TRANSPORT IN A MAGNETIC FIELD

- Carriers which move perpendicular or at an angle with respect to the magnetic field direction, will be deflected from their direction of motion by the Lorentz force. To describe transport in the presence of both electric and magnetic fields, one again needs to solve the BTE, which for the case of homogeneous semiconductors and steady-state conditions is of the form:

$$-\frac{e}{\hbar}\vec{\mathcal{E}}\cdot\nabla_k f - \frac{e}{\hbar}[\vec{v}\times\vec{B}]\cdot\nabla_k f = \left.\frac{\partial f}{\partial t}\right|_{coll.}$$

In the case of $B=0$, the relaxation time approximation gives:

$$f = f_0 + f_A = f_0 + e\tau_m(E)\,\vec{v}\cdot\vec{\mathcal{E}}\,\frac{\partial f_0}{\partial E}$$

- If the distribution function appearing in the Lorentz force term on the LHS of the BTE is approximated with the equilibrium distribution function, then the magnetic field drops completely out of the problem because:

$$\nabla_k f_0 = \hbar\vec{v}\,\frac{\partial f_0}{\partial E}$$

which gives:

$$\frac{e}{\hbar}[\vec{v}\times\vec{B}]\cdot\nabla_k f_0 = e\,\vec{v}\cdot[\vec{v}\times\vec{B}]\,\frac{\partial f_0}{\partial E} = 0$$

- To include magnetic field and avoid the above mentioned problem, we make the ansatz

$$f = f_0 + (\vec{v}\cdot\vec{G})$$

where, as shown later, $\vec{G}$ is independent of $\vec{k}$. When $\vec{B}=0$, then $\vec{G} = e\tau_m\vec{\mathcal{E}}\,\partial f_0/\partial E$. For a general case of $\vec{B}\neq 0$, we get:

$$\nabla_k f = \nabla_k\left(f_0 + \vec{v}\cdot\vec{G}\right) = \nabla_k f_0 + \frac{\hbar}{m^*}\nabla_k(\vec{k}\cdot\vec{G})$$
$$= \hbar\vec{v}\,\frac{\partial f_0}{\partial E} + \frac{\hbar}{m^*}\vec{G}$$

Therefore, for simple parabolic bands, we have:

$$-\frac{e}{\cancel{\hbar}}\cancel{\hbar}\vec{v}\cdot\vec{\mathcal{E}}\,\frac{\partial f_0}{\partial E} - \frac{e}{\cancel{\hbar}}\cancel{\hbar}\vec{v}\cdot[\vec{v}\times\vec{B}]\,\frac{\partial f_0}{\partial E} - \frac{e}{\cancel{\hbar}}\frac{\cancel{\hbar}}{m^*}\vec{G}\cdot[\vec{v}\times\vec{B}] = \left.\frac{\partial f}{\partial t}\right|_{coll.}$$

i.e.
$$-e\,\vec{v}\cdot\vec{\mathcal{E}}\,\frac{\partial f_0}{\partial E} - \underbrace{e\,\vec{v}\cdot[\vec{v}\times\vec{B}]}_{0}\,\frac{\partial f_0}{\partial E} - \frac{e}{m^*}\vec{G}\cdot[\vec{v}\times\vec{B}] = -\frac{\vec{v}\cdot\vec{G}}{\tau_m}$$

To summarize

$$-e(\vec{v}\cdot\vec{\mathcal{E}})\frac{\partial f_0}{\partial E} - \frac{e}{m^*}\vec{v}\cdot(\vec{B}\times\vec{G}) = -\frac{\vec{v}\cdot\vec{G}}{\tau_m}$$

The vector $\vec{G}$ is then calculated from:

$$\vec{G} = e\tau_m(E)\vec{\mathcal{E}}\frac{\partial f_0}{\partial E} + \underbrace{\left(\frac{e\tau_m}{m^*}\right)}_{\mu}(\vec{B}\times\vec{G}) = e\tau_m\vec{\mathcal{E}}\frac{\partial f_0}{\partial E} + \mu(\vec{B}\times\vec{G})$$

- By using the vectors $\vec{\mathcal{E}}$, $\vec{B}$ and $\vec{B}\times\vec{\mathcal{E}}$ as a triad for the representation of $\vec{G}$, i.e.

