Thermal Effects

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Thermal Effects in Semiconductors Can be Separated Into:

- Electro-Thermal Effects
  - Joule Heating
- Thermo-Electric Effects
  - Seebeck Effect
  - Peltier Effect
  - Thomson Effect

ELECTRO-THERMAL EFFECTS

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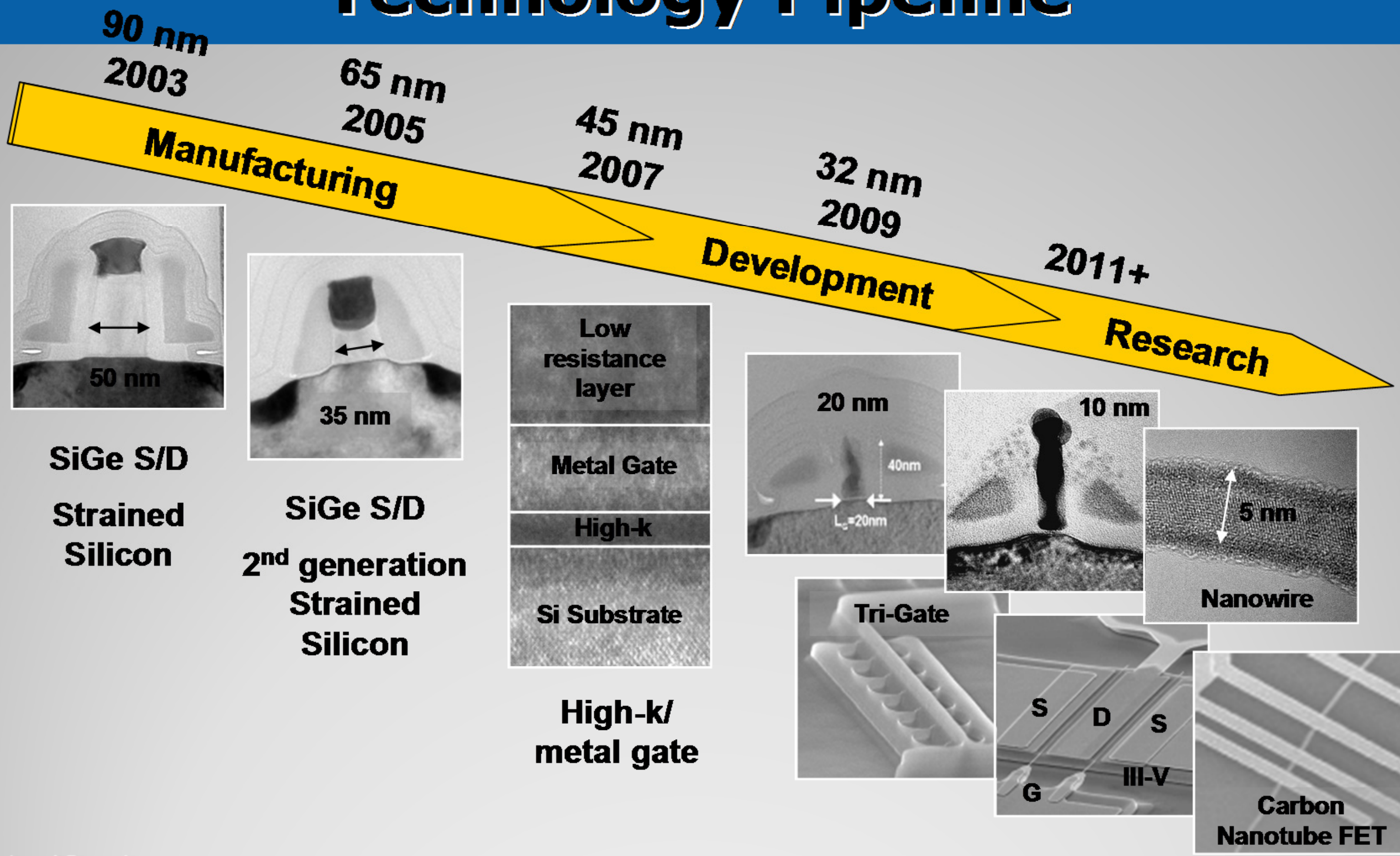
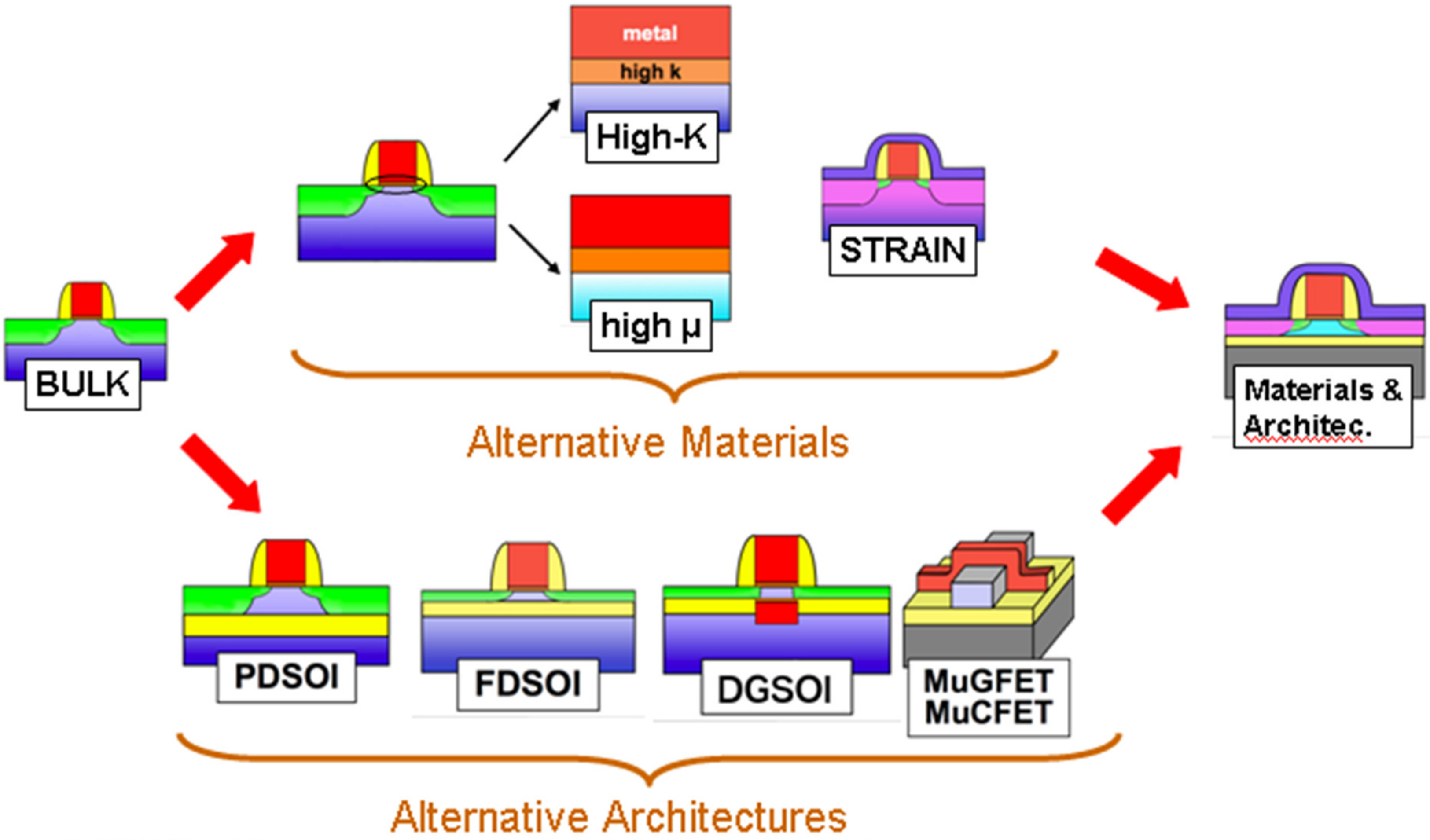
Innovation-Enabled Technology Pipeline



Intel Developer FORUM

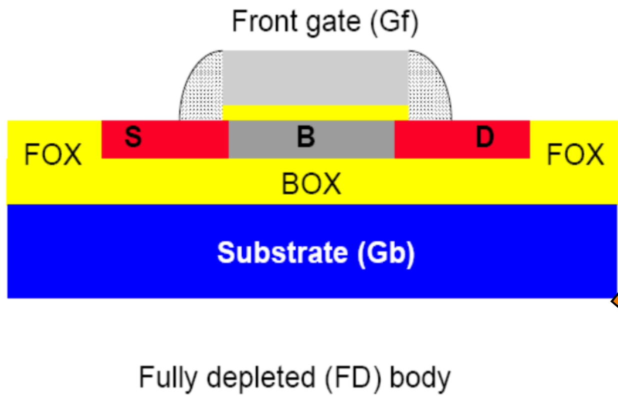
Future options subject to change

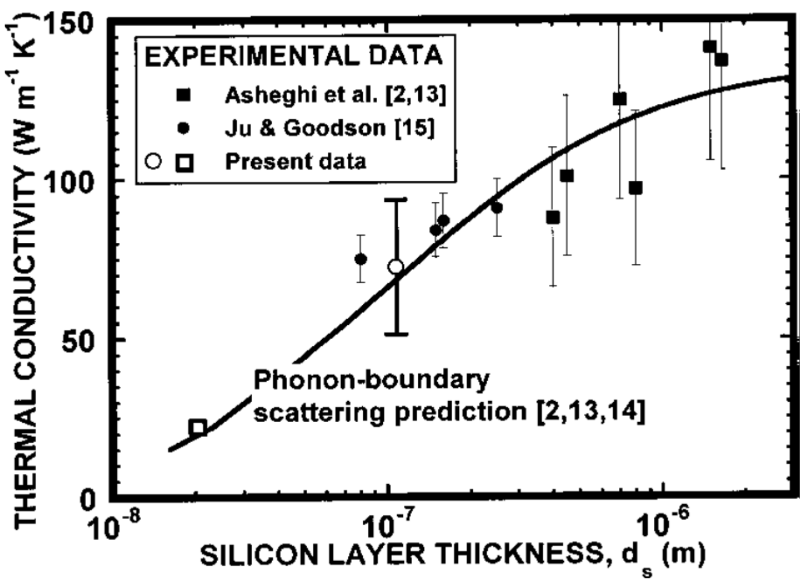
Two avenues for enhancing transistor performance



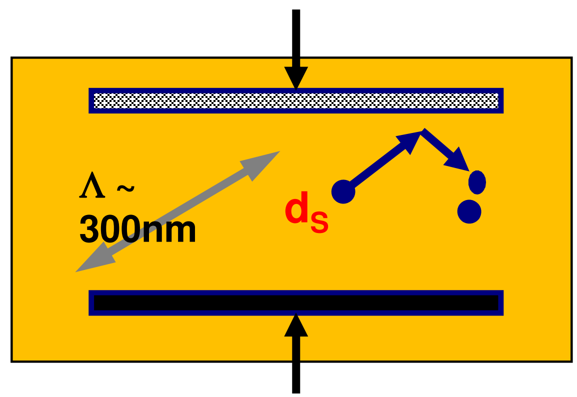
Courtesy of Robert W. Dutton, Stanford University.

Why self-heating effects arise in SOI technology?

