NCN Summer School: July 2011

Near-equilibrium Transport:
Fundamentals and Applications

Lecture 3: Resistance - ballistic to diffusive

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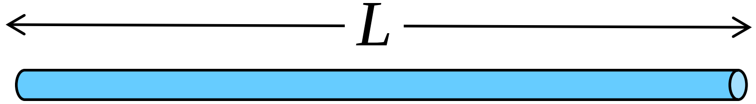
resistors

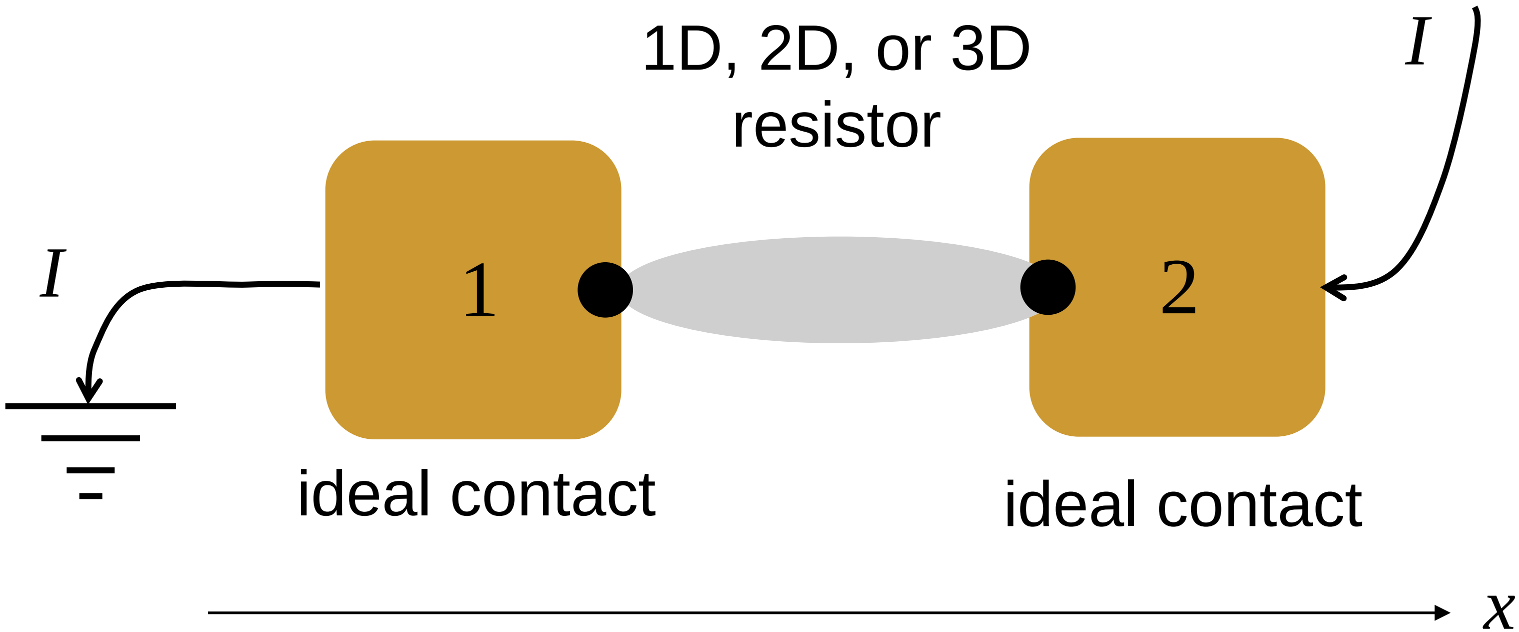


$$R_{1D} = \rho_{1D} L \qquad \rho_{1D} = \frac{1}{n_L q \mu_n}$$

$$R_{2D} = \rho_{2D} \frac{L}{W} \qquad \rho_{2D} = \frac{1}{n_S q \mu_n}$$

$$R_{3D} = \rho_{3D} \frac{L}{A} \qquad \rho_{3D} = \frac{1}{n q \mu_n}$$

Landauer picture



Current is positive when it flows **into** contact 2 (i.e. positive current flows in the -*x* direction).

$$I = GV = \frac{2q^2}{h}\left(\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE\right)V$$

driving "forces" for transport

$$I = \frac{2q}{h}\int T(E)M(E)(f_1 - f_2)dE$$

Differences in occupation, *f*, produce current.

$$(f_1 - f_2) \approx f_1 - \left(f_1 + \frac{\partial f_1}{\partial E_F}\Delta E_F\right) = -\frac{\partial f_1}{\partial E}\Delta E_F$$

Assumes $T_{L1} = T_{L2}$.

but differences in temperature also produce differences in *f* and can, therefore, drive current (thermoelectric effects).

$$\Delta E_F = -q\Delta V = -q(V_2 - V_1)$$

←this lecture

$$\Delta T_L = T_{L2} - T_{L1}$$

(next lecture)

transport regimes

1) Ballistic: $\lambda >> L,\ T \approx 1$

2) Diffusive: $\lambda << L,\ T << 1$

3) Quasi-Ballistic: $\lambda \approx L,\ T < 1$ $\quad T(E) = \dfrac{\lambda(E)}{\lambda(E) + L}$

outline

1) Review

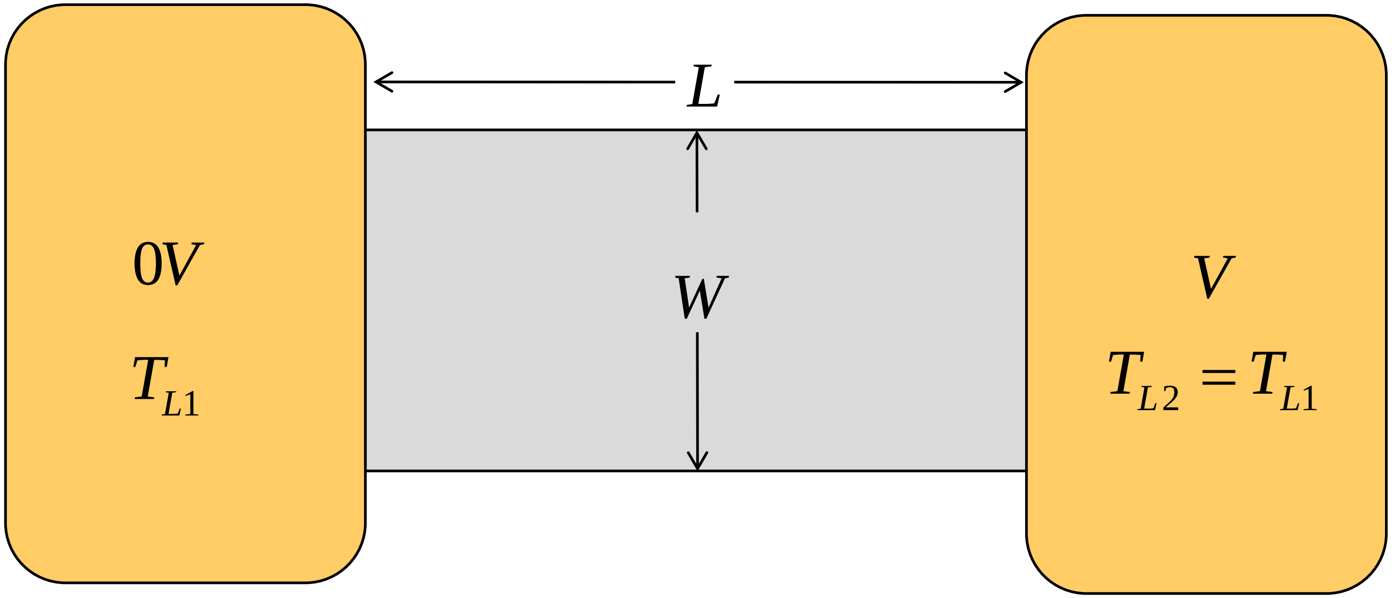
2) 2D ballistic resistors

3) 2D diffusive resistors

4) Discussion

5) Summary

an isothermal 2D resistor

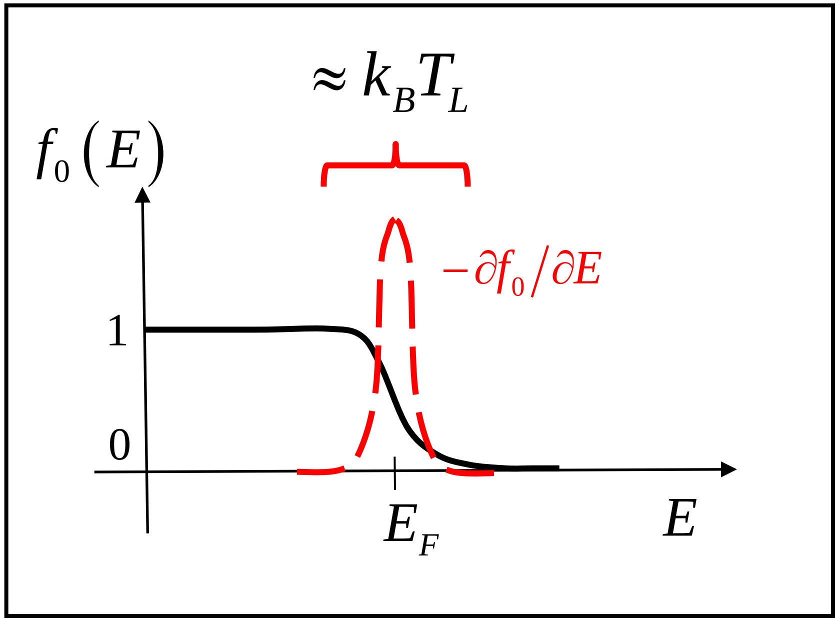


$$G = \frac{1}{R} = \frac{I}{V} = \frac{2q^2}{h}\int T(E)\,M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \qquad (T_{L1} = T_{L2})$$