$$\vec{G} = \alpha\vec{\mathcal{E}} + \beta\vec{B} + \gamma(\vec{B}\times\vec{\mathcal{E}}) \rightarrow \text{Generalized field}$$

with α, β and γ being the unknown coefficient we want to determine, we get:

$$\vec{B}\times\vec{G} = \alpha\vec{B}\times\vec{\mathcal{E}} + \beta\underbrace{\vec{B}\times\vec{B}}_{0} + \gamma\vec{B}\times(\vec{B}\times\vec{\mathcal{E}})$$

$$= \alpha(\vec{B}\times\vec{\mathcal{E}}) + \gamma[(\vec{B}\cdot\vec{\mathcal{E}})\vec{B} - (\vec{B}\cdot\vec{B})\vec{\mathcal{E}}]$$

$$= \alpha(\vec{B}\times\vec{\mathcal{E}}) + \gamma(\vec{B}\cdot\vec{\mathcal{E}})\vec{B} - \gamma B^2\vec{\mathcal{E}}$$

i.e.

$$\alpha\vec{\mathcal{E}} + \beta\vec{B} + \gamma(\vec{B}\times\vec{\mathcal{E}}) = e\tau_m\frac{\partial f_0}{\partial E}\vec{\mathcal{E}} + \alpha\mu(\vec{B}\times\vec{\mathcal{E}}) + \gamma\mu(\vec{B}\cdot\vec{\mathcal{E}})\vec{B} - \gamma\mu B^2\vec{\mathcal{E}}$$

$$\alpha = e\tau_m\frac{\partial f_0}{\partial E} - \gamma\mu B^2$$

$$\beta = \gamma\mu(\vec{B}\cdot\vec{\mathcal{E}})$$

$$\gamma = \alpha\mu \quad \Rightarrow \quad \gamma = \alpha\mu$$

Trigonometric identity:
$\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}$

$$\alpha = e\tau_m\frac{\partial f_0}{\partial E} - \alpha\mu^2B^2 \Rightarrow \alpha(1+\mu^2B^2) = e\tau_m\frac{\partial f_0}{\partial E}$$

$$\alpha = \frac{e\tau_m(\partial f_0/\partial E)}{1+\mu^2B^2}, \quad \gamma = \alpha\mu$$

$$\beta = \gamma\mu(\vec{B}\cdot\vec{\mathcal{E}}) = \alpha\mu^2(\vec{B}\cdot\vec{\mathcal{E}})$$

To summarize:

$$\vec{G} = e\tau_m\frac{\partial f_0}{\partial E}\,\frac{\vec{\mathcal{E}} + \mu^2(\vec{B}\cdot\vec{\mathcal{E}})\vec{B} + \mu(\vec{B}\times\vec{\mathcal{E}})}{1+\mu^2B^2}$$

- For the case when $\vec{B} \perp \vec{\mathcal{E}}$, and assuming $\vec{B} = B\vec{i_z}$ and $\vec{\mathcal{E}} = (\mathcal{E}_x, \mathcal{E}_y, 0)$, we get:

$$\vec{G} = e\tau_m \frac{\partial f_0}{\partial E} \cdot \frac{\vec{\mathcal{E}} + \mu(\vec{B} \times \vec{\mathcal{E}})}{1 + \mu^2 B^2}$$

where: $$\vec{B} \times \vec{\mathcal{E}} = \begin{vmatrix} \vec{i_x} & \vec{i_y} & \vec{i_z} \\ 0 & 0 & B \\ \mathcal{E}_x & \mathcal{E}_y & 0 \end{vmatrix} = -B\mathcal{E}_y \vec{i_x} + B\mathcal{E}_x \vec{i_y}$$

The distribution function is, thus, given by:

$$f = f_0 + \frac{e\tau_m v_x}{1+\mu^2 B^2} \frac{\partial f_0}{\partial E} \mathcal{E}_x + \frac{e\tau_m v_y}{1+\mu^2 B^2} \frac{\partial f_0}{\partial E} \mathcal{E}_y -$$
$$- e\tau_m \frac{\partial f_0}{\partial E} \frac{\mu B \mathcal{E}_y v_x}{1+\mu^2 B^2} + e\tau_m \frac{\partial f_0}{\partial E} \frac{\mu B \mathcal{E}_x v_y}{1+\mu^2 B^2}$$

where: $\mu B = \frac{e\tau_m}{m^*} B = \tau_m(E) \frac{eB}{m^*} = \omega_c \tau_m(E)$, and ω_c is the cyclotron frequency.