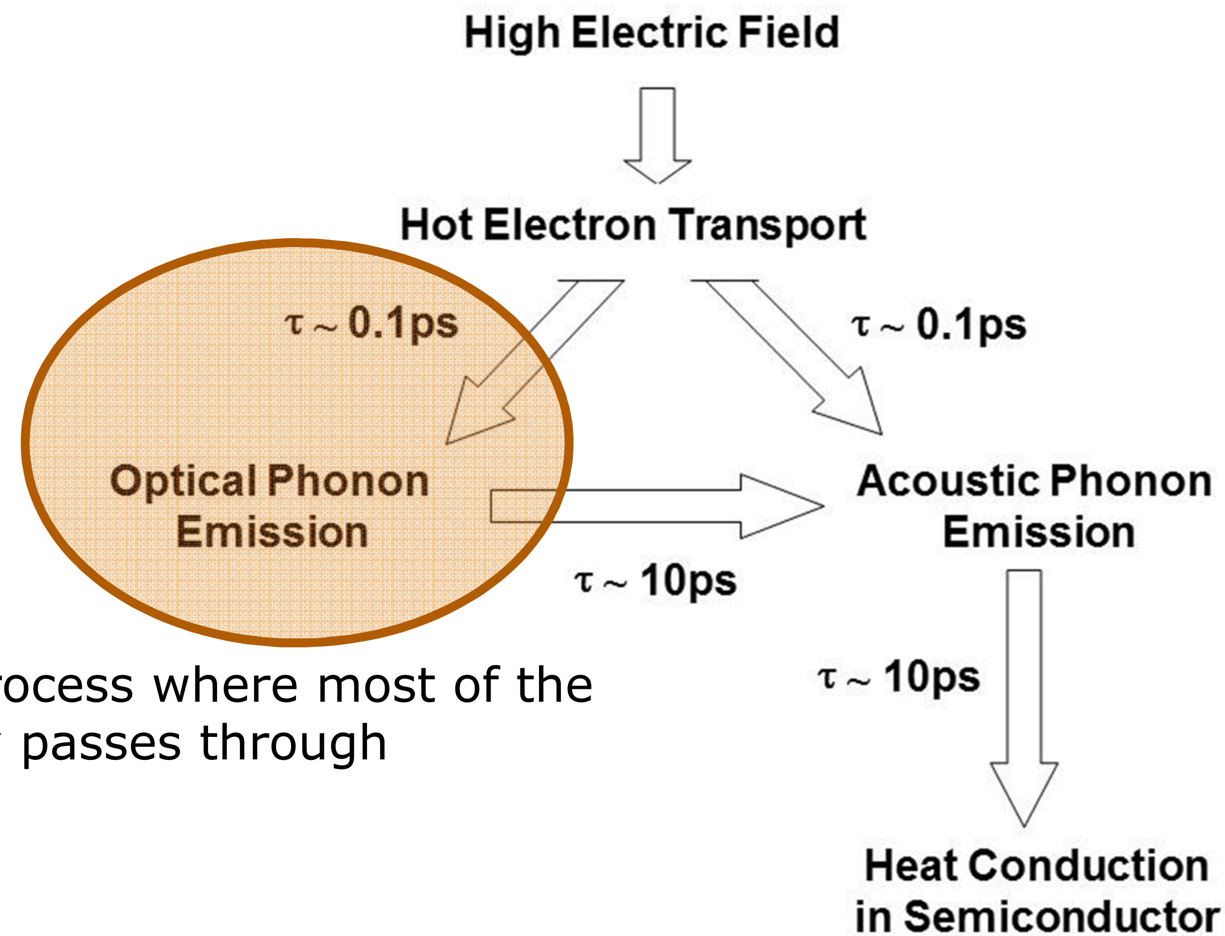




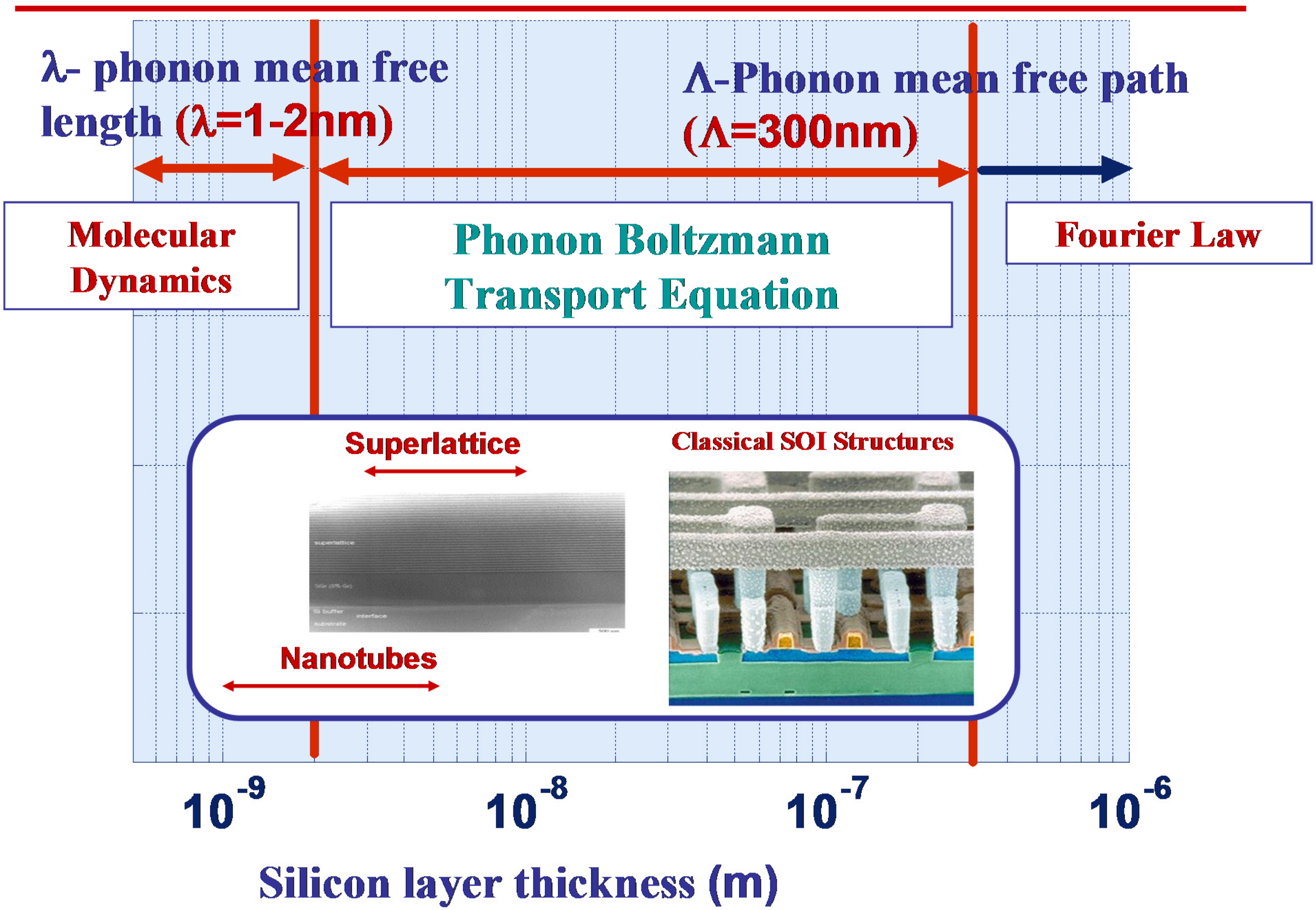
Material	k_{th} (W/mK)
Si	148
Ge	60
Silicides	40
Si (10 nm)	13
SiO_2	1.4



Most likely path for transfer of heat in a semiconductor device:



Fast process where most of the energy passes through


λ- phonon mean free length (λ=1-2nm)
Λ-Phonon mean free path (Λ=300nm)
Molecular Dynamics
Phonon Boltzmann Transport Equation
Fourier Law
Superlattice
Classical SOI Structures
Nanotubes
10^{-9}
10^{-8}
10^{-7}
10^{-6}
Silicon layer thickness (m)

Ways of Treating Electro-Thermal Effects:

- Heat Conduction Equation

- Hydrodynamic Models

- Phonon Boltzmann Equation

Heat Conduction Equation

$$-\nabla\cdot\left[\kappa\nabla T(r,t)\right]+H(r,t)=\rho C_V\frac{\partial T(r,t)}{\partial t}=c\frac{\partial T(r,t)}{\partial t}$$

Within the Joule heating model, the thermal model consists of the heat diffusion equation using a Joule heating term as the source.

$$\mathrm{H}=\mathrm{J}\bullet\mathrm{E}+(R-G)(E_G+3k_BT)$$

Within the electron-lattice scattering model, the thermal system is represented as a single lattice temperature and is considered to be in thermal equilibrium.

$$H=\frac{3\rho k_B}{2}\left(\frac{T_e-T_L}{\tau_{e-L}}\right)$$

Phonon-model. Under thermal non-equilibrium conditions a system of two phonons is used:

$$H=\frac{n}{N_{sim}\Delta t}\sum\left(\hbar\omega_{ems}-\hbar\omega_{abs}\right)$$

Theoretical Models implemented in Silvaco ATLAS Device Simulator

$$\nabla\cdot(\kappa\nabla T) + J\cdot E = 0$$

Temperature dependent
mobilities and diffusion coefficients

ASU Model

$$\left(\frac{\partial}{\partial t}+v_e(\mathrm{k})\cdot\nabla_r+\frac{e}{\hbar}E(\mathrm{r})\cdot\nabla_k\right)f=\sum_{\mathrm{q}}\left\{W_{e,\mathrm{q}}^{\mathrm{k+q}\to\mathrm{k}}+W_{a,-\mathrm{q}}^{\mathrm{k+q}\to\mathrm{k}}-W_{e,-\mathrm{q}}^{\mathrm{k}\to\mathrm{k+q}}-W_{a,\mathrm{q}}^{\mathrm{k}\to\mathrm{k+q}}\right\}$$

$$\left(\frac{\partial}{\partial t}+v_p(q)\cdot\nabla_r\right)g=\sum_{\mathrm{k}}\left\{W_{e,\mathrm{q}}^{\mathrm{k+q}\to\mathrm{k}}-W_{a,\mathrm{q}}^{\mathrm{k}\to\mathrm{k+q}}\right\}+\left(\frac{\partial g}{\partial t}\right)_{p-p}$$

J. Lai and A. Majumdar, "Concurent thermal and electrical modeling of submicrometer silicon devices", J. Appl. Phys. , Vol. 79, 7353 (1996).

$$C_{LO}\frac{\partial T_{LO}}{\partial t}=\frac{3nk_B}{2}\left(\frac{T_e-T_L}{\tau_{e-LO}}\right)+\frac{nm^*v_d^2}{2\tau_{e-LO}}-C_{LO}\left(\frac{T_{LO}-T_A}{\tau_{LO-A}}\right),$$

$$C_A\frac{\partial T_A}{\partial t}=\nabla\cdot(k_A\nabla T_A)+C_{LO}\left(\frac{T_{LO}-T_A}{\tau_{LO-A}}\right)+\frac{3nk_B}{2}\left(\frac{T_e-T_L}{\tau_{e-L}}\right).$$

Meaning of the Various Terms

Energy gain from the electrons

Energy loss to acoustic phonons

$$C_{LO}\frac{\partial T_{LO}}{\partial t}=\frac{3nk_B}{2}\left(\frac{T_e-T_L}{\tau_{e-LO}}\right)+\frac{nm^*v_d^2}{2\tau_{e-LO}}-C_{LO}\left(\frac{T_{LO}-T_A}{\tau_{LO-A}}\right),$$

$$C_A\frac{\partial T_A}{\partial t}=\nabla\cdot(k_A\nabla T_A)+C_{LO}\left(\frac{T_{LO}-T_A}{\tau_{LO-A}}\right)+\frac{3nk_B}{2}\left(\frac{T_e-T_L}{\tau_{e-L}}\right).$$

Heat Diffusion

Gain term due to optical phonons

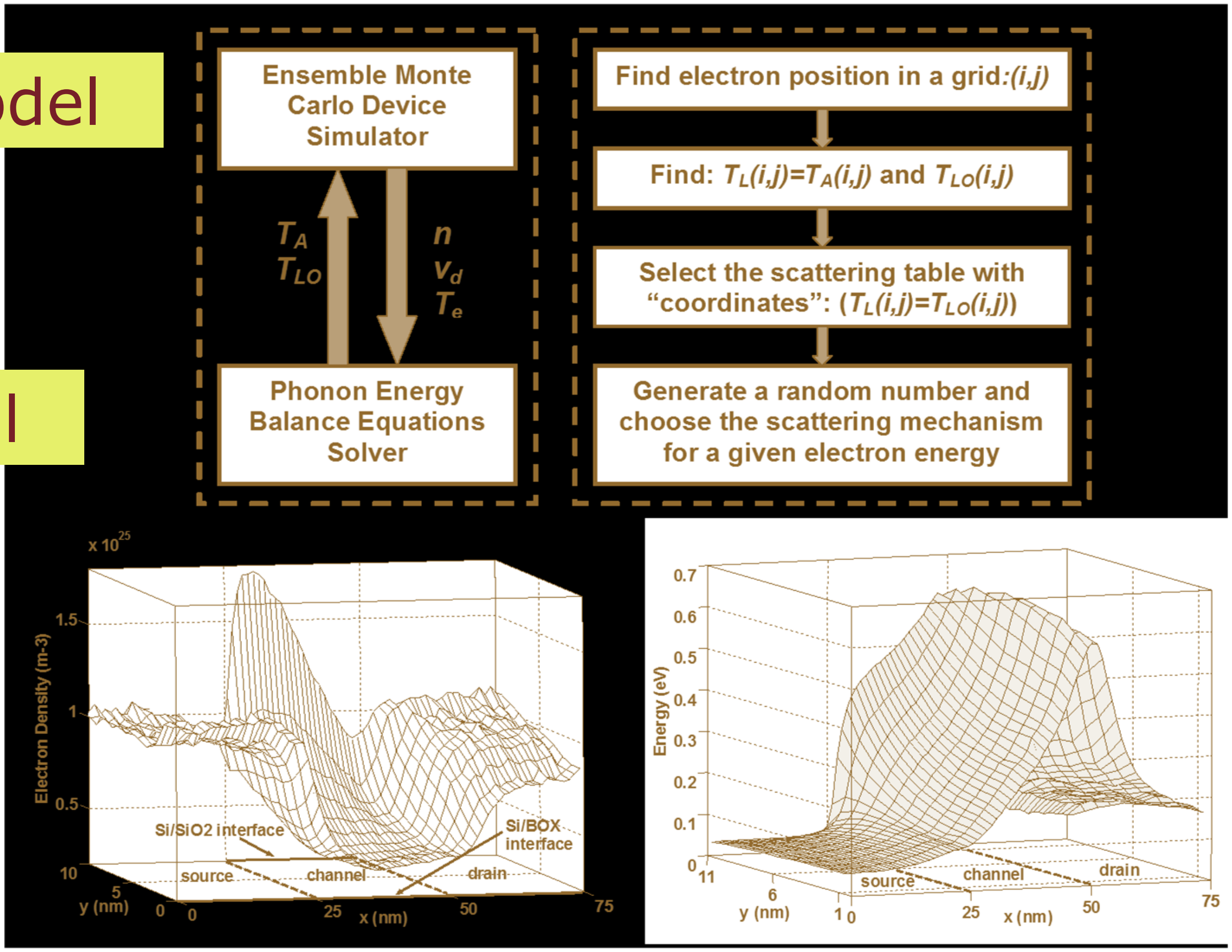
Gain term due to electrons (omitted if acoustic phonon scattering is treated as elastic scattering process)

K. Raleva, D. Vasileska, S. M. Goodnick and M. Nedjalkov, Modeling Thermal Effects in Nanodevices, IEEE Transactions on Electron Devices, vol. 55, issue 6, pp. 1306-1316, June 2008.

