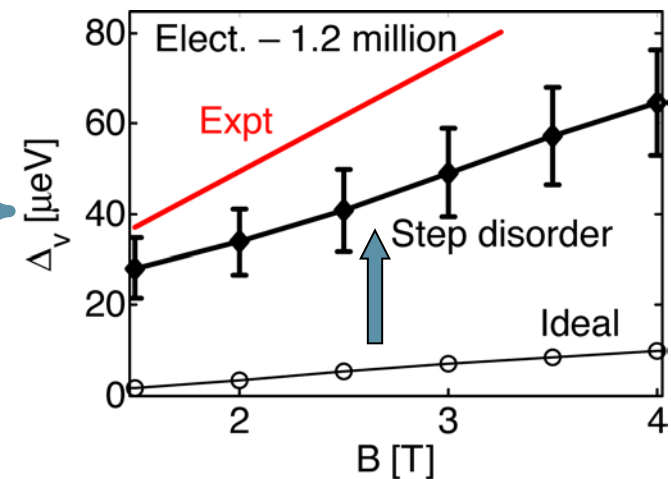
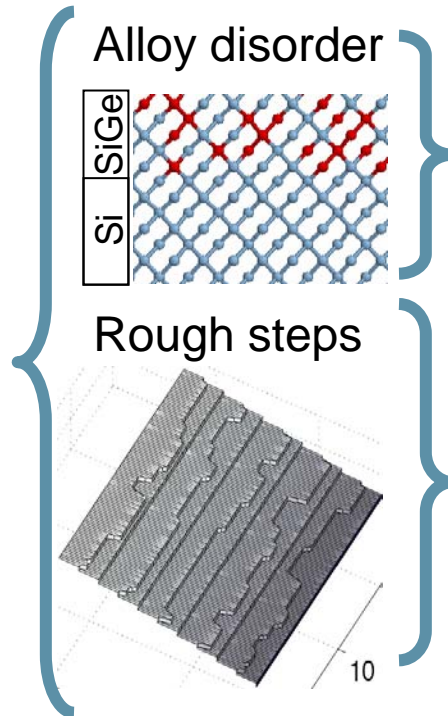
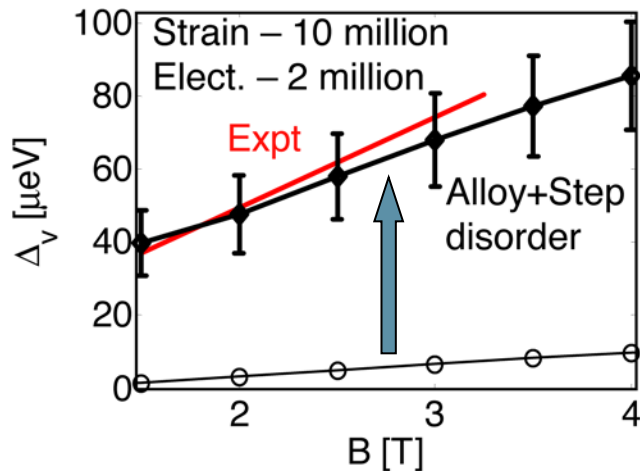
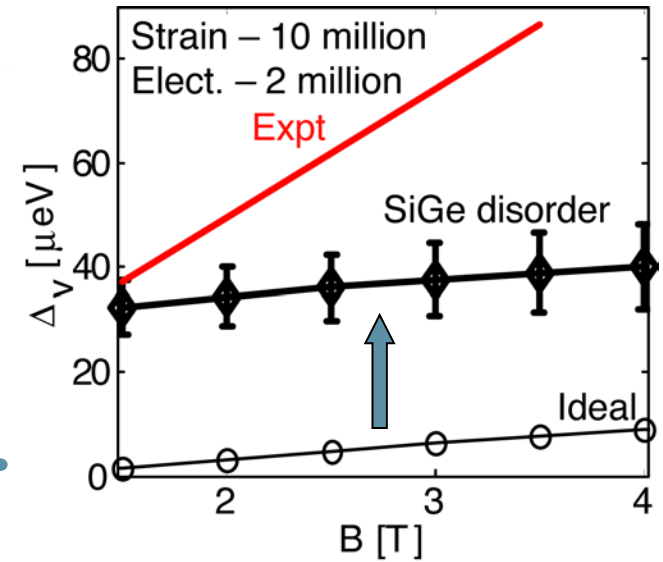
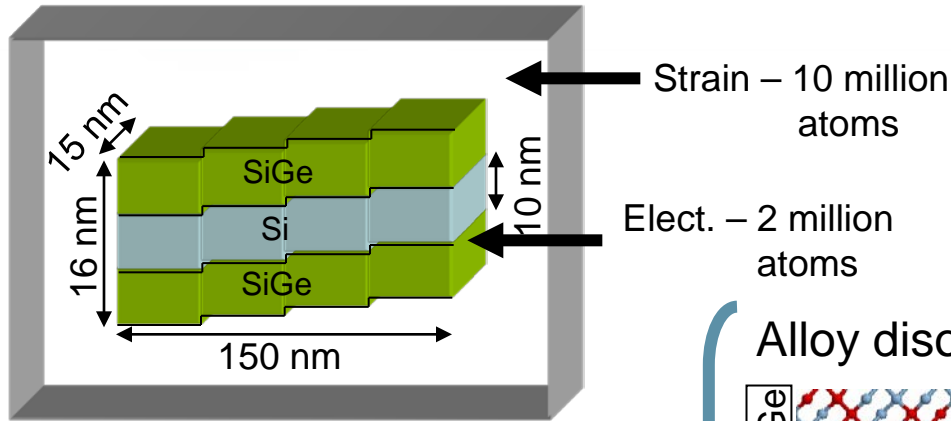
# Effect of disorder on valley-splitting step roughness



Elect. – 1.2 million atoms

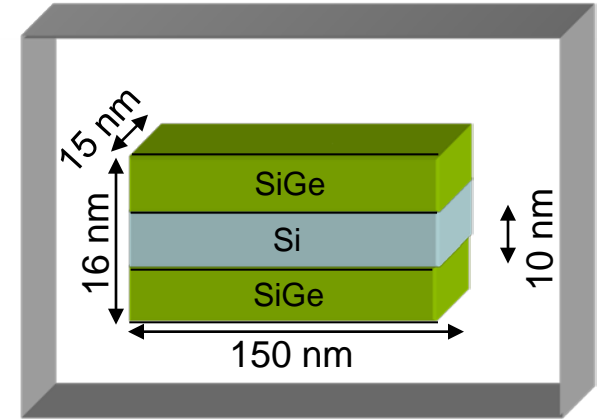
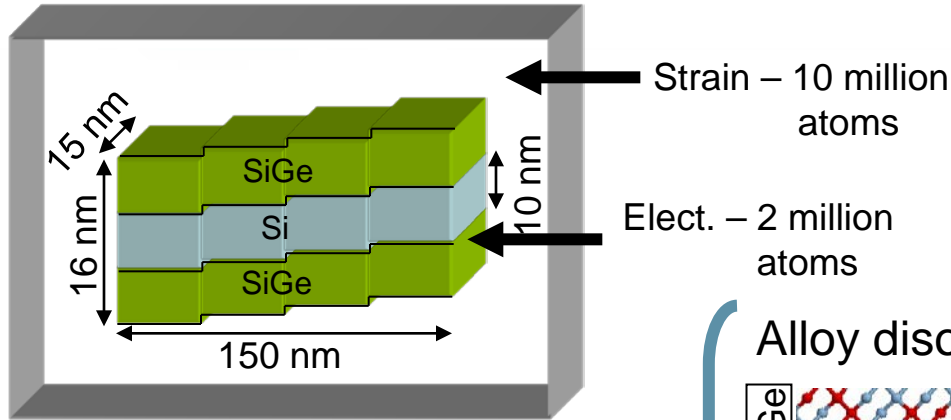


# Effect of disorder on valley-splitting step roughness and alloy disorder

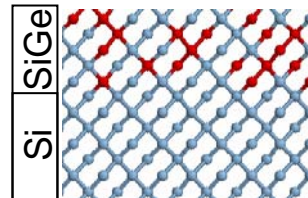


# Effect of disorder on valley-splitting

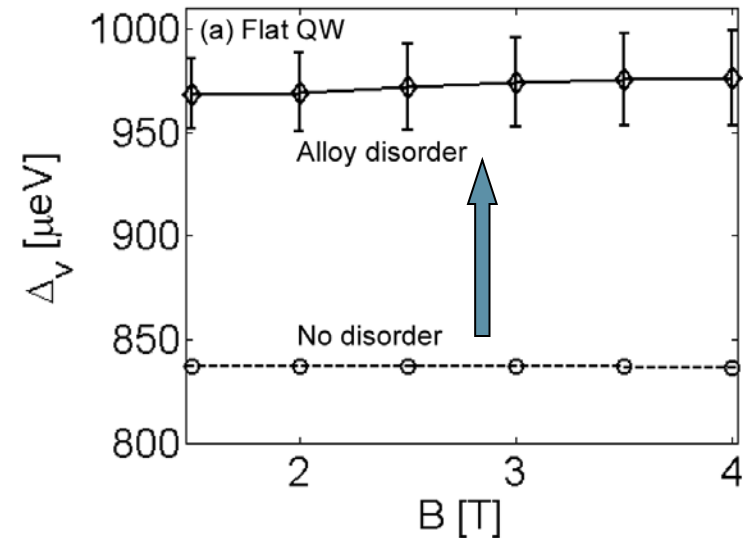
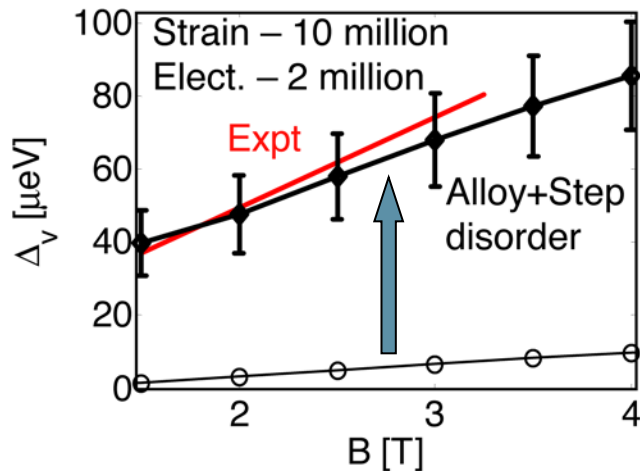
## step roughness and alloy disorder



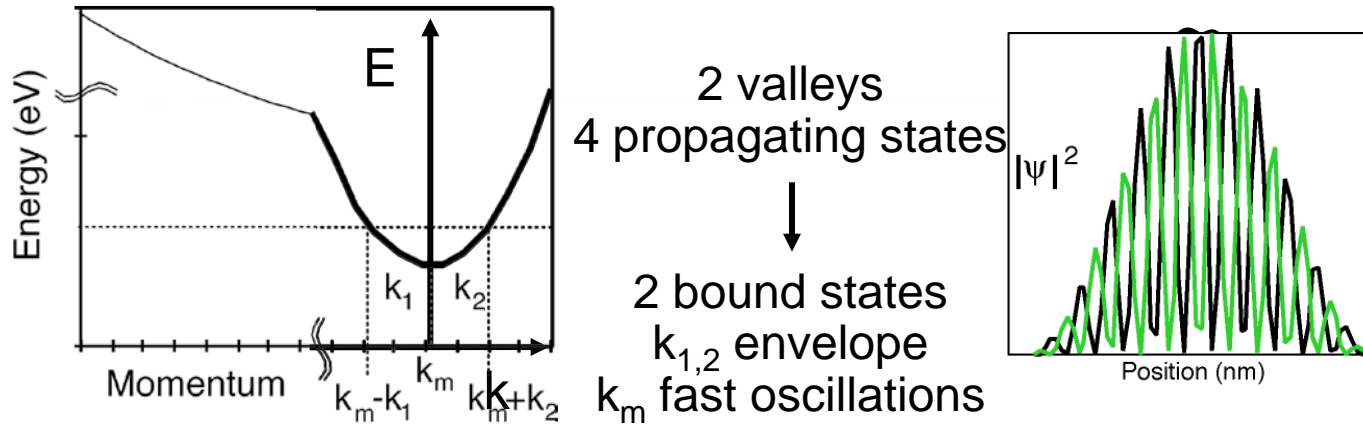
Alloy disorder



Rough steps







Envelope:

$$k_{1/2} \sim a/na$$

a: lattice constant

n: QW layers  $L=na$

Phase control:

$$k_m \sim 0.85 a/a$$

Depends on  
atomic overlaps

Rapid oscillation,  $k_m$

⇒ **sensitivity to atomic details** at the interface

Have demonstrated as critical understanding:

⇒ Disorder in Barriers: Alloy and Step

Appl. Phys. Lett. **90**, 092109 (2007)

Analytical expressions for structures with fields:

JAP, Vol. 97, pg. 113702 (2005).

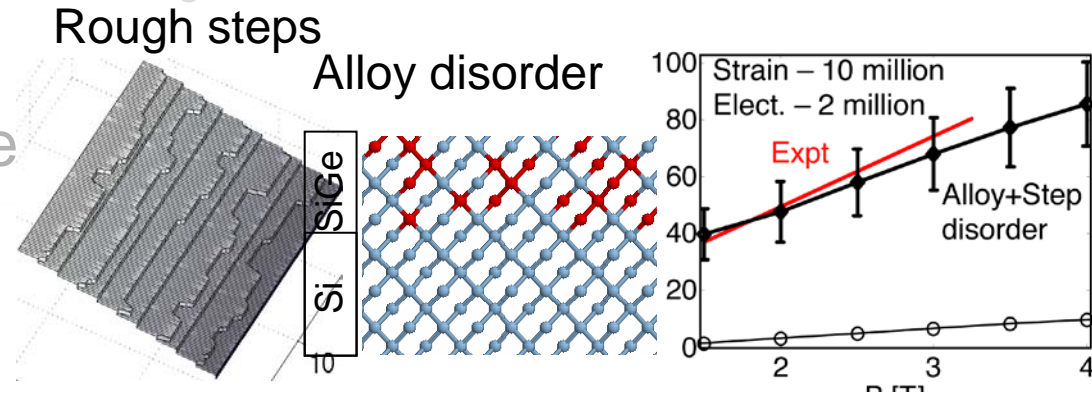
Analytical expressions for finite barriers:

Phys. Rev. B, Vol. 77, 245320 (2007)



# Multi-Million Atom Simulation of Quantum States in Realistically Extended Devices in Silicon

- Motivation
- NEMO Numerical Engine
- Valley splitting
  - » Si quantum wells
  - » Si Quantum Dots
- Single impurities in Si
  - » Metrology – impurity identification
  - » Wavefunction mapping
  - » g-factor engineering
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- Dense impurities in Si – Si:P
  - » Infinite sheets
  - » Infinite wires
  - » Gating, transport through wires
  - » Impurity-based quantum dots

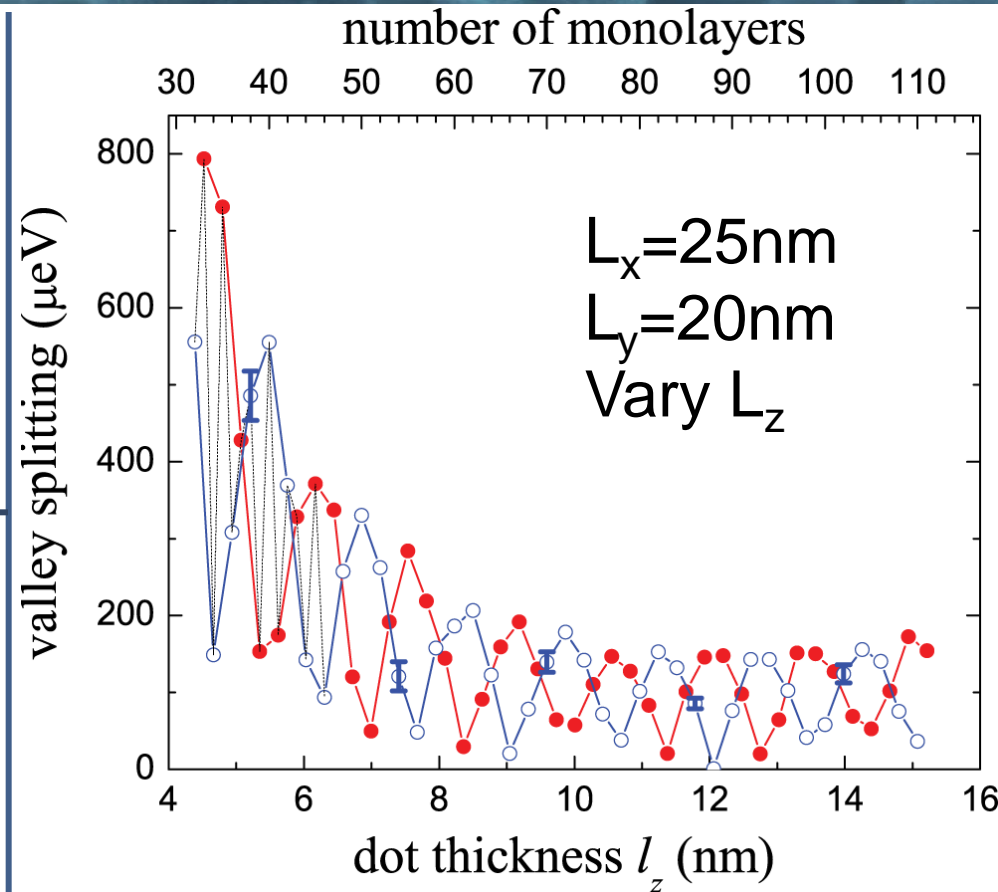
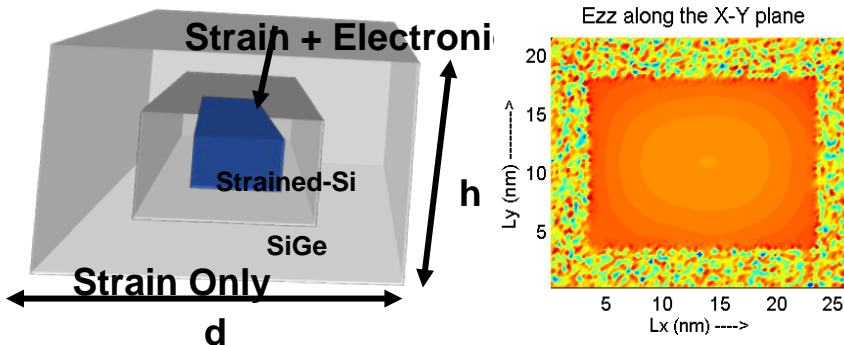


## Objective:

- Prepare experiments (Rokhinson)
- Study realistic/large embedded Si QD
- Aid in understanding and design, buffer randomness, interfaces, Valley Splitting
- Explore size and disorder dependence

## Approach:

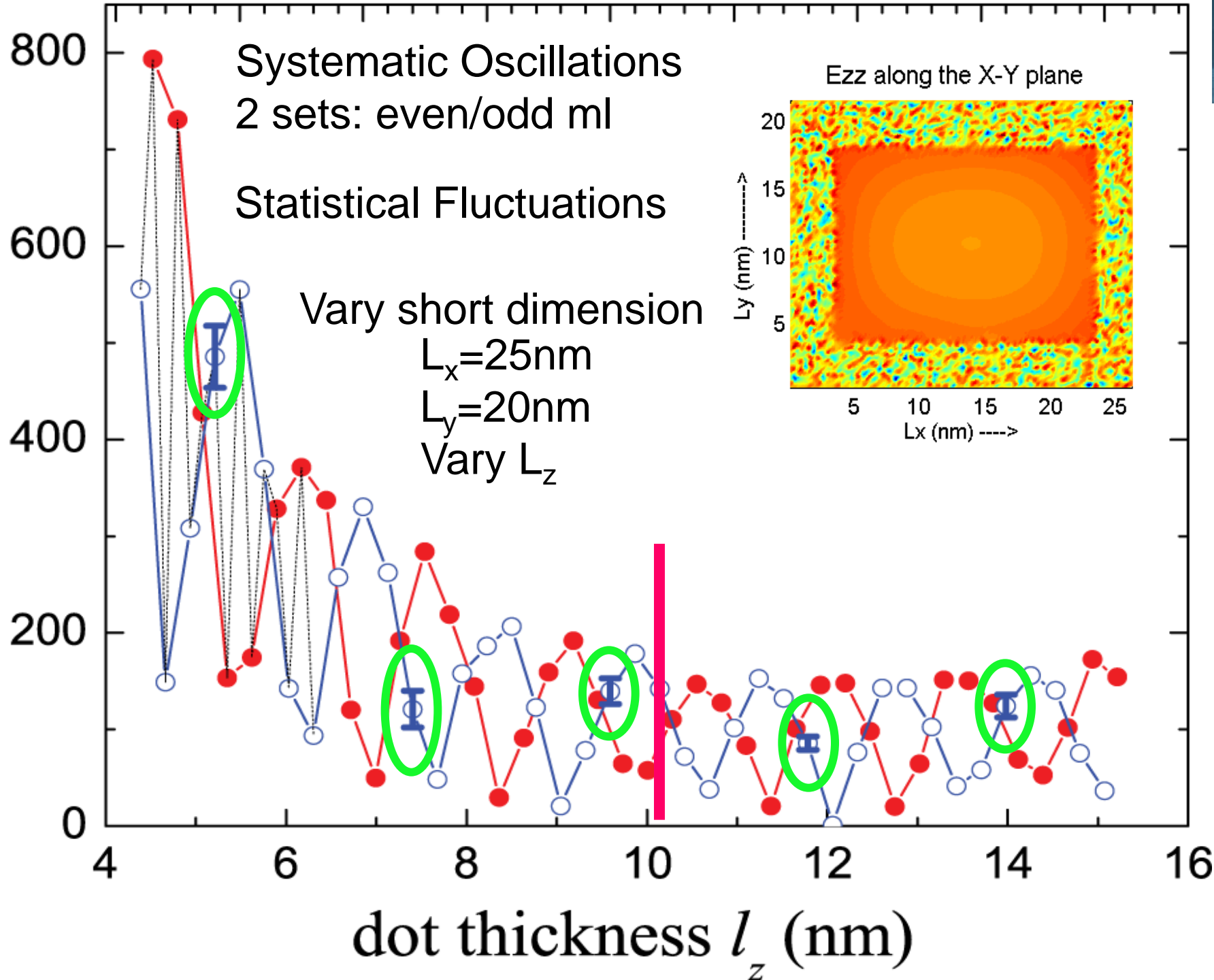
- No NEMO 3D modifications needed
- Strain penetrates 40nm into SiGe => 50 million atom strain sim..
- Typically 365.000 atoms in electronic domain