the ballistic conductance

$$G = \frac{2q^2}{h}\int_{-\infty}^{+\infty} T(E) M(E)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$f_0(E) = \frac{1}{1+e^{(E-E_F)/k_B T_L}}$$



$T(E) = 1$ (ballistic)

$$\left(-\frac{\partial f_0}{\partial E}\right) \approx \delta(E_F)$$

$$G_{ball} = \frac{2q^2}{h} M(E_F)$$

T = 0K ballistic conductance

$$G_{ball} = \frac{2q^2}{h} M\left(E_F\right)$$

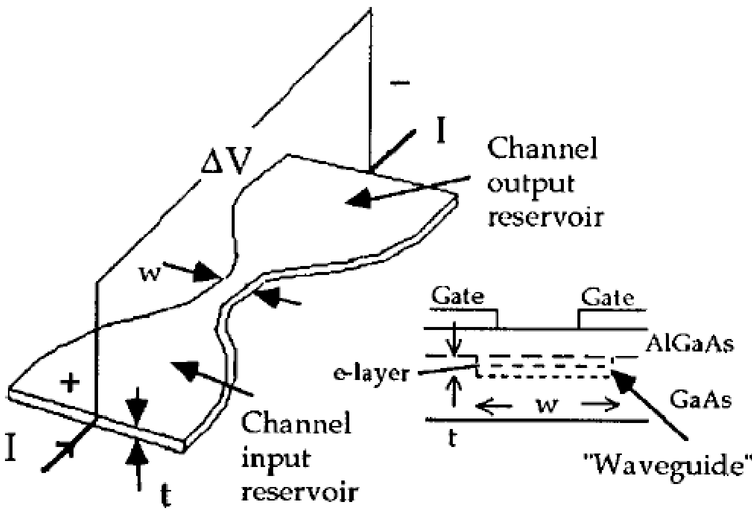
"quantized conductance"

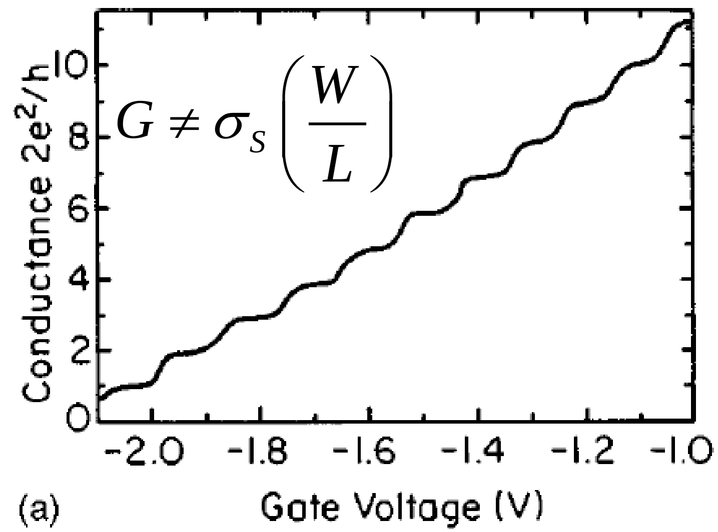
$$R_{ball} = \frac{1}{M\left(E_F\right)} \frac{h}{2q^2} = \frac{12.8\,\mathrm{k}\Omega}{M}$$

Resistance is independent of length, *L*.

If the number of modes is small, we can simply count them.

quantized conductance





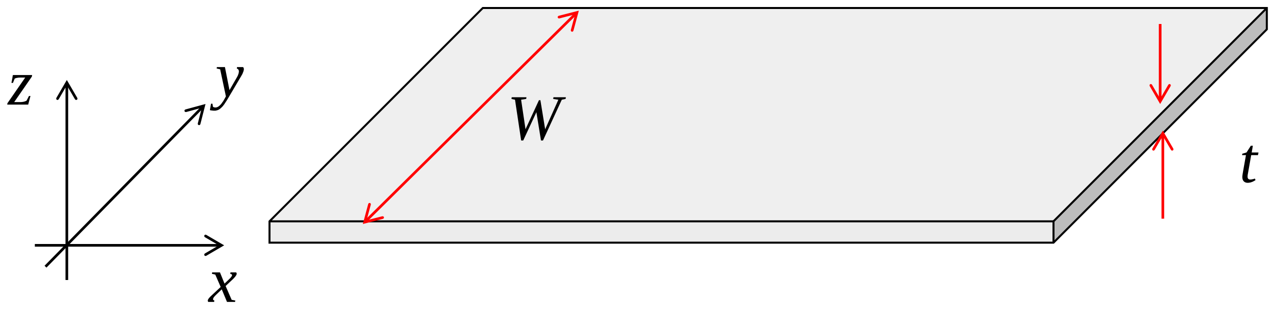
W -->

B. J. van Wees, H. van Houten, C. W. J. Beenakker, J. G. Williamson, L. P. Kouwenhoven, D. van der Marel, and C. T. Foxon, ‘‘Quantized conductance of point contacts in a two-dimensional electron gas,’’ *Phys. Rev. Lett.* **60**, 848–851,1988.

Conductance is quantized in units of $2q^2/h$ - it increases with *W,* but not linearly.

The conductance is finite $L \rightarrow 0$.

for wider resistors, $M(E) \sim W$



$$W M_{2D}(E_F) = W \frac{\sqrt{2m^*(E_F - E_C)}}{\pi\hbar}$$

depends on bandstructure!

$$W M_{2D}(E_F) = \frac{W}{(\lambda_F/2)} = \frac{W k_F}{\pi}$$

$$n_S = \frac{\pi k_F^2}{(2\pi)^2} \times 2$$

$$W M_{2D}(E_F) = W\sqrt{\frac{2n_S}{\pi}}$$

depends only on n_S

conductance in 2D (T > 0K)

$$G_{ball} = \frac{2q^2}{h}\left(\int M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE\right)$$

$$WM_{2D}(E) = Wg_V\,\frac{\sqrt{2m^*(E-E_C)}}{\pi\hbar}$$

$$f_0(E) = \frac{1}{1+e^{(E-E_F)/k_BT_L}}$$

$$\left(-\frac{\partial f_0}{\partial E}\right) = +\left(-\frac{\partial f_0}{\partial E_F}\right)$$

$$G_{ball} = \frac{2q^2}{h}\left(+\frac{\partial}{\partial E_F}\right)\left(\int M(E)\,f_0(E)\,dE\right)$$

conductance in 2D (T > 0K)

$$G_{ball} = \frac{2q^2}{h}\left(+\frac{\partial}{\partial E_F}\right)\left(\int M(E)\, f_0(E)\, dE\right)$$

$$f_0(\eta) = \frac{1}{1+e^{\eta-\eta_F}}$$

$$M(E) = Wg_V \frac{\sqrt{2m^* k_B T_L \eta}}{\pi\hbar}$$

$$dE = k_B T_L d\eta$$

$$\frac{\partial}{\partial E_F} = k_B T_L \frac{\partial}{\partial \eta_F}$$

$$WM_{2D}(E) = Wg_V \frac{\sqrt{2m^*(E-E_C)}}{\pi\hbar}$$

$$f_0(E) = \frac{1}{1+e^{(E-E_F)/k_B T_L}}$$

$$f_0(E) = \frac{1}{1+e^{(E-E_C+E_C-E_F)/k_B T_L}}$$

$$\eta = \frac{E-E_C}{k_B T_L} \qquad \eta_F = \frac{E_F-E_C}{k_B T_L}$$

conductance in 2D (T > 0K)