Exchange of Variables Between the Two Modules

Particle Model

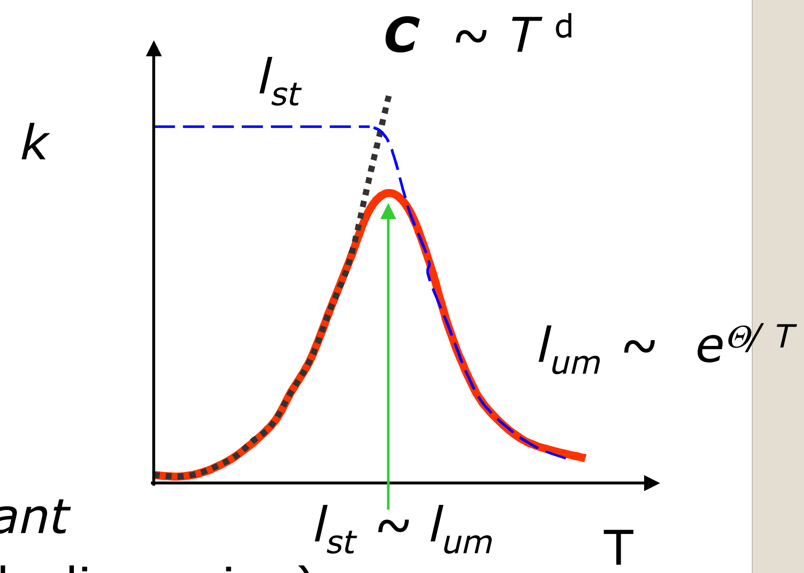
Fluid Model



Thermal Conductivity: $k = k_e + k_p$

$$k_p = \frac{1}{3} C\, v\, l$$

Specific heat — Sound velocity — Phonon *mfp*



Specific heat :

If $T > \Theta$, $C \sim constant$

If $T << \Theta$, $C \sim T^d$ (d: dimension)

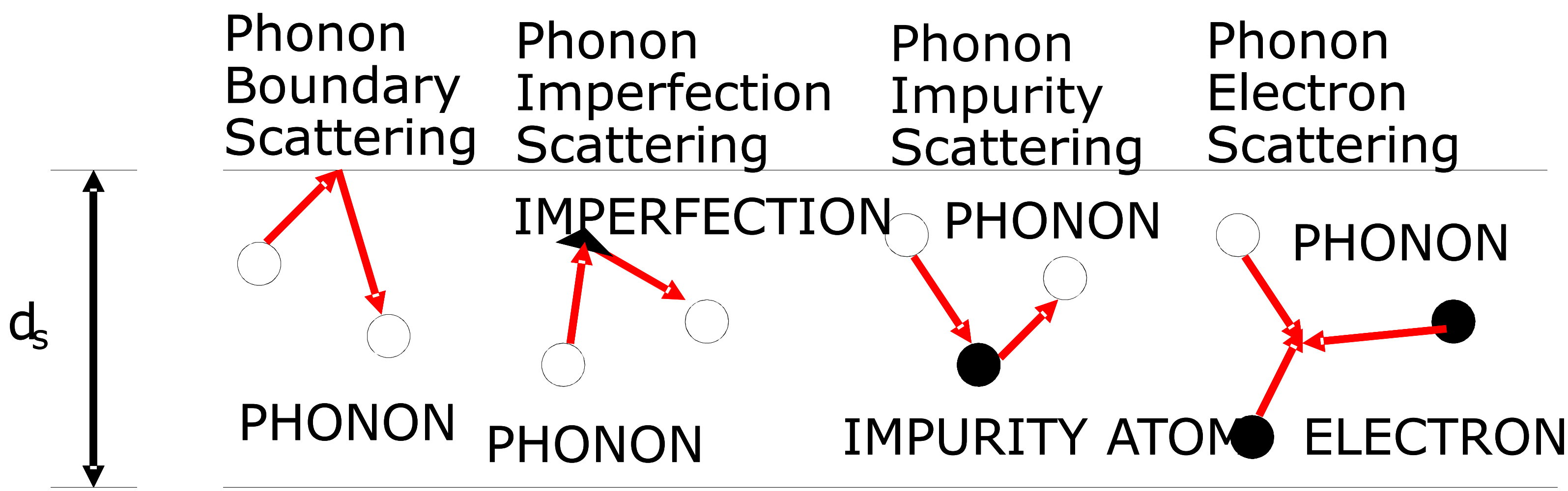
Mean free path:

$$\frac{1}{l} = \frac{1}{l_{st}} + \frac{1}{l_{um}}$$

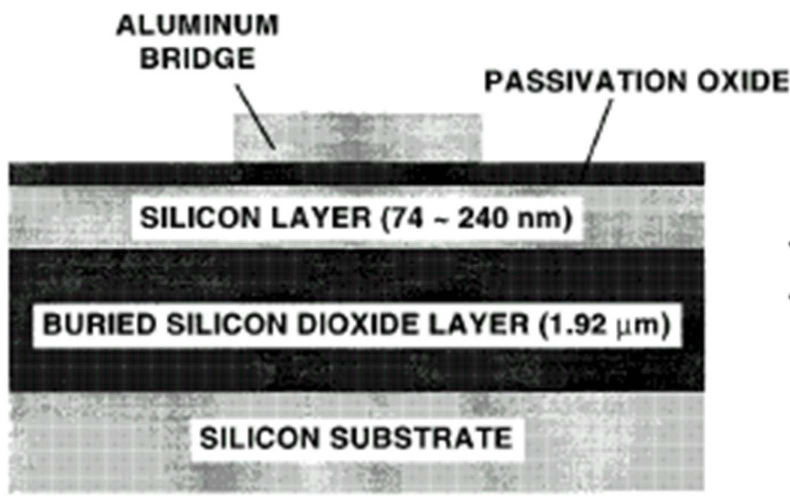
Static scattering (phonon -- defect, boundary): $l_{st} \sim constant$

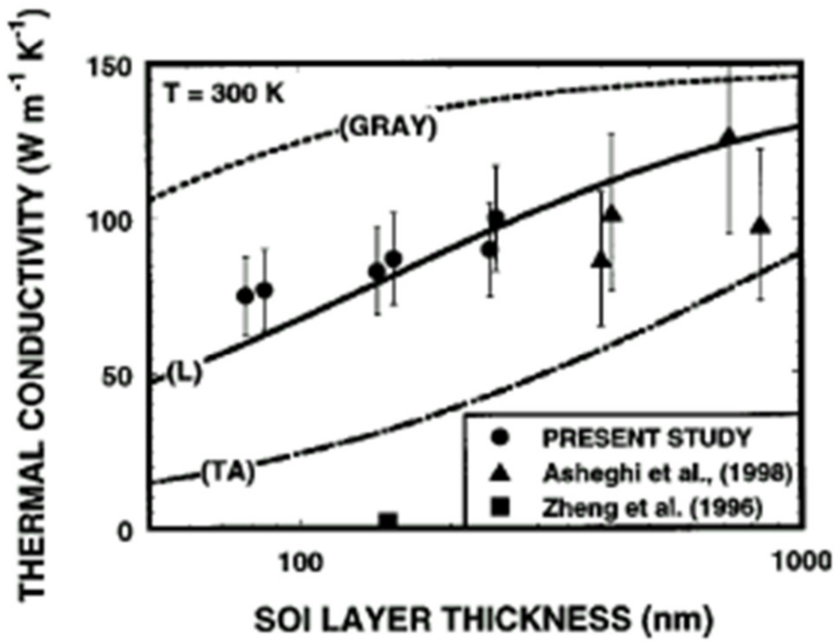
Umklapp phonon scattering: $l_{um} \sim e^{\Theta/T}$

Various factors that affect the thermal conductivity value



1. Ashegi, Leung, Wong, Goodson, *Appl. Phys. Lett.* 71, 1798 (1997)
2. Ju and Goodson, *Appl. Phys. Lett.* 74, 3005 (1999)



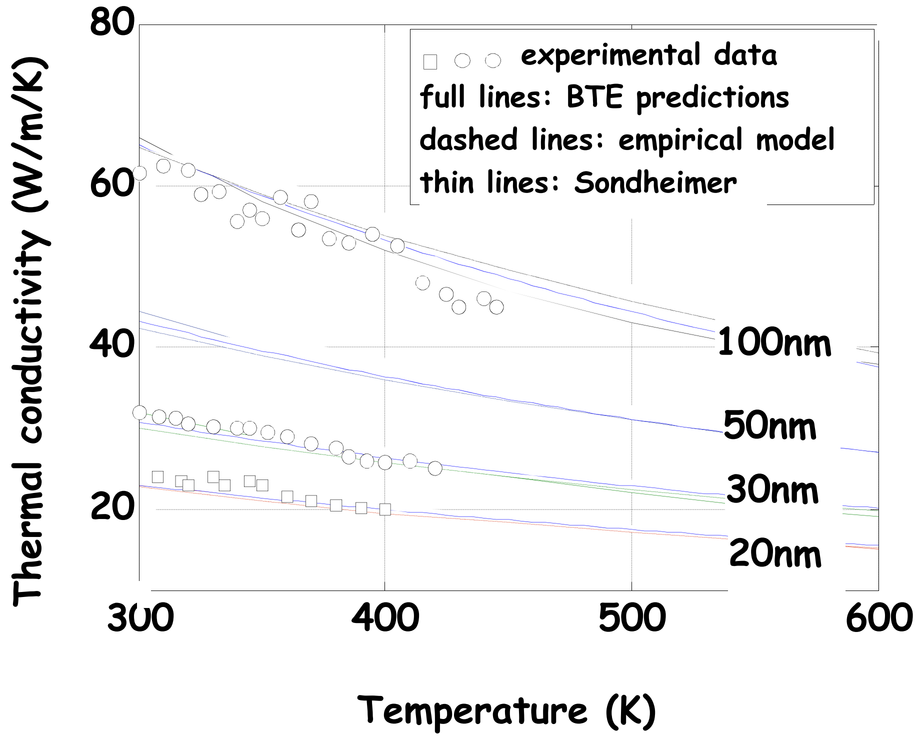


- Experimental data are from Asheghi and coworkers
- Simulated data are empirical fit by Vasileska and co-workers that follows from the Sondheimer theory and perfectly explains the experimental data of Asheghi and co-workers