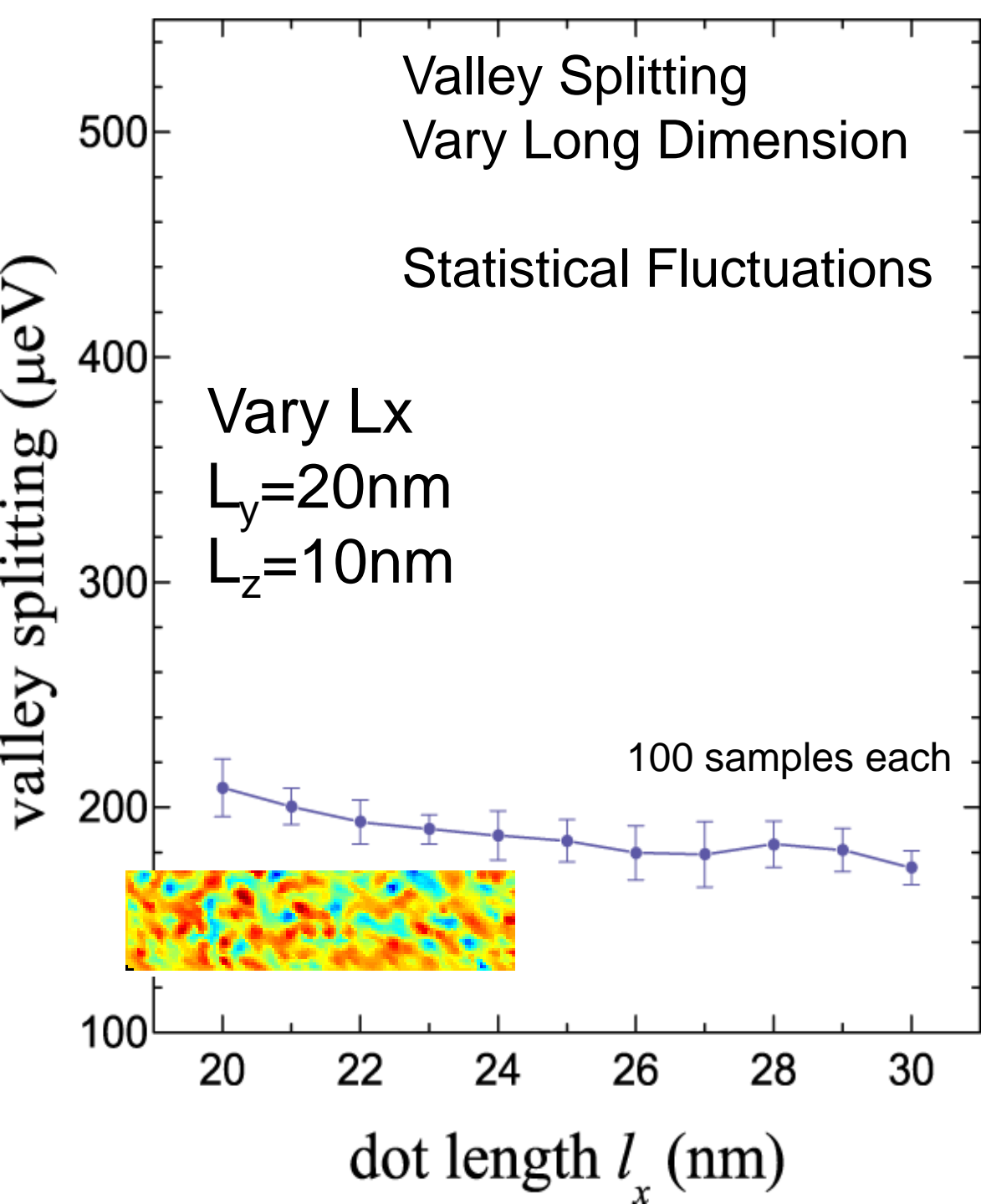


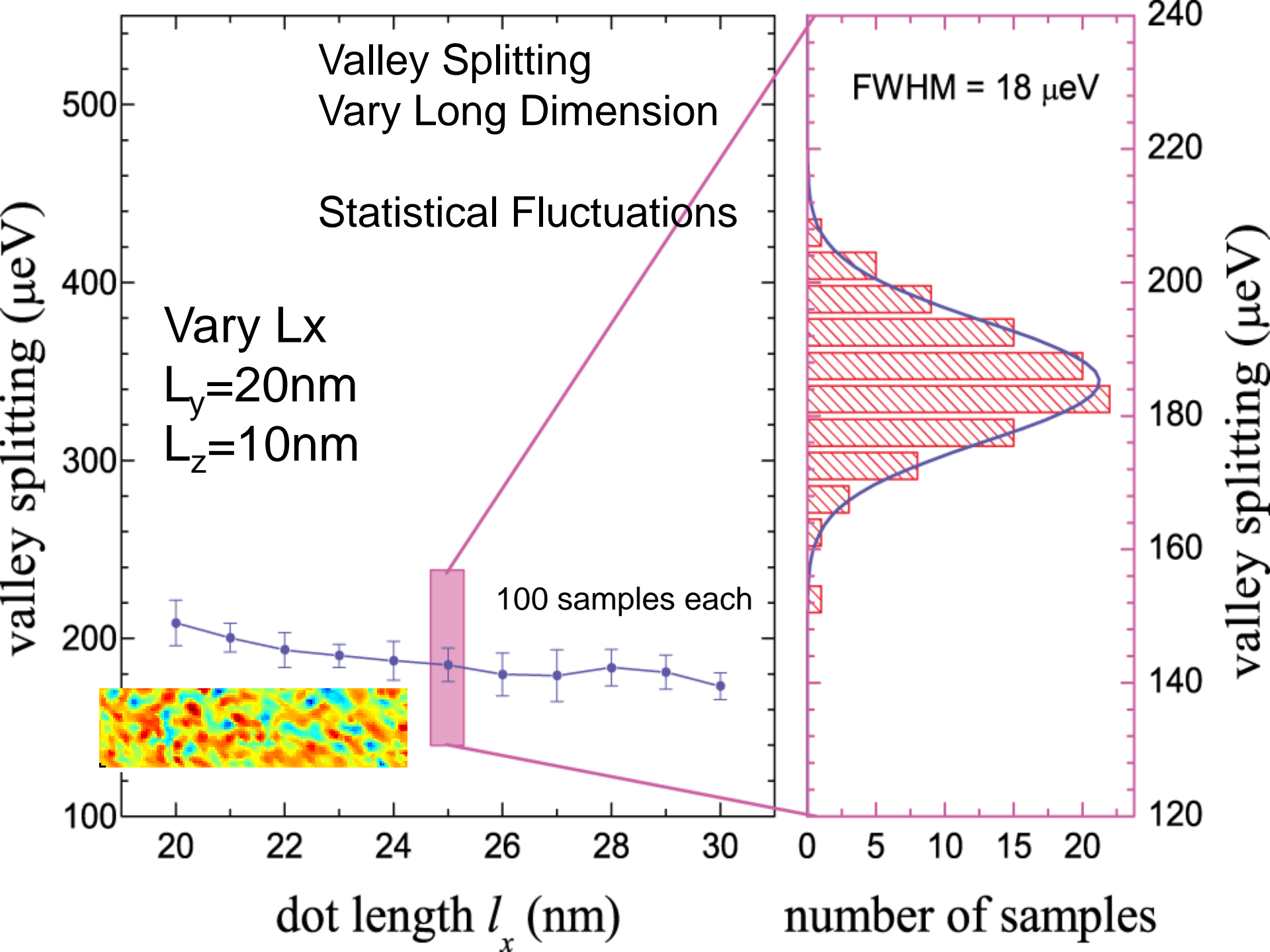
## Results / Impact:

- Fluctuations with quantum dot size like 1D

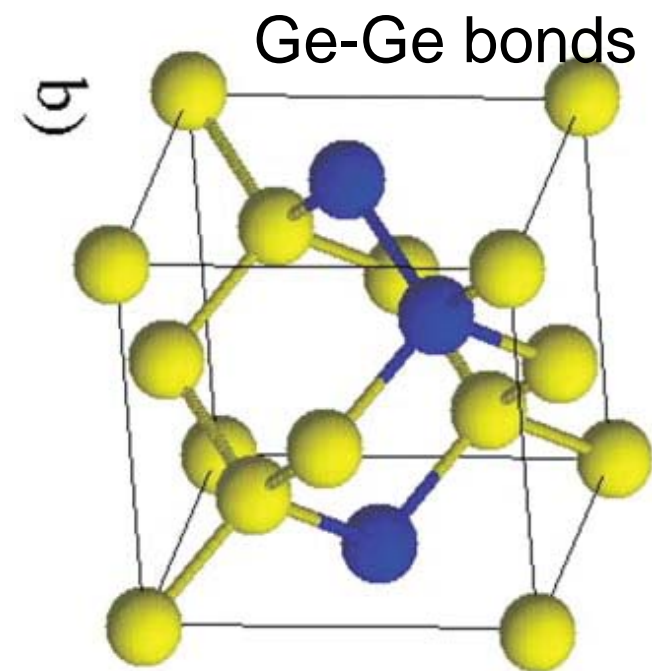
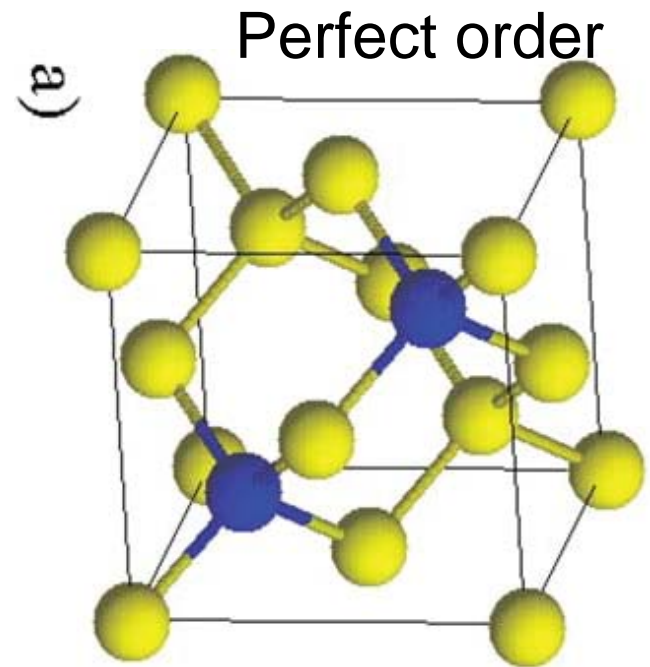
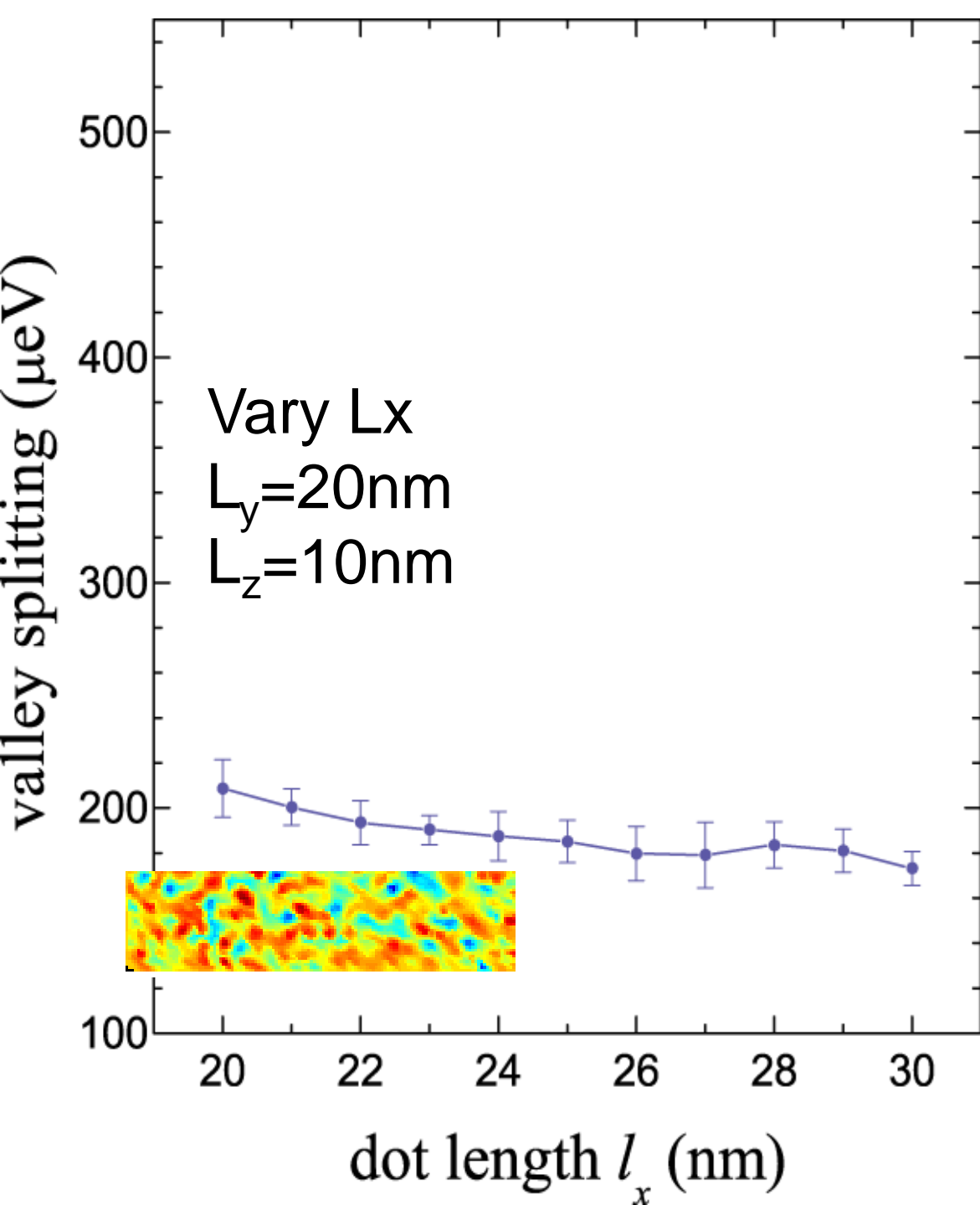
valley splitting ( $\mu\text{eV}$ )

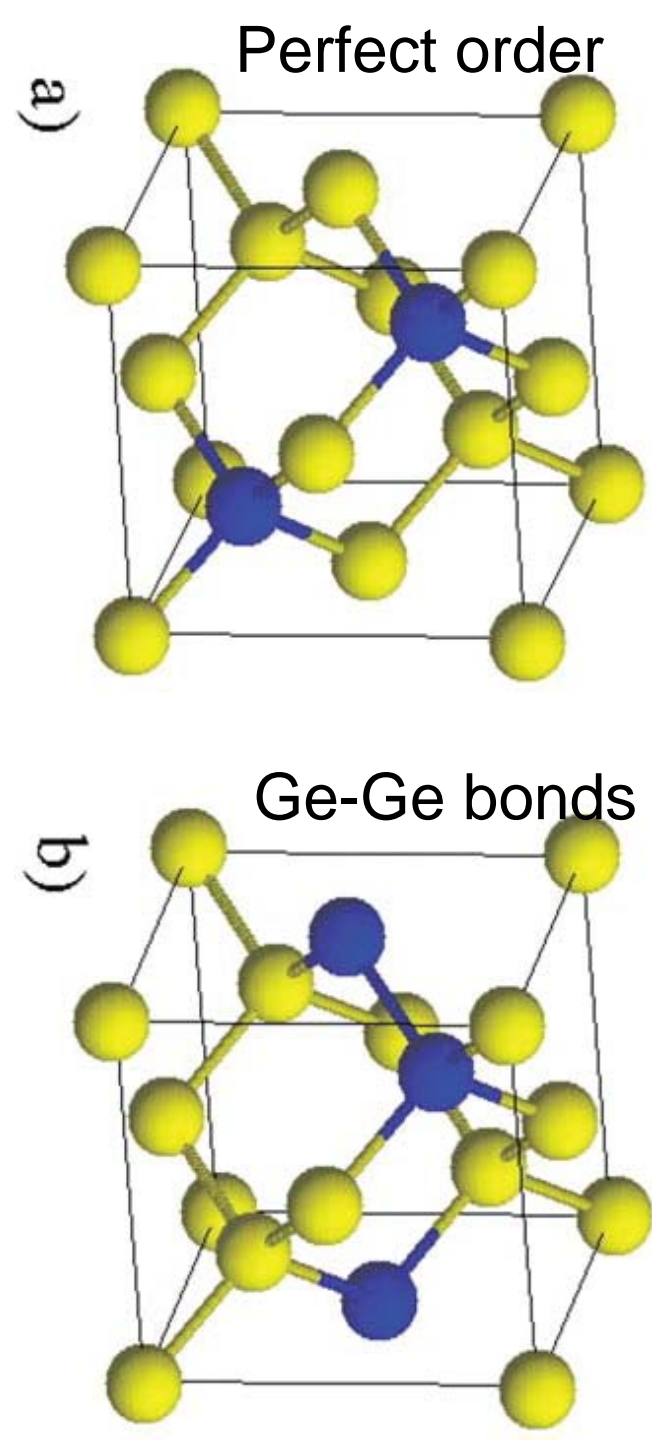
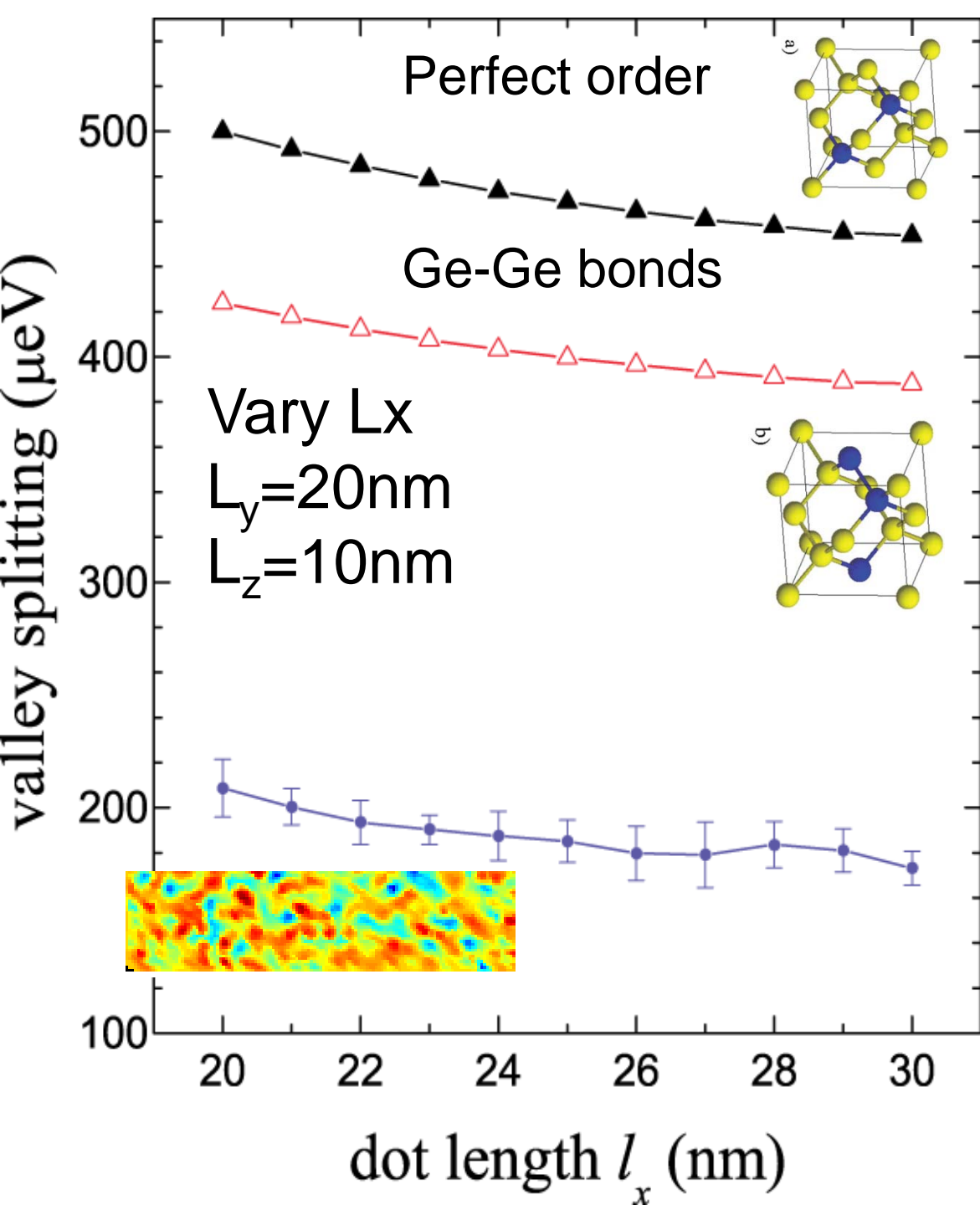


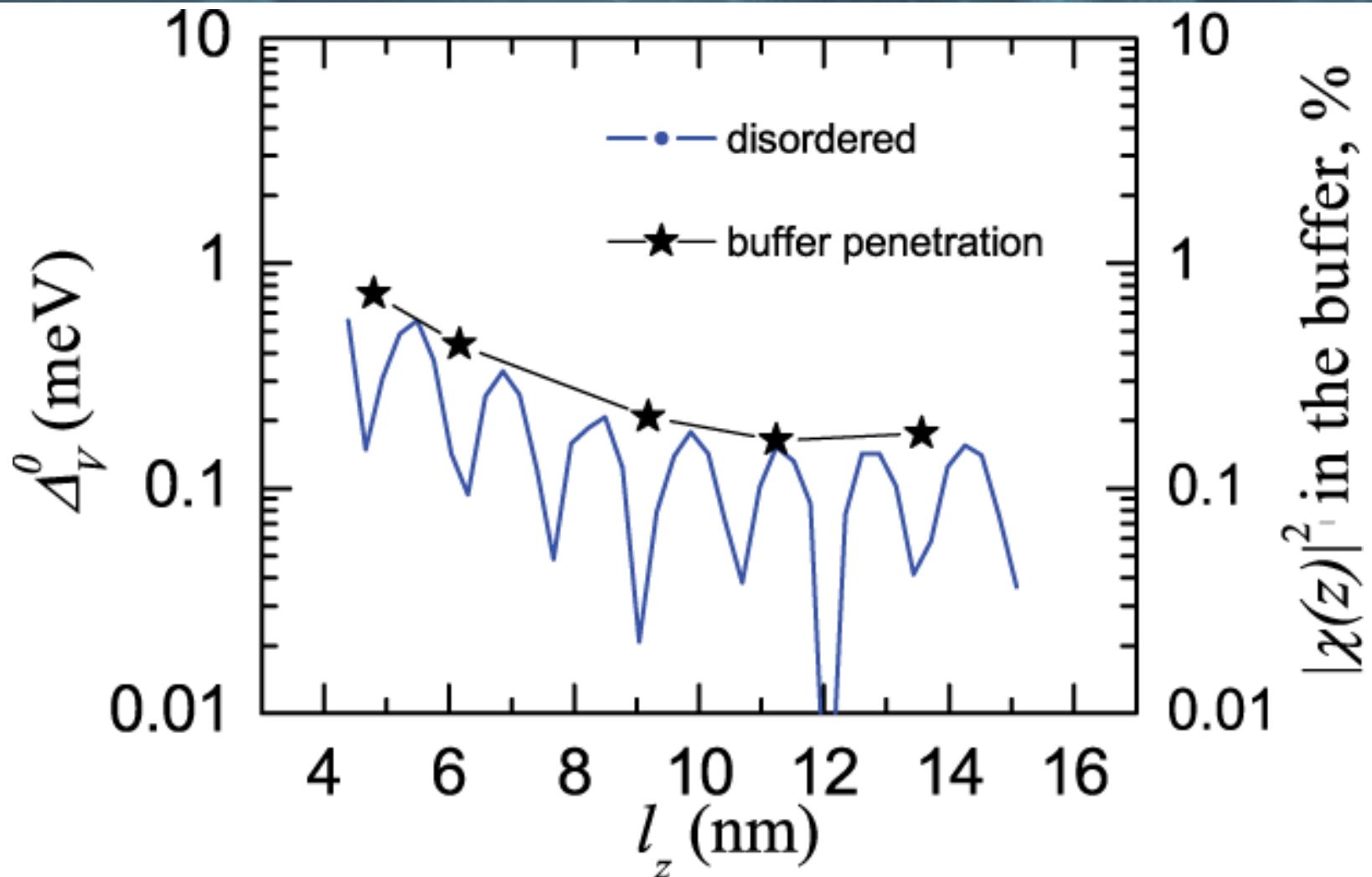




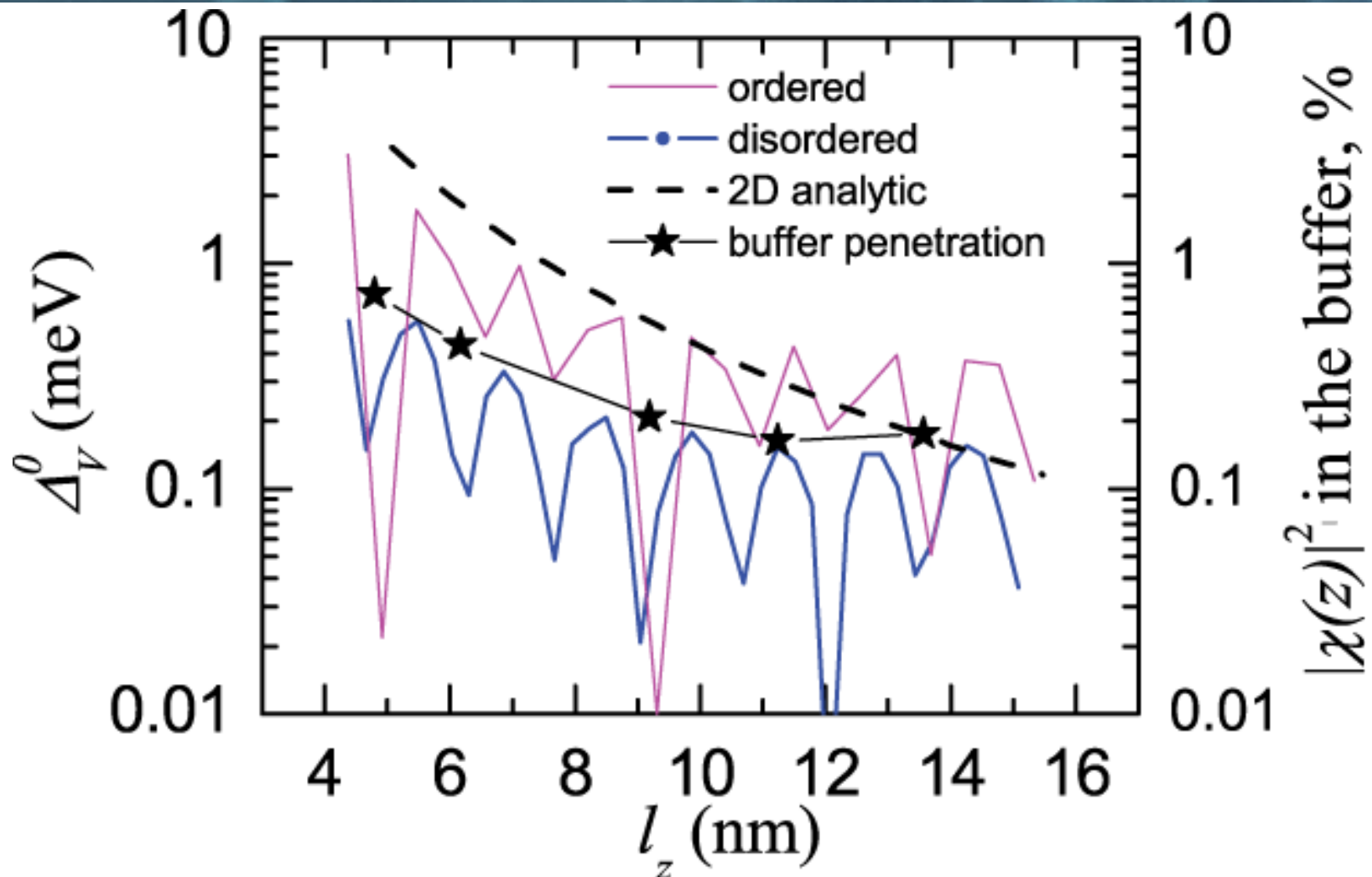








**Valley Splitting is an INTERFACE DOMINATED phenomenon**



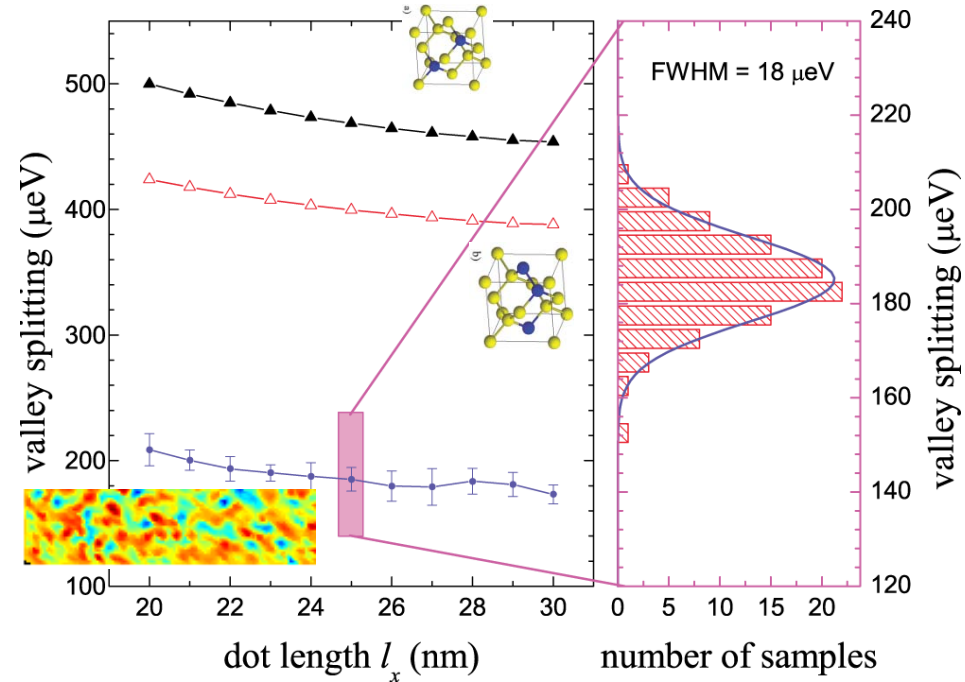
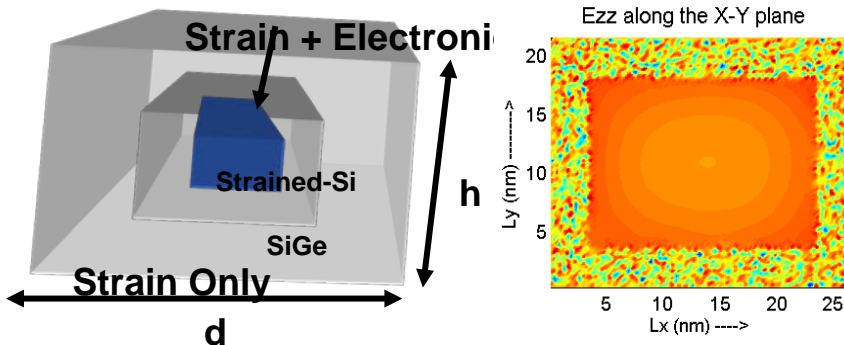
**Valley Splitting is an INTERFACE DOMINATED phenomenon**  
**Analytical formulas help, but cannot really guide the decision process**