$$G_{ball} = \frac{2q^2}{h}\frac{W\sqrt{2m^* k_B T_L}}{\pi\hbar}\left(\frac{\partial}{\partial \eta_F}\right)\int_0^\infty \frac{\sqrt{\eta}}{1+e^{\eta-\eta_F}}d\eta$$

$$G_{ball} = \frac{2q^2}{h}\frac{W\sqrt{2m^* k_B T_L}}{\pi\hbar}\frac{\sqrt{\pi}}{2}\left(\frac{\partial}{\partial \eta_F}\right)\mathcal{F}_{1/2}(\eta_F)$$

$$G_{ball} = \frac{2q^2}{h}\frac{W\sqrt{2m^* k_B T_L}}{\pi\hbar}\frac{\sqrt{\pi}}{2}\mathcal{F}_{-1/2}(\eta_F)$$

$$\mathcal{F}_{1/2}(\eta_F) \equiv \frac{2}{\sqrt{\pi}}\int_0^\infty \frac{\eta^{1/2}d\eta}{1+e^{\eta-\eta_F}}$$

$$\eta = \frac{E-E_C}{k_B T_L} \qquad \eta_F = \frac{E_F-E_C}{k_B T_L}$$

$$\frac{d\mathcal{F}_{1/2}}{d\eta_F} \rightarrow \mathcal{F}_{-1/2}$$

conductance in 2D (T > 0K)

$$G_{ball} = \frac{2q^2}{h} W \langle M_{2D} \rangle$$

$$\langle M_{2D} \rangle = \frac{\sqrt{2m^* k_B T_L}}{\pi\hbar} \frac{\sqrt{\pi}}{2} \mathcal{F}_{-1/2}(\eta_F) = M_{2D}(k_B T_L) \frac{\sqrt{\pi}}{2} \mathcal{F}_{-1/2}(\eta_F)$$

$$\eta_F = \frac{E_F - E_C}{k_B T_L} \qquad n_S = N_{2D} \mathcal{F}_0(\eta_F) = \left(g_V \frac{m^* k_B T_L}{\pi\hbar^2} \right) \mathcal{F}_0(\eta_F)$$

Fermi-Dirac integrals

$$\mathcal{F}_j(\eta_F) = \frac{1}{\Gamma(j+1)} \int_0^{\infty} \frac{\eta^j d\eta}{1+e^{\eta-\eta_F}}$$

$$\Gamma(n) = (n-1)!$$ (*n* integer)

$$\Gamma(1/2) = \sqrt{\pi}$$

$$\Gamma(p+1) = p\Gamma(p)$$

$$\mathcal{F}_j(\eta_F) \to e^{\eta_F} \quad \eta_F << 1$$

$$(E_F - E_C)/k_B T << 1$$

$$\frac{d\mathcal{F}_j}{d\eta_F} = \mathcal{F}_{j-1}$$

don't confuse with…. $F_j(\eta_F) = \int_0^{+\infty} \frac{\eta^j d\eta}{1+e^{\eta-\eta_F}}$

For an introduction to Fermi-Dirac integrals, see: "Notes on Fermi-Dirac Integrals," Ed., by R. Kim and M. Lundstrom) https://www.nanohub.org/resources/5475

conductance and carrier density

In conventional semiconductor theory, it is common to write:

$$G_{2D} = n_S q \mu_n \frac{W}{L} \qquad G_{2D} \propto n_S$$

What is the relation between n_S and G_{2D} for a ballistic resistor?

conductance in 2D

$$G_{2D}^{ball} = W\frac{2q^2}{h}\langle M_{2D}\rangle$$

T_L=0 K:

$$\langle M_{2D}\rangle = M_{2D}\left(E_F\right) = \sqrt{2n_S/\pi}$$

$$G_{2D}^{ball} = W\frac{2q^2}{h}\sqrt{\frac{2n_S}{\pi}}$$

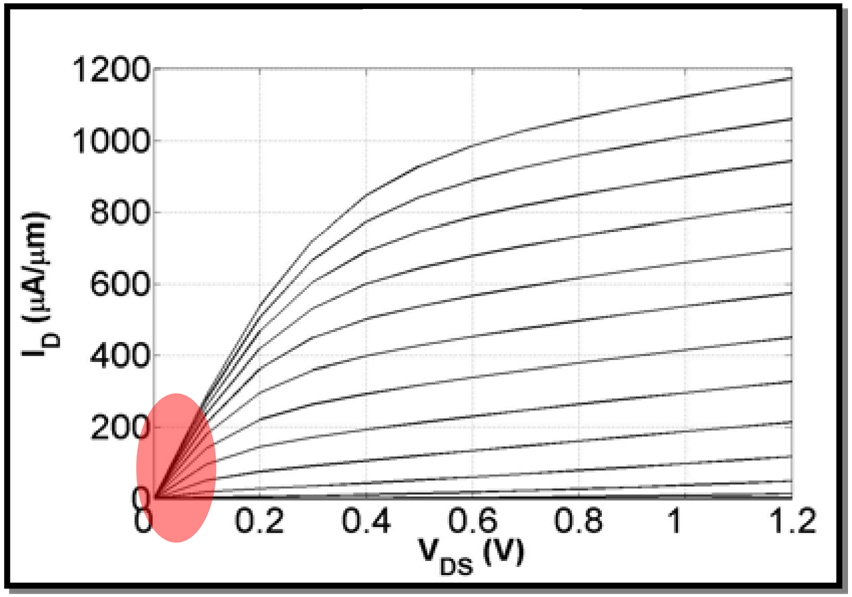
$$G_{2D}^{ball} \propto \sqrt{n_S}$$

Non-degenerate conditions:

$$G_{2D}^{ball} = Wqn_s\frac{\upsilon_T}{2k_BT_L/q}$$

$$G_{2D}^{ball} \propto n_s$$

example: nanoscale MOSFETs



(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

$$L \approx 60 \text{ nm}$$

$$\mu_n \approx 260 \text{ cm}^2/\text{V-s}$$

$$n_S \approx 6.7 \times 10^{12} \text{ cm}^{-2}$$

$$R_{CH} \approx 215\Omega\text{-}\mu\text{m}$$

Question: What is the ballistic resistance of this transistor?

example: nanoscale MOSFETs

ballistic resistance (MB statistics to keep it simple)

$$G_{ball} = Wqn_s \frac{\upsilon_T}{2k_B T_L / q}$$

$$\upsilon_T = \sqrt{\frac{2k_B T_L}{\pi m^*}} \approx 1.2\times 10^7 \text{ cm/s}$$

$$R_{ball} = 1/G_{ball} \approx 40\ \Omega\text{-}\mu\text{m}$$

$$L \approx 60 \text{ nm}$$

$$\mu_n \approx 260 \text{ cm}^2/\text{V-s}$$

$$n_S \approx 6.7\times 10^{12} \text{ cm}^{-2}$$

$$R_{CH} \approx 215\ \Omega\text{-}\mu\text{m}$$

$$m^* \approx 0.19 m_0$$

$$T_L = 300\text{K}$$

$$R_{CH} \approx 5 R_{ball}$$

not ballistic /
not diffusive

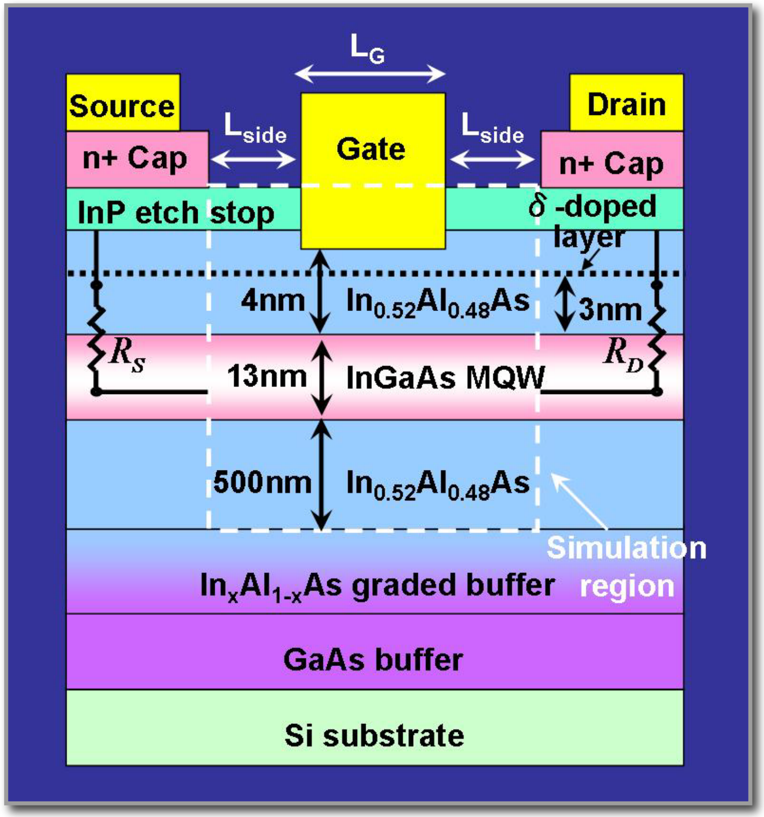
C. Jeong, D. A. Antoniadis and M. Lundstrom, “On Back-Scattering and Mobility in Nanoscale Silicon MOSFETs,” *IEEE Trans. Electron Dev.*, **56**, pp. 2762-2769, 2009.

exercise: nanoscale MOSFETs

Re-do the calculation with Fermi-Dirac statistics.