- Using the averaging procedure described when calculating the conductivity in the RTA, we get:

$$J_x = \frac{ne^2}{m^*} \left[\left\langle\!\left\langle \frac{\tau_m}{1+\omega_c^2\tau_m^2} \right\rangle\!\right\rangle \mathcal{E}_x - \omega_c \left\langle\!\left\langle \frac{\tau_m^2}{1+\omega_c^2\tau_m^2} \right\rangle\!\right\rangle \mathcal{E}_y \right]$$

$$J_y = \frac{ne^2}{m^*} \left[\omega_c \left\langle\!\left\langle \frac{\tau_m^2}{1+\omega_c^2\tau_m^2} \right\rangle\!\right\rangle \mathcal{E}_x + \left\langle\!\left\langle \frac{\tau_m}{1+\omega_c^2\tau_m^2} \right\rangle\!\right\rangle \mathcal{E}_y \right]$$

Hence, instead of the simple average over the relaxation time, we now need to perform a very complicated averaging procedure. The conductivity tensor is, thus, of the form:

$$\sigma(B) = \begin{bmatrix} \alpha_0 & -\omega_c\beta_0 \\ \omega_c\beta_0 & \alpha_0 \end{bmatrix}; \text{ where } \begin{cases} \alpha_0 = \frac{ne^2}{m^*} \left\langle\!\left\langle \frac{\tau_m}{1+\omega_c^2\tau_m^2} \right\rangle\!\right\rangle \\ \beta_0 = \frac{ne^2}{m^*} \left\langle\!\left\langle \frac{\tau_m^2}{1+\omega_c^2\tau_m^2} \right\rangle\!\right\rangle \end{cases}$$

Note that the conductivity tensor $\sigma(B)$ satisfies the Onsager relation:

$$\sigma_{ij}(B) = \sigma_{ji}(-B)$$

The law of nature expressed by the above equation is a cornerstone of the irreversible thermodynamics.

(a) Weak magnetic field case

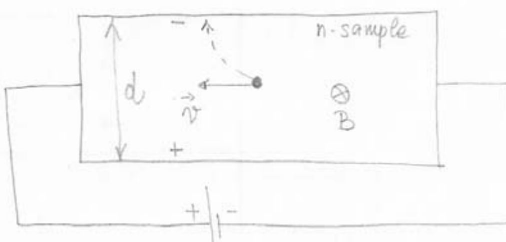
- The weak magnetic field case is defined by the condition:

$$\omega_c \tau_m(E) \ll 1 .$$

In this case:

$$\begin{cases} J_x = \dfrac{ne^2}{m^*}\left[\langle\langle \tau_m \rangle\rangle \mathcal{E}_x - \omega_c \langle\langle \tau_m^2 \rangle\rangle \mathcal{E}_y \right] \\ J_y = \dfrac{ne^2}{m^*}\left[\omega_c \langle\langle \tau_m^2 \rangle\rangle \mathcal{E}_x + \langle\langle \tau_m \rangle\rangle \mathcal{E}_y \right] \end{cases}$$

- Consider now a semiconductor sample in which the current is flowing in the x-direction. Due to the non-vanishing electric field in the y-direction, a Hall voltage will develop.



The Hall voltage is calculated by assuming $J_y = 0$, which gives:

$$J_y = 0 \Rightarrow \omega_c \langle\langle \tau_m^2 \rangle\rangle \mathcal{E}_x = -\langle\langle \tau_m \rangle\rangle \mathcal{E}_y$$

$$\tan\theta_H = \frac{\mathcal{E}_y}{\mathcal{E}_x} = -\omega_c \frac{\langle\langle \tau_m^2 \rangle\rangle}{\langle\langle \tau_m \rangle\rangle} = -\omega_c \langle\langle \tau_m \rangle\rangle \frac{\langle\langle \tau_m^2 \rangle\rangle}{\langle\langle \tau_m \rangle\rangle^2}$$

$$\tan\theta_H = -\frac{eB}{m^*} \langle\langle \tau_m \rangle\rangle \frac{\langle\langle \tau_m^2 \rangle\rangle}{\langle\langle \tau_m \rangle\rangle^2} = -\mu_{eff} B r_H$$

where: θ_H is the Hall angle

$r_H = \langle\langle \tau_m^2 \rangle\rangle / \langle\langle \tau_m \rangle\rangle^2$ is the Hall scattering factor

