High - Field Transport

(A) General considerations on high-field transport; velocity saturation and velocity overshoot effects

○ In all examples that we considered so far, which involved the RTA, it was assumed that the system is near thermal equilibrium. Also, the symmetric component of the distribution function was assumed to be equal to the thermal equilibrium distribution function. The asymmetric component of the distribution function then equals to (for parabolic band structure):

$$f_A = -e\tau_m \, \mathcal{E}_z \, v_z \, \frac{\partial f_o}{\partial E} = -\frac{e\tau_m \, \mathcal{E}_z}{m^*} \, \frac{\partial f_o}{\partial v_z}$$

i.e.

$$f = f_o + f_A = f_o - \frac{e\tau_m \, \mathcal{E}_z}{m^*} \, \frac{\partial f}{\partial v_z}$$

If we now substitute back for f in the term $\partial f / \partial v_z$, we will generate the series:

$$f = f_o - \frac{e\tau_m \, \mathcal{E}_z}{m^*} \, \frac{\partial f_o}{\partial v_z} + \left(\frac{e\tau_m}{m^*}\right)^2 \mathcal{E}_z^2 \, \frac{\partial^2 f_o}{\partial v_z^2} + \cdots$$

For not too large values for $\mathcal{E}_z$, where the drift velocity is much smaller than the thermal velocity, the above series will converge. Also, using the definition

$$v_d = \frac{\int_{-\infty}^{+\infty} v_z \, f \, d^3v}{\int f_o \, d^3v}$$

for the average drift velocity along the electric field direction, and taking into account that those terms that are even powers of v_z will vanish (terms, such as $\partial^2 f_o / \partial v_z^2$ and $\partial^4 f_o / \partial v_z^4$), we arrive at:

$$v_d = \mu(\mathcal{E}_z) \, \mathcal{E}_z = \mu_o \left(\mathcal{E}_z + \beta \mathcal{E}_z^3 + \cdots \right).$$

This leads to the following series expansion for the field-dependent mobility:

$$\mu(\mathcal{E}_z) = \mu_0 + \mu_0 \beta \mathcal{E}_z^2 + \cdots$$

low-field mobility   deviation from Ohm's law

Based on the fact how many terms are needed to accurately describe the field-dependent mobility, we can make the following two classifications:

(a) If the series expansion may be terminated with the $\beta \mathcal{E}_z^2$ terms in the mobility expression, the carriers are considered to be WARM.

(b) If one has to retain more terms in the series, or if the series expansion is not possible at all, the carriers are called HOT.

Note that warm or hot carriers exist when current flows in the sample. If we only have built-in electric fields, such as those that occur in a pn-junction, the carriers remain in thermal equilibrium with the lattice as a result of the scattering events.

- For qualitative understanding of the field dependence of the carrier drift velocity or mobility, let us consider a semiconductor in which phonon scattering (nonpolar optical or intervalley phonons) dominates. If we have warm carriers in the system, then a Maxwell-Boltzmann distribution function with electron temperature Te larger than the lattice temperature T can be assumed, i.e.

$$f(E) \propto \exp(-E/k_B T_e)$$

Now consider the following:

→ The rate of absorption of phonons, which moves carriers from state E to state $E + \hbar\omega_0$ is given by:

$$A f(E) N_0$$

→ The rate of emission of phonons, which moves carriers from state $E + \hbar\omega_0$ to state E is given by:

$$A f(E + \hbar\omega_o)(N_o + 1)$$

Therefore, the ratio of the phonon absorption and phonon emission rates is:

$$\frac{f(E + \hbar\omega_o)}{f(E)} \frac{N_o + 1}{N_o} = \frac{e^{-(E + \hbar\omega_o)/k_B T_e}}{e^{-E/k_B T_e}} \frac{N_o + 1}{N_o} = e^{-\hbar\omega_o/k_B T_e} \left(\frac{N_o + 1}{N_o}\right)$$

Using Bose-Einstein statistics to calculate the phonon occupancy, we get:

$$\frac{N_o + 1}{N_o} = \frac{\dfrac{1}{\exp(\hbar\omega_o/k_B T) - 1} + 1}{\dfrac{1}{\exp(\hbar\omega_o/k_B T) - 1}} = 1 + \exp\left(\frac{\hbar\omega_o}{k_B T}\right) - 1 = \exp\left(\frac{\hbar\omega_o}{k_B T}\right)$$