Then estimate the number of conduction channels.

40 nm channel length III-V FETs



D. H. Kim and J. A. del Alamo, "Lateral and Vertical Scaling of $In_{0.7}Ga_{0.3}As$ HEMTs for Post-Si-CMOS Logic Applications," *IEEE TED, **55**, pp. 2546-2553, 2008.*

$$\mu_n \approx 10{,}000 \text{ cm}^2/\text{V-s}$$

$$D_n = \frac{\upsilon_T \lambda_0}{2}$$

$$\upsilon_T = \sqrt{\frac{2k_B T}{\pi m^*}} = 2.7\times 10^7 \text{ cm/s}$$

$$\left(m^* = 0.041 m_0\right)$$

$$\lambda_0 \approx 200 \text{ nm}$$

$$\lambda_0 > L$$

outline

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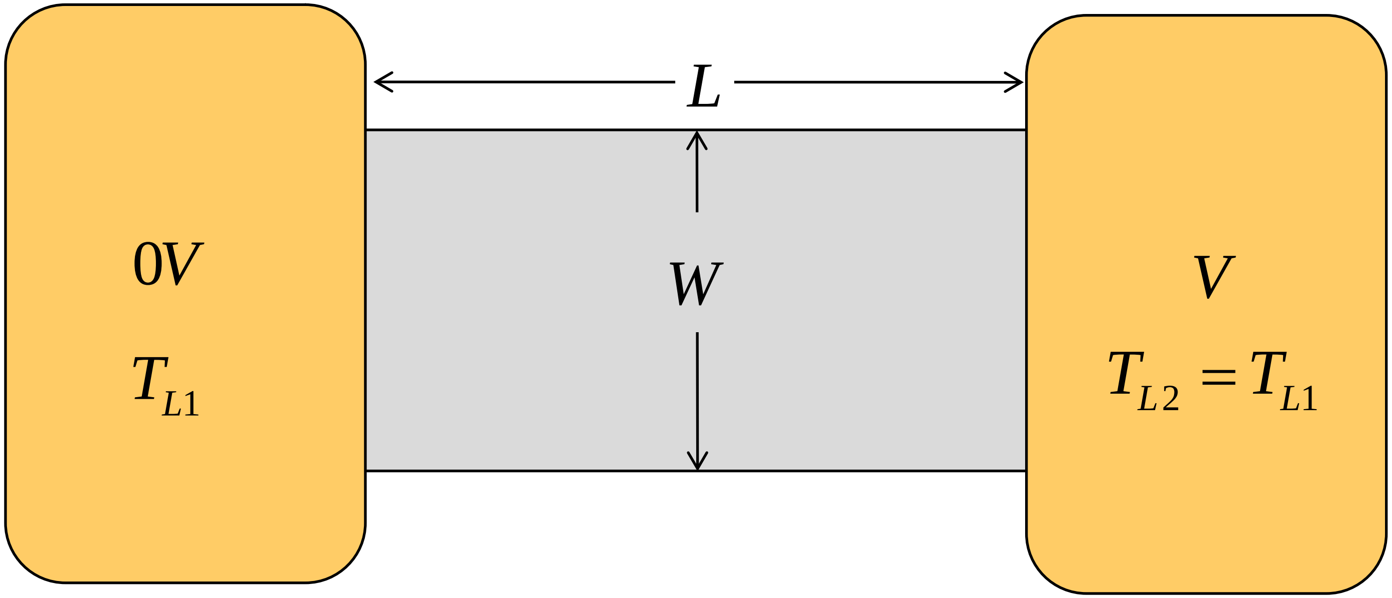
an isothermal 2D resistor



$$G = \frac{2q^2}{h}\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \qquad (T_{L1} = T_{L2})$$

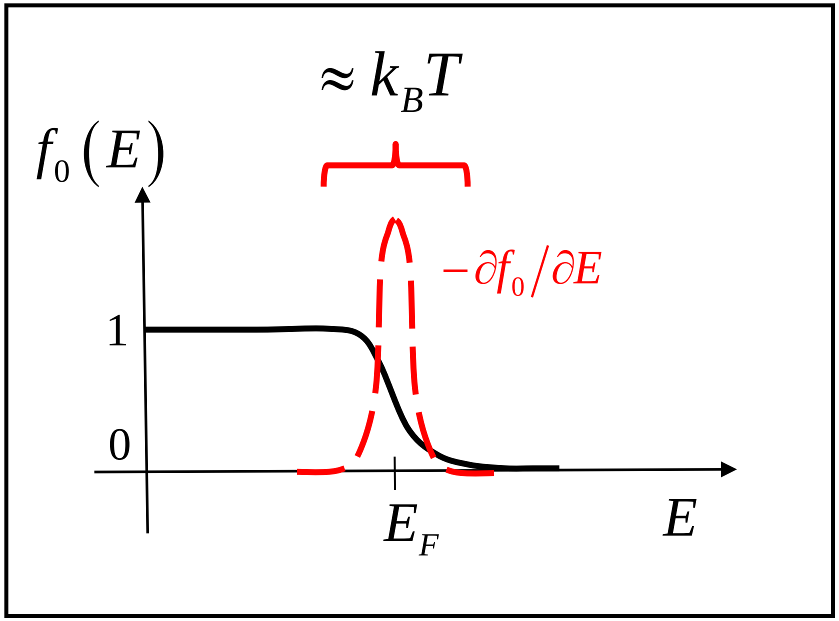
the T_L = 0 diffusive conductance

$$G = \frac{2q^2}{h} \int_{-\infty}^{+\infty} T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$T(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

$$G = \frac{2q^2}{h} T(E_F) M(E_F)$$

$$f_0(E) = \frac{1}{1 + e^{(E - E_F)/k_B T_L}}$$



$$\left(-\frac{\partial f_0}{\partial E} \right) \approx \delta(E_F)$$

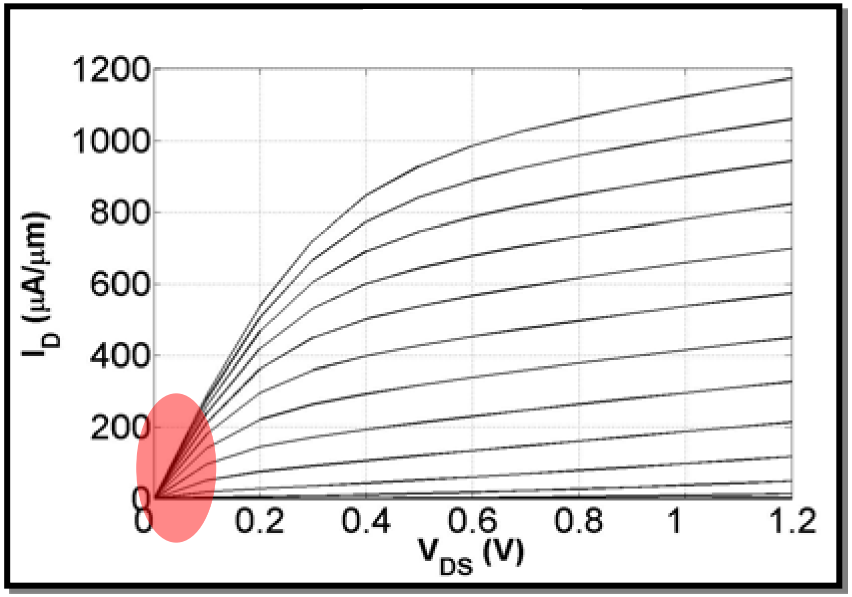
ballistic to diffusive

$$G = \frac{2q^2}{h} \int_{-\infty}^{+\infty} T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$T = \frac{\lambda(E_F)}{\lambda(E_F) + L}$$

$$\boxed{G = \frac{\lambda(E_F)}{\lambda(E_F) + L} G_{ball} \qquad R = \left(1 + \frac{L}{\lambda(E_F)} \right) R_{ball}}$$

example: nanoscale MOSFETs



(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

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$$n_S \approx 6.7 \times 10^{12} \text{ cm}^{-2}$$

$$R_{CH} \approx 215\Omega\text{-}\mu\text{m}$$

$$R_{CH} \approx 5R_{ball}$$

$$R_{CH} = \left(1 + L/\lambda_0\right) R_{ball} \approx 5R_{ball} \rightarrow \lambda_0 \approx 15 \text{ nm}$$