$\mu_{eff} = e\langle\langle \tau_m \rangle\rangle / m^*$ is the effective mobility.

- To get an idea for the magnitude of the Hall scattering factor, which takes into account the energy spread of the carriers, we will assume:

$$\tau_m = \tau_0 \left(\frac{E}{k_B T}\right)^s$$

Then, for non-degenerate semiconductors:

$$\langle\langle \tau_m \rangle\rangle = \tau_0 \; \frac{\Gamma(s+5/2)}{\Gamma(5/2)}$$

and:

$$\langle\langle \tau_m^2 \rangle\rangle = \frac{\int_0^\infty E^{3/2} \tau_0^2 \left(\frac{E}{k_BT}\right)^{2s} \left(-\frac{A}{k_BT}\right) e^{-E/k_BT} dE}{\int_0^\infty E^{3/2} \left(-\frac{A}{k_BT}\right) e^{-E/k_BT} dE} = \tau_0^2 \; \frac{\Gamma(2s+5/2)}{\Gamma(5/2)}$$

The Hall scattering factor is then equal to:

$$r_H = \frac{\langle\langle \tau_m \rangle\rangle^2}{\langle\langle \tau_m \rangle\rangle^2} = \frac{\cancel{\tau_0^2}\, \Gamma(2s+5/2)/\cancel{\Gamma(5/2)}}{\cancel{\tau_0^2}\, \Gamma^2(s+5/2)/\Gamma^2(5/2)} = \frac{\Gamma(5/2)\Gamma(2s+5/2)}{[\Gamma(s+5/2)]^2}$$

where:

$$\Gamma(5/2) = \frac{3}{2} \cdot \frac{1}{2} \Gamma\left(\frac{1}{2}\right) = \frac{3\sqrt{\pi}}{4}$$

Therefore:
$$r_H = \frac{3\sqrt{\pi}}{4} \frac{\Gamma(2s+5/2)}{\Gamma^2(s+5/2)}$$

Examples:

(a) Acoustic phonon scattering, for which $s = -1/2$.

$$r_H = \frac{3\sqrt{\pi}}{4} \cdot \frac{\Gamma(5/2-1)}{\Gamma^2(5/2-1/2)} = \frac{3\sqrt{\pi}}{4} \frac{\Gamma(3/2)}{\Gamma^2(2)} = \frac{3\pi}{8} = 1.18$$

(b) For ionized impurity scattering, $s = 3/2$, which gives:

$$r_H = \frac{3\sqrt{\pi}}{4} \frac{\Gamma(3+5/2)}{\Gamma^2\left(\frac{3}{2}+\frac{5}{2}\right)} = \frac{315\pi}{512} = 1.93$$

- The Hall coefficient is defined as:

$$R_H = \frac{\mathcal{E}_y}{J_x B} = -\frac{\mu_{eff} B r_H \mathcal{E}_x}{J_x B} = -\mu_{eff} r_H \frac{1}{J_x/\mathcal{E}_x}$$

where:

$$J_x = \frac{ne^2}{m^*}\left[\langle\langle \tau_m \rangle\rangle \mathcal{E}_x + \omega_c \langle\langle \tau_m^2 \rangle\rangle \frac{\omega_c \langle\langle \tau_m^2 \rangle\rangle}{\langle\langle \tau_m \rangle\rangle} \mathcal{E}_x\right]$$

$$= \frac{ne^2 \langle\langle \tau_m \rangle\rangle}{m^*} \mathcal{E}_x \left[1 + \omega_c^2 \frac{\langle\langle \tau_m^2 \rangle\rangle^2}{\langle\langle \tau_m \rangle\rangle^2}\right] \approx \frac{ne^2 \langle\langle \tau_m \rangle\rangle}{m^*} \mathcal{E}_x$$

which gives:

$$R_H = -\mu_{eff}\, r_H \frac{1}{ne^2 \langle\langle \tau_m \rangle\rangle} m^* = - r_H \frac{\cancel{e \langle\langle \tau_m \rangle\rangle}}{\cancel{m^*}} \frac{\cancel{m^*}}{ne^{\cancel{2}} \cancel{\langle\langle \tau_m \rangle\rangle}} = - \frac{r_H}{ne}$$