## Objective:

- Prepare experiments (Rokhinson)
- Study realistic/large embedded Si QD
- Aid in understanding and design, buffer randomness, interfaces, Valley Splitting
- Explore size and disorder dependence

## Approach:

- No NEMO 3D modifications needed
- Strain penetrates 40nm into SiGe => 50 million atom strain sim..
- Typically 365.000 atoms in electronic domain



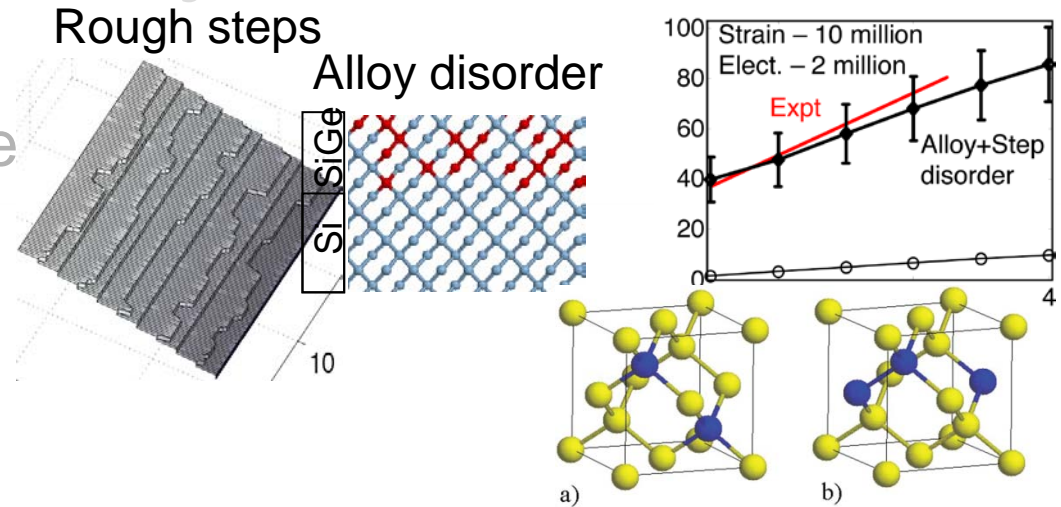
## Results / Impact:

- Fluctuations with quantum dot size like 1D in short dimensions
  - VS controllable in long dimensions
  - Disorder and Order matters
  - Analytical formulas cannot provide quantitative experimental guidance
- APL, Vol. 93, pg. 112102 (2008),



# Multi-Million Atom Simulation of Quantum States in Realistically Extended Devices in Silicon

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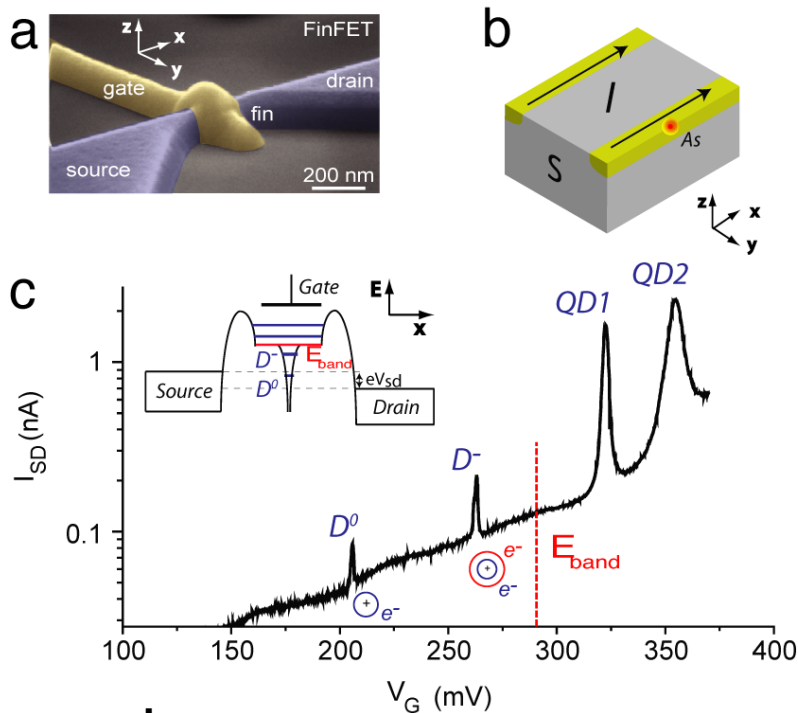


# Metrology with multimillion atom simulations

## Excited States are Critical!

### Objective:

- Support single impurity spectroscopy

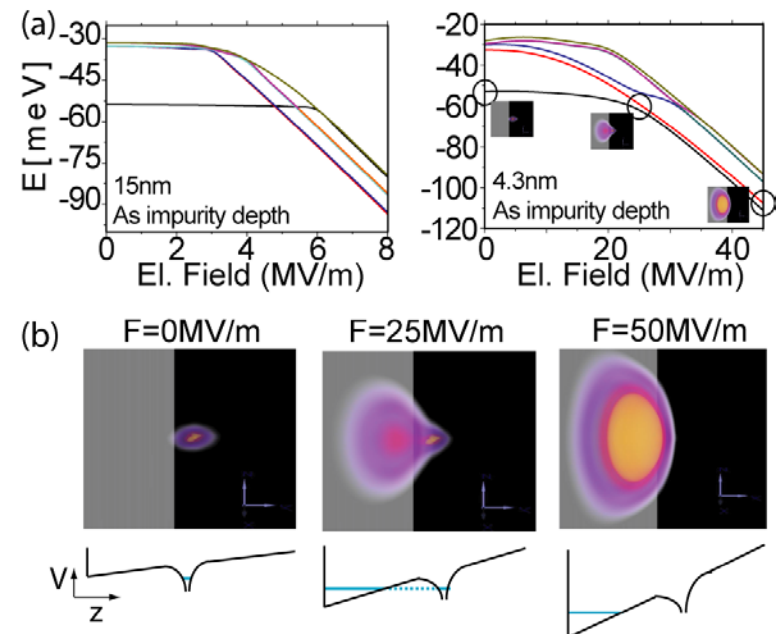
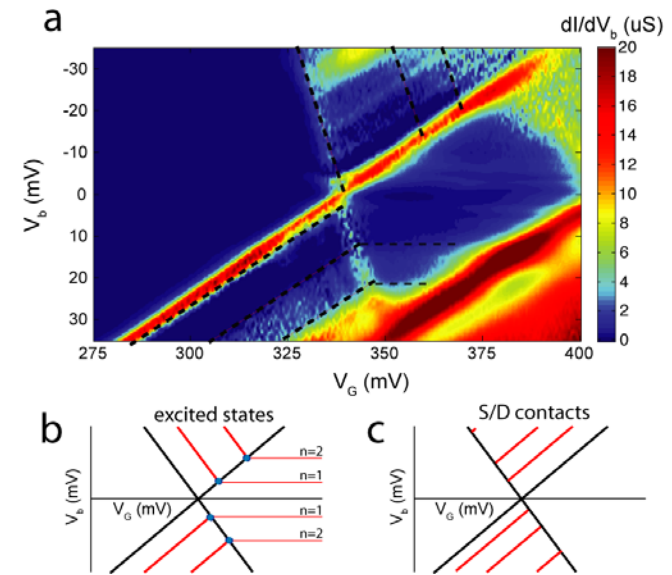


### Approach:

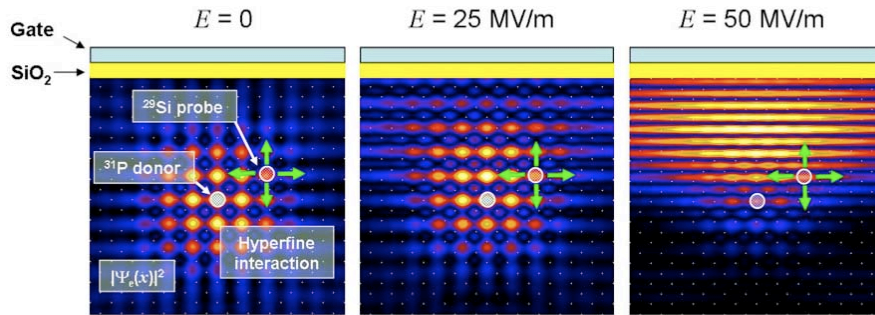
- Experiments & NEMO3D (on 3 continents!)

### Results / Impact:

- Identify Impurities as As not P
- Determine depth of impurities from interface
- Nature Physics, V4, p 656 (2008)



Need for high precision control of WF in QC



Hyperfine Interaction Tensor (A)

$$A_{ij} = \gamma_I \gamma_S \hbar^2 \left( \frac{8\pi}{3} |\Psi(0)|^2 \delta_{ij} + \left\langle \Psi \left| \frac{3x_i x_j - r^2 \delta_{ij}}{r^5} \right| \Psi \right\rangle \right)$$

$\gamma_{I,S}$  = gyromagnetic ratio,  $x_{ij} = (x, y, z)$

Originally measured in 1969

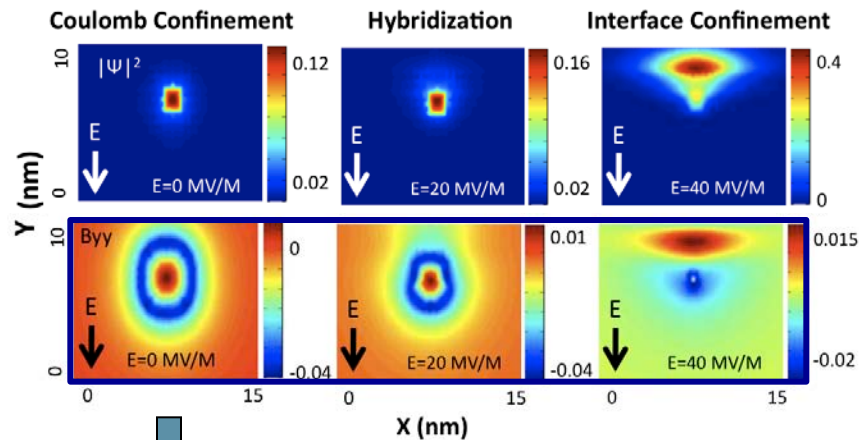
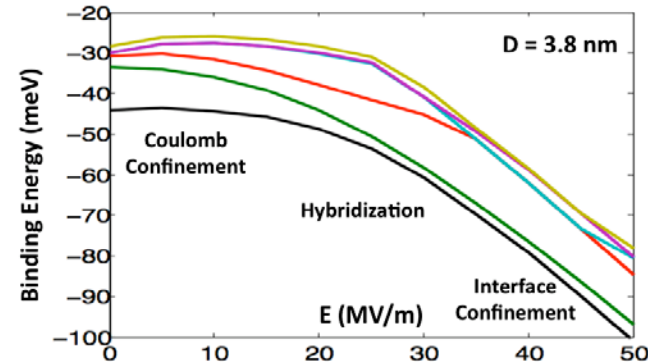
ESR Experiments can measure  $A_{ij}$

➔ Indirect measure of WF

Park, PRL (2009)

arXiv:0902.1515v1  
Gerhard Klimeck

Stark Shift Spectrum of a P donor at 3.8nm from an oxide interface in Si



Corresponding Hyperfine Maps in the forms of  $B_{yy}$  tensor component

## Objective:

- Study Stark Shift of the g-factor for an isolated donor w.r.t. expt in Si and Ge.
- g-factor shift for interface-donor system.
- Probes spin-orbit effects with electric fields and symmetry change.
- Effect of relative orientations of B and E field.

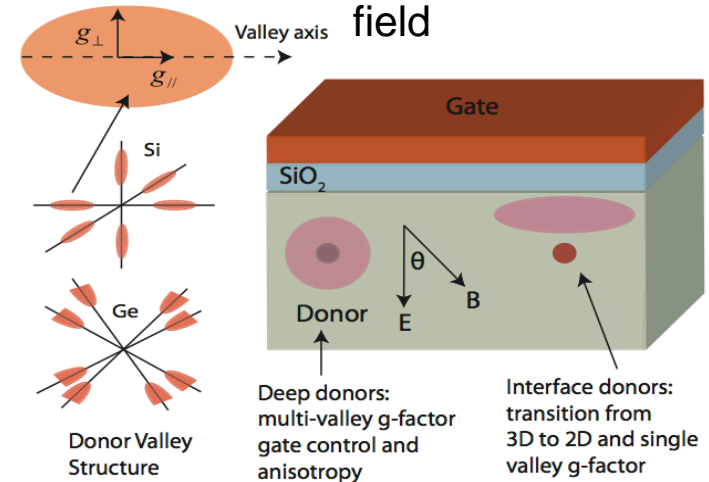
## Approach:

- The 20 band nearest neighbor  $sp^3d^5s^*$  spin model captures SO interaction of the host.
- Same atom p orbital energy between spin orbitals.
- g-factor obtained from orbital and spin angular momentum operators.
- Donor wfs with E-field are obtained from NEMO

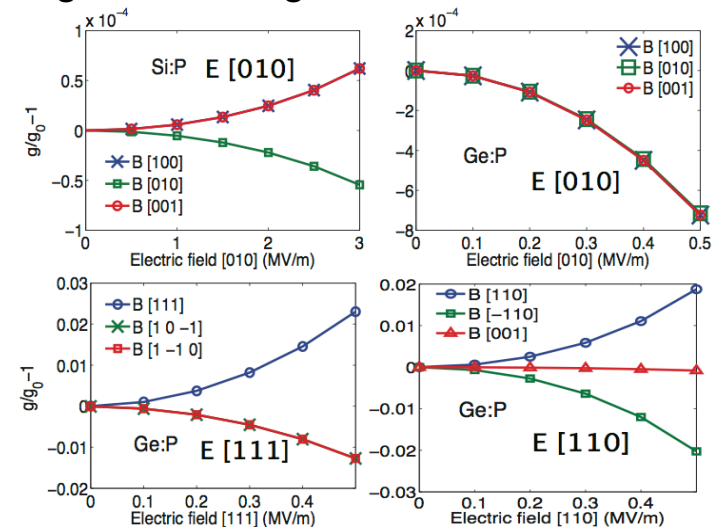
## Results / Impact:

- Quadratic Stark coefficient from TB agrees with measured value in magnitude.
- Relative E and B orientations seem to affect g-factor shift. Linear for shallow donors.
- Stark shift of Zeemann splitting may become comparable to hyperfine shift at modest B fields.
- Understanding of the B field dynamics of the donor electron.

## WF and g-factor of donor engineering by EM field



## Change of donor g-factor for various EM-field



R Rahman in Phys Rev. B (2009)

[arXiv:0905.3200v1](https://arxiv.org/abs/0905.3200v1)



## Objective:

- Investigate CTAP in realistic setting.
- Include Si full band-structure, TCAD gates, interfaces, excited states, cross-talk.
- Verify that adiabatic path exists: 3 donor device.

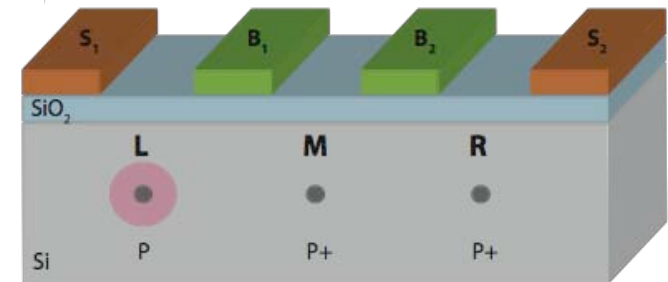
## Approach:

- TCAD gates coupled with a 3 donor TB. Hamiltonian to obtain molecular states.
- Simulate "small" 3 M atom device rapidly to demonstrate concept
- Compute time of 4-5 hours on 40 procs.
- Many bias configurations to fine tune gate voltages to explore the CTAP. regime.

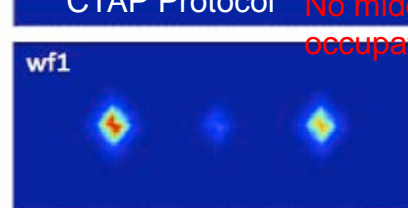
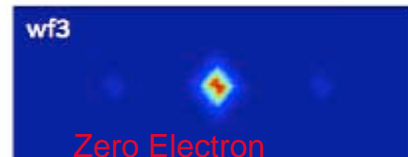
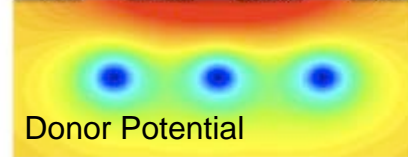
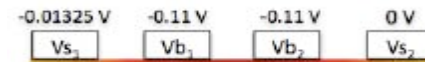
## Results / Impact:

- 15nm separated donors to explore rapidly => significant gate cross talk
- => Determination of gate bias control tedious
- Demonstrated that the CTAP regime exists for a 3 donor test device.

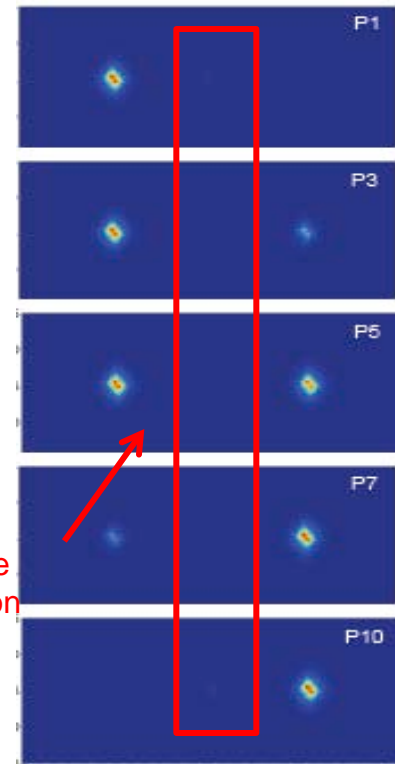
## Schematic of the CTAP device



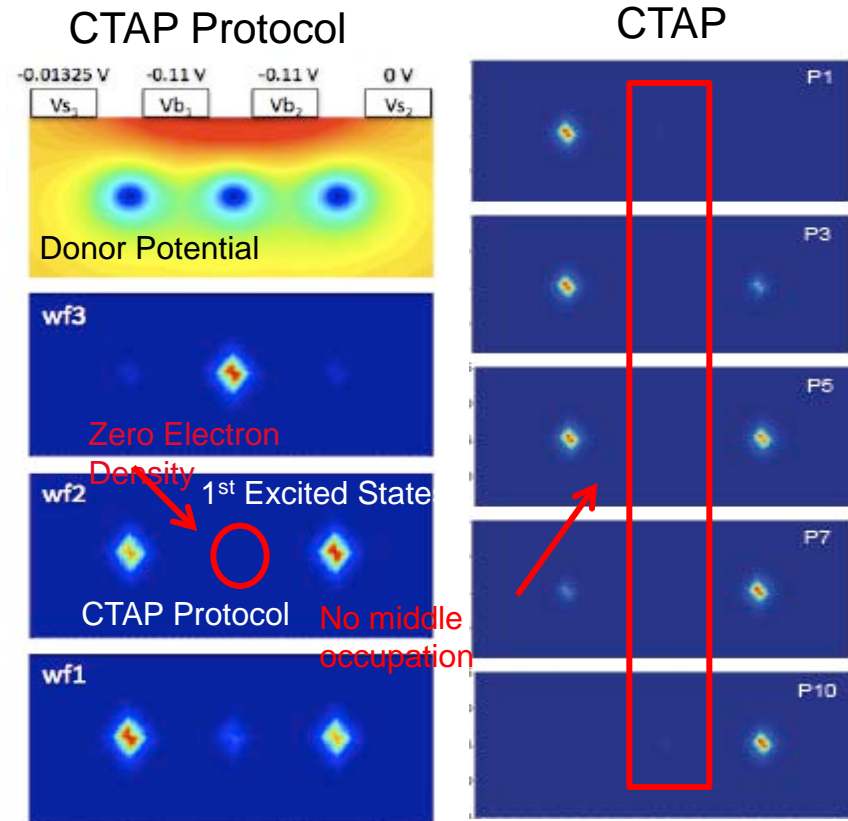
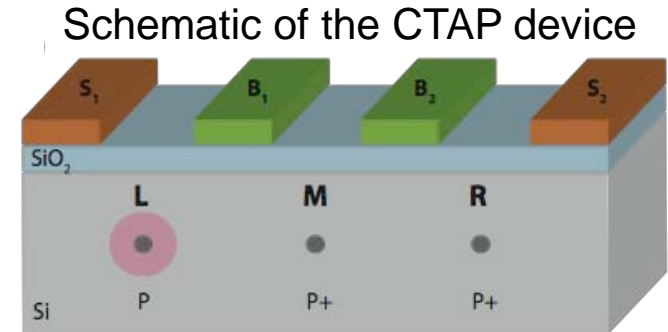
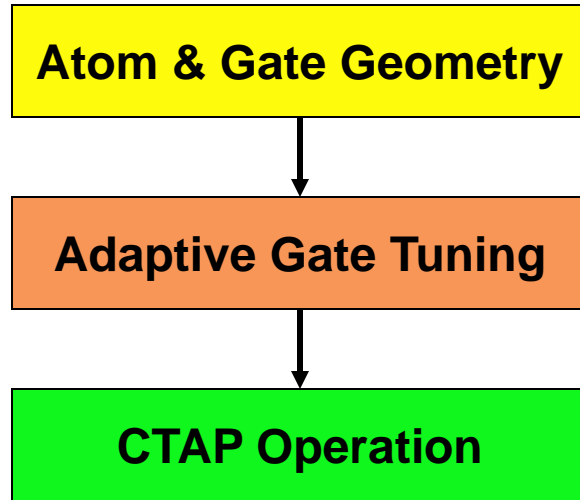
## CTAP Protocol



## CTAP







## Results / Impact:

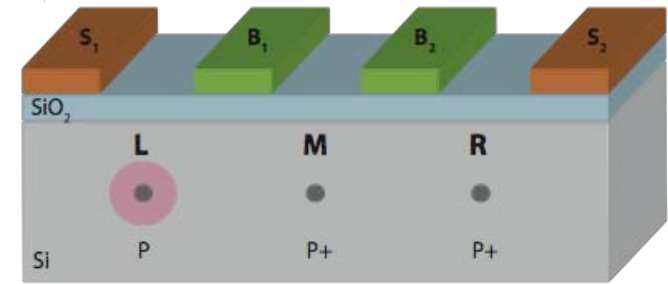
- 15nm separated donors to explore rapidly  
=> significant gate cross talk  
=> Determination of gate bias control tedious
- Demonstrated that the CTAP regime exists for a 3 donor test device.