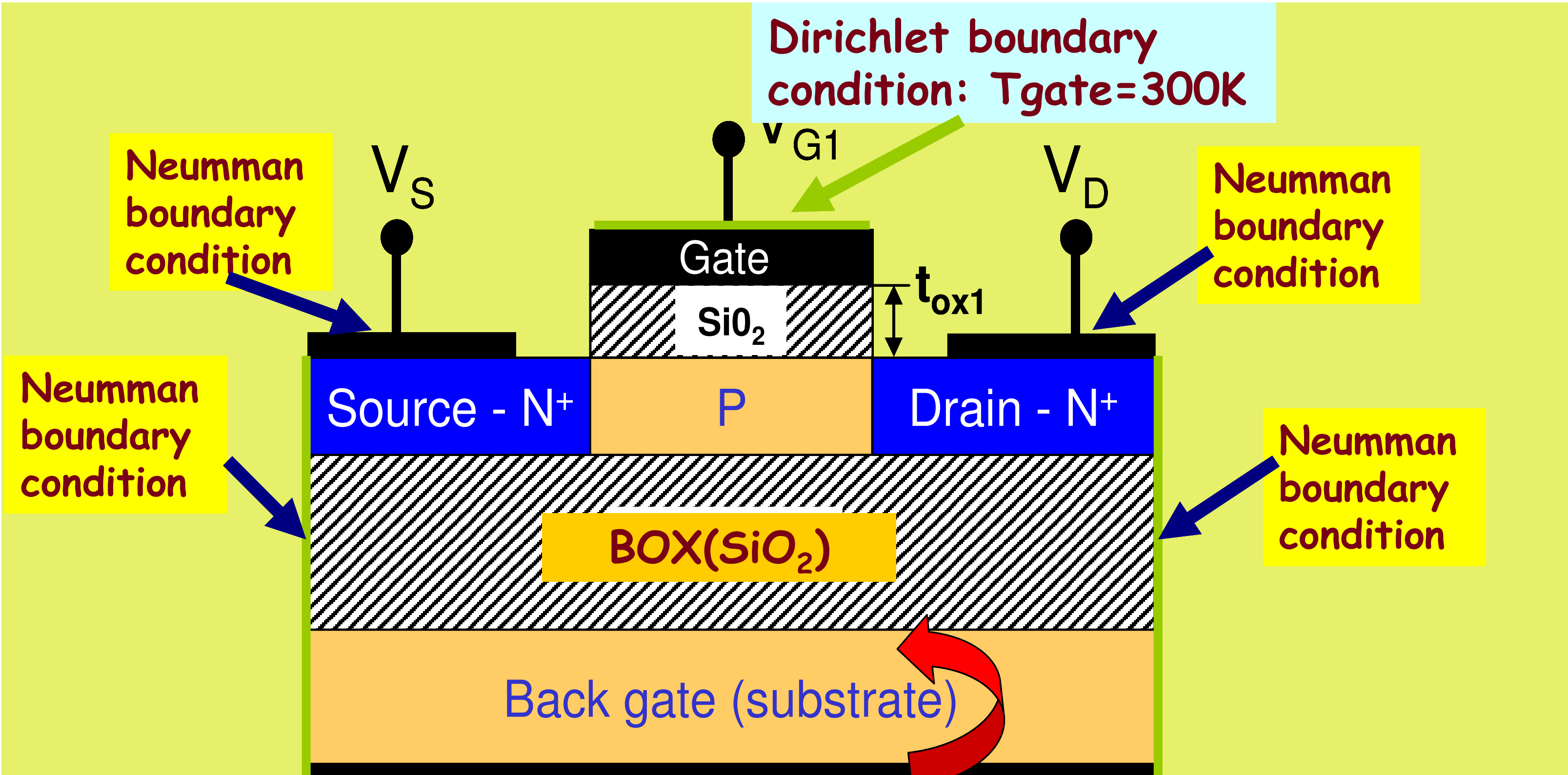
$$\kappa(z) = \kappa_0(T)\int_0^{\pi/2} \sin^3\theta \left\{1-\exp\left(-\frac{a}{2\lambda(T)\cos\theta}\right)\cosh\left(\frac{a-2z}{2\lambda(T)\cos\theta}\right)\right\}d\theta$$

$$\lambda(T) = \lambda_0(300/T)$$

$$\kappa_0(T) = \frac{135}{a+bT+cT^2} \quad \mathrm{W/m/K}$$

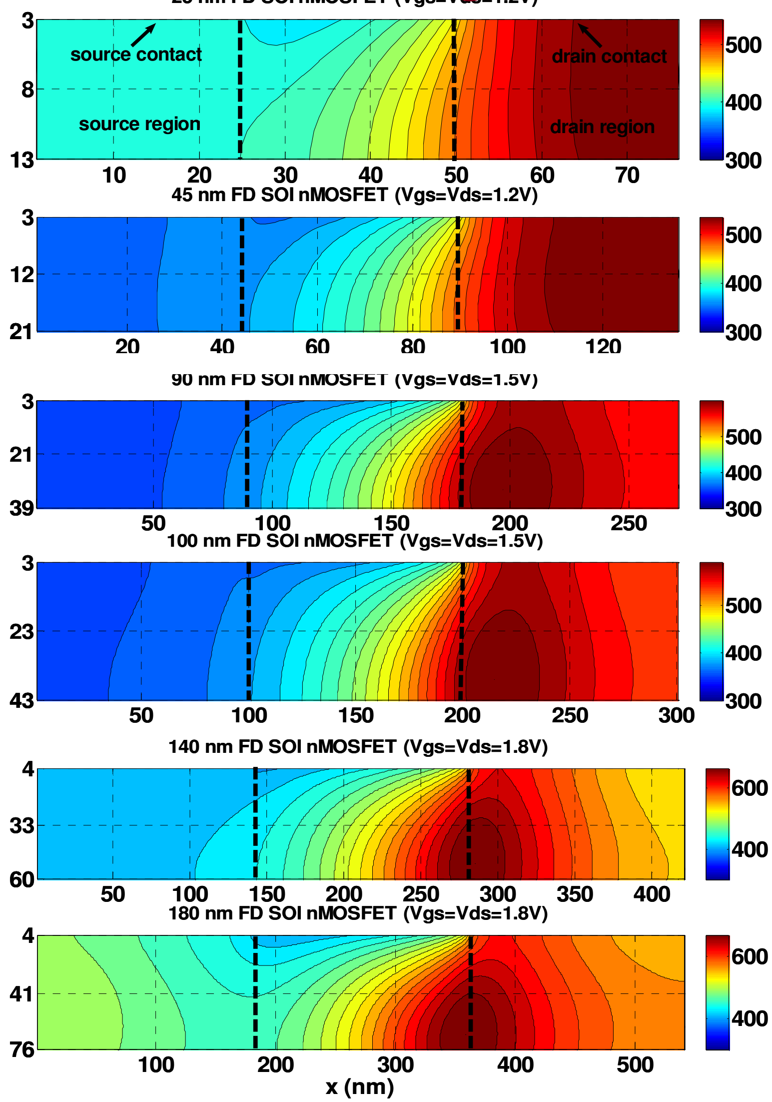


D. Vasileska, K. Raleva and S. M. Goodnick, Electrothermal Studies of FD SOI Devices That Utilize a New Theoretical Model for the Temperature and Thickness Dependence of the Thermal Conductivity, IEEE Transactions on Electron Devices, Vol. 57, pp. 726 – 728 (2010).

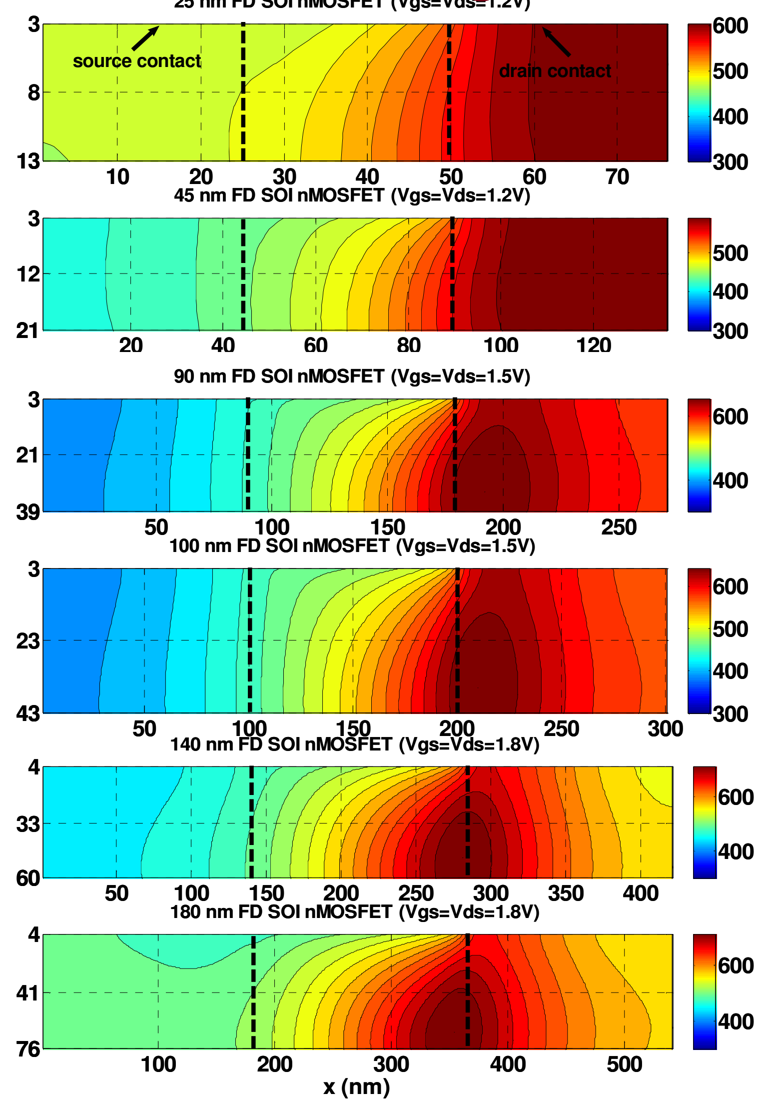


The presence of the bottom silicon substrate does not affect either the electrical or the thermal characteristics of the structure being considered.