Therefore, in the weak field approximation :

$$R_H = \begin{cases} -\frac{r_H}{ne} < 0 & \text{for n-type semiconductors} \\ +\frac{r_H}{pe} > 0 & \text{for p-type semiconductors} \end{cases}$$

The sign of the Hall coefficient can be used to determine the type of the semiconductor (whether it is n-type or p-type).

- The Hall mobility is defined as:

$$\mu_H = |R_H|\,\sigma = \frac{r_H}{\cancel{n}\cancel{q}} \frac{\cancel{n}e^2 \langle\langle \tau_m \rangle\rangle}{m^*} = \mu_{eff}\, r_H$$

drift or effective mobility

The Hall mobility is different from the drift mobility by the Hall scattering factor. Usually, it is the Hall mobility which is measured rather than the drift mobility. From the observed temperature dependence of the Hall mobility, one can get an idea of the most probable energy dependence of τ_m, from which the Hall factor r_H is calculated. The effective, or drift mobility, is then equal to:

$$\mu_n = \mu_{eff} = \mu_H / r_H < \mu_H$$

- The Hall voltage is given by:

$$V_H = \mathcal{E}_y\, d = J_x B R_H\, d = - J_x B d \frac{r_H}{ne}$$

where d is the thickness of the sample.

- For the case when both electrons and holes are present in the system, we have:

$$\left.\begin{aligned} J_{xe} &= \frac{ne^2}{m_e^*}\left[\langle\langle \tau_{me} \rangle\rangle \mathcal{E}_x - W_{ce} \langle\langle \tau_{me}^2 \rangle\rangle \mathcal{E}_y\right] \\ J_{ye} &= \frac{ne^2}{m_e^*}\left[W_{ce} \langle\langle \tau_{me}^2 \rangle\rangle \mathcal{E}_x + \langle\langle \tau_{me} \rangle\rangle \mathcal{E}_y\right] \end{aligned}\right\} \text{for electrons}$$

$$\left.\begin{aligned} J_{xh} &= \frac{pe^2}{m_h^*}\left[\langle\langle \tau_{mh} \rangle\rangle \mathcal{E}_x + W_{ch} \langle\langle \tau_{mh}^2 \rangle\rangle \mathcal{E}_y\right] \\ J_{yh} &= \frac{pe^2}{m_h^*}\left[-W_{ch} \langle\langle \tau_{mh}^2 \rangle\rangle \mathcal{E}_x + \langle\langle \tau_{mh} \rangle\rangle \mathcal{E}_y\right] \end{aligned}\right\} \text{for holes}$$

Using: $\frac{ne^2 \langle\langle \tau_{me} \rangle\rangle}{m_e^*} = ne \frac{e \langle\langle \tau_{me} \rangle\rangle}{m_e^*} = ne\mu_e$

and:

$$\frac{ne^2}{m_e^*}\, \omega_{ce} \langle\langle \tau_{me}{}^2 \rangle\rangle = ne\, \frac{e}{m_e^*}\, \frac{eB}{m_e^*}\, r_e \langle\langle \tau_m \rangle\rangle^2 = nBer_e\mu_e{}^2$$

we have:

$$\left.\begin{aligned} J_{xe} &= ne\mu_e \left[\mathcal{E}_x - Br_e\mu_e \mathcal{E}_y\right] \\ J_{ye} &= ne\mu_e \left[Br_e\mu_e \mathcal{E}_x + \mathcal{E}_y\right] \end{aligned}\right\} \text{ for electrons}$$

$$\left.\begin{aligned} J_{xh} &= pe\mu_h \left[\mathcal{E}_x + Br_h\mu_h \mathcal{E}_y\right] \\ J_{yh} &= pe\mu_h \left[-Br_h\mu_h \mathcal{E}_x + \mathcal{E}_y\right] \end{aligned}\right\} \text{ for holes}$$