Atom & Gate Geometry

Adaptive Gate Tuning

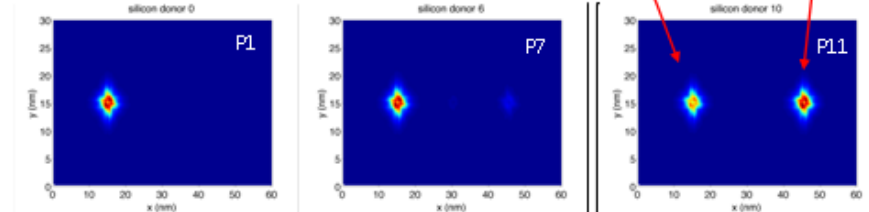
CTAP Operation

## Schematic of the CTAP device

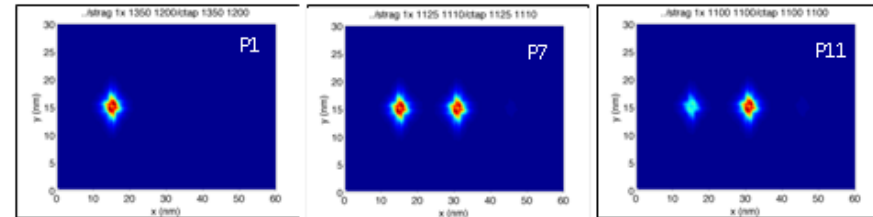


## NEMO3D Results

Unstraggled case: *Rahman, et al., PRB 80, 035302 (2009)*



Straggled case: 1 atomic site displacement in 100 direction (3 million atom calculation)



Next: project NEMO3D results on to 3x3 matrix and get  $V_{gate}$  dependence of elements

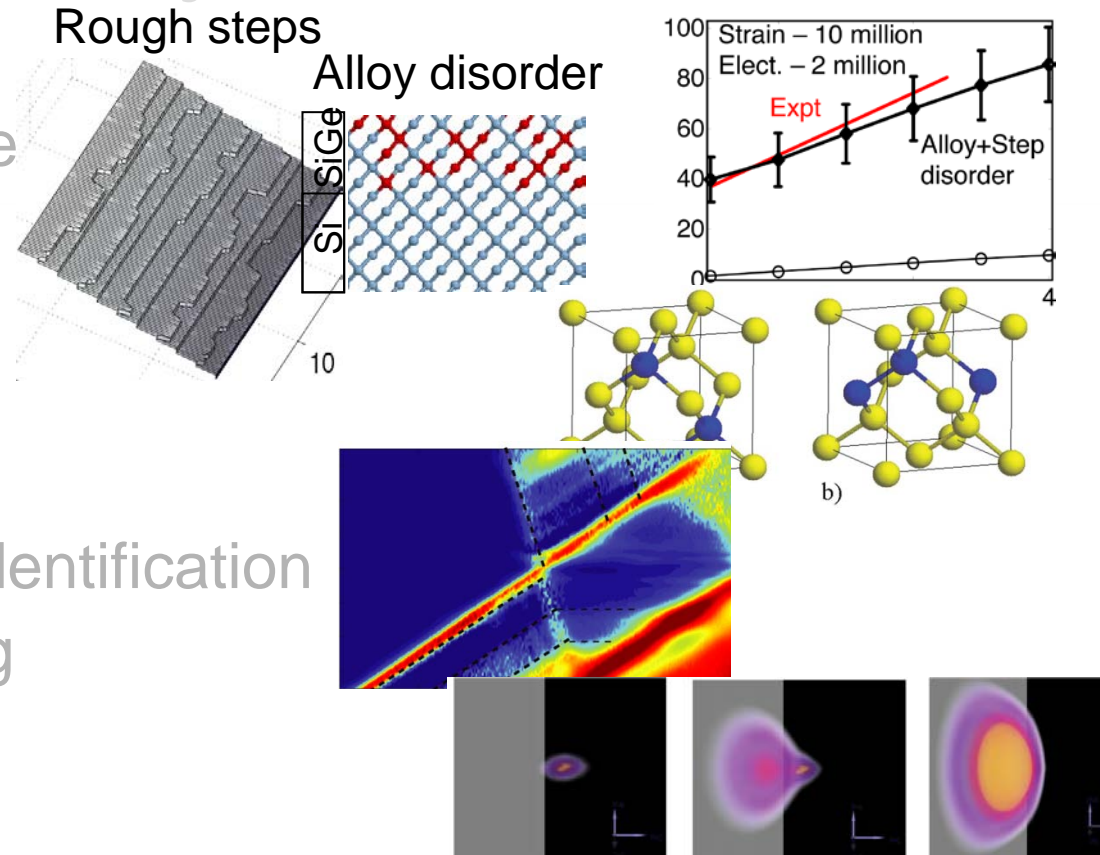
Sandia / Purdue / Melbourne

## Results / Impact:

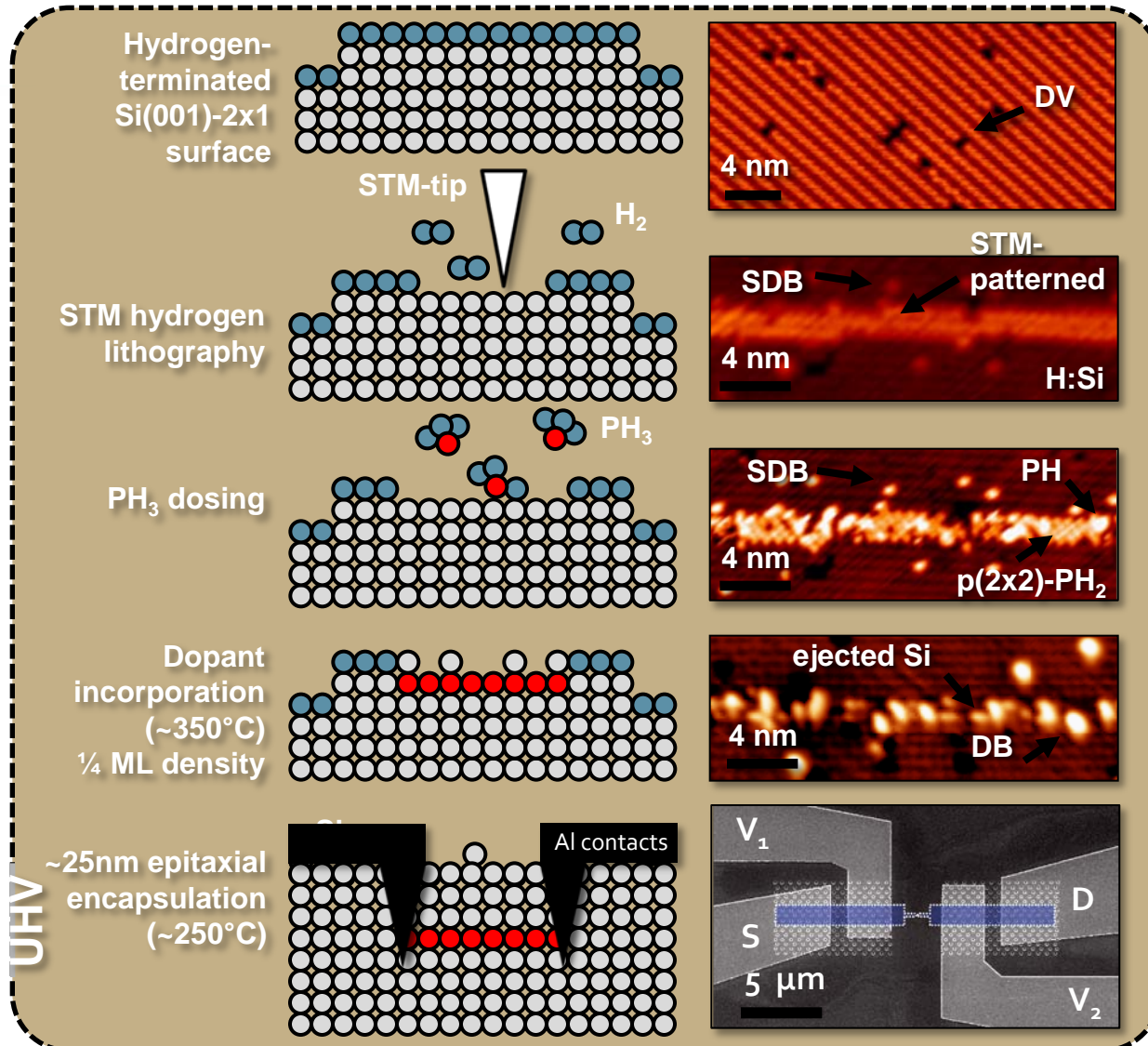
- 15nm separated donors to explore rapidly  
=> significant gate cross talk  
=> Determination of gate bias control tedious
- Demonstrated that the CTAP regime exists for a 3 donor test device.
- Initial straggled case in high cross talk regime indicates a different bias pathway
- Indicative of general need for individual characterization of qubit transport mechanisms
- Simulations enable development of bias control algorithms in actual experiments (e.g. charge sensing)

# Multi-Million Atom Simulation of Quantum States in Realistically Extended Devices in Silicon

- Motivation
- NEMO Numerical Engine
- Valley splitting
  - » Si quantum wells
  - » Si Quantum Dots
- Single impurities in Si
  - » Metrology – impurity identification
  - » Wavefunction mapping
  - » g-factor engineering
  - » CTAP
- Dense impurities in Si – Si:P
  - » Infinite sheets
  - » Infinite wires
  - » Gating, transport through wires
  - » Impurity-based quantum dots



# Atomic-Precision Doping by STM



Slide courtesy of  
Simmons Group



## Motivation:

- Understand available experimental data
- Guide experimental set-ups

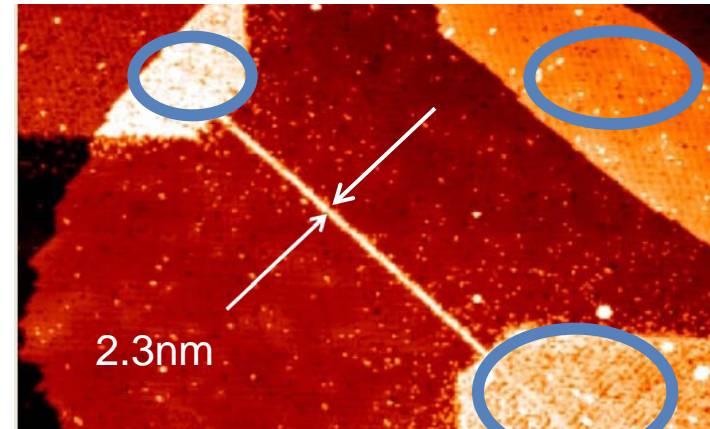
## Problems:

- Validate / confirm our approach to other theories
- Incorporate extended devices
- Understand metallic behavior
- Explain effects of
  - Doping density
  - Wire size / orientation
  - Electrical gating

## Approach:

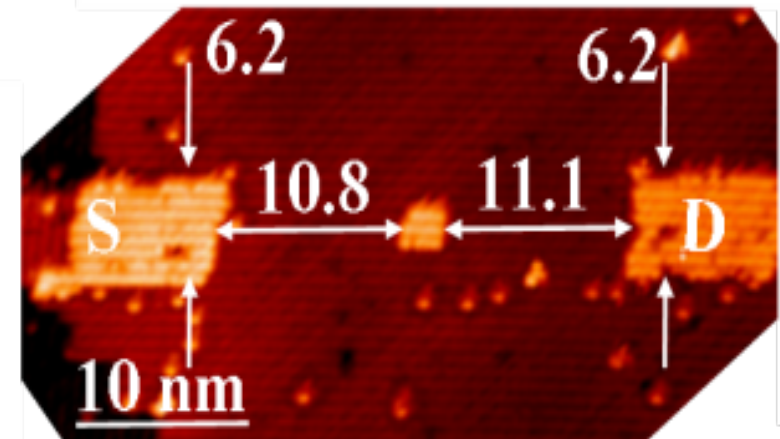
- NEMO 3D with tight binding
- Charge-potential self-consistency

## • P $\delta$ -doped Si (Si:P) Wire



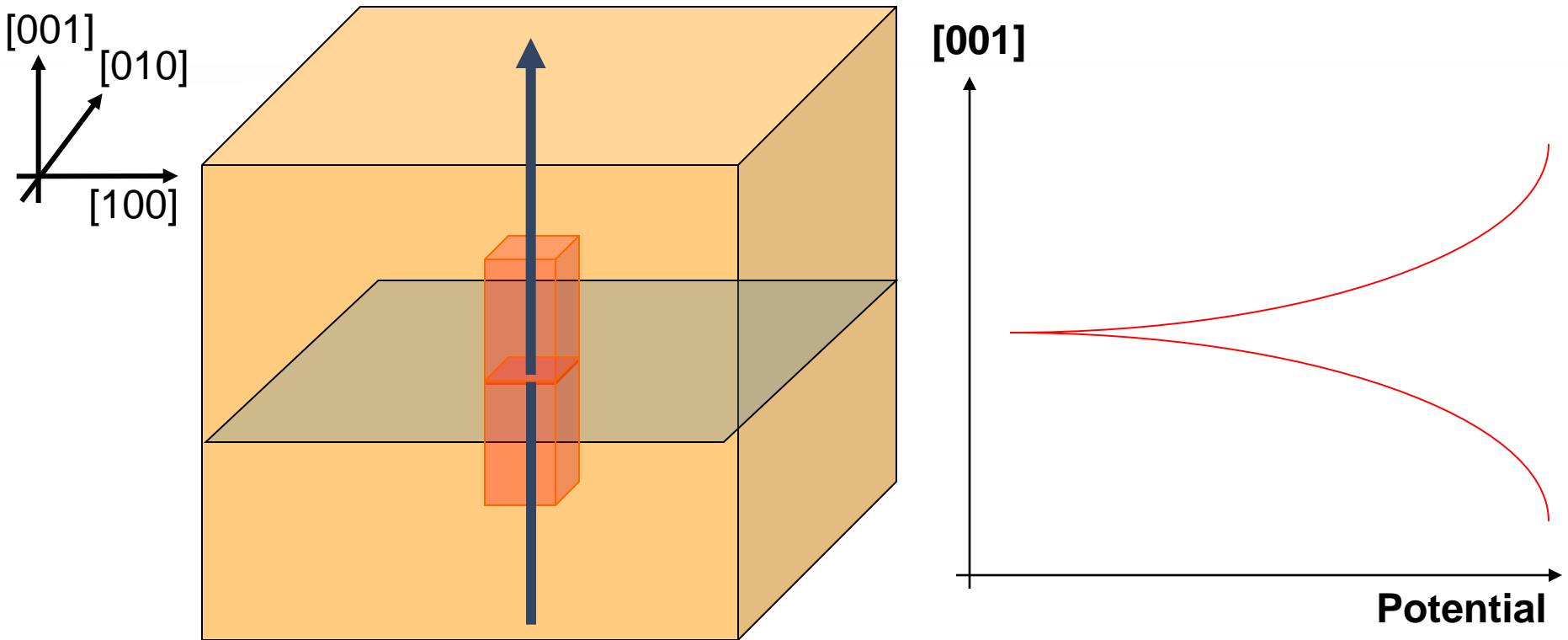
examine large contacts first

## • Si:P Planar quantum Dots



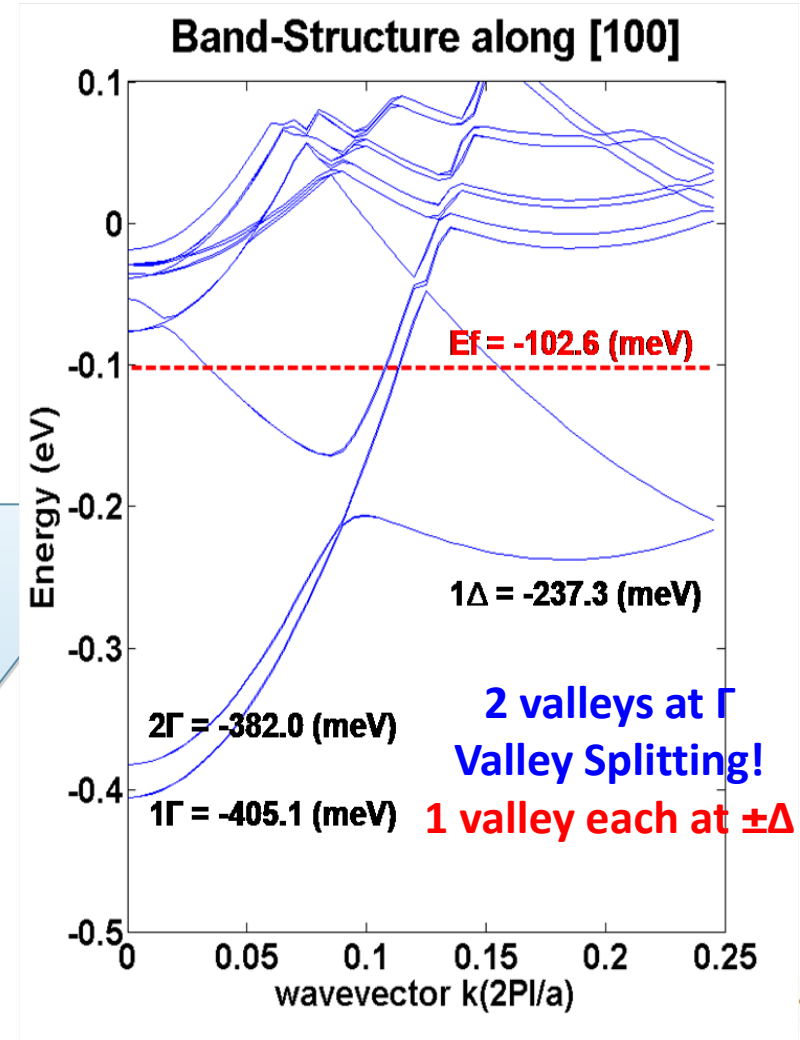
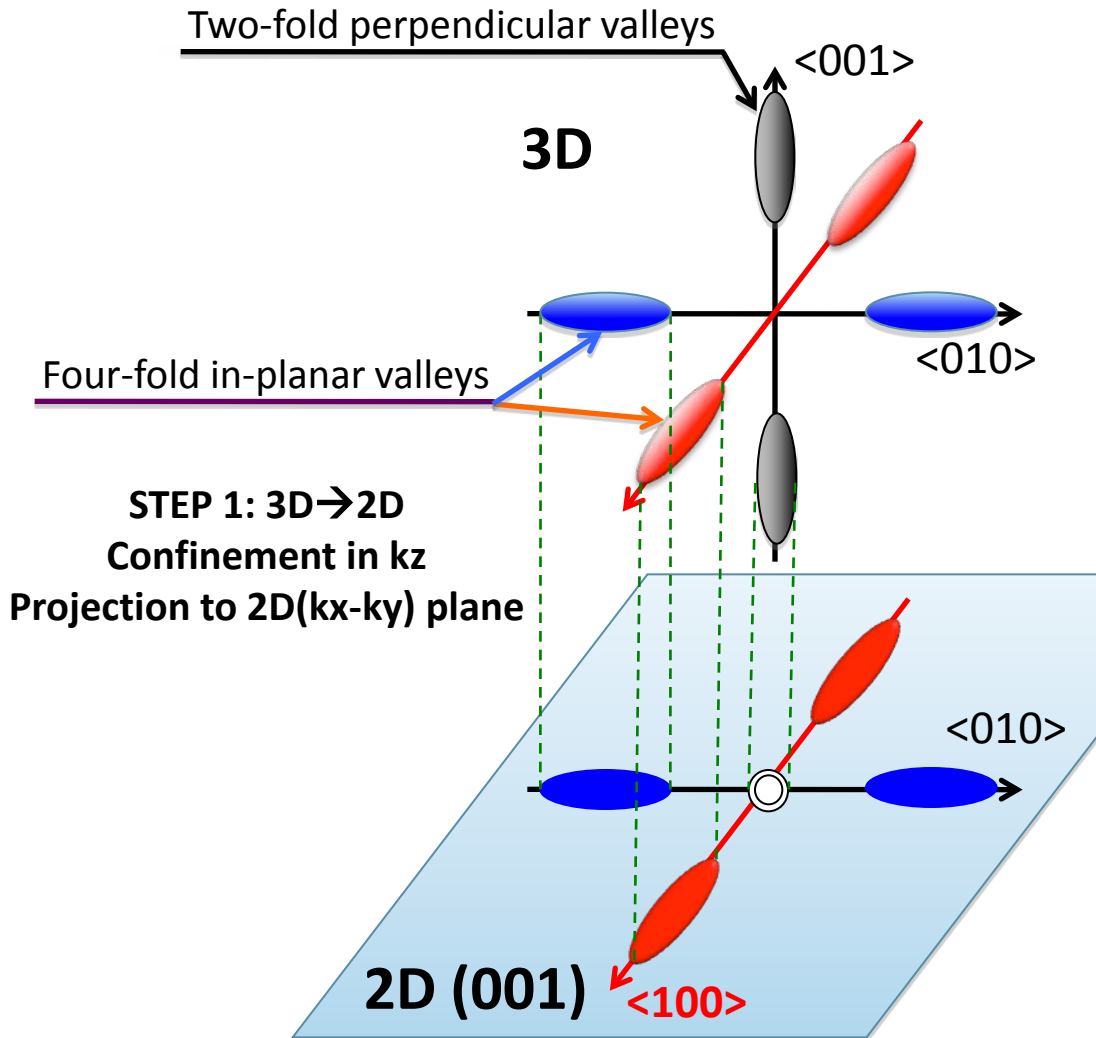


## Contact Regions - Infinite Square Well



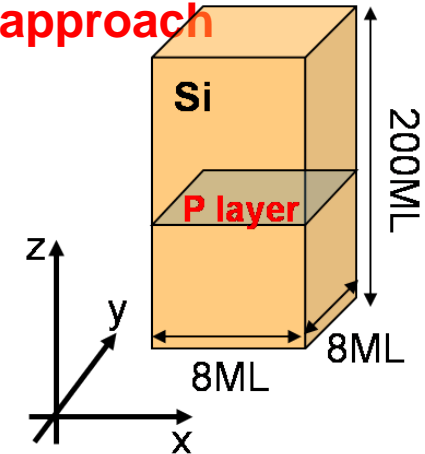
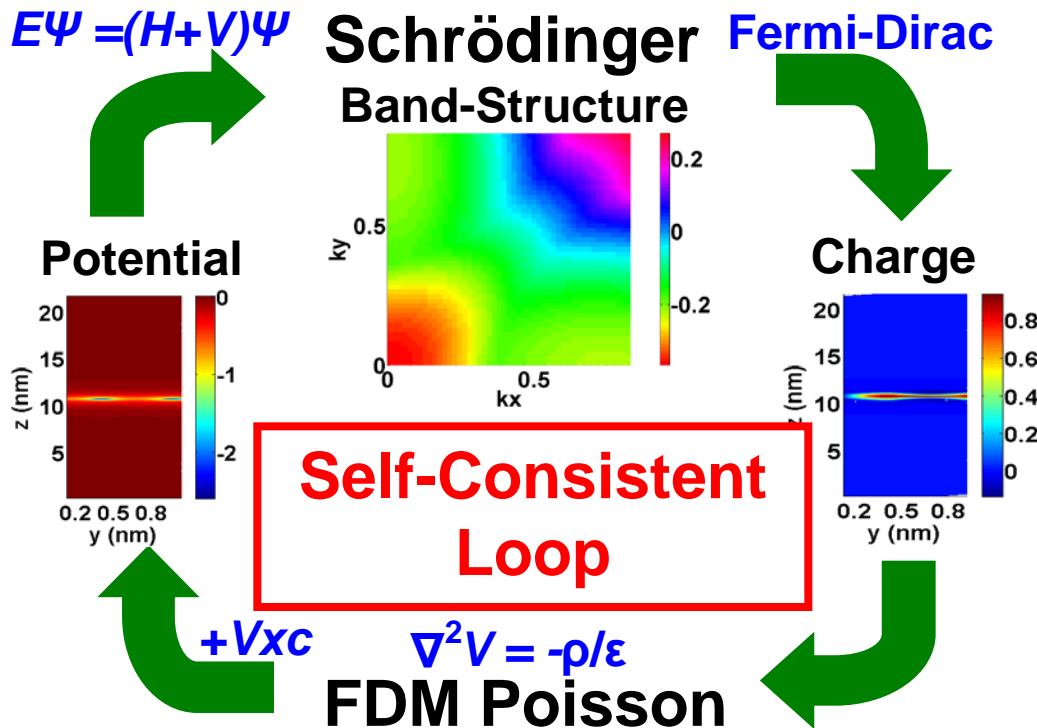
- Extremely thin P-doped layer in Si bulk.
- Confinement happens with impurity potential along [001].
- Need approximation for efficient modeling → Quantum Well Approach.
- Assume periodic boundary condition along [100]/[010]

- 3D→2D→1D projection of Si [100] nanowire



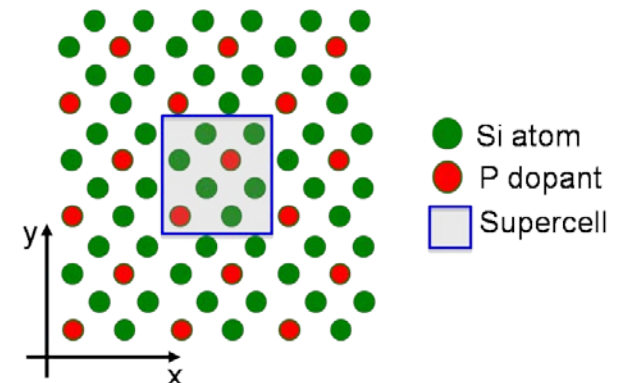
# Charge-Potential Self-Consistency

- ¼ ML Doping = 2 impurity ions in the super cell ~  $1.7 \times 10^{14}$  (cm<sup>2</sup>)
- Metal-like system : Charge-Potential Self-consistency
- **Schrödinger-Poisson + Exchange/Correlation with LDA approach**