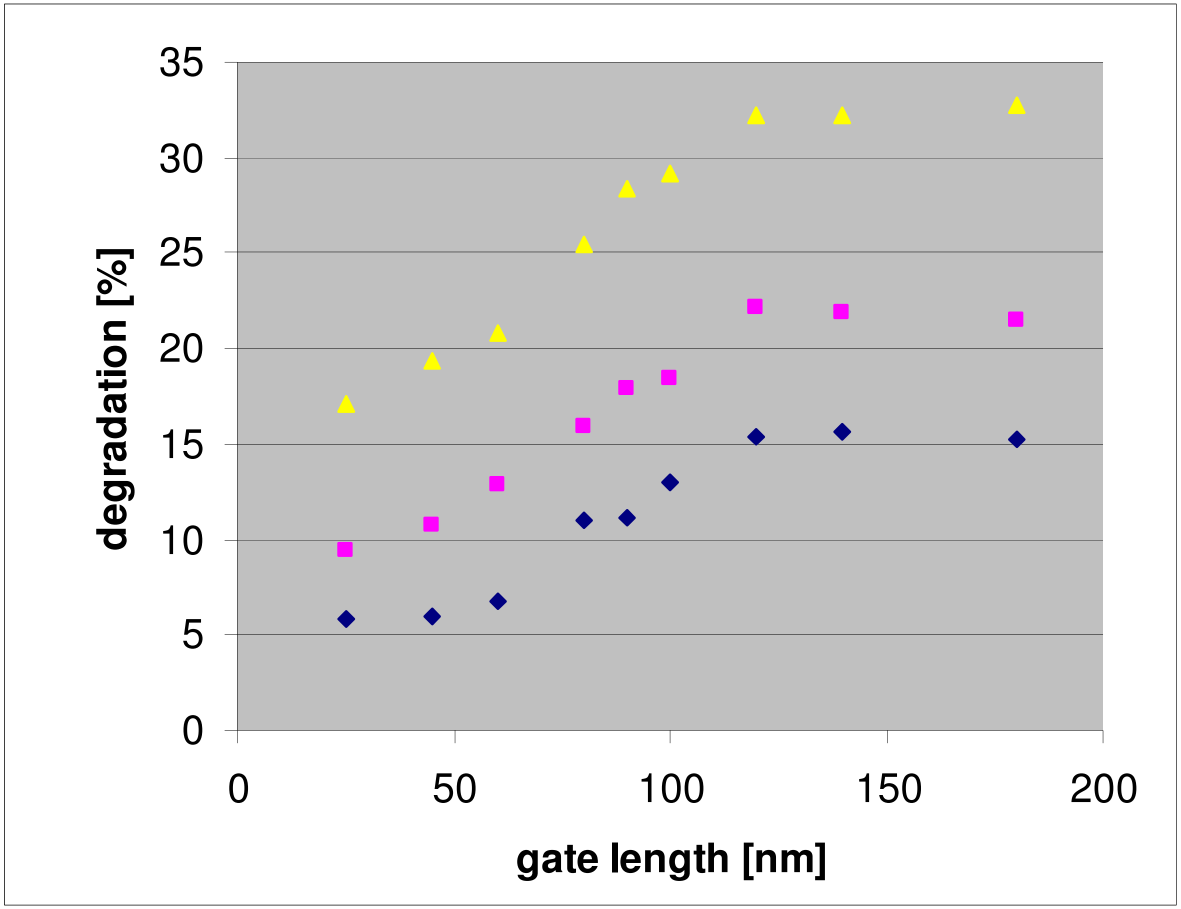
T=300K on gate



T=400K on gate

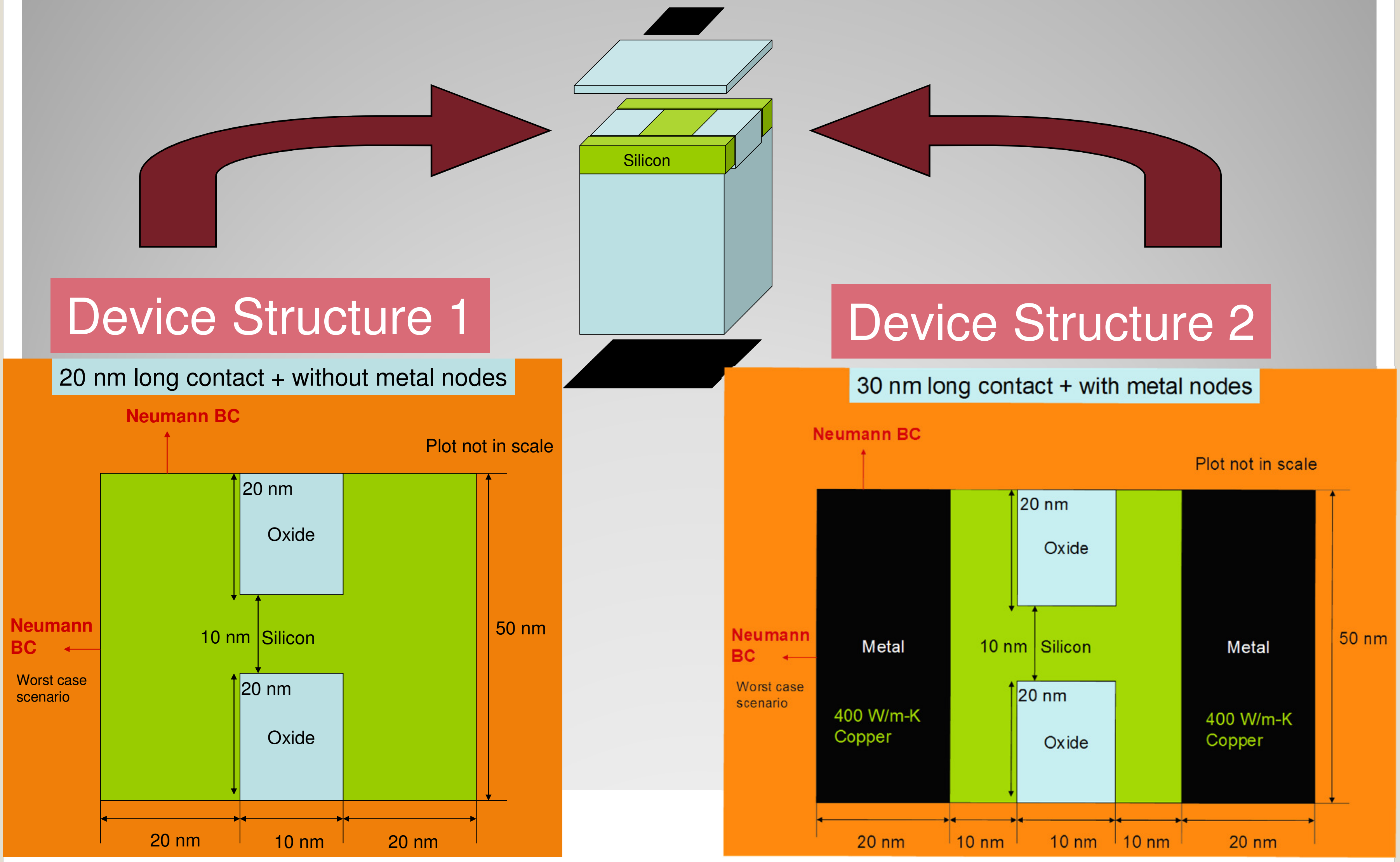


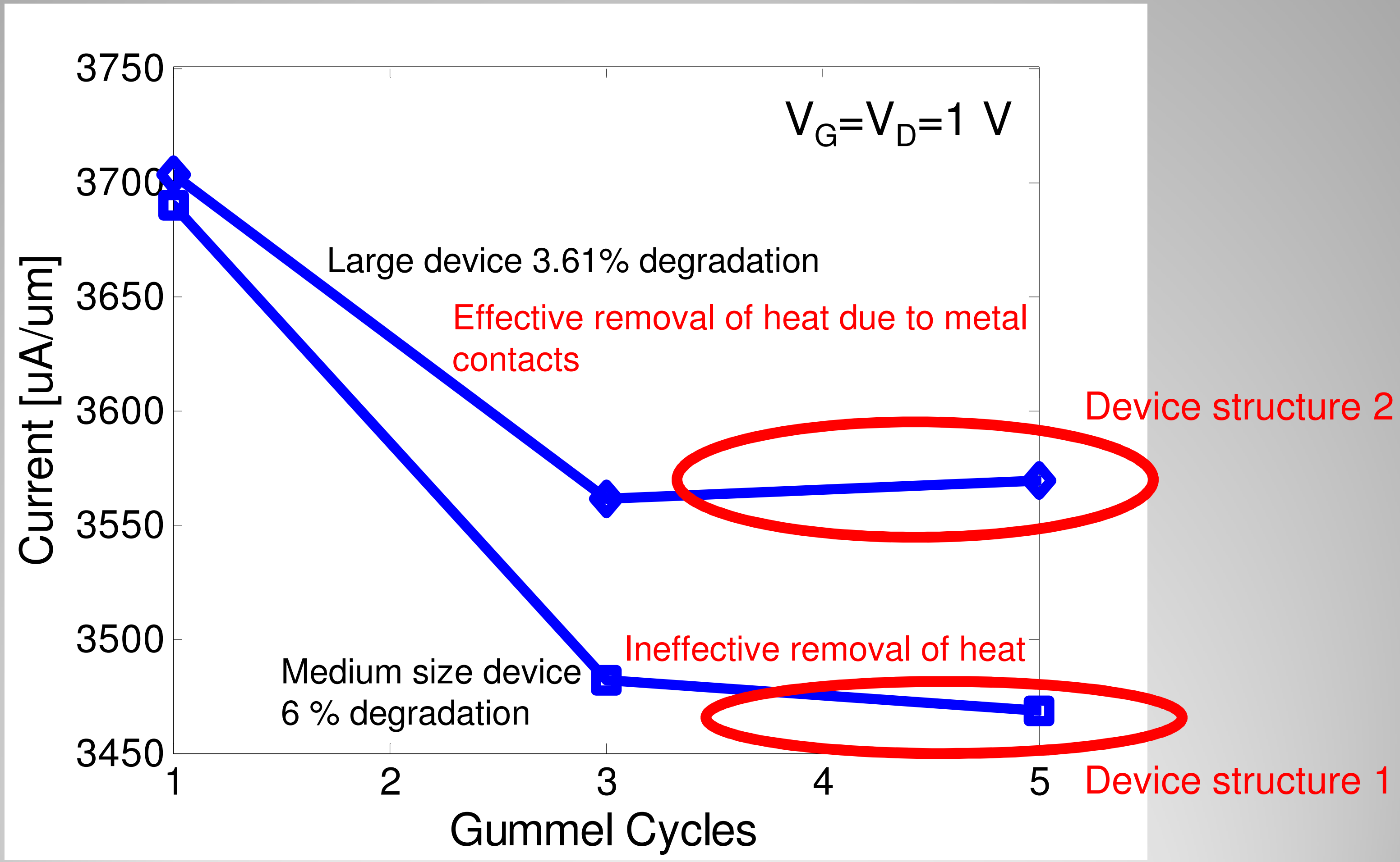
Amount of self-heating across different technology generations



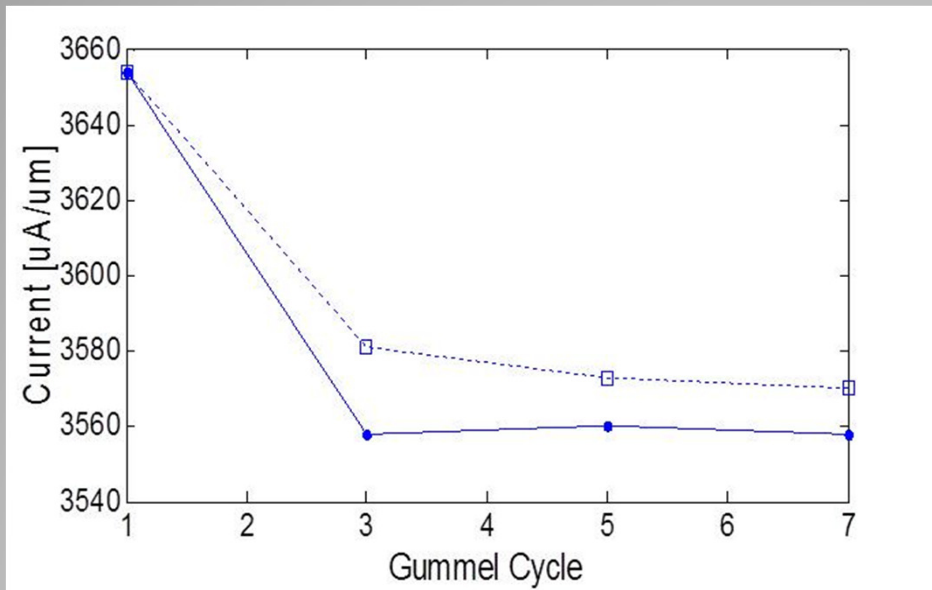
D. Vasileska, K. Raleva and S. M. Goodnick, Self-Heating Effects in Nano-Scale FD SOI Devices: The Role of the Substrate, Boundary Conditions at Various Interfaces and the Dielectric Material Type for the BOX, *IEEE Trans. Electron Devices*, Vol. 56, No. 12, pp. 3064-3071 (2009).

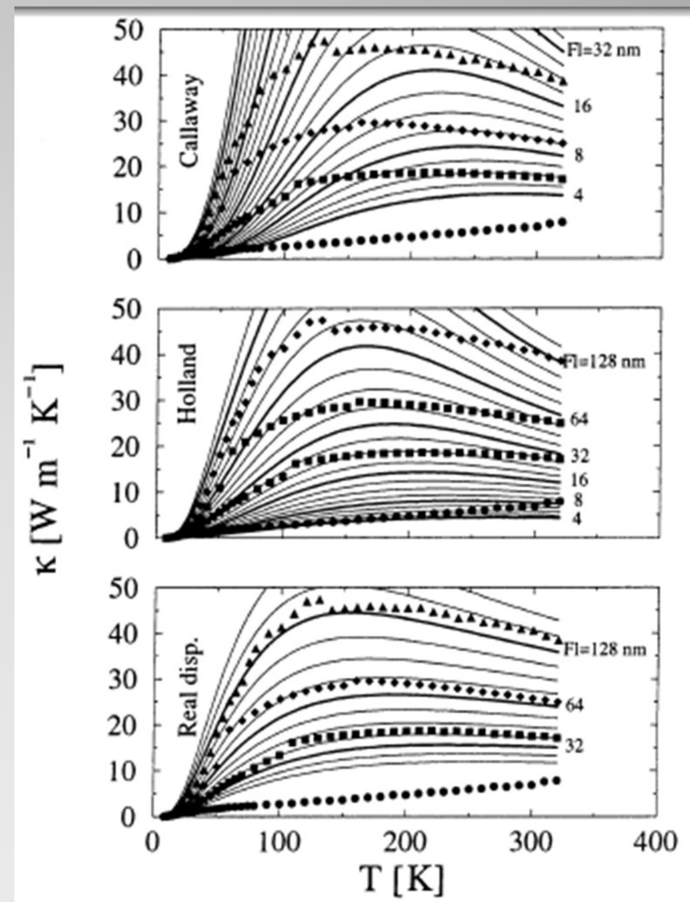
Structures Being Considered





D. Vasileska, A. Hossain and S. M. Goodnick, "The Role of the Source and Drain Contacts on Self-Heating Effect in Nanowire Transistors", *ECS Transactions*, Volume 31, No. 1, from the *25th Symposium on Microelectronics Technology and Devices*, pp. 83-91 (September 2010).





Mingo N, Yang L, Li D, Majumdar A, Predicting the thermal conductivity of Si and Ge nanowires, Nano Leters, Vol. 3, pp. 1713-1716 (2003).

THERMO-ELECTRIC EFFECTS

SEEBECK EFFECT
PELTIER EFFECT
THOMSON EFFECT

- The **thermoelectric effect** is the direct conversion of temperature differences to electric voltage and vice versa.
- Simply put, a thermoelectric device creates a voltage when there is a different temperature on each side, and when a voltage is applied to it, it creates a temperature difference.
- This effect can be used to generate electricity, to measure temperature, to cool objects, or to heat them.
- Because the direction of heating and cooling is determined by the sign of the applied voltage, thermoelectric devices make very convenient temperature controllers.