From the condition that $J_y = 0$, we get:

$$J_y = J_{ye} + J_{yh} = ne\mu_e \left[Br_e\mu_e \mathcal{E}_x + \mathcal{E}_y\right] + pe\mu_h\left[-Br_h\mu_h \mathcal{E}_x + \mathcal{E}_y\right]$$

$$= n\cancel{e}b\cancel{\mu_h}\left[Br_e b\mu_h \mathcal{E}_x + \mathcal{E}_y\right] + p\cancel{e\mu_h}\left[-Br_h\mu_h \mathcal{E}_x + \mathcal{E}_y\right] = 0$$

where $b = \mu_e/\mu_h$. Therefore, the Hall angle equals to:

$$Br_e b^2 n\mu_h \mathcal{E}_x + nb\mathcal{E}_y - Br_h p\mu_h \mathcal{E}_x + p\mathcal{E}_y = 0$$

$$B\mu_h\left[nr_e b^2 - pr_h\right]\mathcal{E}_x + (nb+p)\mathcal{E}_y = 0$$

$$\frac{\mathcal{E}_y}{\mathcal{E}_x} = \tan\theta_H = B\mu_h \cdot \frac{pr_h - nr_e b^2}{nb+p}$$

Using:

$$J_x = J_{xe} + J_{xh} = enb\mu_h(\mathcal{E}_x - r_e B b\mu_h \mathcal{E}_y) + pe\mu_h(\mathcal{E}_x + Br_h\mu_h\mathcal{E}_y)$$

$$\approx e\mu_h(nb+p)\mathcal{E}_x$$

we can calculate the Hall coefficient:

$$R_H = \frac{\mathcal{E}_y}{J_x B} = \cancel{\mathcal{E}_x\mu_h}\,\frac{pr_h - nr_e b^2}{nb+p} \cdot \frac{1}{e\cancel{\mu_h\mathcal{E}_x}(nb+p)} = \frac{pr_h - nr_e b^2}{e(nb+p)^2}$$

$$R_H = -\frac{1}{e}\,\frac{nr_e b^2 - pr_h}{(nb+p)^2}$$

This last result suggests that the Hall coefficient changes sign when $p = nb^2$, rather than at the intrinsic carrier concentration n_i. Also:

- for n-type samples: $R_H = \frac{1}{ep}\,\frac{r_h - (n/p)r_e b^2}{(1+\frac{n}{p}b)^2} \xrightarrow[\substack{n/p>1 \\ b\gg 1}]{} < 0$ always
- for p-type samples, R_H changes sign.

(B) Strong magnetic field case and two types of carriers

- We now assume that we have both electrons and holes present in the system and we have arbitrarily large magnetic field. For simplicity, we will also assume that the scattering mechanisms are energy independent, which implies $r_h = r_e = 1$. Then:

$$\begin{cases} J_{ex} = \sigma_{e1}\mathcal{E}_x - \sigma_{e2}\mathcal{E}_y \\ J_{ey} = \sigma_{e2}\mathcal{E}_x + \sigma_{e1}\mathcal{E}_y \end{cases} \quad \text{where: } \sigma_{e1} = \frac{ne\mu_e}{1+\mu_e^2B^2} \text{ and } \sigma_{e2} = B\mu_e\sigma_{e1}$$

$$\begin{cases} J_{hx} = \sigma_{h1}\mathcal{E}_x + \sigma_{h2}\mathcal{E}_y \\ J_{hy} = -\sigma_{h2}\mathcal{E}_x + \sigma_{h1}\mathcal{E}_y \end{cases} \quad \text{where: } \sigma_{h1} = \frac{pe\mu_h}{1+\mu_h^2B^2} \text{ and } \sigma_{h2} = B\mu_h\sigma_{h1}$$