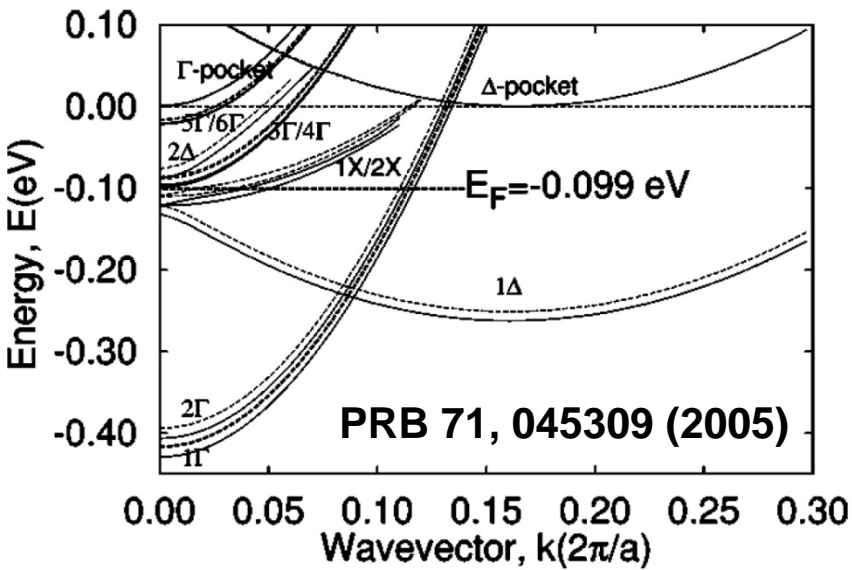
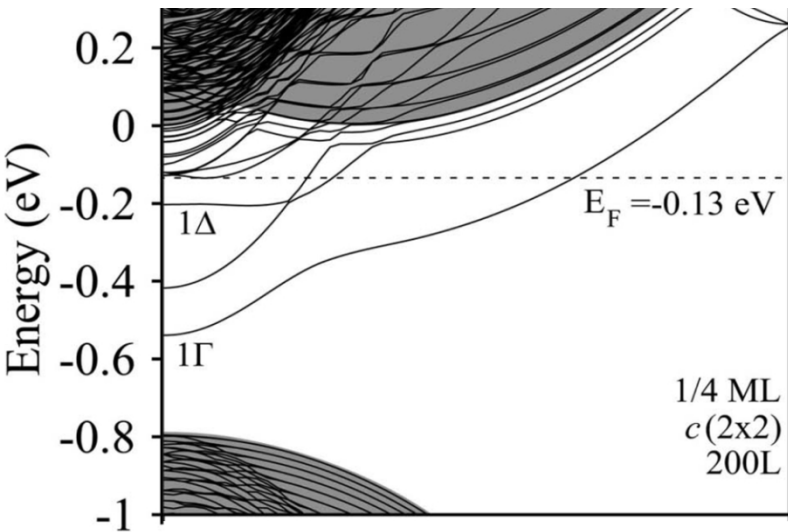


(Periodic along x, y and z)

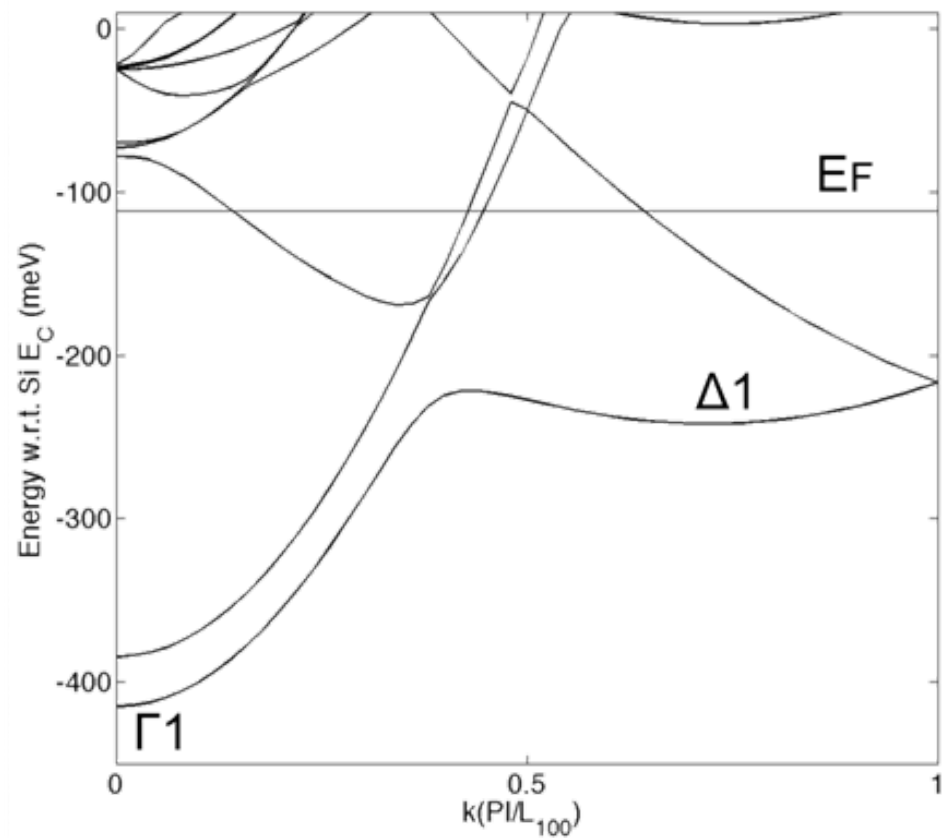
Atomistic Representation of the P  $\delta$ -layer

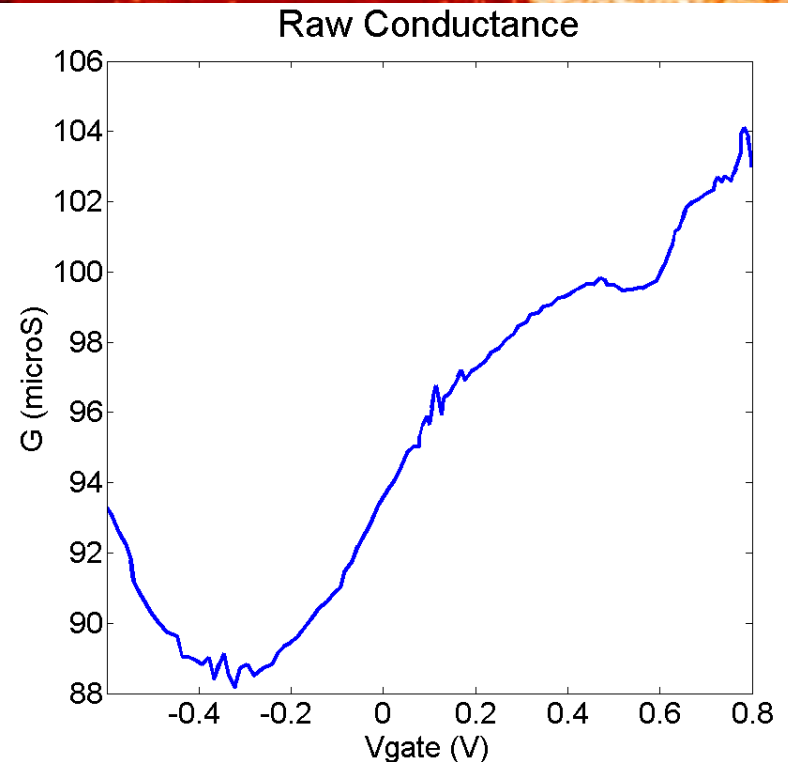
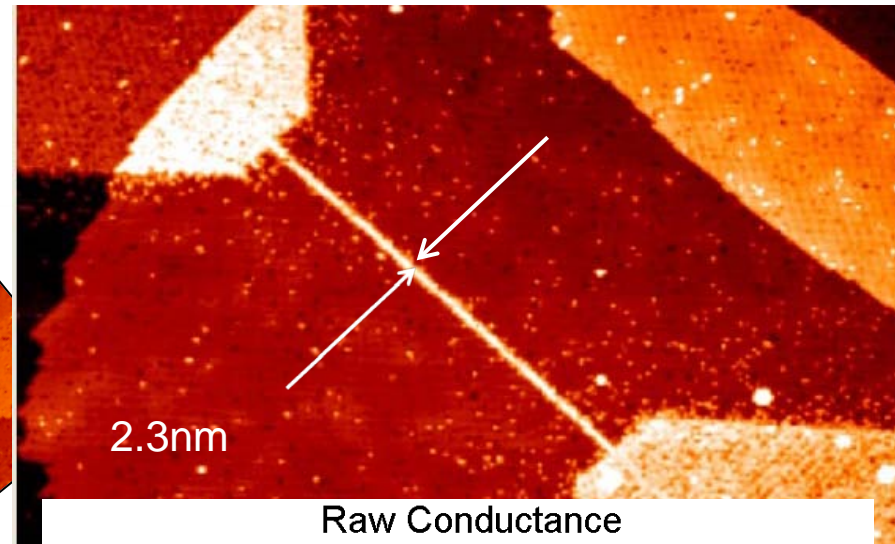
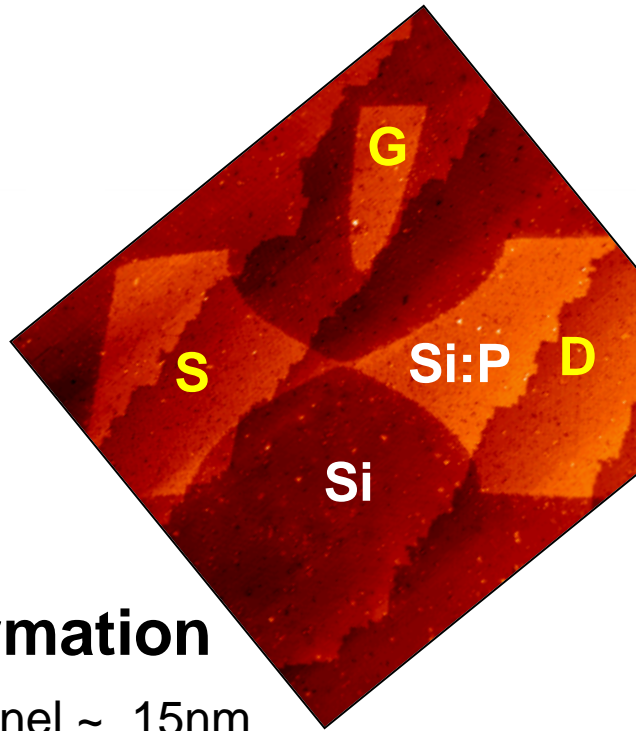


PRB 79, 033204 (2009)



Case	$\Gamma_1$ (meV)	$E_F$ (meV)
Carter <i>et al</i>	- 540	- 130
Qian <i>et al</i>	- 410	- 99
Ours	- 415	- 111





## Device Information

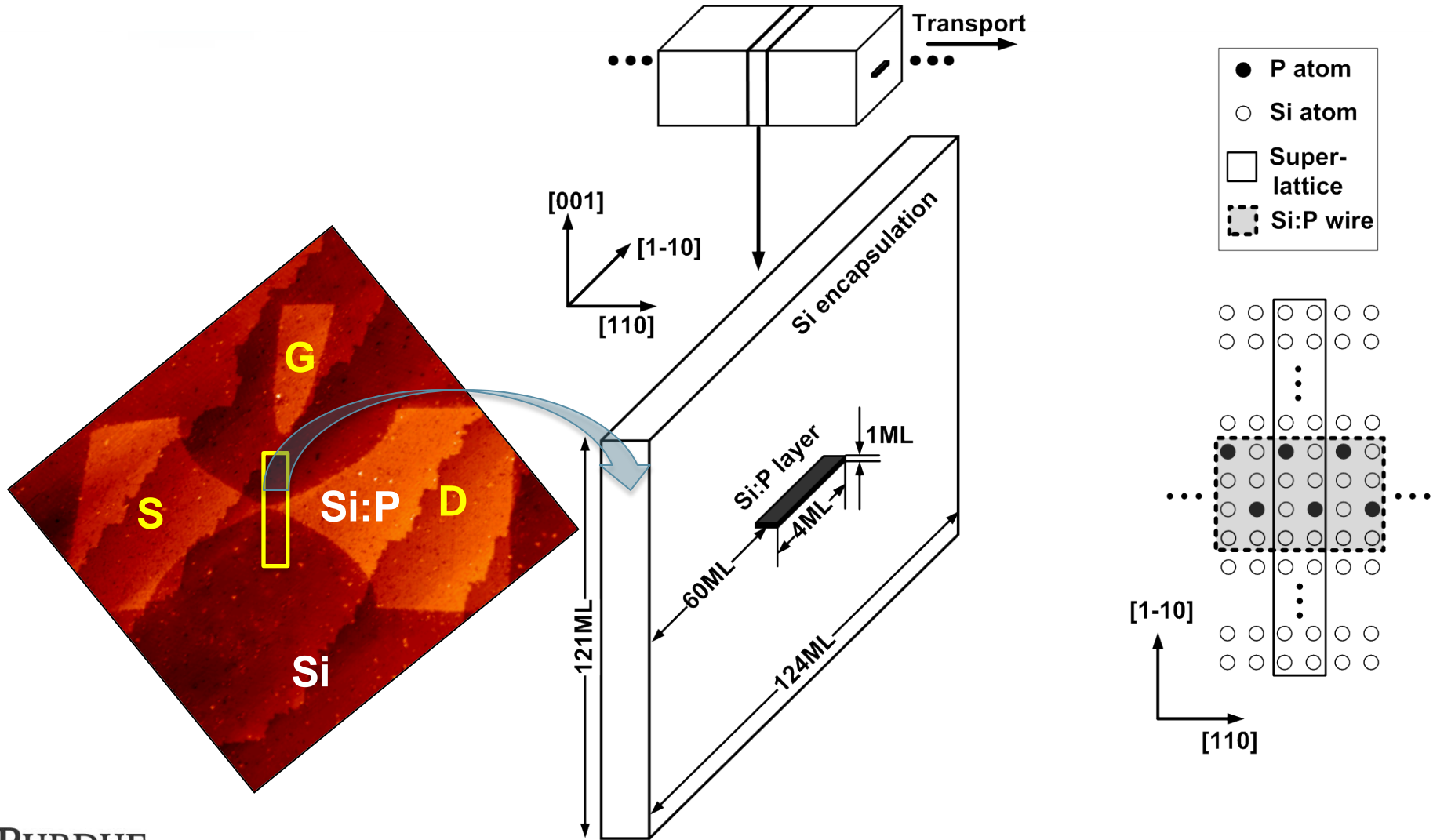
- Effective Channel ~ 15nm
- From Gate to Channel ~ 35nm
- T~ 1.2(K). Doping Constant ~ 1/4ML

## Problem to be solved

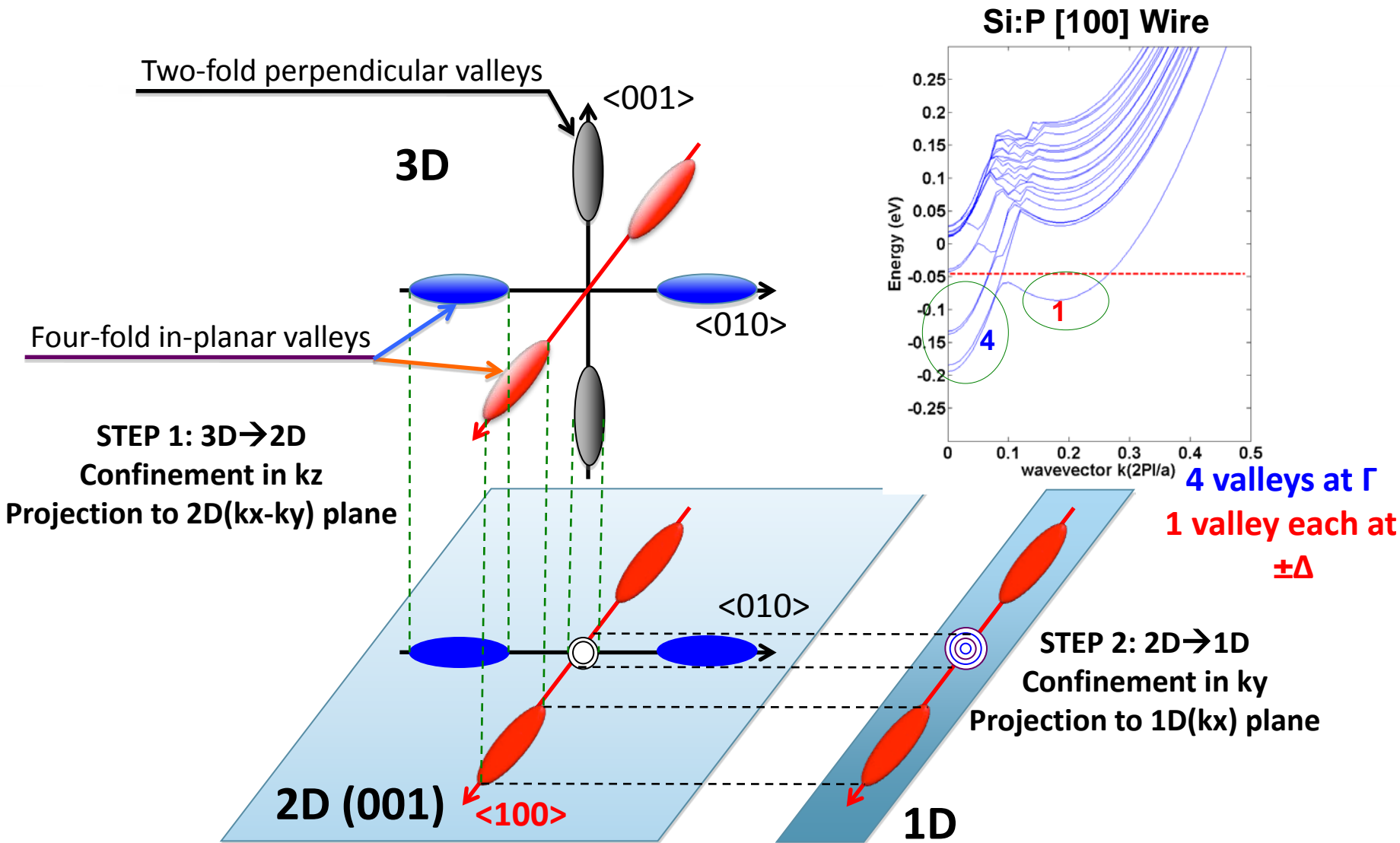
- Gate-dependent S-to-D Conduction Behavior
- Conduction through Donor-bands



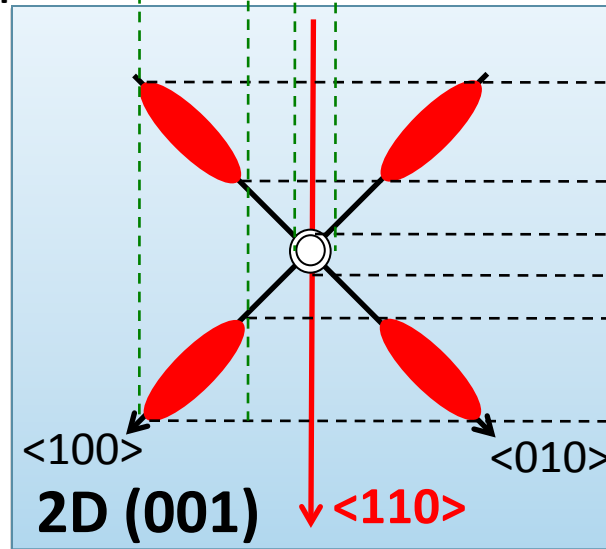
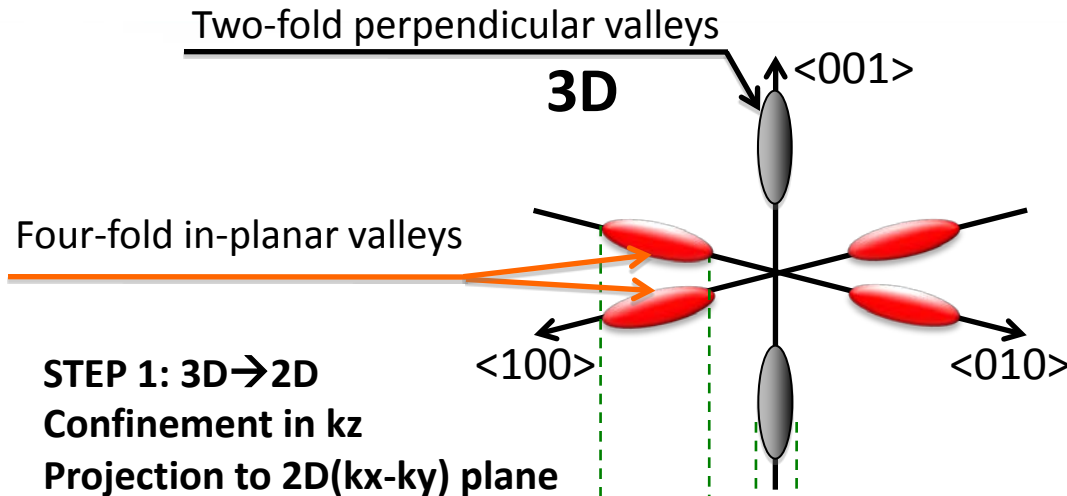
**Need to reduce computational domain size.**



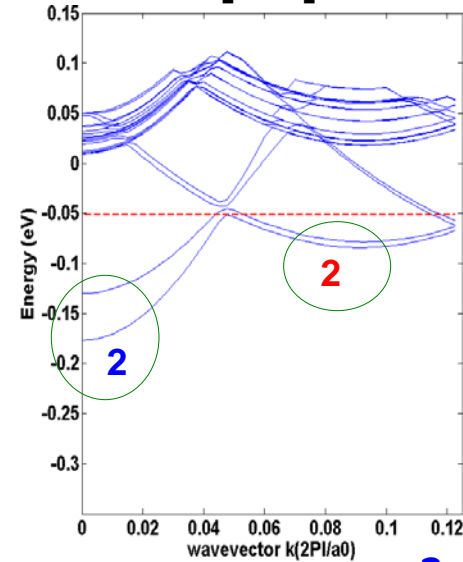
- 3D → 2D → 1D projection of Si [100] nanowire