This leads to the following estimate of the ratio of phonon absorption and phonon emission processes:

$$\frac{f(E + \hbar\omega_o)}{f(E)} \left(\frac{N_o + 1}{N_o}\right) = \exp\left[\frac{\hbar\omega_o}{k_B T} - \frac{\hbar\omega_o}{k_B T_e}\right] = \exp\left[\frac{\hbar\omega_o}{k_B T}\left(1 - \frac{T}{T_e}\right)\right]$$

For $T_e > T$ the exponential factor is positive and this means that the emission process will dominate. Because of this, v_d will start to increase more slowly and eventually velocity saturation is achieved. This, in turn, means that the mobility of the carriers will decrease with increasing the electric field.
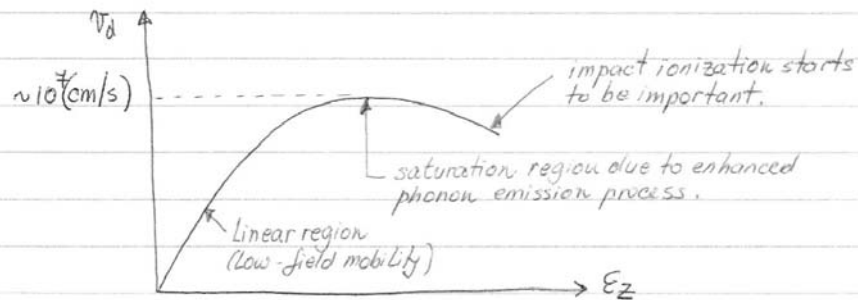
- For quantitative description of $\mu(\mathcal{E})$, i.e. the coefficient β, we consider the energy balance equation, which is nothing more but a statement for the conservation of energy in the system, i.e.

$$\underbrace{e\,\vec{v_d}\cdot\vec{\mathcal{E}}}_{\substack{\text{energy gained}\\\text{by the field}} } = -\underbrace{\left\langle \frac{\partial E}{\partial t}\right\rangle_{coll}}_{\text{energy loss due to energy relaxation mechanisms}}$$

For energy independent relaxation times, and using RTA, we have:

$$\left\langle \frac{\partial E}{\partial t}\right\rangle_{coll} = \frac{\langle E(T_e)\rangle - \langle E(T)\rangle}{\tau_E} = \frac{3}{2}k_B\,\frac{T_e - T}{\tau_E}$$

In this case, the energy balance equation becomes:

$$-e\,\mu(\mathcal{E}_z)\,\mathcal{E}_z^2 = -\frac{3}{2}\frac{k_B}{\tau_E}(T_e - T) \quad \Rightarrow \quad \boxed{e\,\mu(\mathcal{E}_z)\,\mathcal{E}_z^2 = \frac{3k_B}{2\tau_E}(T_e - T)}$$

In general, the momentum relaxation time τ_m depends upon the energy, i.e. $\tau_m = \tau_0\,E^s$. Since $\mu = e\tau_m/m^*$, that means that we can use the following series expansion for $\mu(\mathcal{E}_z)$, i.e. $\mu(T_e)$:

$$\mu(T_e) = \mu(T) + \left(\frac{\partial \mu}{\partial T}\right)(T_e - T) + \cdots = g + (T_e - T)g' + \cdots$$
$$\cong g\left[1 + (T_e - T)\,g'/g\right]$$

Also, from $\mu(T_e) = \mu_0(1 + \beta\mathcal{E}_z^2) = g(1 + \beta\mathcal{E}_z^2)$, we get:

$$g(1 + \beta\mathcal{E}_z^2) \cong g\left[1 + (T_e - T)g'/g\right] \quad \Rightarrow \quad \boxed{\beta\mathcal{E}_z^2 = (T_e - T)\frac{g'}{g}}$$

Substituting this result into the energy conservation expression, we get:

$$e\underset{\approx g}{\underbrace{\mu(\mathcal{E}_z)}}\frac{1}{\beta}\cancel{(T_e - T)}\frac{g'}{g} = \frac{3k_B}{2\tau_E}\cancel{(T_e - T)}$$