- Traditionally, the term *thermoelectric effect* or *thermoelectricity* encompasses three separately identified effects, the **Seebeck effect**, the **Peltier effect**, and the **Thomson effect**.
- The Peltier and Seebeck effects are the most basic of the three and are essentially the inverses of one another, for which reason the thermoelectric effect may also be called the **Peltier–Seebeck effect**.
- **Joule heating**, the heat that is generated whenever a voltage difference is applied across a resistive material, is somewhat related, though it is not generally termed a thermoelectric effect (and it is usually regarded as being a loss mechanism or non-ideality in thermoelectric devices).
- **The Peltier–Seebeck and Thomson effects are reversible, whereas Joule heating is not.**

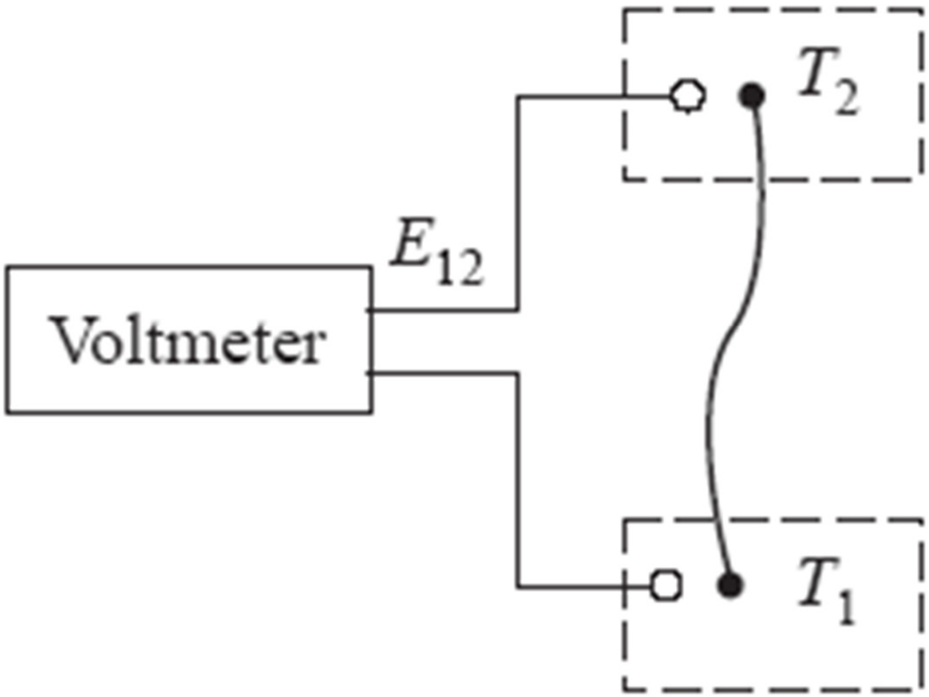
1. SEEBECK EFFECT

Dragica Vasileska

- The Seebeck effect is manifest as a voltage potential that occurs when there is a temperature gradient along the length of a conductor.
- This temperature-induced electrical potential is called an electromotive force and abbreviated as EMF.
- The macroscopic manifestation of EMF is due to the rearrangement of the free electrons in the conductor.
- When the temperature and all other environmental variables that might affect the wire are uniform, the most probable distribution of the free electrons is uniform.
- The presence of a temperature gradient causes a redistribution of the free electrons, which results in a nonuniform distribution of the electric charge on the conductor.

- Above submicron length scales, the charge distribution does not depend on the geometry, e.g. cross-section shape or length, of the conductor.
- As a practical consequence of the charge distribution, the conductor exhibits a variation of voltage potential (the EMF) that is directly related to the temperature gradient imposed on the conductor.
- As the EMF is uniquely related to the temperature gradient, the Seebeck effect can be used to measure temperature.

Conceptual experiment to exhibit the Seebeck



$$E_{12} = \overline{\sigma}\left(T_2 - T_1\right)$$

where $\overline{\sigma}$ is the average Seebeck coefficient for the wire material over the temperature range $T_1 \leq T \leq T_2$.

- In general, the Seebeck coefficient is a function of temperature.
- A more precise and versatile relationship should be developed.
- Consider an experiment where T_1 is fixed, and T_2 is varied.
- For practical thermocouple materials the relationship between E and T is continuous.
- For sufficiently small change ΔT_2 in T_2, the EMF indicated by the voltmeter will change by a corresponding small amount ΔE_{12}.
- As ΔT_2 and ΔE_{12} are small, it is reasonable to linearize the EMF response as

$$E_{12} + \Delta E_{12} = \overline{\sigma}\left(T_2 - T_1\right) + \sigma \Delta T_2$$

where $\sigma(T_2)$ is the value of the Seebeck coefficient at T_2.

- The change in EMF only depends on the value of the Seebeck coefficient at T_2 because T_1 is held fixed.

$$E_{12} = \overline{\sigma}\left(T_2 - T_1\right)$$

$$E_{12} + \Delta E_{12} = \overline{\sigma}\left(T_2 - T_1\right) + \sigma \Delta T_2$$

Subtract first Equation from second Equation to get

$$\Delta E_{12} = \sigma \Delta T_2$$

$$\sigma\left(T_2\right) = \frac{\Delta E_{12}}{\Delta T_2}$$

$$\sigma(T) = \lim_{\Delta T \to 0} \frac{\Delta E}{\Delta T}$$

The general definition of the Seebeck Coefficient

$$\sigma(T) = \frac{dE}{dT}$$

Derivations

- In the discussion of RTA we introduced the concept of electrical conductivity assuming no spatial gradients in the system. In general, the term that involves spatial derivative of the distribution function in the BTE leads to:
  - Diffusion effects, when the electron density is position dependent
  - A variety of thermoelectric effects when the temperature and/or the Fermi energy are spatially varying functions.

General case – variation in both T and E_F

$$f = f_0 + f_A$$

$$f_0 = \frac{1}{1+e^{\theta}}, \quad \theta = \frac{E_C + E_k - E_F}{k_B T}$$

$$\nabla_r f_s = \frac{\partial f_s}{\partial \theta}\nabla_r \theta = \frac{\partial f_s}{\partial E_k}\frac{\partial E_k}{\partial \theta}\nabla_r \theta = T\nabla_r \left(\frac{E_C + E_k - E_F}{T}\right)\frac{\partial f_s}{\partial E_k}$$

Substituting into BTE gives

$$Tv \cdot \nabla_r \left(\frac{E_C + E_k - E_F}{T}\right)\frac{\partial f_s}{\partial E_k} - e\varepsilon \cdot v\frac{\partial f_s}{\partial E_k} = -\frac{f_A}{\tau_m}$$

Solving for the asymmetric component of the DF gives:

$$f_A = e\tau_m v\cos\theta\left[\varepsilon_z + \frac{1}{eT}(E_k - F_n)\frac{\partial T}{\partial z} + \frac{\partial}{\partial z}\left(\frac{F_n}{e}\right)\right]\frac{\partial f_s}{\partial E_k}$$

$$J_z = \frac{ne^2}{m}\left[\langle\langle\tau_m\rangle\rangle\varepsilon_z + \frac{1}{eT}\left(\langle\langle E\tau_m\rangle\rangle - \langle\langle\tau_m\rangle\rangle F_n\right)\frac{\partial T}{\partial z} + \langle\langle\tau_m\rangle\rangle\frac{\partial}{\partial z}\left(\frac{F_n}{e}\right)\right]$$

$$= \sigma_{zz}\varepsilon_z + \sigma_{zz}\left[\frac{1}{eT}\left(\frac{\langle\langle E\tau_m\rangle\rangle}{\langle\langle\tau_m\rangle\rangle} - F_n\right)\frac{\partial T}{\partial z} + \frac{\partial}{\partial z}\left(\frac{F_n}{e}\right)\right]$$

$$\Rightarrow \varepsilon_z = \rho_{zz}J_z + Q\frac{\partial T}{\partial z} - \frac{\partial}{\partial z}\left(\frac{F_n}{e}\right)$$

$$\Rightarrow Q = -\frac{1}{eT}\left(\frac{\langle\langle E\tau_m\rangle\rangle}{\langle\langle\tau_m\rangle\rangle} - F_n\right) \rightarrow Seebeck\ Coefficient$$

When the momentum relaxation time obeys power-law dependence. then

$$Q = -\frac{k_B}{e}\left(s + 5/2 - \frac{E_F - E_C}{k_B T} \right), \quad E_F - E_C = -k_B T \ln\left(\frac{N_C}{n} \right)$$

Seeback coefficient is:

- negative for n-type sample
- positive for p-type sample

2. PELTIER EFFECT - 1834

Dragica Vasileska

- The ***Peltier effect*** explains a voltage generated in a junction of two metal wires.
- Peltier found that the junctions of dissimilar metals were heated or cooled, depending upon the direction in which an electrical current passed through them.
- Heat generated by current flowing in one direction was absorbed if the current was reversed.
- The Peltier effect is found to be proportional to the first power of the current, not to its square, as is the irreversible generation of heat caused by resistance throughout the circuit.
- Absorption or production of heat at a current-carrying junction is an indication that an electromotive force (emf) exists at the junction.
- Suppose the direction of the main current is the same as that produced by the junction emf, π.
- Each current carrier, q, is accelerated as it crosses the junction, gaining energy : $U = \pi q$.

- No moving parts make them very reliable; approximately 10^5 hrs of operation at 100 degrees Celsius, longer for lower temps (Goldsmid,1986).
- Ideal when precise temperature control is required.
- Ability to lower temperature below ambient.
- Heat transport controlled by current input.
- Able to operate in any orientation.
- Compact size make them useful for applications where size or weight is a constraint.
- Ability to alternate between heating and cooling.
- Excellent cooling alternative to vapor compression coolers for systems that are sensitive to mechanical vibration.

Why are TE Coolers Used for Cooling?

Disadvantages

- Able to dissipate limited amount of heat flux.
- Lower coefficient of performance than vapor-compression systems.
- Relegated to low heat flux applications.
- More total heat to remove than without a TEC.

$$Q_{removed} = Q_{chip} + W_{TEC}$$

(Simons and Chu, 2000)

- Electronic
- Medical
- Aerospace
- Telecommunications

Which Industries Use TE Cooling?

(TE Tecnology, Inc., http://www.tetech.com/techinfo/)

Cooling:

- Electronic enclosures
- Laser diodes
- Laboratory instruments
- Temperature baths
- Refrigerators
- Telecommunications equipment
- Temperature control in missiles and space systems
- Heat transport ranges vary from a few milliwatts to several thousand watts, however, since the efficiency of TE devices are low, smaller heat transfer applications are more practical.

What are Some Applications?

(TE Tecnology, Inc.,http://www.tetech.com/techinfo/)

- ***Peltier Effect-*** when a voltage or DC current is applied to two dissimilar conductors, a circuit can be created that allows for continuous heat transport between the conductor's junctions. The Seebeck Effect- is the reverse of the Peltier Effect. By applying heat to two different conductors a current can be generated. The Seebeck Coefficient is given by:

$$\alpha = \frac{\varepsilon_x}{dT / dx}$$

where ε is the electric field.

- The current is transported through charge carriers (opposite the hole flow or with electron flow).
- Heat transfer occurs in the direction of charge carrier movement.

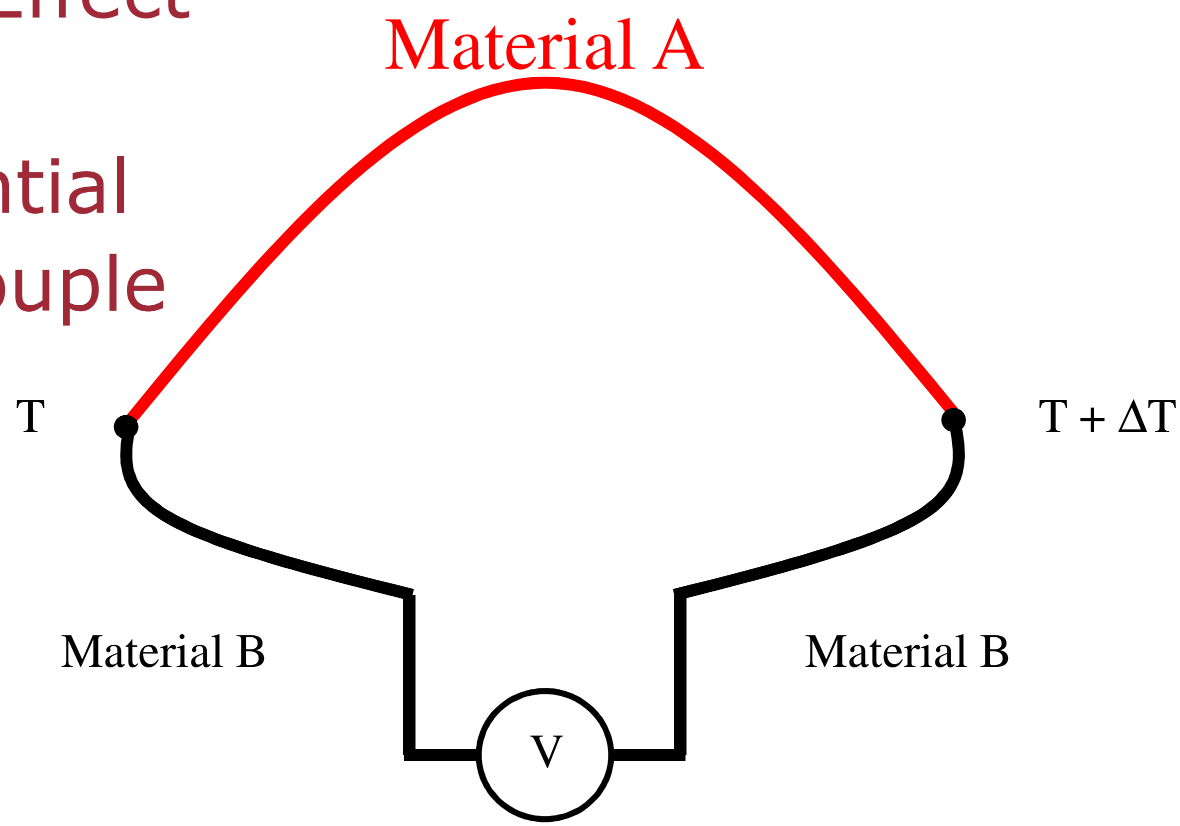
Basic Principles

(Tellurex, www.tellurex.com)