- 3D→2D→1D projection of Si [110] nanowire

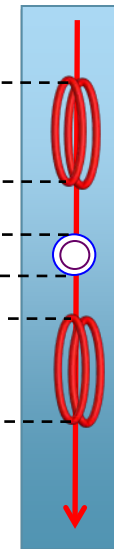


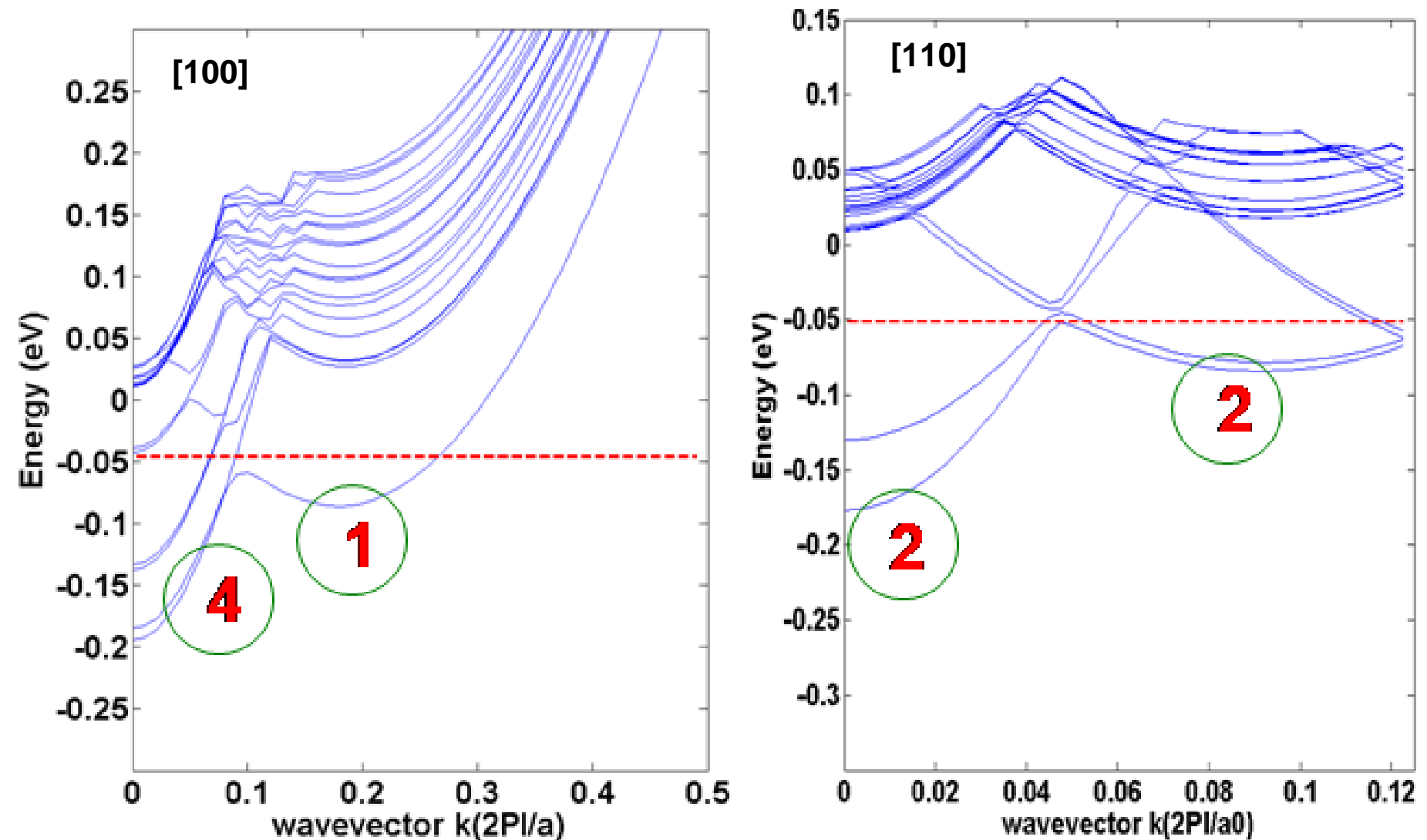
Si:P [110] Wire



2 valleys at  $\Gamma$   
2 valley each at  $\pm\Delta$

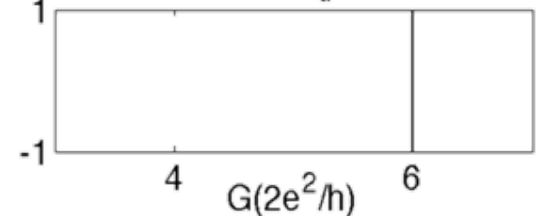
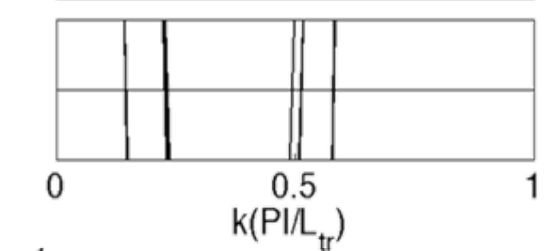
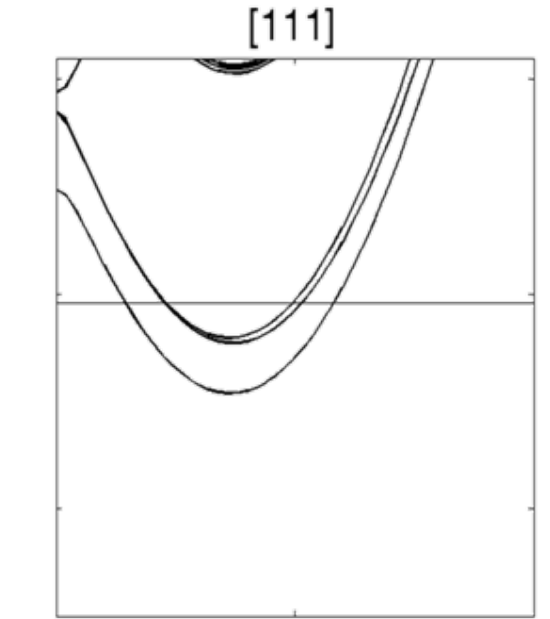
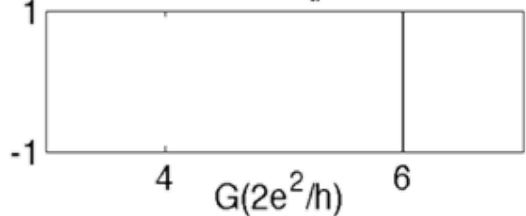
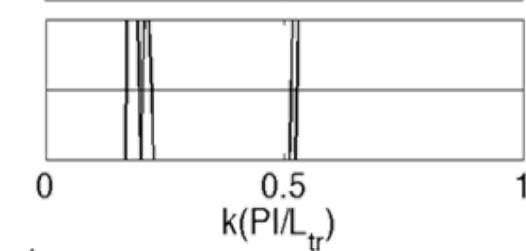
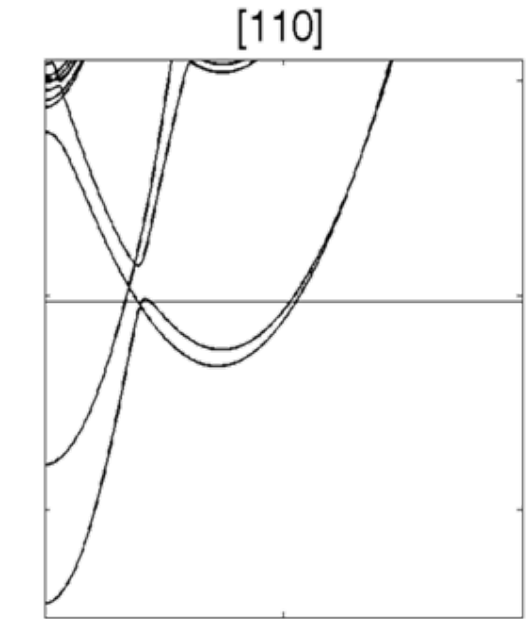
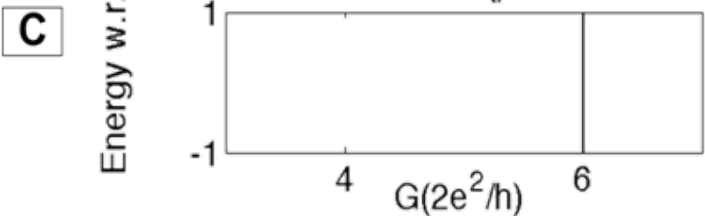
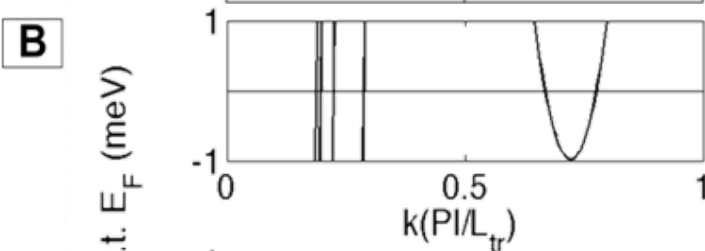
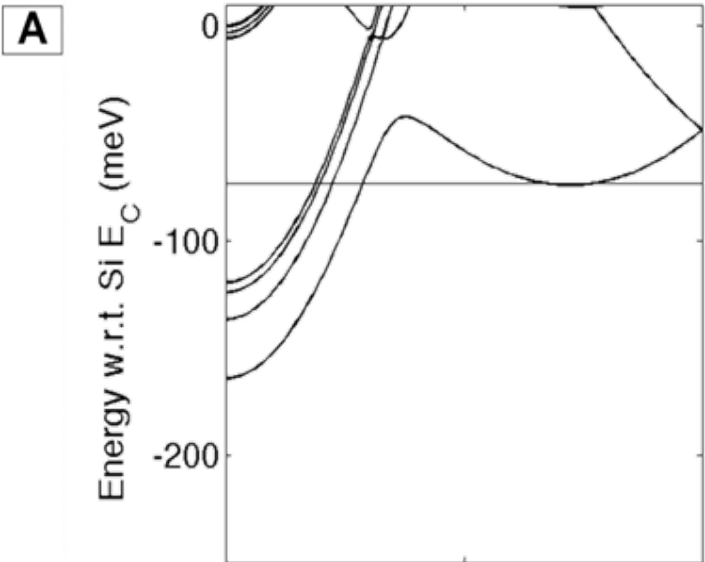
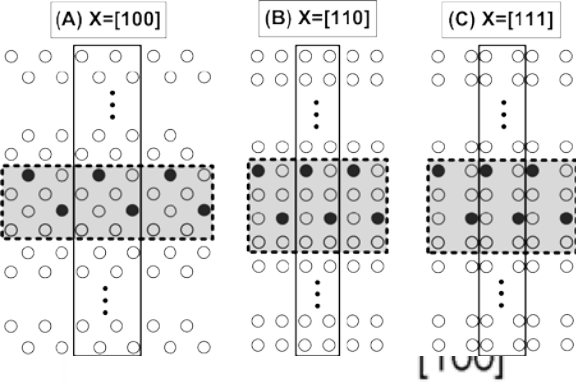
STEP 2: 2D→1D  
Confinement in  $k_y$   
Projection to 1D( $k_x$ ) plane





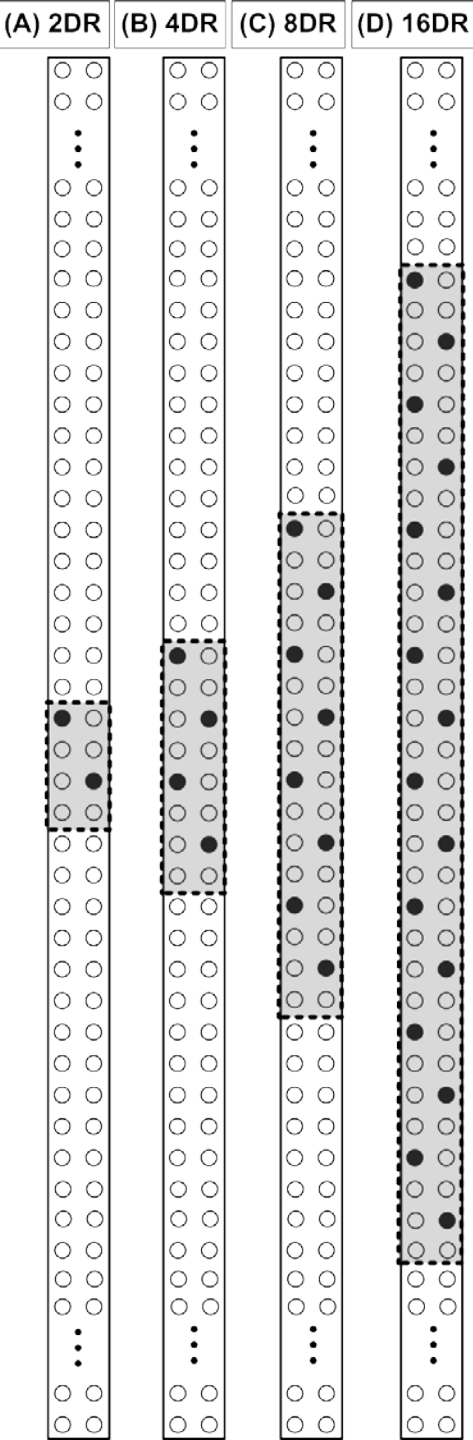
Wires are metallic with:  
 $G(\text{Ballistic}) \sim 6(2e^2/h)$

# Crystal direction Dependence [100],[110],[111]

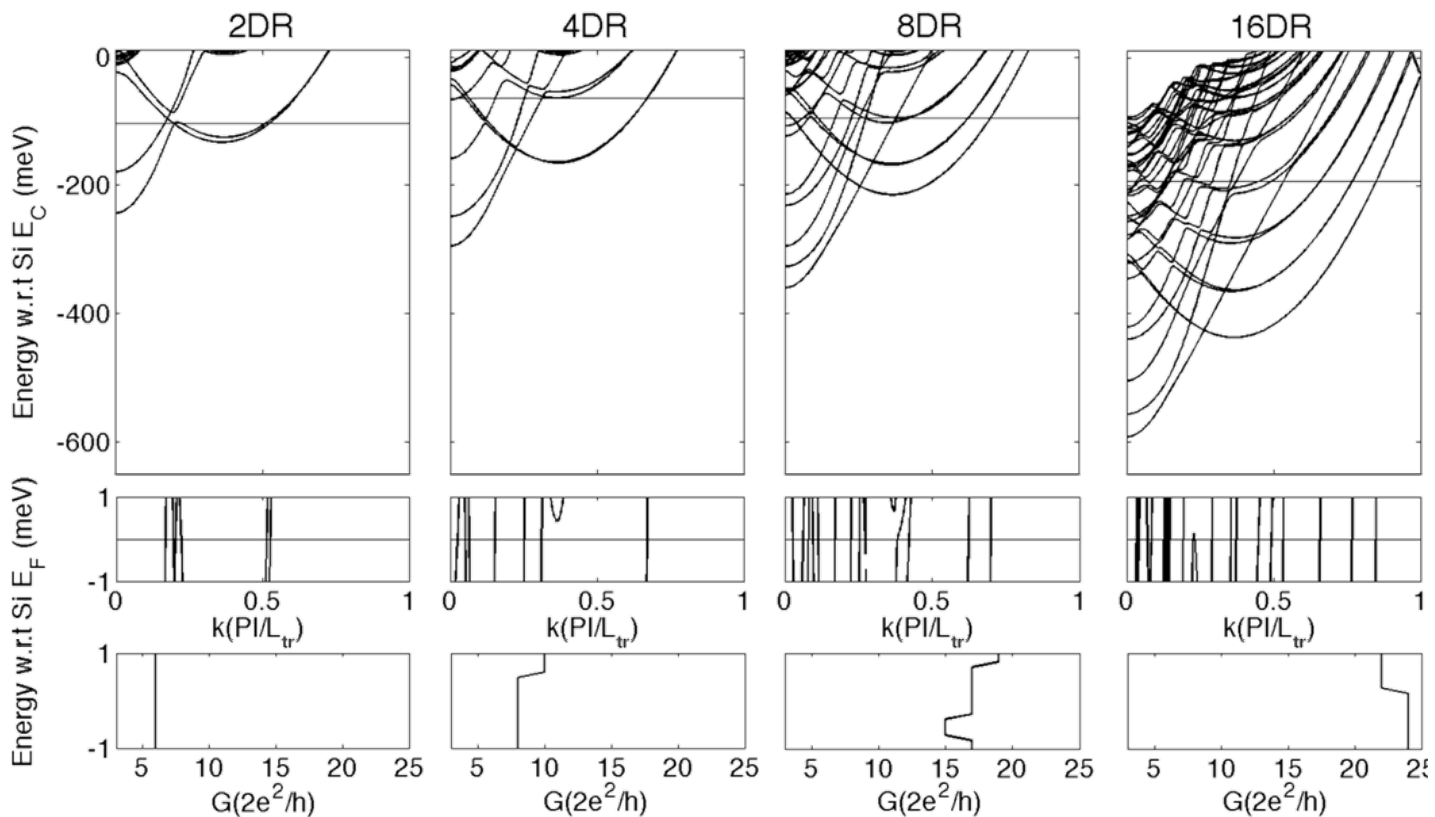




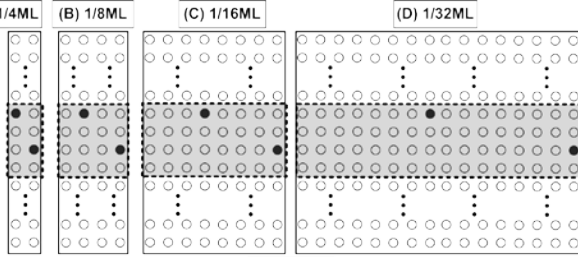
# Width Dependence



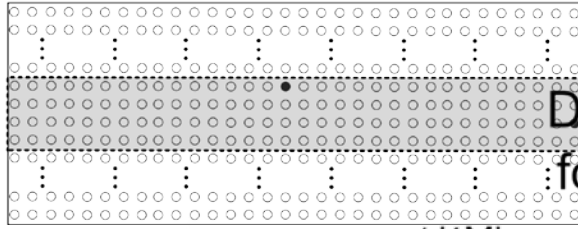
Dependence of bandstructures on width  
for [110] transport and 1/4ML doping



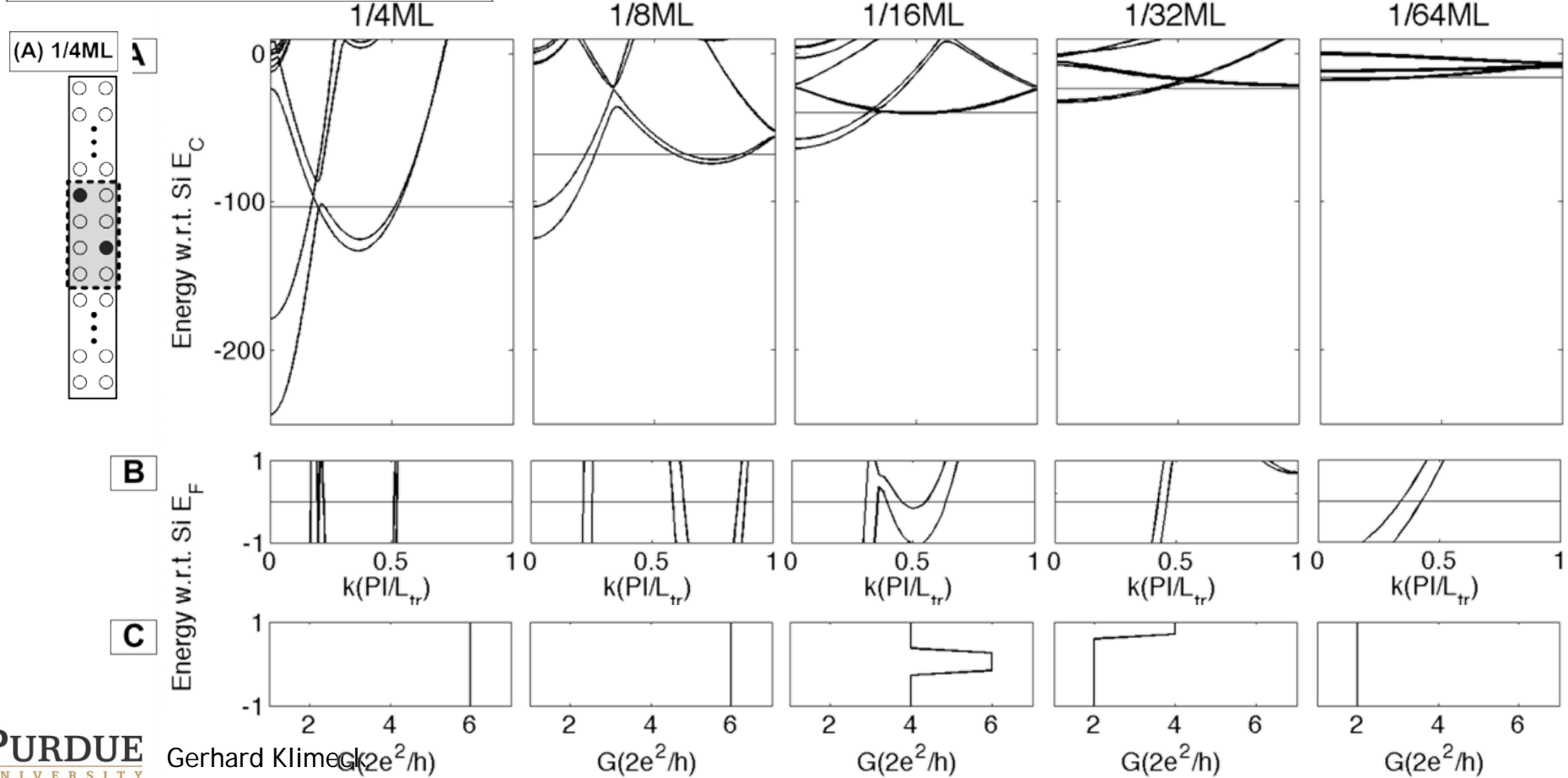
# Doping Dependence



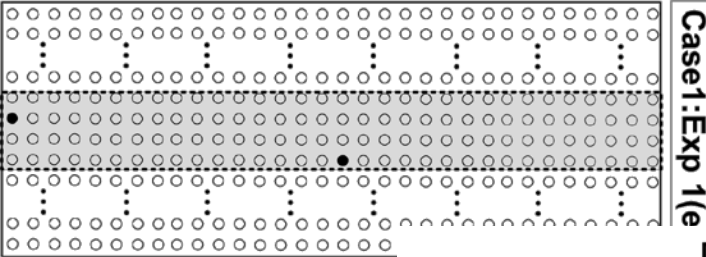
(E) 1/64ML



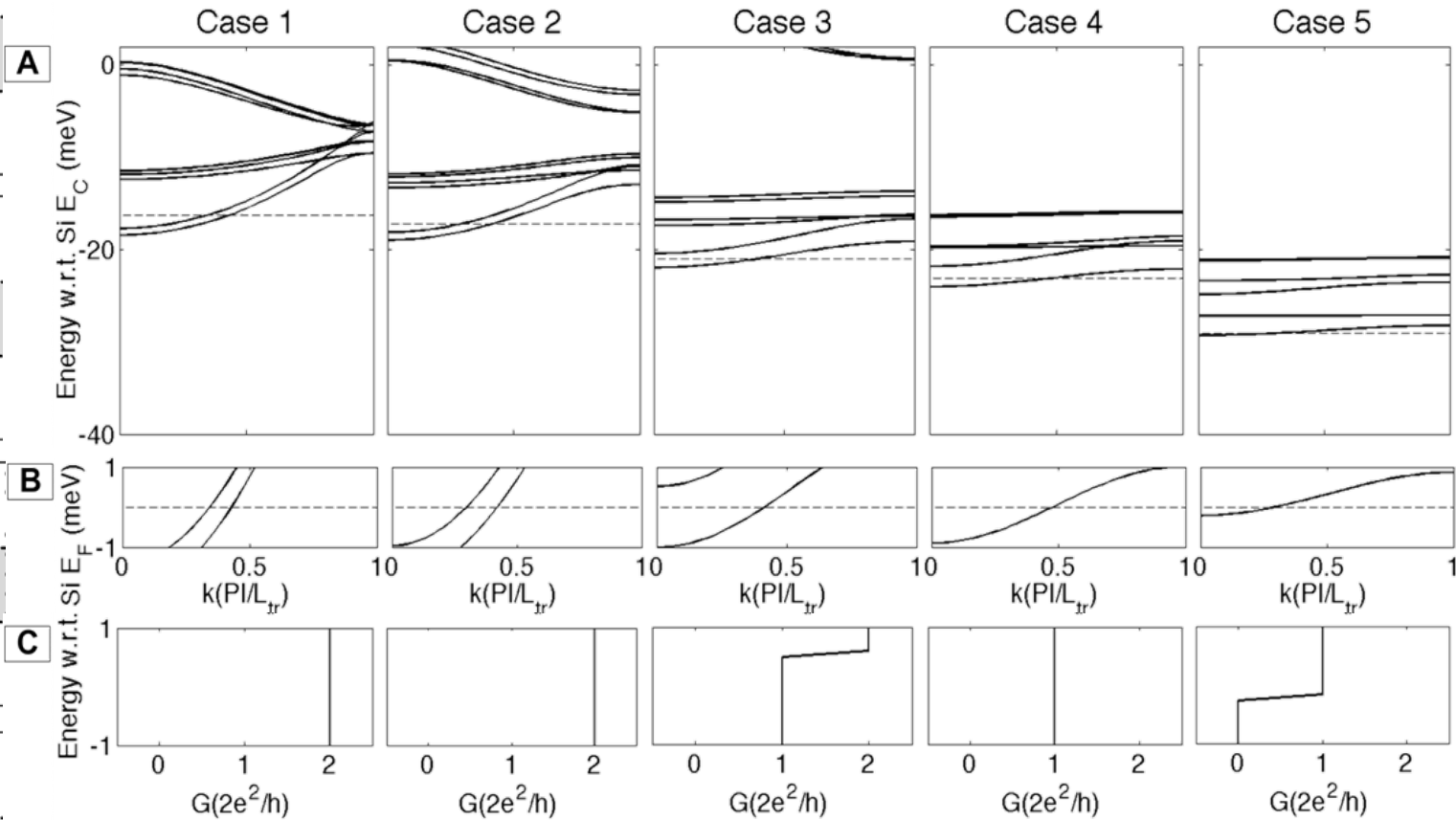
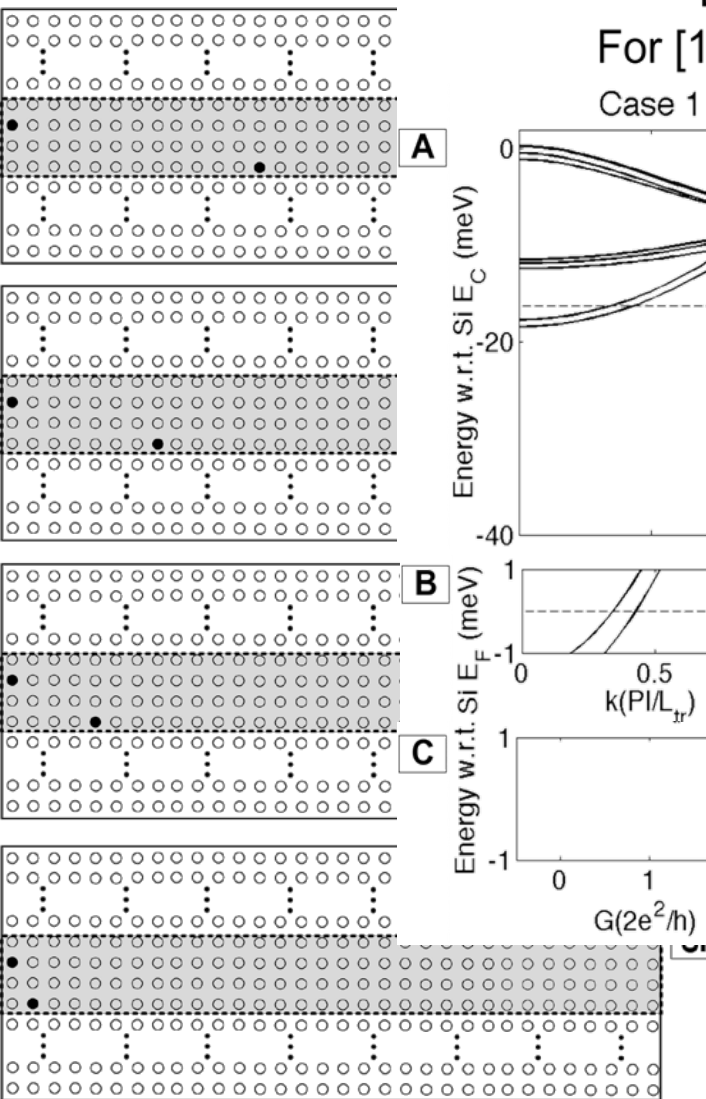
Dependence of bandstructures on doping for [110] transport and 2 dimer row width



# Dopant Placement Details

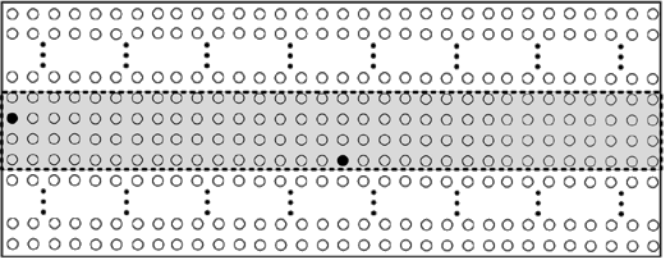


Dependence of bandstructures on dopant profile  
For [110] transport, 2 dimer row width and 1/64 ML doping