C. Jeong, D. A. Antoniadis and M. Lundstrom, "On Back-Scattering and Mobility in Nanoscale Silicon MOSFETs," *IEEE Trans. Electron Dev.*, **56**, pp. 2762-2769, 2009.

outline

1) Review

2) 2D ballistic resistors

3) 2D diffusive resistors

4) **Discussion**

   -about mobility
   -ways to write the conductivity
   -a 1D example
   -power dissipation
   -voltage drop
   -resistors in 1D, 2D, 3D

5) Summary

i) about mobility

$$G_{2D} = \frac{2q^2}{h} \int T(E) W M_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

(ballistic to diffusive)

$$G_{diff} = n_S q \mu_n \frac{W}{L}$$

(diffusive)

definition of mobility:

$$\mu_{app} \equiv \frac{1}{n_S} \frac{2q}{h} \int T(E) L M_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

not:

$$\mu_n = \frac{q \tau_m}{m^*}$$

in the diffusive limit

definition of mobility:

$$\mu_n \equiv \frac{1}{n_S}\frac{2q}{h}\int T(E)\,LM_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$T(E) = \frac{\lambda(E)}{\lambda(E)+L} \rightarrow \frac{\lambda(E)}{L}$$

$$\mu_n \equiv \frac{1}{n_S}\frac{2q}{h}\int \lambda(E)M_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$\mu_n = \frac{q\tau_m}{m^*}$$

ballistic to diffusive…

definition of **apparent** mobility:

$$\mu_{app} \equiv \frac{1}{n_S}\frac{2q}{h}\int T(E)\,L\,M_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$T(E)L = \frac{\lambda(E)L}{\lambda(E)+L} \rightarrow \frac{1}{\lambda_{app}(E)}$$

$$\frac{1}{\lambda_{app}(E)} \equiv \frac{1}{\lambda(E)} + \frac{1}{L}$$

$$\mu_{app} \equiv \frac{1}{n_S}\frac{2q}{h}\int \lambda_{app}(E)M_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$\mu_n = \frac{q\tau_m}{m^*}$$

T_L = 0K apparent mobility

definition of apparent mobility:

$$\mu_{app} \equiv \frac{1}{n_S}\frac{2q}{h}\int T(E)LM_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$\mu_{app}(T_L = 0\,\mathrm{K}) = \frac{1}{n_S}\frac{2q}{h}T(E_F)LM_{2D}(E_F)$$

$$\frac{1}{\mu_{app}} = \frac{1}{\mu_n} + \frac{1}{\mu_{ball}}$$

$$T(E_F) = \frac{\lambda(E_F)}{\lambda(E_F)+L}$$

$$M_{2D} = \frac{h}{4}\left\langle \upsilon_x^+ \right\rangle D_{2D} \qquad \left\langle \upsilon_x^+ \right\rangle = \frac{2}{\pi}\upsilon_F$$

$$n_S = \int_{E_C}^{E_F} D_{2D}(E)\,dE = \frac{m^*}{\pi\hbar^2}(E_F - E_C)$$

$$= D_{2D}(E_F - E_C)$$

Example: "ballistic mobility" in 2D at T_L = 0K

$$G_{2D} = \frac{2q^2}{h} T\left(E_F\right) M_{2D}\left(E_F\right) = n_S q \mu_{app} \frac{W}{L}$$

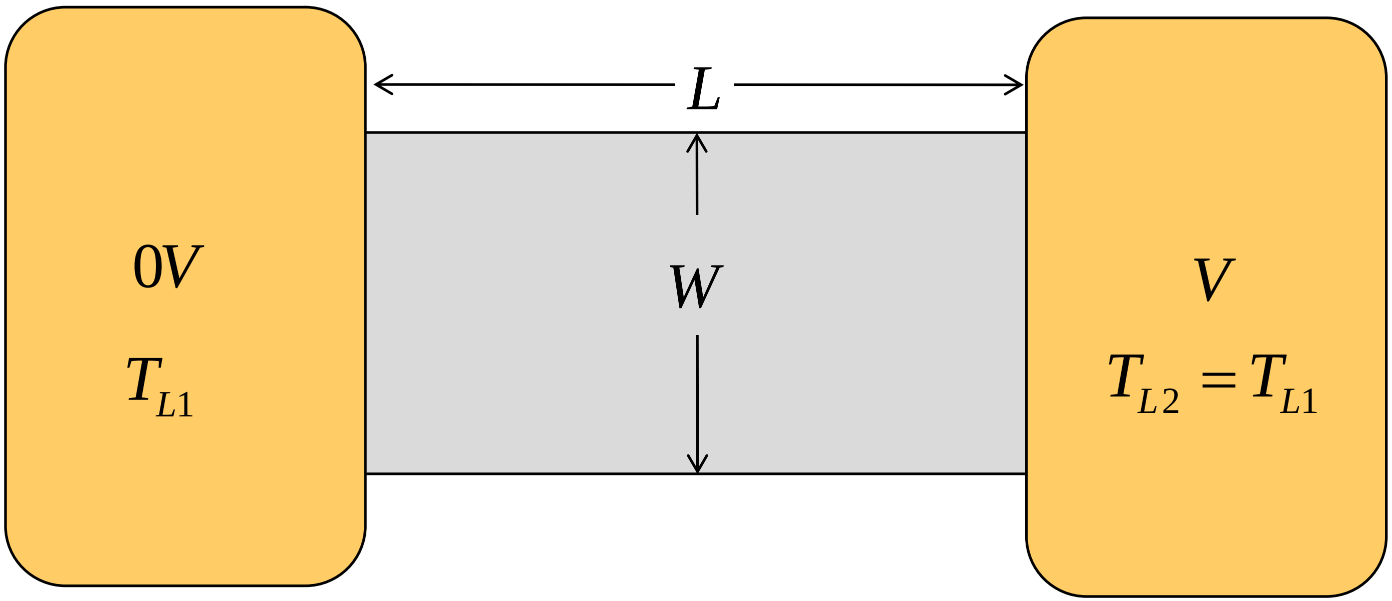
$$\mu_n = \frac{D_n}{\left(E_F - E_C\right)/q}$$

$$\frac{1}{\mu_{app}} = \frac{1}{\mu_n} + \frac{1}{\mu_{ball}} \qquad \mu_{ball} = \frac{D_{ball}}{\left(E_F - E_C\right)/q}$$

$$D_n = \frac{\left\langle \upsilon_x^+ \right\rangle \lambda\left(E_F\right)}{2} \qquad D_{ball} = \frac{\left\langle \upsilon_x^+ \right\rangle L}{2}$$

M.S. Shur, "Low Ballistic Mobility in GaAs HEMTs," *IEEE Electron Dev. Lett*., **23**, 511-513, 2002

physical interpretation



$$\frac{1}{\lambda_{app}(E)} \equiv \frac{1}{\lambda(E)} + \frac{1}{L}$$

ii) ways to write the 2D conductivity

at T_L = 0 K for a diffusive conductor, we have:

$$G_{2D}^{diff} = \frac{2q^2}{h}\lambda\left(E_F\right)M_{2D}\left(E_F\right)\frac{W}{L} = \sigma_S\frac{W}{L}$$

so we can write the 2D conductivity (sheet resistance) as:

$$\sigma_S = \frac{2q^2}{h}\lambda\left(E_F\right)M_{2D}\left(E_F\right)$$

But this result is often written differently.

other ways to write the 2D conductivity

$$\sigma_S = \frac{2q^2}{h}\lambda\left(E_F\right)M_{2D}\left(E_F\right)$$

$$\sigma_S = q^2 D_{2D}\left(E_F\right)\frac{\upsilon^2\left(E_F\right)\tau\left(E_F\right)}{2}$$

$$\sigma_S = q^2 D_{2D}\left(E_F\right)D_n\left(E_F\right)$$

Lecture 2:

$$M_{2D}\left(E\right) = \frac{h}{4}\left\langle \upsilon_x^+ \right\rangle D_{2D}\left(E\right)$$

$$\left\langle \upsilon_x^+ \right\rangle = \frac{2}{\pi}\upsilon$$

Lecture 6:

$$\lambda\left(E_F\right) = \frac{\pi}{2}\upsilon\left(E_F\right)\tau\left(E_F\right)$$