Seebeck Effect

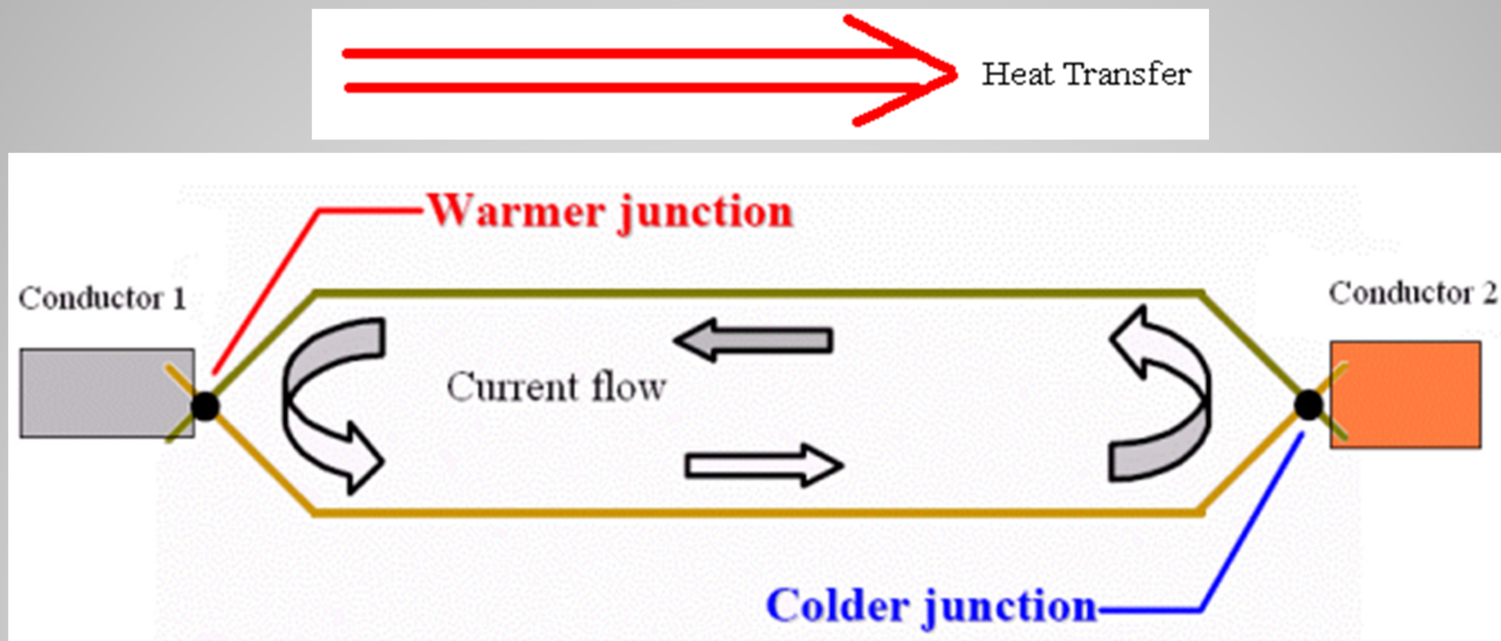
Differential Thermocouple

$$\alpha = \frac{V_{ab}}{\Delta T}$$



Seeback Effect

- Applying a current (e^- carriers) transports heat from the warmer junction to the cooler junction.



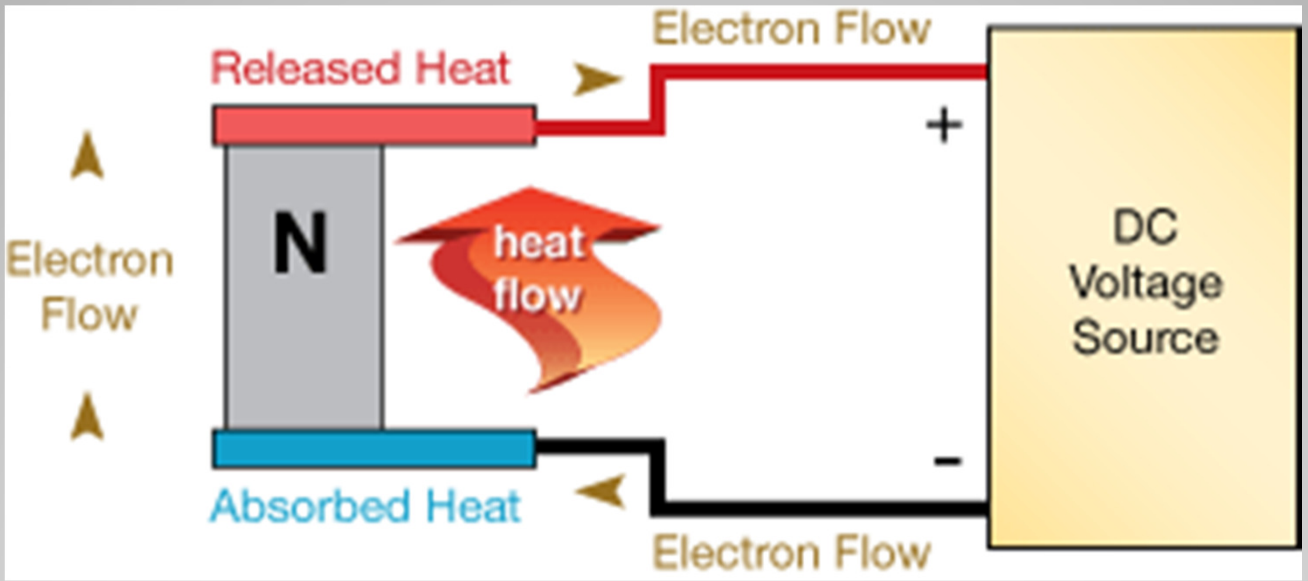
Basic Principles of Peltier Effect

(Tellurex, www.tellurex.com)

- A typical thermoelectric cooling component is shown on the next slide. Bismuth telluride (a semiconductor), is sandwiched between two conductors, usually copper. A semiconductor (called a pellet) is used because they can be optimized for pumping heat and because the type of charge carriers within them can be chosen. The semiconductor in this examples N type (doped with electrons) therefore, the electrons move towards the positive end of the battery.
- The semiconductor is soldered to two conductive materials, like copper. When the voltage is applied heat is transported in the direction of current flow.

Basic Principles

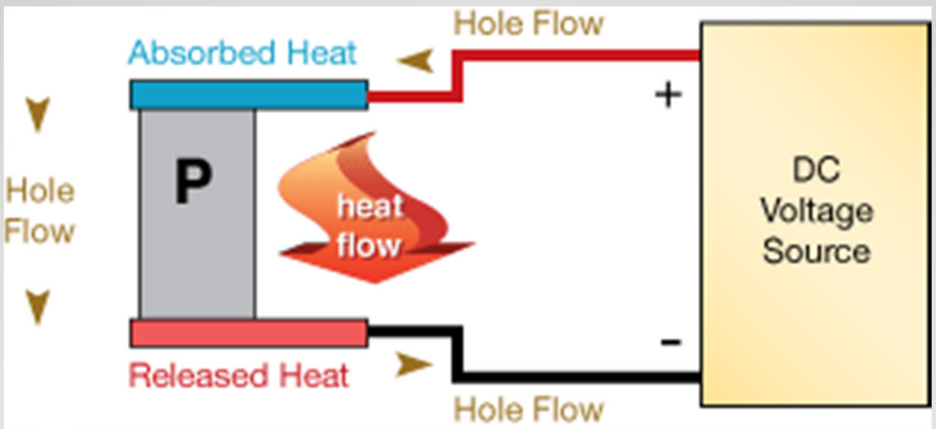
(Tellurex, www.tellurex.com)



Basic Principles

(Tellurex, www.tellurex.com)

- When a p type semiconductor (doped with holes) is used instead, the holes move in a direction opposite the current flow. The heat is also transported in a direction opposite the current flow and in the direction of the holes. Essentially, the charge carriers dictate the direction of heat flow.



Basic Principles

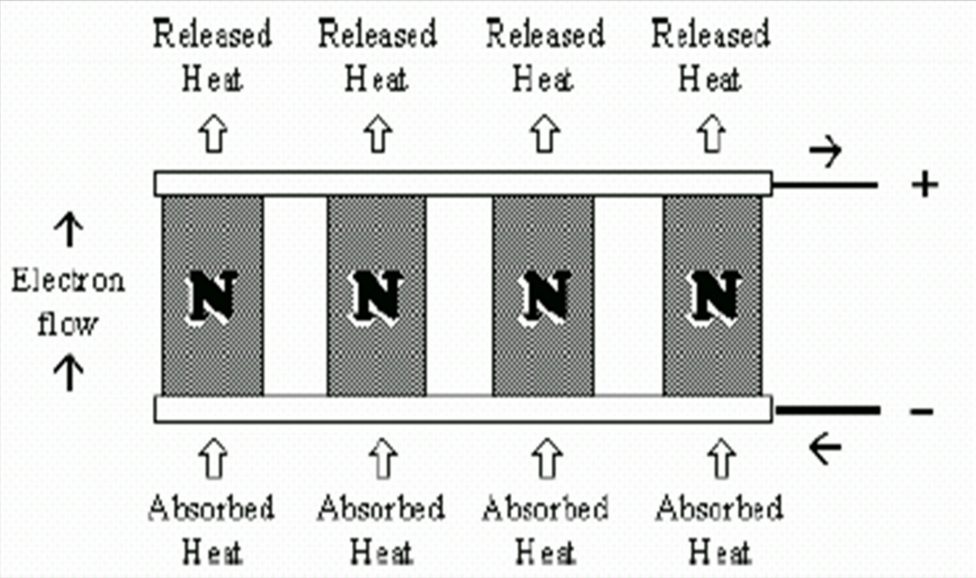
(Tellurex, www.tellurex.com)

- Electrons can travel freely in the copper conductors but not so freely in the semiconductor.
- As the electrons leave the copper and enter the hot-side of the p-type, they must fill a "hole" in order to move through the p-type. When the electrons fill a hole, they drop down to a lower energy level and release heat in the process.
- Then, as the electrons move from the p-type into the copper conductor on the cold side, the electrons are bumped back to a higher energy level and absorb heat in the process.
- Next, the electrons move freely through the copper until they reach the cold side of the n-type semiconductor. When the electrons move into the n-type, they must bump up an energy level in order to move through the semiconductor. Heat is absorbed when this occurs.
- Finally, when the electrons leave the hot-side of the n-type, they can move freely in the copper. They drop down to a lower energy level and release heat in the process.

Method of Heat Transport

http://www.tetech.com/techinfo/#1

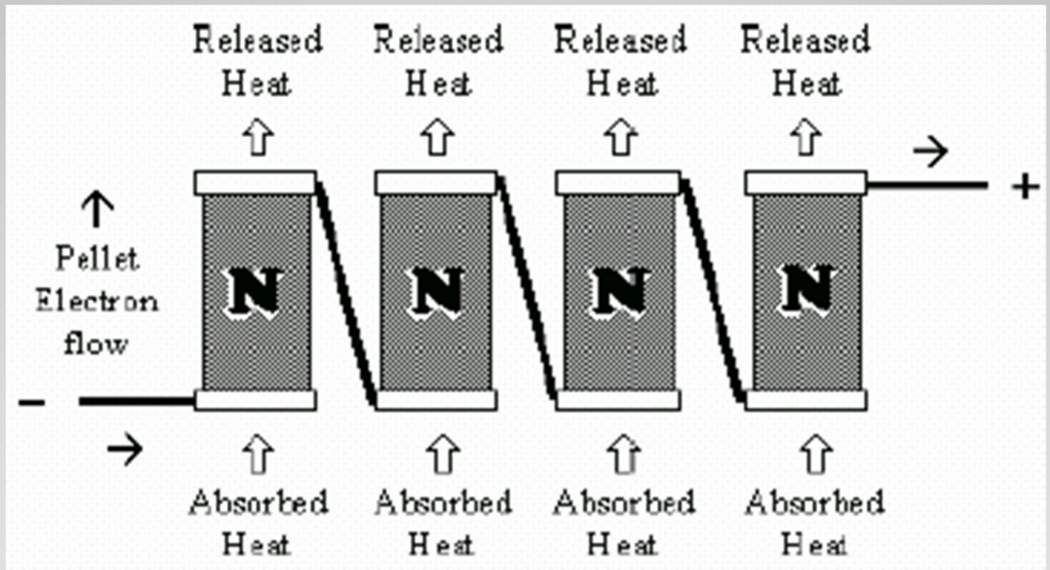
- To increase heat transport, several p type or n type thermoelectric(TE) components can be hooked up in parallel.
- However, the device requires low voltage and therefore, a large current which is too great to be commercially practical.



Basic Principles

(Tellurex, www.tellurex.com)

- The TE components can be put in series but the heat transport abilities are diminished because the interconnectings between the semiconductor creates thermal shorting.