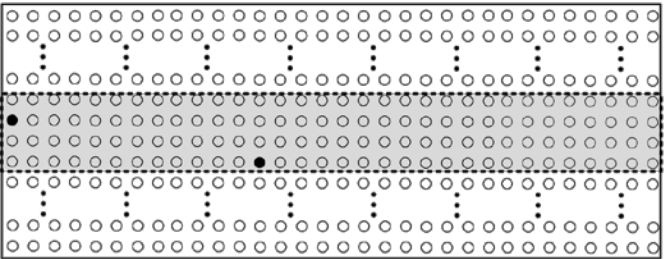
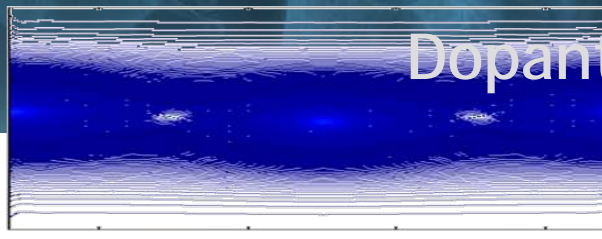




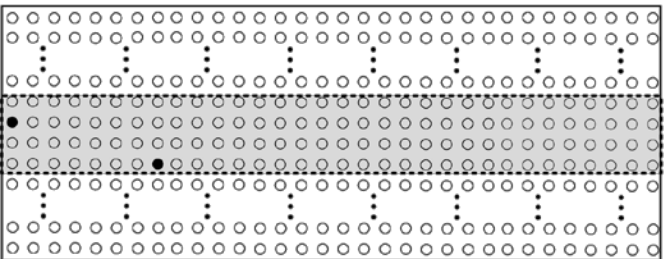
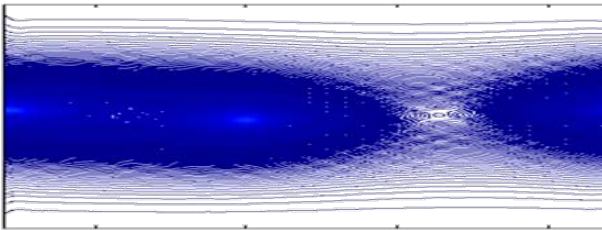
# Dopant Placement Details



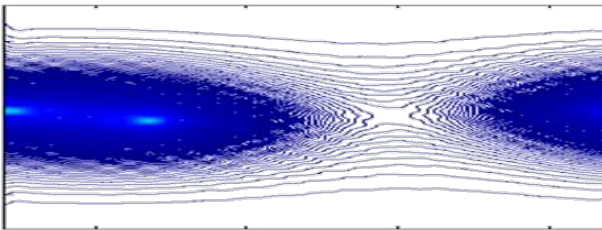
Case 1: Exp 1(e)



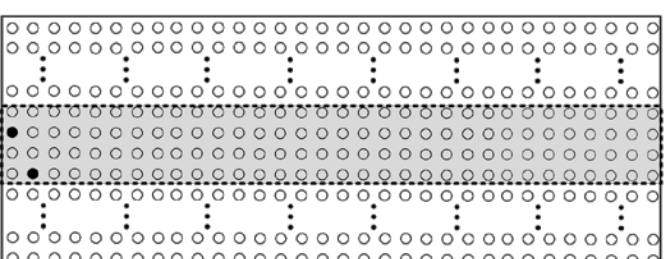
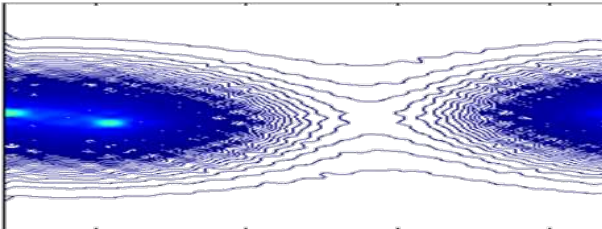
Case 2



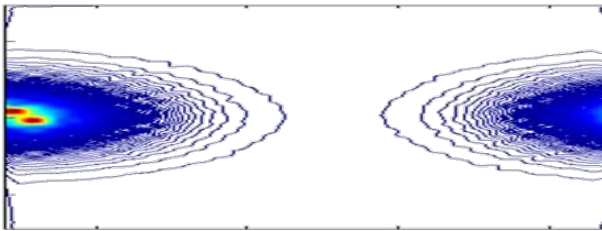
Case 3



Case 4



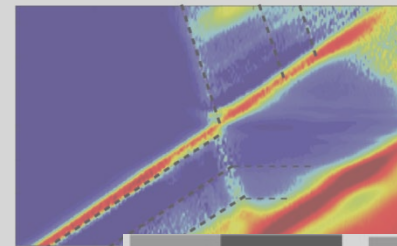
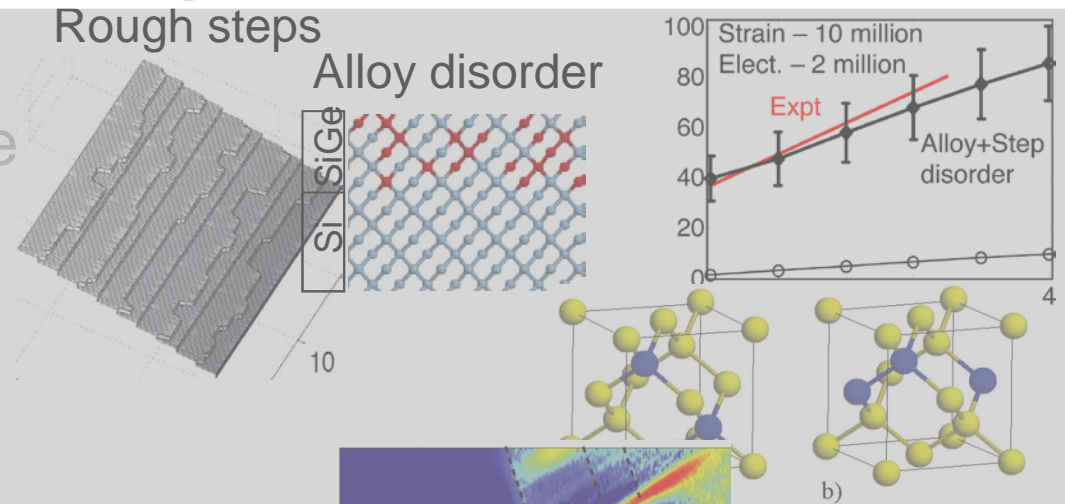
Case 5



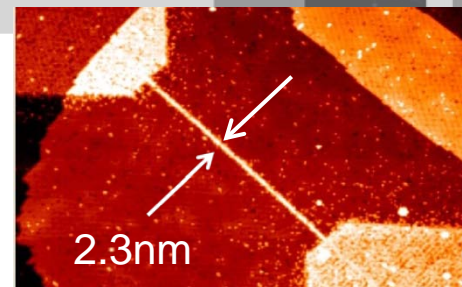
- Charge distribution on  $\delta$ -doping layer

# Multi-Million Atom Simulation of Quantum States in Realistically Extended Devices in Silicon

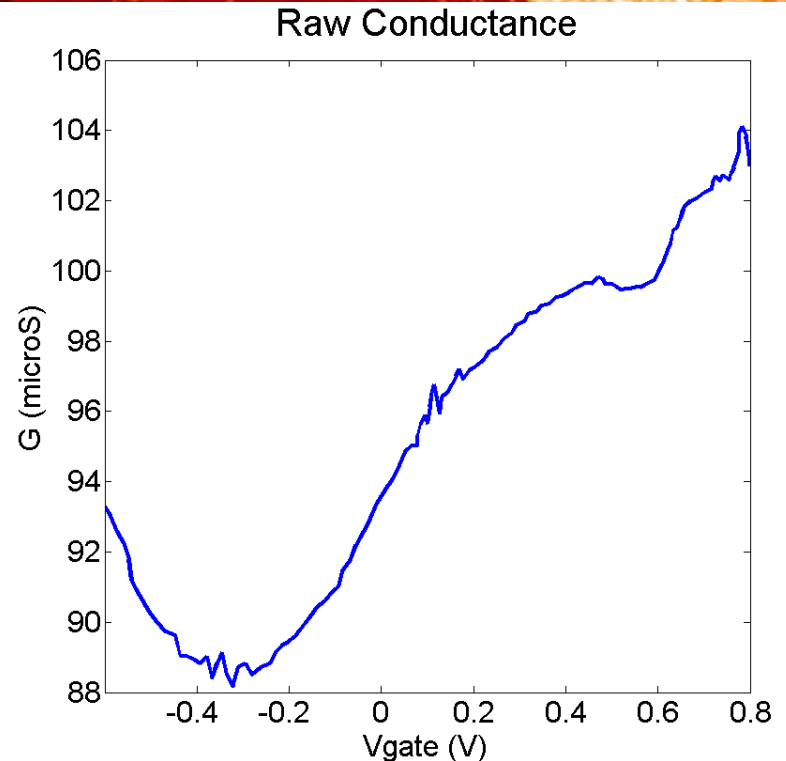
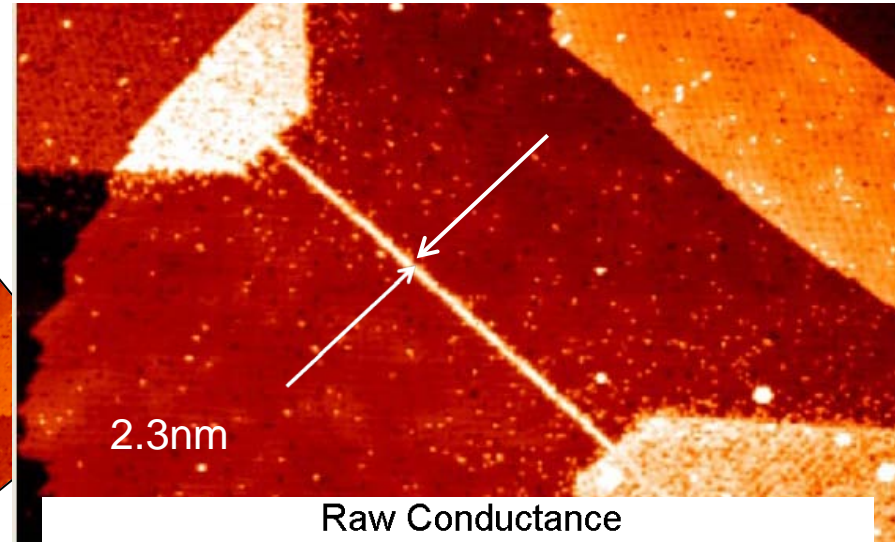
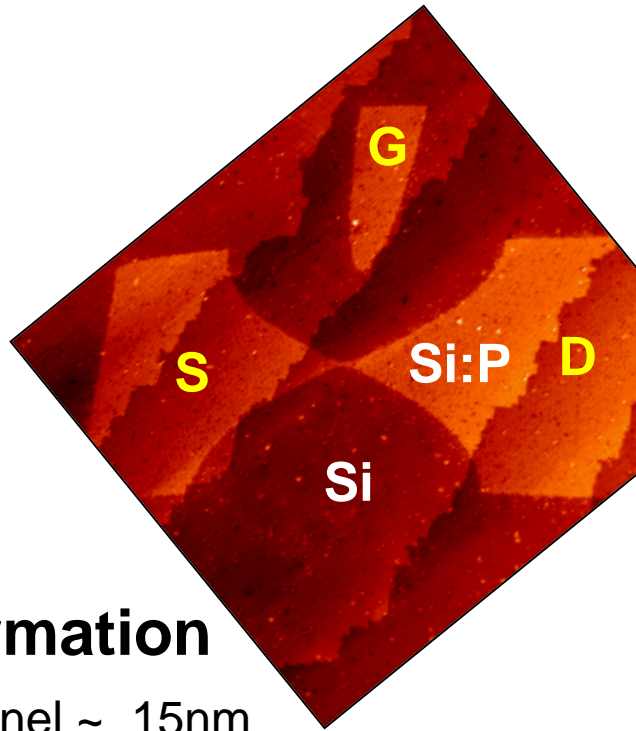
- Motivation
- NEMO Numerical Engine
- Valley splitting
  - » Si quantum wells
  - » Si Quantum Dots
- Single impurities in Si
  - » Metrology – impurity identification
  - » Wavefunction mapping
  - » g-factor engineering
  - » CTAP



- Dense impurities in Si – Si:P
  - » Infinite sheets
  - » Infinite wires
  - » Gating, transport through wires
  - » Impurity-based quantum dots





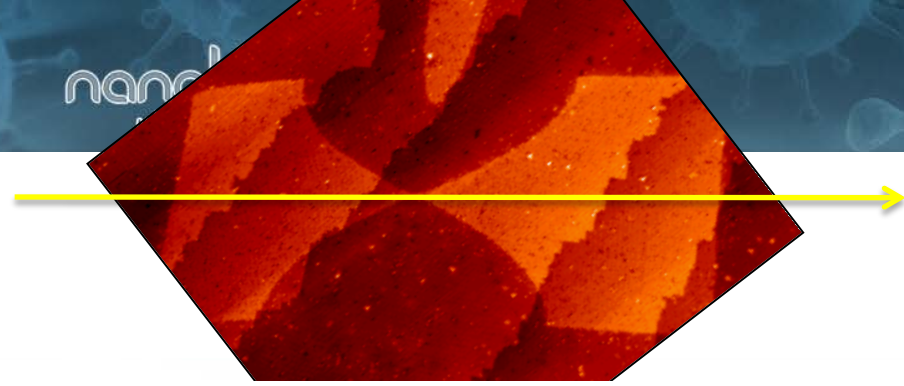


## Device Information

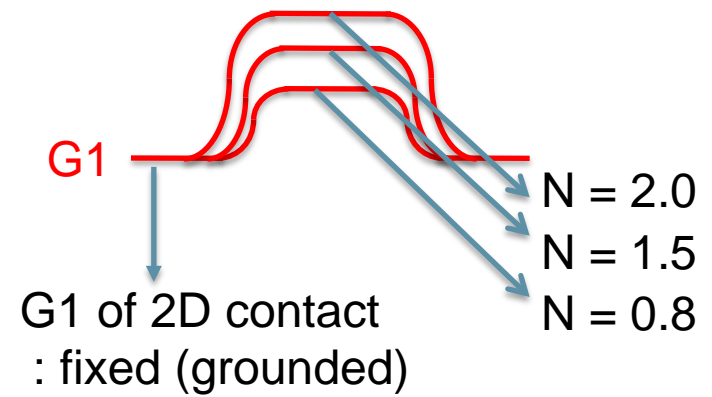
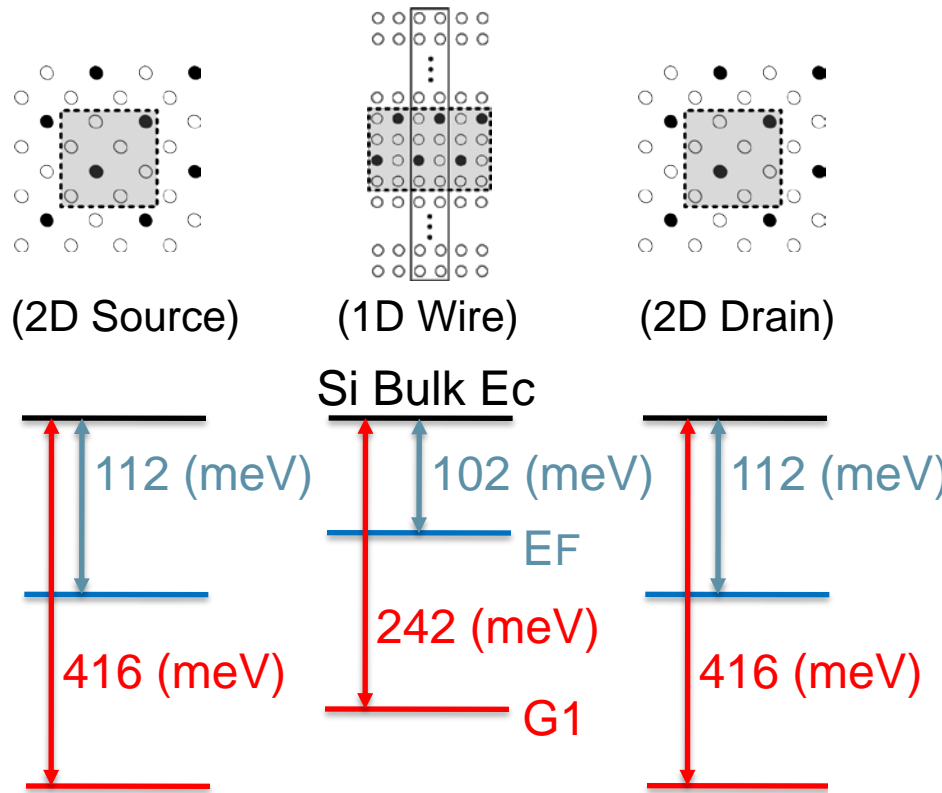
- Effective Channel ~ 15nm
- From Gate to Channel ~ 35nm
- T~ 1.2(K). Doping Constant ~ 1/4ML

## Problem to be solved

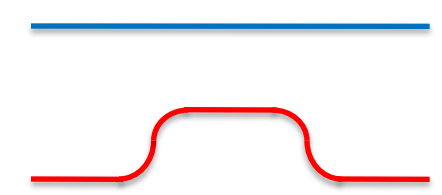
- Gate-dependent S-to-D Conduction Behavior
- Conduction through Donor-bands



## The Local-bandstructure along transport direction.

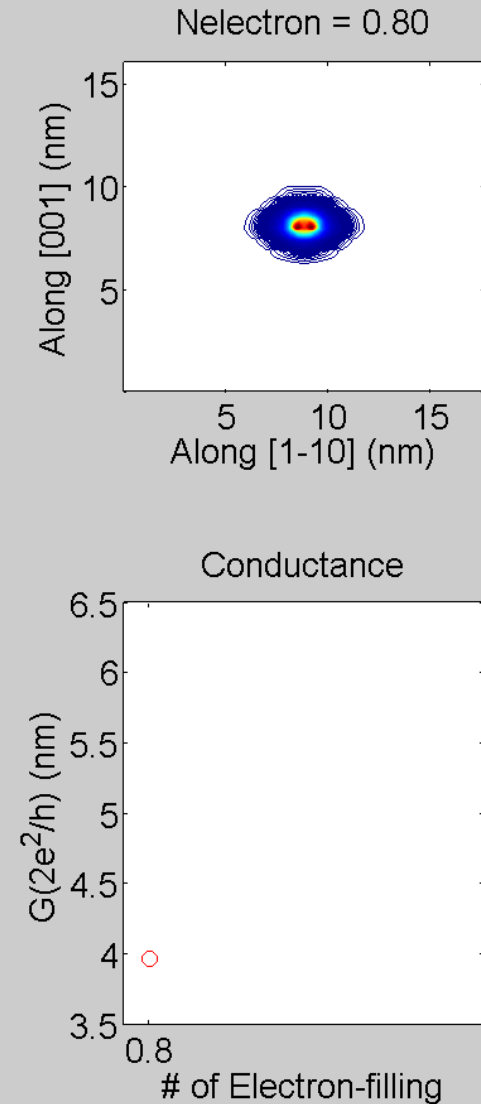
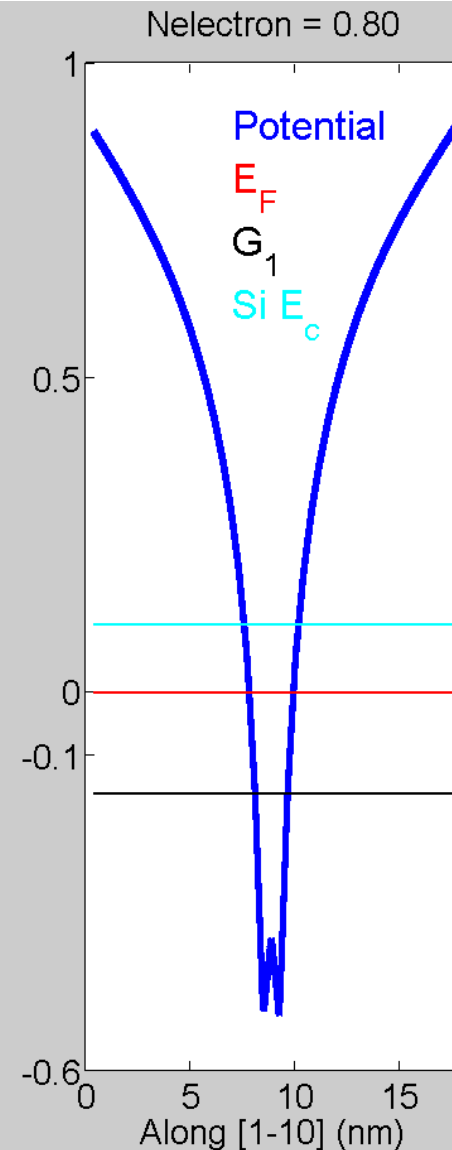
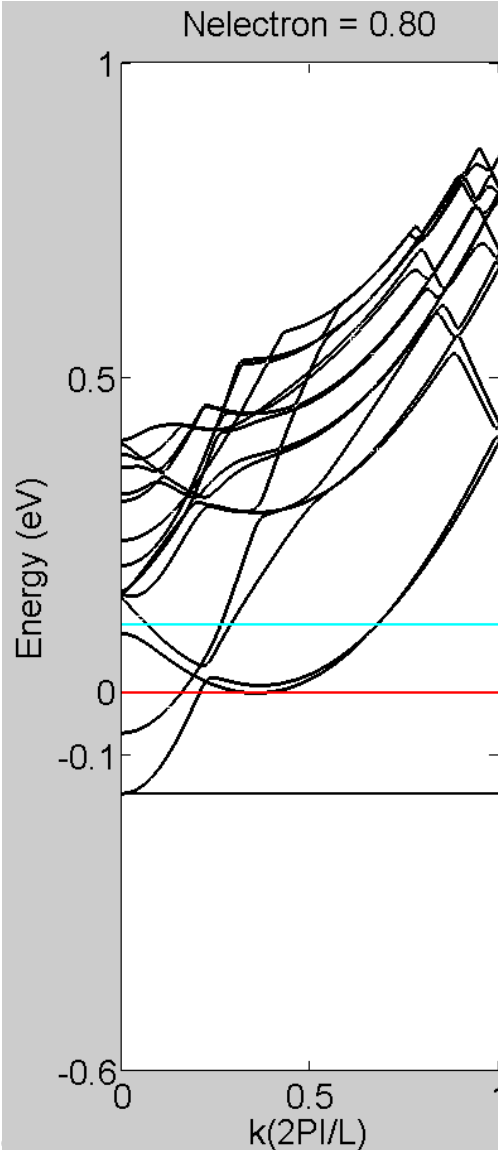
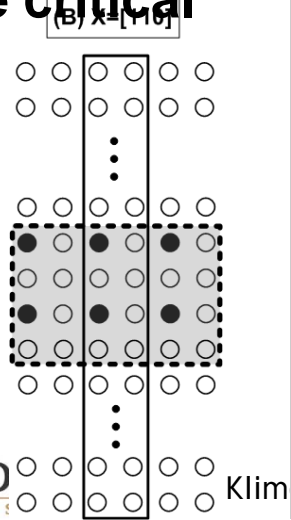


Align EF

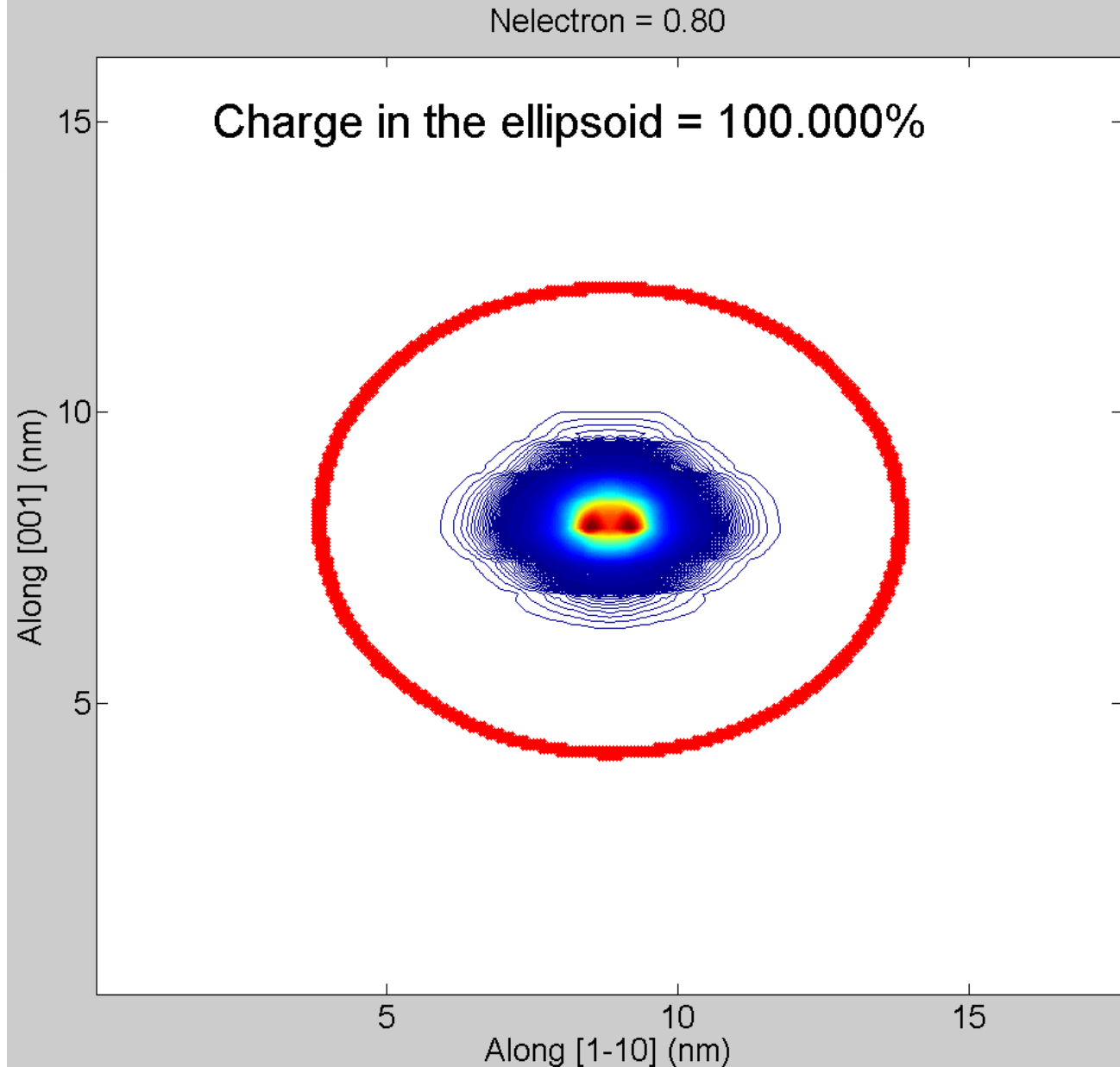
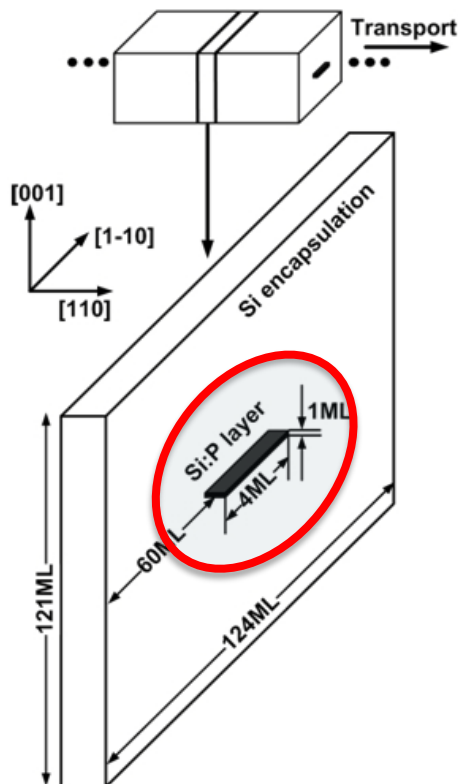