Define:

$$D_n\left(E_F\right) = \upsilon^2\left(E_F\right)\tau\left(E_F\right)/2$$

one more way…

$$\sigma_S = \frac{2q^2}{h} \lambda(E_F) M_{2D}(E_F)$$

$$\sigma_S = q^2 D_{2D}(E_F) \frac{\upsilon^2(E_F)\tau(E_F)}{2}$$

$$\frac{1}{2} m^* \upsilon(E_F)^2 = (E_F - E_C)$$

$$n_S = D_{2D}(E_F - E_C)$$

$$\sigma_S = n_S q \mu_n(E_F)$$

$$\mu_n = q\tau(E_F)/m^*$$

general expression for 2D conductivity…

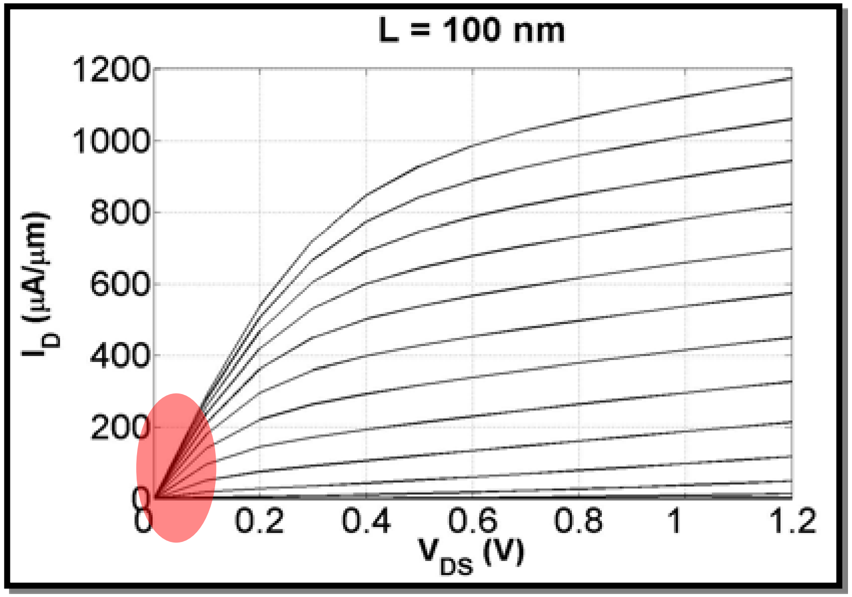
$$\sigma_S = \frac{2q^2}{h}\int \lambda(E) M_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$\sigma_S = \frac{2q^2}{h}\int \sigma_s'(E) dE$$

$$\sigma_S' = \frac{2q^2}{h}\lambda(E) M_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right)$$

"differential conductivity" units are $1/(\Omega\text{-J})$

example: nanoscale FETs



(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

$$L \approx 60 \text{ nm}$$

$$\mu_n \approx 260 \text{ cm}^2/\text{V-s}$$

$$n_S \approx 1\times 10^{13} \text{ cm}^{-2}$$

$$\mu_{ball} \approx \frac{\upsilon_T L}{2}\frac{1}{k_B T/q} \approx 1400 \text{ cm}^2/\text{V-s}$$

$$\mu_{eff} = \left(\frac{1}{\mu_n} + \frac{1}{\mu_{ball}}\right)^{-1} \approx \mu_n$$

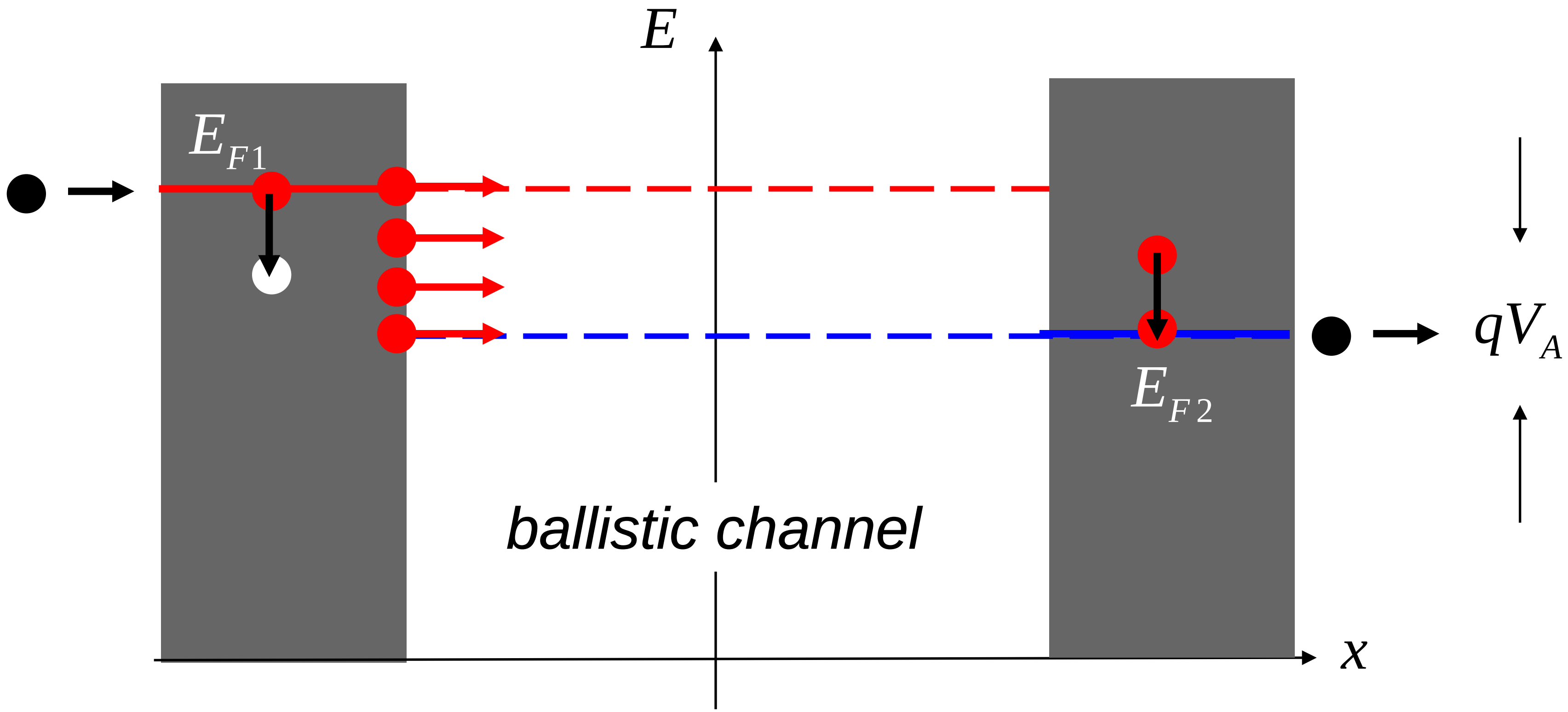
iii) power dissipation in a ballistic resistor

$$P_D = IV = GV^2 = V^2 \big/ R$$

Where is the power dissipated in a ballistic resistor?