The energy relaxation time τ_E is, thus, given by:

$$\tau_E = \frac{3k_B\beta}{2eg'}$$

Now using: $\dfrac{d(\ln g)}{d(\ln T)} = \dfrac{dg/g}{dT/T} = \dfrac{T}{g}\dfrac{dg}{dT} = T\dfrac{g'}{g}$, we have:

$$\frac{1}{g'} = \frac{T}{g}\frac{1}{d(\ln g)/d(\ln T)}$$

This finally leads to the following expression for the energy relaxation time τ_E:

$$\boxed{\tau_E \approx \frac{3k_B\beta T}{2eg}\frac{1}{d(\ln g)/d(\ln T)} = \frac{T\beta}{7740\,°K/V}\frac{1}{\mu_0}\left(\frac{d(\ln g)}{d(\ln T)}\right)^{-1}}$$

When the momentum relaxation time is of the form $\tau_m = \tau_0 E^r$, this gives: $\mu(T) = \mu_0(k_B T)^r$, i.e.

$$\frac{d(\ln g)}{d(\ln T)} = \frac{d(\ln \mu_0 + r\ln(k_B T))}{d(\ln T)} = r$$

Therefore:

$$\boxed{\tau_E = \frac{T}{7740\,°K/V}\cdot\frac{\beta}{\mu_0 r}}$$

Examples:

$\mu_0 = 10^4\ cm^2/Vs \implies |\beta| = 10^{-4}\ cm^{-2}V^{-2}$, $\tau_E \approx 10^{-10}$ sec.

Also, for scattering mechanisms with $r \gtrless 0$, we get:

(a) $r < 0$, $\beta < 0 \implies \mu\downarrow$ with $\varepsilon_z\uparrow$ (acoustic)

(b) $r > 0$, $\beta > 0 \implies \mu\uparrow$ with $\varepsilon_z\uparrow$ (ionized impurities)

The first measurements for the field dependence of the mobility, i.e. drift velocity have been done by Ryder and Shockley, on n-type Ge samples at different temperatures.

(*) A technique of measuring the mobility is the time-of-flight experiment: A pulsed beam of $\approx 50\,keV$ electrons is focussed on one side of a trap-free conductor and the time of arrival of the carriers at the opposite side of the conductor is observed.
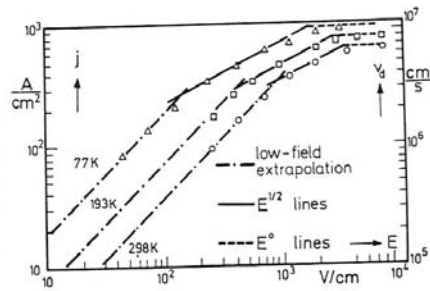
Fig. 4.31. Current density in n-type germanium as a function of electric field intensity at various lattice temperatures (after [4.70])
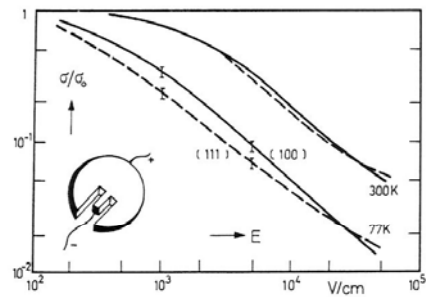

Fig. 4.32. Conductivity of n-type germanium as a function of electric field intensity for two crystallographic directions of the applied field and for two temperatures. The inset shows the sample shape (after [4.71])
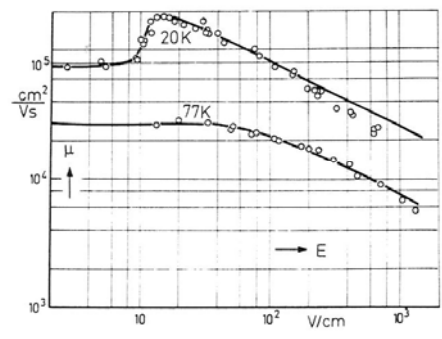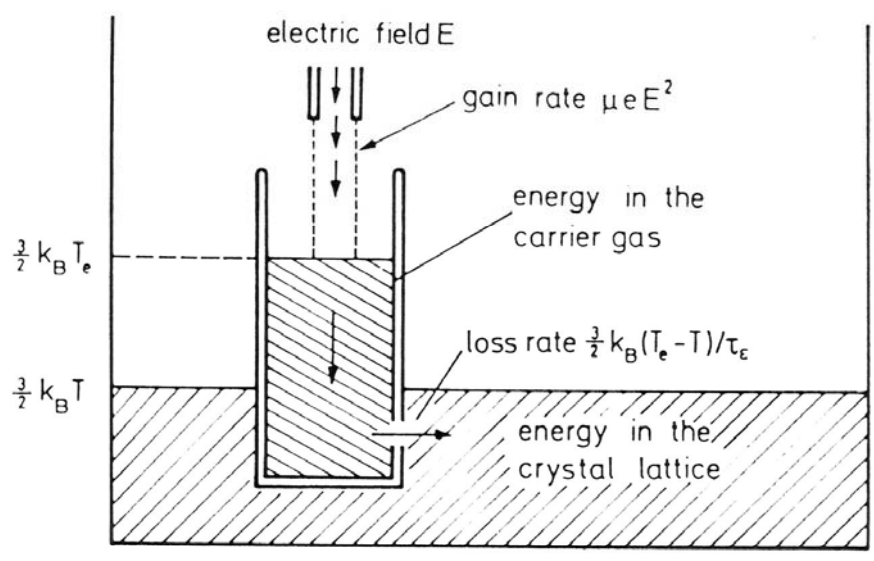