- The Hall coefficient is again found from the condition $J_y = 0$, which in this case gives:

$$J_y = J_{ey} + J_{hy} = (\sigma_{e2} - \sigma_{h2})\mathcal{E}_x + (\sigma_{e1} + \sigma_{h1})\mathcal{E}_y$$

$$= B(\mu_e\sigma_{e1} - \mu_h\sigma_{h1})\mathcal{E}_x + (\sigma_{e1} + \sigma_{h1})\mathcal{E}_y = 0$$

i.e.
$$\frac{\mathcal{E}_y}{\mathcal{E}_x} = B\,\frac{\mu_h\sigma_{h1} - \mu_e\sigma_{e1}}{\sigma_{e1} + \sigma_{h1}} = \frac{\sigma_{h2} - \sigma_{e2}}{\sigma_{h1} + \sigma_{e1}}$$

and the Hall coefficient equals to:

$$R_H = \frac{\mathcal{E}_y}{J_x B} = \frac{\frac{\sigma_{h2} - \sigma_{e2}}{\sigma_{h1} + \sigma_{e1}} \cdot \frac{1}{B}}{(\sigma_{e1} + \sigma_{h1}) + (\sigma_{h2} - \sigma_{e2})\frac{\sigma_{h2} - \sigma_{e2}}{\sigma_{h1} + \sigma_{e1}}} = \frac{\sigma_{h2} - \sigma_{e2}}{(\sigma_{e1} + \sigma_{h1})^2 + (\sigma_{h2} - \sigma_{e2})^2} \cdot \frac{1}{B}$$

where:

$$\sigma_{h2} - \sigma_{e2} = B(\mu_h\sigma_{h1} - \mu_e\sigma_{e1}).$$

For high magnetic fields B, $\sigma_{e1} + \sigma_{h1} \ll |\sigma_{h2} - \sigma_{e2}|$, which leads to:

$$R_H \approx \frac{1}{B}\frac{1}{\sigma_{h2} - \sigma_{e2}} = -\frac{1}{B}\frac{1}{\sigma_{e2} - \sigma_{h2}}$$

where:

$$\sigma_{e2} - \sigma_{h2} = B(\mu_e\sigma_{e1} - \mu_h\sigma_{h1}) = B\left(\mu_e\frac{ne\mu_e}{1+\mu_e^2B^2} - \mu_h\frac{pe\mu_h}{1+\mu_h^2B^2}\right)$$

$$= B\left[ne\frac{\mu_e^2}{1+\mu_e^2B^2} - pe\frac{\mu_h^2}{1+\mu_h^2B^2}\right] \cong B\frac{1}{B^2}(n-p)e = \frac{(n-p)e}{B}$$

In this case:
$$R_H \approx -\frac{1}{\cancel{B}}\frac{1}{\frac{e}{\cancel{B}}(n-p)} = -\frac{1}{e}\frac{1}{n-p} = \frac{1}{e(p-n)}$$

This last result that $R_H \approx \frac{1}{e}\frac{1}{p-n}$, for high magnetic fields, is the same as the one obtained in degenerate metals where scattering occurs at the Fermi surface.

→ Note also that the Hall coefficient R_H does not depend upon the scattering mechanisms in the system. The reason for this is the following: If B is large, $w_c = eB/m^*$ is also large and $T_c = 2\pi/w_c$ is very small. Therefore, the electrons or holes do several cycles before they scatter, which makes the scattering events irrelevant. Note also that this is experimentally-observed behavior.

(c) Magnetoresistance

The magnetic field also influences the conductivity of the sample. Let $J_y = 0$, which gives:

$$J_x = \left[(\sigma_{e1} + \sigma_{h1}) + \frac{(\sigma_{h2} - \sigma_{e2})^2}{\sigma_{e1} + \sigma_{h1}}\right]\mathcal{E}_x = \sigma_{xx}\,\mathcal{E}_x$$

where:
$$\sigma_{xx} = \sigma_{e1} + \sigma_{h1} + \frac{(\sigma_{h2} - \sigma_{e2})^2}{\sigma_{e1} + \sigma_{h1}}$$

and:
$$\begin{cases} \sigma_{e1} + \sigma_{h1} = \dfrac{ne\mu_e}{1+\mu_e^2B^2} + \dfrac{pe\mu_h}{1+\mu_h^2B^2} \\[2ex] \sigma_{h2} - \sigma_{e2} = -B\left[ne\dfrac{\mu_e^2}{1+\mu_e^2B^2} - pe\dfrac{\mu_h^2}{1+\mu_h^2B^2}\right] \end{cases}$$