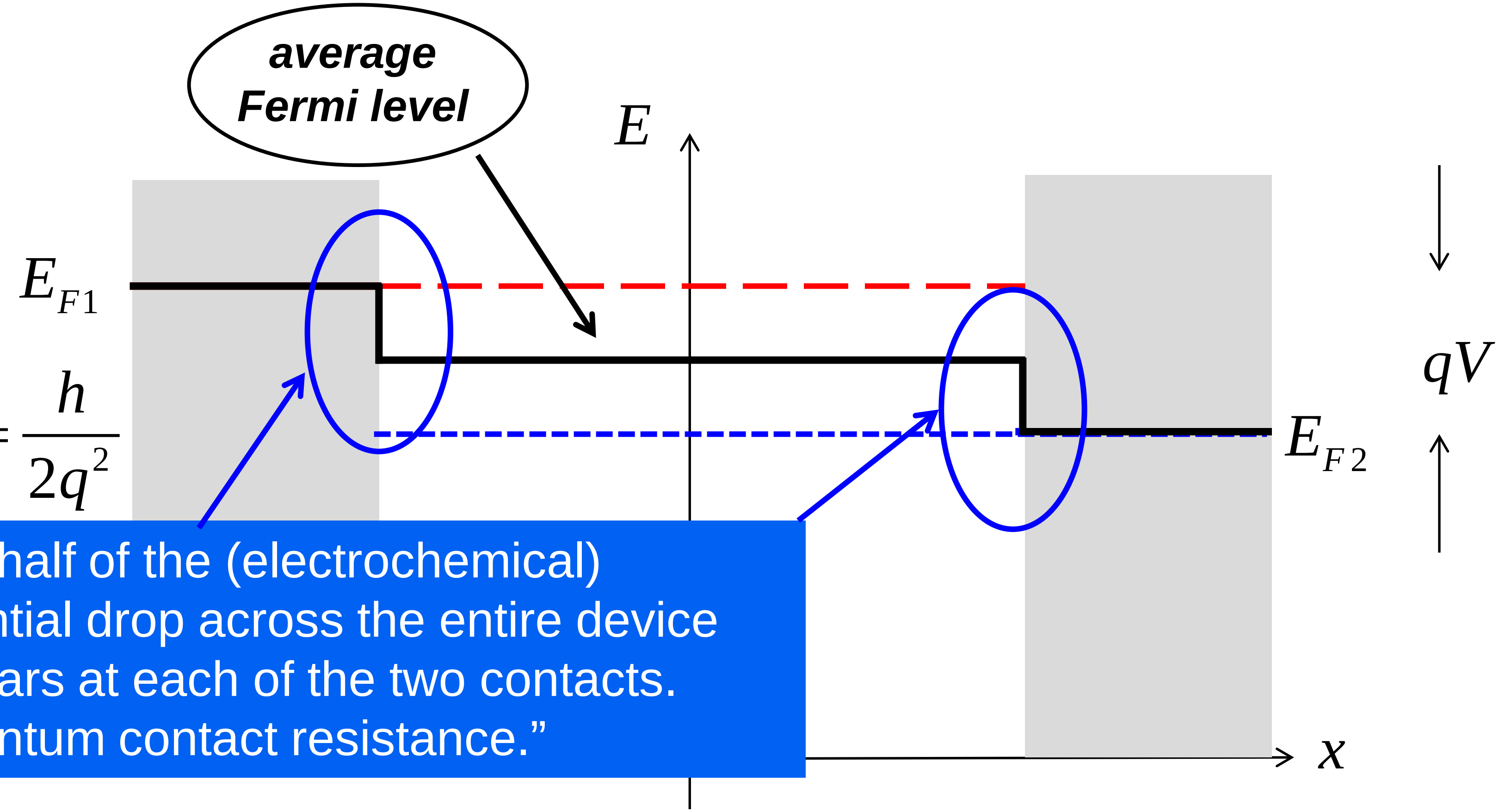
Answer: In the two contacts.

power dissipation in a ballistic resistor



dissipation occurs in the contacts

iv) where is the voltage drop?

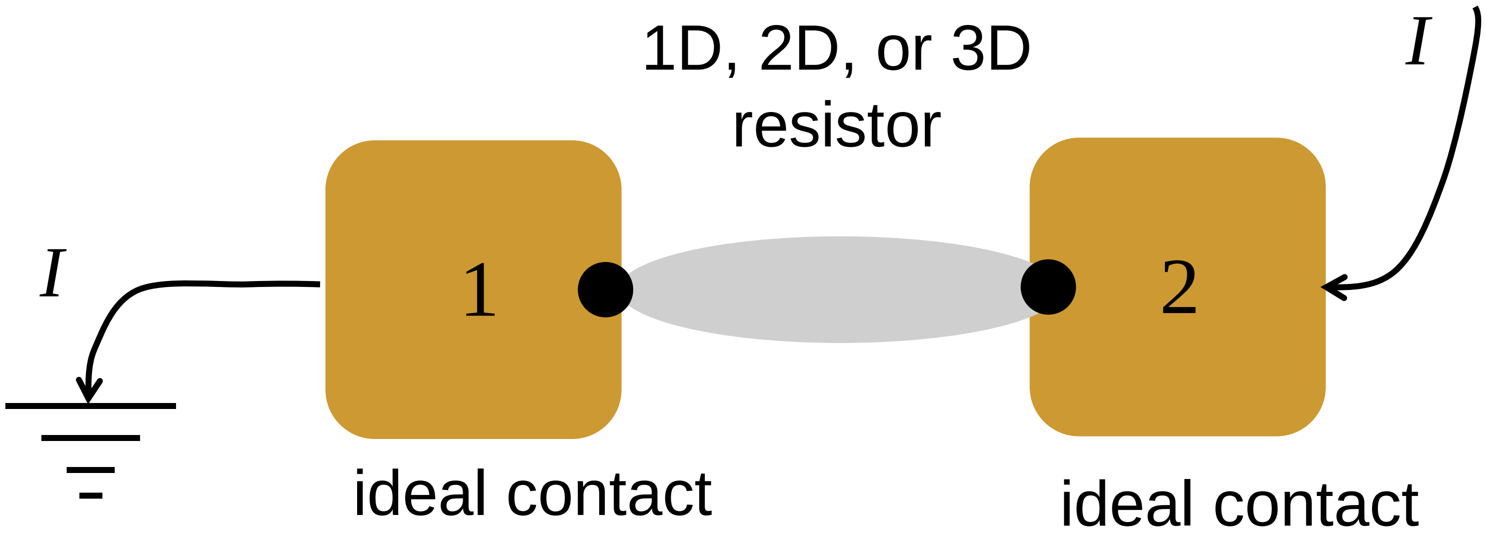


$$R_Q = \frac{h}{2q^2}$$

One-half of the (electrochemical) potential drop across the entire device appears at each of the two contacts. "Quantum contact resistance."

S. Datta, *Electronic Conduction in Mesoscopic Systems*, Chpt. 2, Cambridge, 1995

v) 1D and 3D resistors



$$G = \frac{2q^2}{h}\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

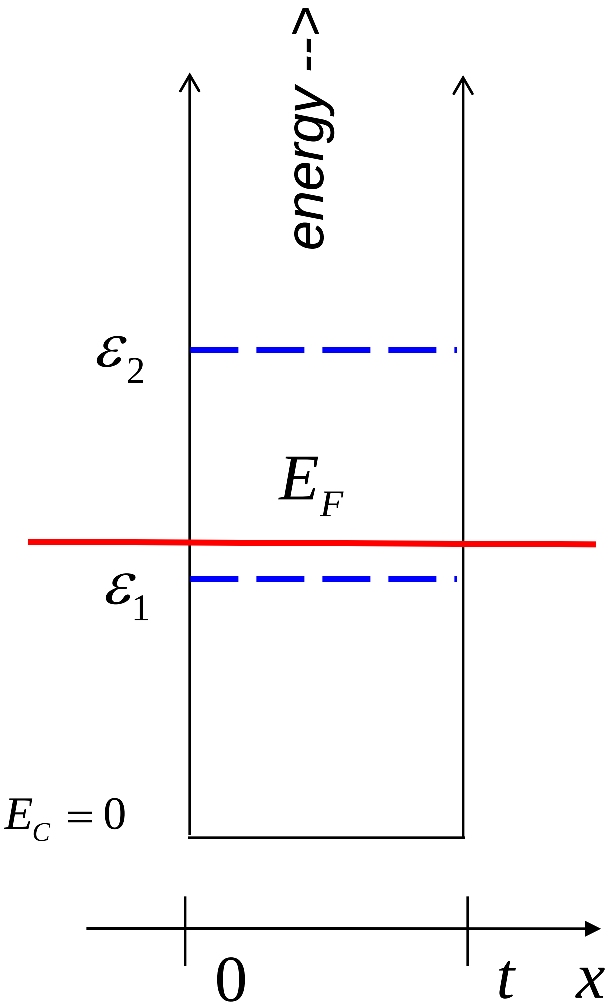
This expression describes 1D, 2D, or 3D resistors.

quantum confinement

$$\frac{d^2\psi(x)}{dx^2} + k^2\psi = 0 \qquad k^2 = \frac{2m^* E}{\hbar^2}$$

$$\psi(x) = \sin k_n x$$

$$\varepsilon_n = \frac{\hbar^2 k_n^2}{2m^*} = \frac{\hbar^2 n^2 \pi^2}{2m^* a^2}$$



modes or channels

$$\varepsilon_n = \frac{\hbar^2 n^2 \pi^2}{2m^* W^2}$$

1) Each occupied subband, is a channel for current to flow.