Fig. 4.33. Mobility of n-type germanium as a function of electric field intensity at 20 and 77 K (after [4.70])

Schematic representation of the carrier energy balance indicating the energy gain from the applied electric field and the energy loss ~~due~~ to the crystalline lattice. For $E \neq 0$, the mean carrier energy $3k_BT_e/2$ for a non-degenerate gas is always larger than the mean thermal energy: $3k_BT/2$. Also notice the small specific heat of the carriers and the large specific heat of the lattice.

○ Another effect that occurs during transients, because of a sudden application of an electric field, is the VELOCITY OVERSHOOT EFFECT. The velocity overshoot effect can be explained by considering the momentum balance equation, of the form

$$m^* \frac{dv_{dz}}{dt} = -e \, \mathcal{E}_z - \underbrace{\frac{m^* v_{dz}}{\tau_m}}_{\text{friction introduced by scattering}}$$

The above equation neglects diffusion effects and works quite well for the case when we have strong, uniform electric fields.

If the electric field is applied at $t=0$, we have:

$$m^* \frac{dv_{dz}}{dt} + \frac{m^* v_{dz}}{\tau_m} = -e \, \mathcal{E}_z$$

$$\frac{dv_{dz}}{dt} + \frac{v_{dz}}{\tau_m} = -\frac{e \, \mathcal{E}_z}{m^*}$$

The solution of this equation is of the form

$$v_{dz}(t) = -\frac{e \, \mathcal{E}_z \, \tau_m}{m^*} + A \, e^{-t/\tau_m}$$

$$v_{dz}(0) = A - \frac{e \, \mathcal{E}_z \, \tau_m}{m^*} = 0 \implies v_{dz}(t) = \frac{e \, \mathcal{E}_z \, \tau_m}{m^*} \left(e^{-t/\tau_m} - 1 \right)$$

This last expression suggests that steady-state is achieved in a time interval on the order of the momentum relaxation time. Note that we have assumed here that τ_m is not a function of time, which is not really true in realistic situations, in which τ_m is not a constant but decreases as carriers are accelerated by gaining energy from the field. Hence, the actual velocity has to be between the two limits displayed on the figure on the next page.

V_{dz}

τ_{m1}

$\tau_{m2} < \tau_{m1}$

t
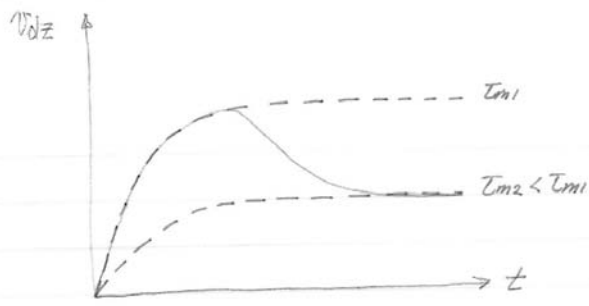
(a) The velocity overshoot in Si occurs because of the rather different momentum and energy relaxation times. More precisely $\tau_E \gg \tau_m$, i.e. the electron temperature rises much more slowly than the average velocity.

(b) For GaAs materials, the velocity overshoot effect occurs because of the onset of intervalley transfer, i.e. transfer of carriers from the Γ to the L-valley, in which the effective mass is much larger.