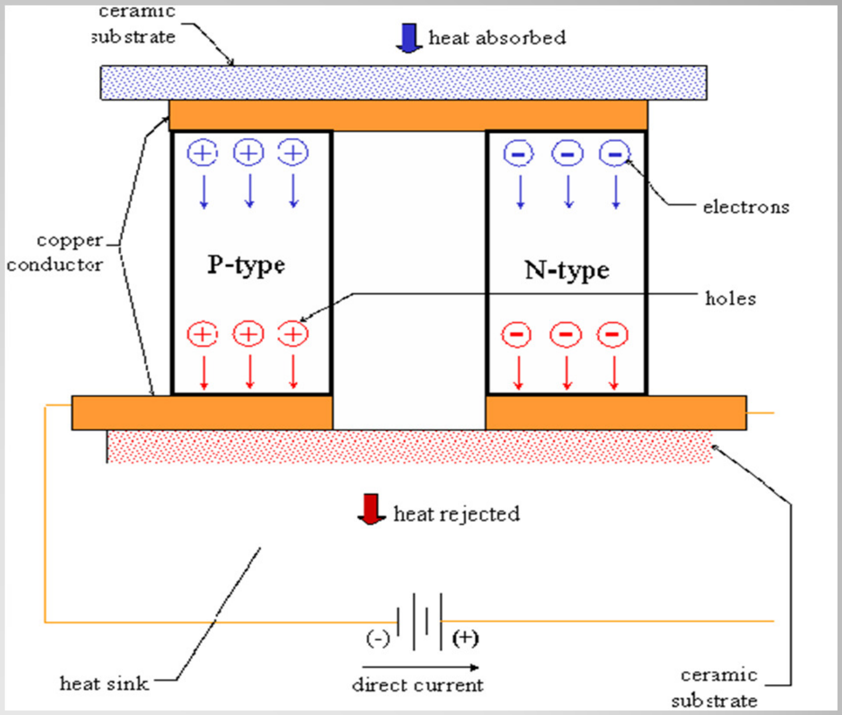


Basic Principles

(Tellurex, www.tellurex.com)

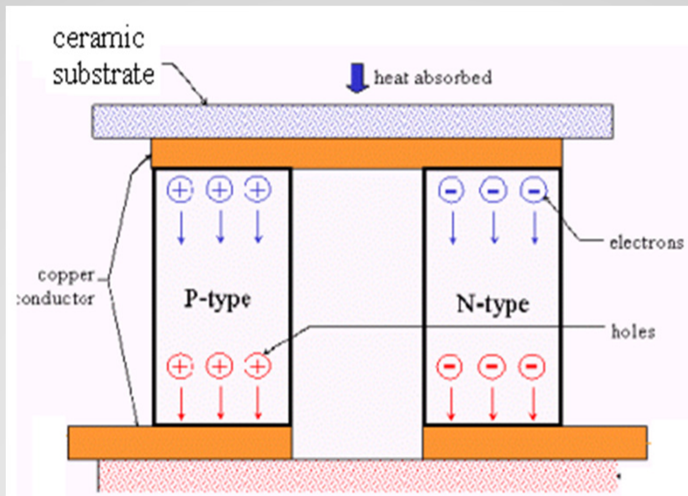
Basic Principles

- The most efficient configuration is where a p and n TE component is put electrically in series but thermally in parallel . The device to the right is called a couple.
- One side is attached to a heat source and the other a heat sink that convects the heat away.
- The side facing the heat source is considered the cold side and the side facing the heat sink the hot side.



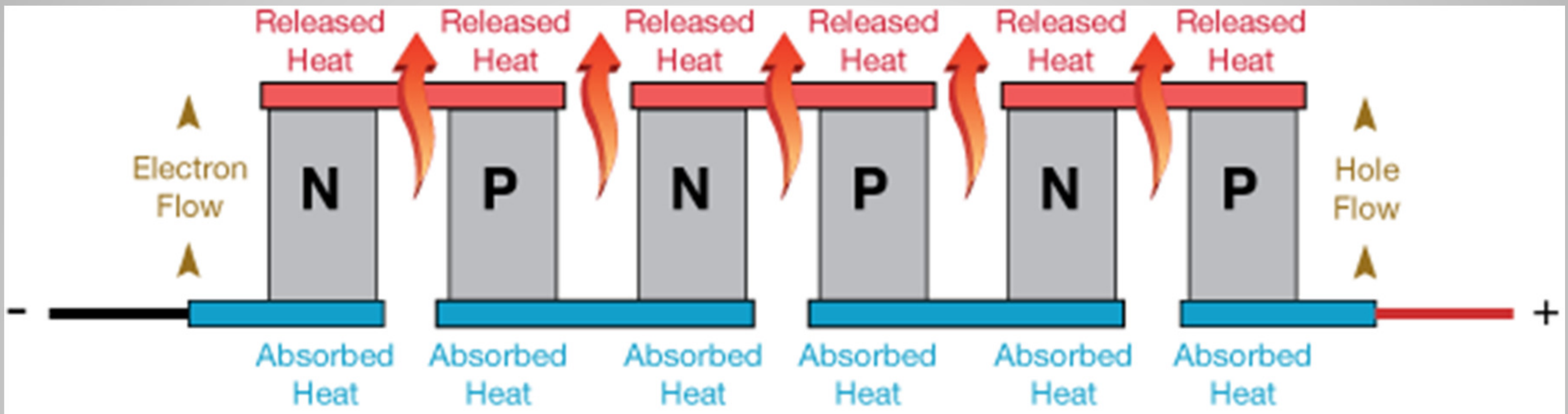
(TE Tecnology, Inc. , http://www.tetech.com/techinfo/)

- Between the heat generating device and the conductor must be an electrical insulator to prevent an electrical short circuit between the module and the heat source.
- The electrical insulator must also have a high thermal conductivity so that the temperature gradient between the source and the conductor is small.
- Ceramics like alumina are generally used for this purpose. (Rowe, 1995).



Basic Principles

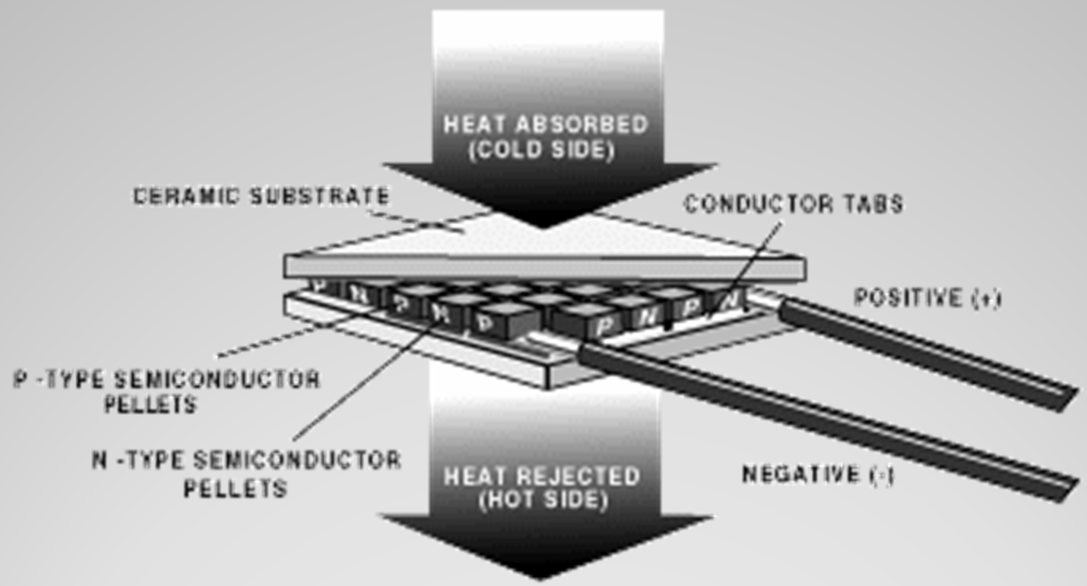
- The most common devices use 254 alternating p and n type TE devices.
- The devices can operate at 12-16 V at 4-5 amps. These values are much more practical for real life operations.



Basic Principles

(Tellurex, www.tellurex.com)

- An entire assembly:



Basic Principles

(Tellurex., http://www.tellurex.com/12most.html)

Figure of Merit

- If there is no temperature gradient in the system, then $\Delta T=0$ and the sample voltage $V_A=0$
- If constant current is applied to the sample, then the sample Voltage is $V_A = V_{IR} = IR_s$
  - Constant current leads to thermal gradient dT which, in turn leads to

    A) Power $Q_p=SIT$
    B) Voltage $V_{TE}=SdT$

- The total voltage in the sample is:
- $V_A = V_{IR} + V_{TE}$

The ZT factor is defined as

$$ZT = V_A/V_{IR} - 1 = V_{TE}/V_{IR}$$

$$ZT = SdT/RI$$

Under the adiabatic conditions the heat pumped by the Peltier Effect equals to the heat carried by thermal conduction. Then

$$SIT = \kappa A dT/L$$

The ZT factor is then given by:

$$ZT = S^2T\,\sigma/\kappa$$

Figure of Merit ZT

Thermal Power Flow

A proper definition for the heat current is suggested by the thermodynamic relation:

$$dU = dQ + E_F dN$$

where: dU -> increase in internal energy
dQ -> increase in heat
$E_F dN$ -> increase in internal energy when a small number of carriers is added at a constant temperature

$$J_Q = J_U - E_F J_n$$

Particle current density

Heat flux

Flux of internal energy

Thus:

$$J_Q = \frac{1}{V}\sum_k \left[E_{C0} + E_k - E_F\right] v f_A$$

$$\Updownarrow$$

$$J_{Qz} = \frac{ne\langle\langle\tau_m\rangle\rangle}{m}\frac{\partial}{\partial z}\left(\frac{E_F}{e}\right)\left[\frac{\langle\langle E\tau_m\rangle\rangle}{\langle\langle\tau_m\rangle\rangle} - F_n\right] + \frac{n}{mT}\left\langle\left\langle\tau_m\left(E_k - F_n\right)^2\right\rangle\right\rangle\frac{\partial T}{\partial z}$$

$$= \frac{1}{e}\left[\frac{\langle\langle E\tau_m\rangle\rangle}{\langle\langle\tau_m\rangle\rangle} - F_n\right] J_z + \frac{n}{mT}\left[-\frac{\langle\langle E\tau_m\rangle\rangle^2}{\langle\langle\tau_m\rangle\rangle} + \langle\langle E^2\tau_m\rangle\rangle^2\right]\frac{\partial T}{\partial z}$$

$$\Updownarrow$$

Peltier coefficient

$$\pi = \frac{1}{e}\left[\frac{\langle\langle E\tau_m\rangle\rangle}{\langle\langle\tau_m\rangle\rangle} - F_n\right] = -TQ \rightarrow \textit{Kelvin}\ \text{Relation}$$

$$J_{Qz} = \pi J_z - \kappa\frac{\partial T}{\partial z} \rightarrow$$

$$\kappa = -\frac{n}{mT}\left[-\frac{\langle\langle E\tau_m\rangle\rangle^2}{\langle\langle\tau_m\rangle\rangle} + \langle\langle E^2\tau_m\rangle\rangle^2\right]$$

Thermal conductivity

When the momentum relaxation time obeys power-law dependence. then

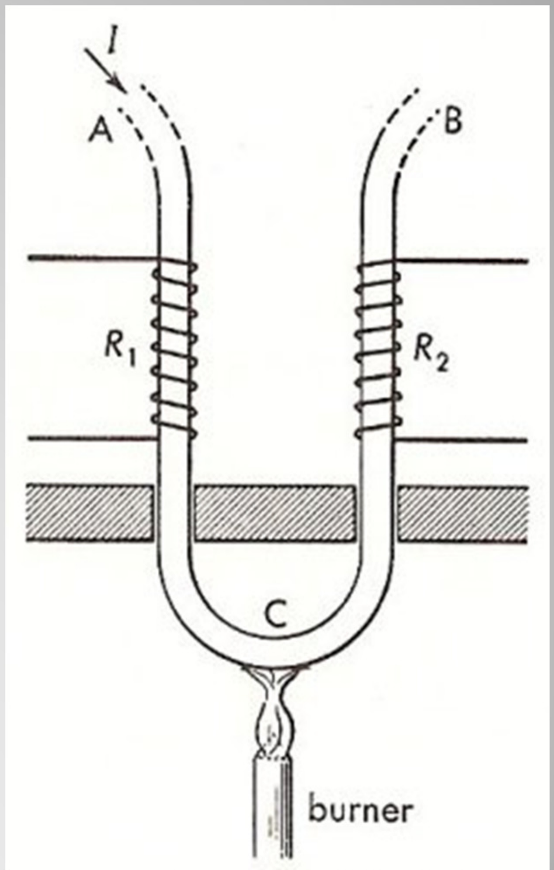
$$\kappa = T\left(\frac{k_B}{e}\right)^2 (s+5/2)$$

3. THOMSON EFFECT

Dragica Vasileska

In 1851 William Thomson (later Lord Kelvin) was led by thermodynamic reasoning to conclude that sources of electromotive force (emf) exist in a thermoelectric circuit in addition to those located at the junctions.

An emf would arise within in a *single* conductor whenever a temperature gradient was present.



The Thomson effect : 1851