(1) Small magnetic field case

For small magnetic fields: $\sigma_{e1} + \sigma_{h1} \approx ne\mu_e + pe\mu_h$
$\sigma_{h2} - \sigma_{e2} \approx B(ne\mu_e^2 - pe\mu_h^2)$

which gives:
$$\sigma_{xx} \approx ne\mu_e + pe\mu_h - \frac{B^2(ne\mu_e^2 - pe\mu_h^2)^2}{ne\mu_e + pe\mu_h} = \sigma_{xx}(B)$$

The zero-field conductivity is:
$$\sigma_{xx}(0) \approx ne\mu_e + pe\mu_h$$

which gives:
$$\frac{\Delta\sigma_{xx}}{\sigma_{xx}(0)} = \frac{\sigma_{xx}(B) - \sigma_{xx}(0)}{\sigma_{xx}(0)} \cong -B^2\frac{(ne\mu_e^2 - pe\mu_h^2)^2}{(ne\mu_e + pe\mu_h)^2}$$

The longitudinal conductivity of the sample decreases quadratically with the magnetic field.

(2) Large magnetic field case

For large magnetic fields, we have:

$$\sigma_{e1} + \sigma_{h1} \approx \frac{ne}{\mu_e B^2} + \frac{pe}{\mu_h B^2}$$

$$\sigma_{h2} - \sigma_{e2} \approx -\cancel{B}\left[\frac{ne}{B^{\cancel{2}}} - \frac{pe}{B^{\cancel{2}}}\right] \approx -\frac{1}{B}(n-p)e$$

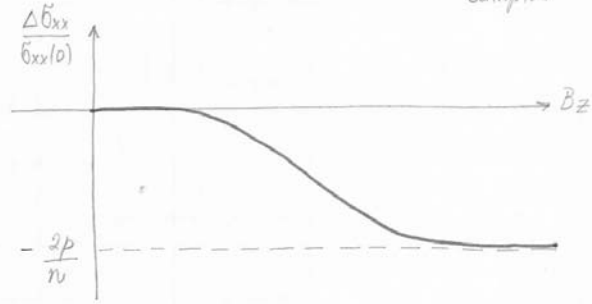
The magnetoconductance change is then equal to:

$$\frac{\Delta\sigma_{xx}}{\sigma_{xx}(0)} \approx \frac{\cancelto{0}{\frac{ne}{\mu_e B^2}} + \cancelto{0}{\frac{pe}{\mu_h B^2}} - (ne\mu_e + pe\mu_h) + \frac{1}{\cancel{B^2}}(n-p)^2 e^2 \cdot \frac{\cancel{B^2}}{(ne/\mu_e + pe/\mu_h)}}{ne\mu_e + pe\mu_h}$$

$$\approx \frac{(n-p)^2 e^2}{\left(\frac{ne}{\mu_e} + \frac{pe}{\mu_h}\right)(ne\mu_e + pe\mu_h)} - 1 = \frac{(n-p)^2}{\left(\frac{n}{\mu_e} + \frac{p}{\mu_h}\right)(n\mu_e + p\mu_h)} - 1$$

For $\mu_e \approx \mu_h$ (which is rarely satisfied), we have:

$$\frac{\Delta\sigma_{xx}}{\sigma_{xx}(0)} \approx \frac{(n-p)^2 - (n+p)^2}{(n+p)^2} = -\frac{4np}{(n+p)^2} \xrightarrow[\text{samples}]{n\text{-type}} -\frac{4np}{n^2} = -4\frac{p}{n}$$



For $\mu_e \ll \mu_h$, which is more realistic case:

$$\frac{\Delta\sigma_{xx}}{\sigma_{xx}(0)} \approx \frac{(n-p)^2}{n^2} - 1 = \frac{n^2 - 2np + p^2}{n^2} - 1 = 1 - \frac{2p}{n} + \left(\frac{p}{n}\right)^2 - 1 \approx -\frac{2p}{n}$$

This case is plotted in the figure.

- At low temperatures in strong magnetic fields, quantum effects occur with the formation of Landau levels. In degenerate semiconductors, there is an oscillatory which is known as Shubnikov-de Haas effect.