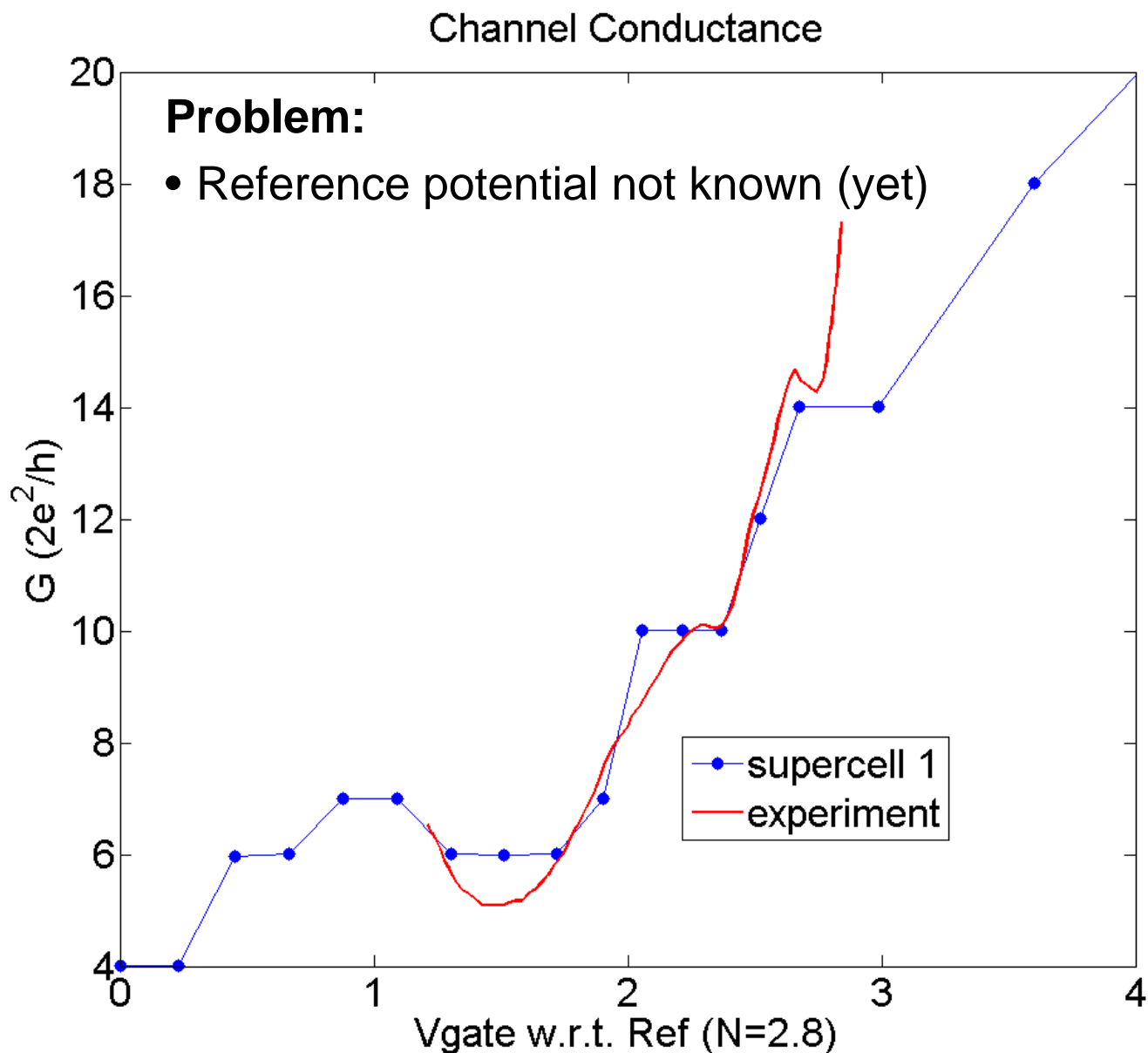
Electrostatic potential is spatially widely reaching  
 The region is too large (right now) for full self-consistent calculations

- We fix # of charge filling => Obtain  $E_F$  and potential
- Neutral at  $N=2$
- **Dispersion depends on on the electron filling**
- Like usual Si nanowires
- **Electrostatics are critical**

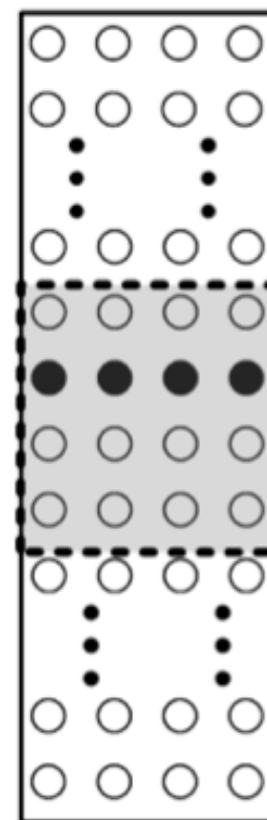


- Charge expands in diameter
- Ellipsoid :  
10nm axis along [1-10]  
8nm axis along [001]



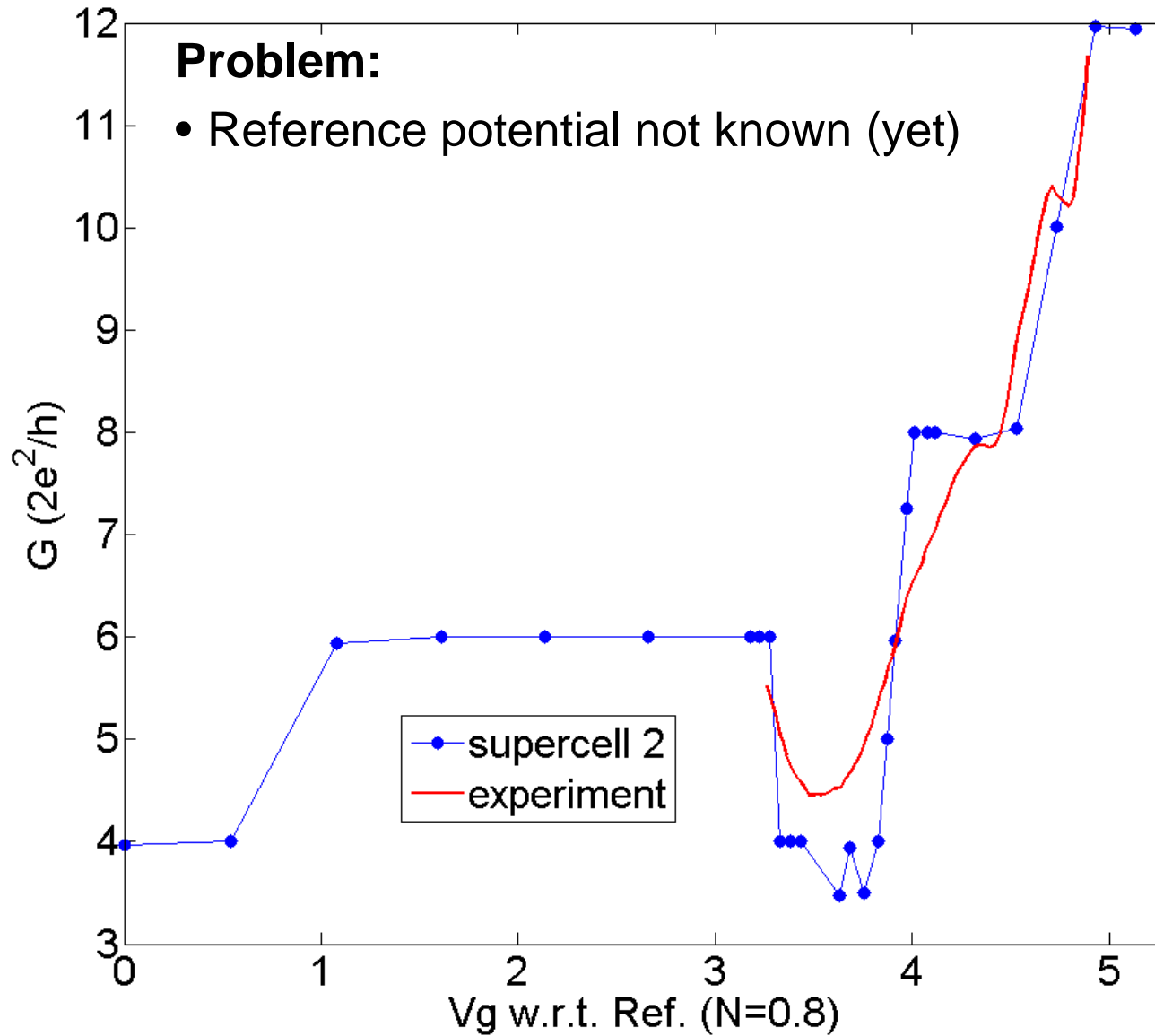


**Super-cell configuration**

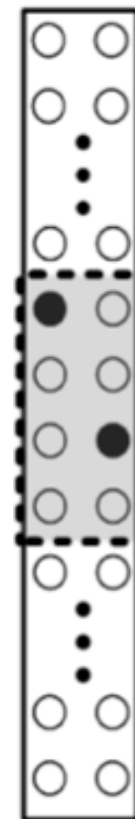


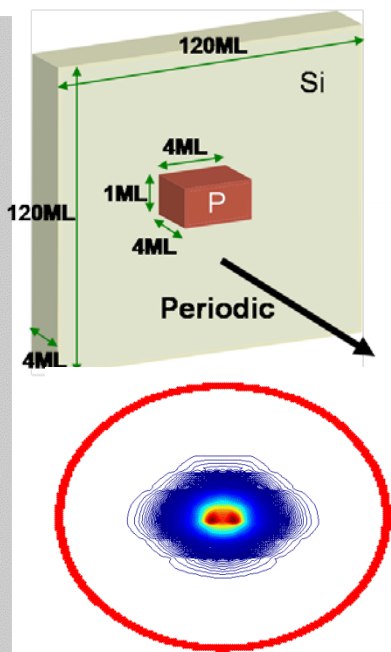
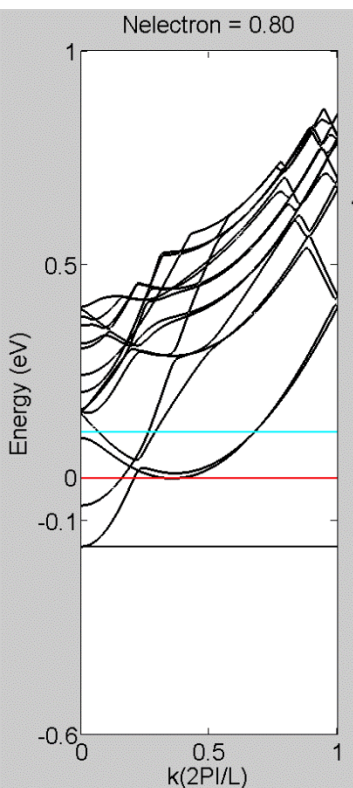


### Channel Conductance



### Super-cell configuration





## Results – Numerical Experiments :

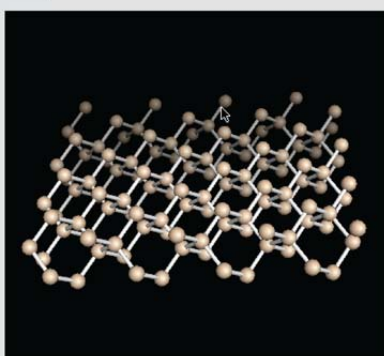
- Direction dependence:
  - Dispersion (strong)
  - Conductance (weak)
- Doping, width, configurations:
  - Dispersion and conductance (strong)
- Strong gate / charge dependence!

## Problems:

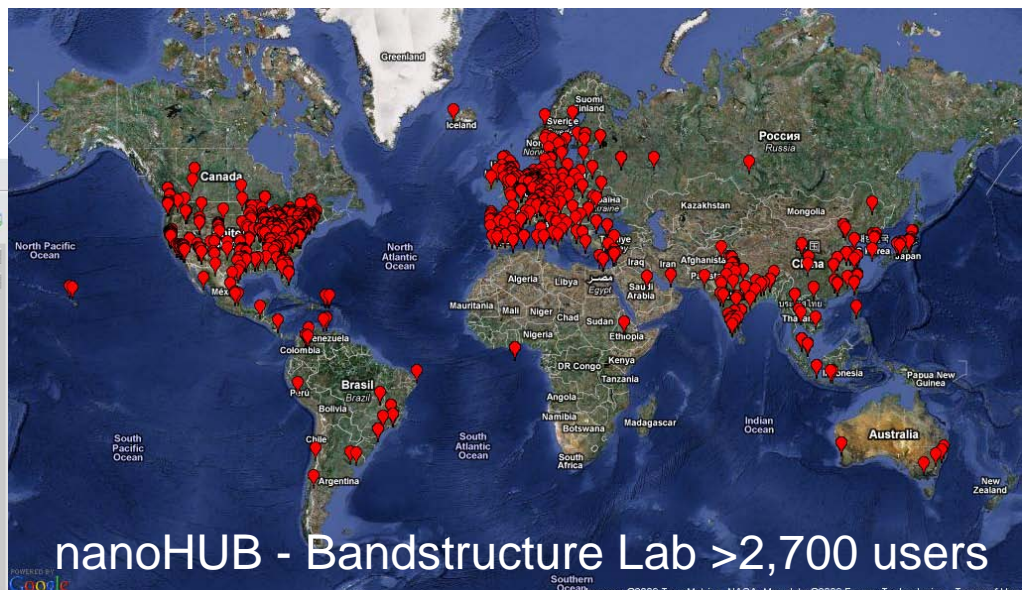
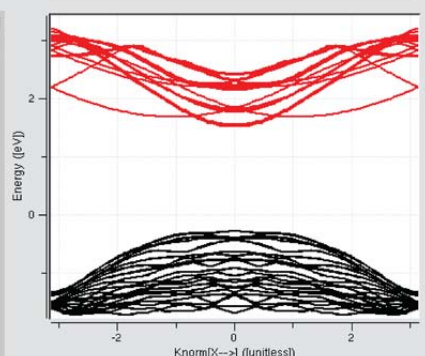
- Need better control over the spatially extended electrostatic potential

1 Device Structure → 2 TB Parameters → 3 Analysis → 4 Simulate

Result: Unitcell Structure



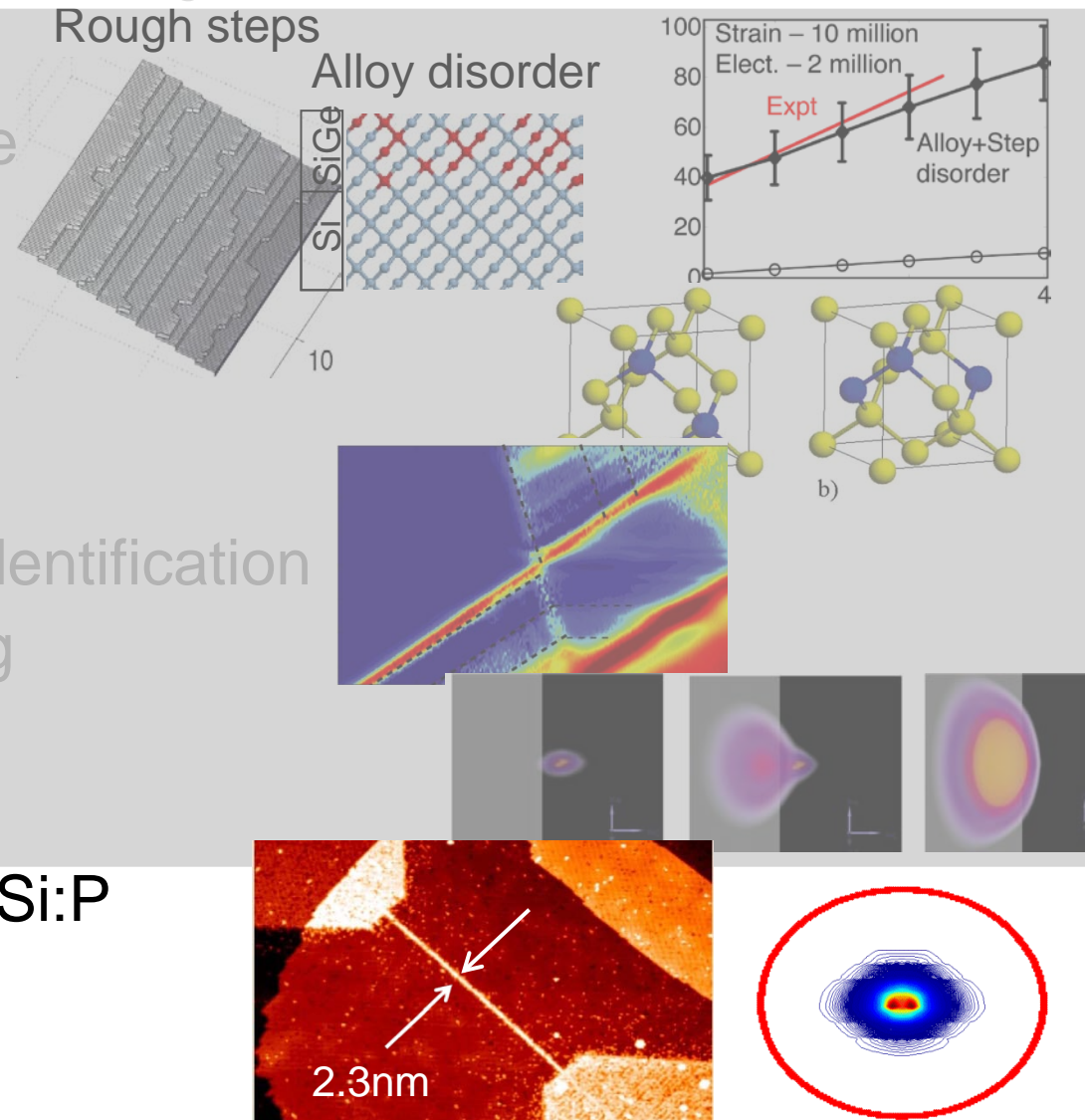
Result: Wire Bandstructure



nanoHUB - Bandstructure Lab >2,700 users

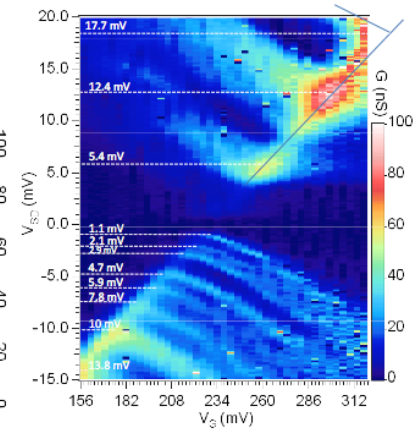
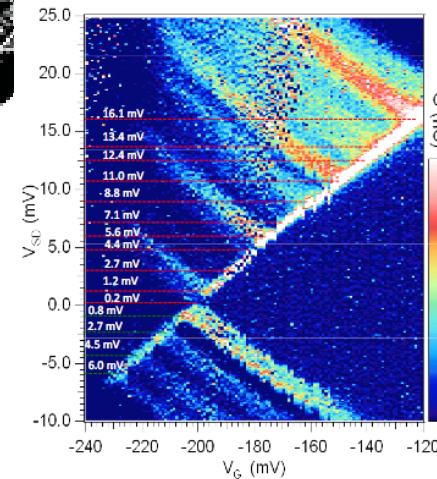
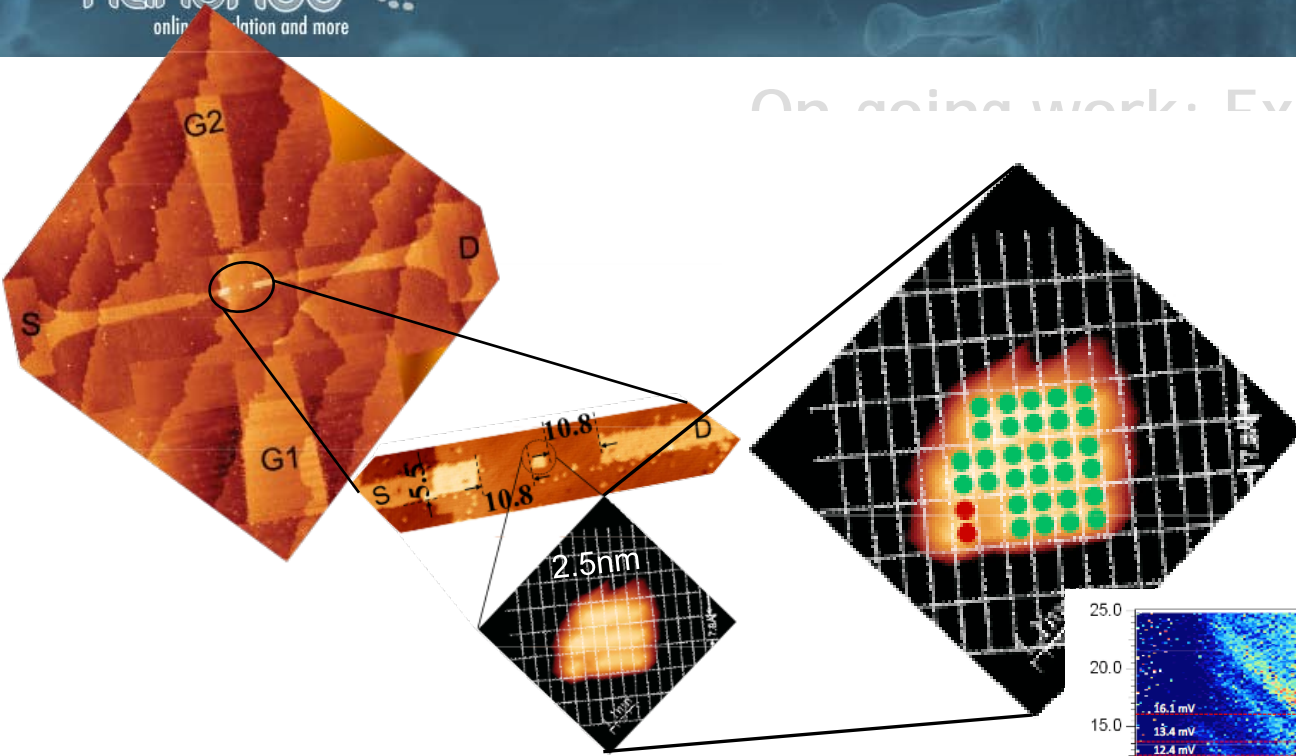
# Multi-Million Atom Simulation of Quantum States in Realistically Extended Devices in Silicon

- Motivation
- NEMO Numerical Engine
- Valley splitting
  - » Si quantum wells
  - » Si Quantum Dots
- Single impurities in Si
  - » Metrology – impurity identification
  - » Wavefunction mapping
  - » g-factor engineering
  - » CTAP
- Dense impurities in Si – Si:P
  - » Infinite sheets
  - » Infinite wires
  - » Gating, transport through wires
  - » Impurity-based quantum dots





## On going work: Exploring Si:P QD, D85



- Identify the correct state of the system
  - » Number of donors (#P) in the dot
  - » Number of filled electrons (#e) in the dot
  - » Correct estimate of the excited spectrum from the Coulomb diamond.

- Enable:
  - » Numerical experiments – one could not do otherwise
    - ✓ Experimentally
    - ✓ Analytically
  - » **Realistically extended** devices
  - » Quantitative explanation of experiments
    - ✓ Valley splitting in quantum wells
    - ✓ Single impurities
    - Not yet .... Dense impurity channels
- Strain and electrostatics are far-reaching into the crystal
- Disorder at the interface is critical
- Valley splitting is interface dominated - sensitive to the details of the interface!
- Scattered Impurity placement in dense Si:P systems does not affect performance radically