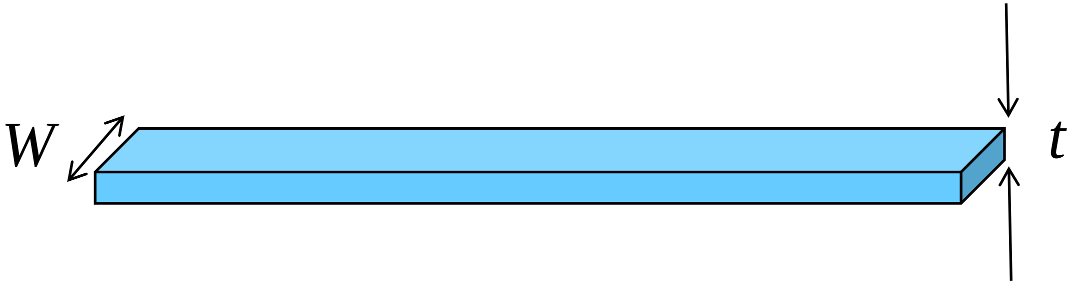
2) If *t* is small, the subbands are widely spaced, and we can count them.

3) If *W* is large, the subbands are closely spaced, and *M*(*E*) ~ W.

4) To include all subbands:

$$M(E) = \sum_{n=1}^{N} W \frac{\sqrt{2m^*(E-\varepsilon_n)}}{\pi\hbar}$$

1D



$$\varepsilon_n = \frac{\hbar^2 n^2 \pi^2}{2m^* W^2}$$

1) If *W* and *t* are small, all subbands are widely spaced, and we just count them.

2) For a 1D resistor: $M(E) = \text{no. of subbands at energy}, E$

3D

$$\varepsilon_n = \frac{\hbar^2 n^2 \pi^2}{2m^* W^2}$$

1) Is W and t are large, all of the subbands are closely spaced.

2) For a 3D resistor: $M(E) = A\frac{m^*}{2\pi\hbar^2}(E - E_C)$

1D, 2D, and 3D resistors

$$G = \frac{2q^2}{h}\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

1) In 1D (quantum confinement in two dimensions):

$$M(E) = \text{no. of subbands at energy}, E$$

2) In 2D (quantum confinement in one dimension):

$$M(E) = \sum_{n=1}^{N} W g_V \sqrt{2m^*(E-\varepsilon_n)} \Big/ \pi\hbar$$

3) In 3D (no quantum confinement):

$$M(E) = A g_V \frac{m^*}{2\pi\hbar^2}(E - E_C)$$

1D, 2D, and 3D resistors

$$G = \frac{2q^2}{h}\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$G = \frac{2q^2}{h}\langle\langle T\rangle\rangle\langle M\rangle$$

$$\langle M\rangle = \int M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$\langle\langle T\rangle\rangle = \frac{\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE}{\int M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE}$$

For a constant mfp:

$$\langle\langle T\rangle\rangle = \frac{\lambda_0}{\lambda_0 + L}$$

1D resistors

$$G = \frac{2q^2}{h}\frac{\lambda_0}{\lambda_0 + L}\langle M \rangle$$

$$\langle M \rangle = \sum_i \mathcal{F}_{-1}(\eta_{Fi})$$

$$\eta_{Fi} = \frac{E_F - \varepsilon_i}{k_B T_L}$$

(parabolic energy bands)
(constant mfp)

T_L = 0 K:

$$G = \frac{2q^2}{h}\frac{\lambda_0}{\lambda_0 + L} \times \text{no. of subbands at } E_F$$

MB statistics:

$$G_{1D} = n_L q \mu_{app} \frac{1}{L}$$

$$\mu_{app} = D_n / k_B T_L / q$$

$$D_n = \lambda_{app} \upsilon_T / 2$$

$$\upsilon_T = \sqrt{2k_B T_L / \pi m^*}$$

$$1/\lambda_{app} = (1/\lambda_0 + 1/L)$$

2D resistors

$$G = \frac{2q^2}{h} \frac{\lambda_0}{\lambda_0 + L} \langle M \rangle$$

$$\langle M \rangle = \frac{\sqrt{\pi}}{2} W M_{2D} \left(k_B T_L \right) \sum_i \mathcal{F}_{-1/2} \left(\eta_{Fi} \right)$$

$$M_{2D} \left(k_B T_L \right) = g_V \frac{\sqrt{2m^* k_B T_L}}{\pi \hbar}$$

$$\eta_{Fi} = \frac{E_F - \varepsilon_i}{k_B T_L}$$

(parabolic energy bands)

(constant mfp)

***T_L* = 0 K:**

$$G = \frac{2q^2}{h} \frac{\lambda_0}{\lambda_0 + L} \times W M_{2D} \left(E_F \right)$$

MB statistics:

$$G_{2D} = n_S q \mu_{app} \frac{W}{L}$$

$$\mu_{app} = D_n / k_B T_L / q$$

$$D_n = \lambda_{app} \upsilon_T / 2$$

$$\upsilon_T = \sqrt{2 k_B T_L / \pi m^*}$$

$$1/\lambda_{app} = \left(1/\lambda_0 + 1/L \right)$$

3D resistors

$$G = \frac{2q^2}{h} \frac{\lambda_0}{\lambda_0 + L} \langle M \rangle$$

$$\langle M \rangle = A M_{3D}\left(k_B T_L\right) \mathcal{F}_0\left(\eta_f\right)$$

$$M_{3D}\left(k_B T_L\right) = g_V \frac{m^* k_B T_L}{2\pi\hbar^2}$$

$$\eta_F = \frac{E_F - E_C}{k_B T_L}$$

(parabolic energy bands)

(constant mfp)

T_L = 0 K:

$$G = \frac{2q^2}{h} \frac{\lambda_0}{\lambda_0 + L} \times A M_{3D}\left(E_F\right)$$

MB statistics:

$$G_{3D} = nq\mu_{app} \frac{A}{L}$$

$$\mu_{app} = D_n / k_B T_L / q$$

$$D_n = \lambda_{app} \upsilon_T / 2$$

$$\upsilon_T = \sqrt{2 k_B T_L / \pi m^*}$$

$$1/\lambda_{app} = \left(1/\lambda_0 + 1/L\right)$$

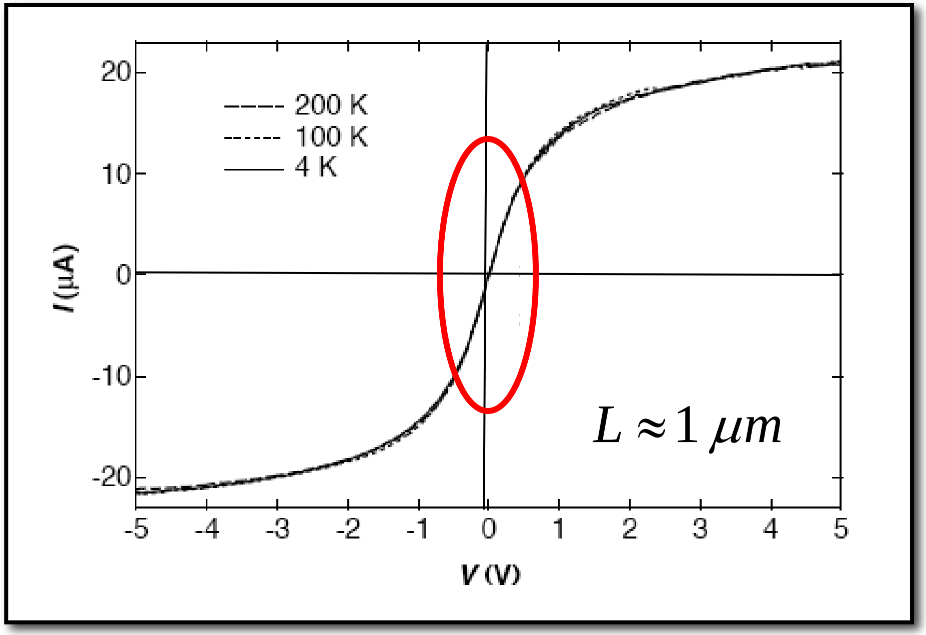
vi) 1D: example: low-field transport in metallic CNTs

$$G_{1D} = \frac{\Delta I}{\Delta V} = \frac{22\ \mu A}{1.0\ V} = 22\ \mu S$$

$$G_B = \frac{4q^2}{h} = 154\ \mu S \qquad (g_V = 2)$$

$$G_{1D} = \frac{4q^2}{h}\frac{\lambda_0}{\lambda_0 + L}$$

$$\lambda_0 \approx 167\ \text{nm} << L$$



Zhen Yao, Charles L. Kane, and Cees Dekker, "High-Field Electrical Transport in Single-Wall Carbon Nanotubes," *Phys. Rev. Lett*., **84**, 2941-2944, 2000.

outline

1) Review

2) 2D ballistic resistors

3) 2D diffusive resistors

4) Discussion

5) Summary

near-equilibrium ballistic transport

1) Conductors display a finite resistance, even in the absence of scattering.

2) The quantum contact resistance is quantized, and the quantum of resistance is $h/2q^2$

3) The ballistic resistance sets a lower limit to device resistance and is becoming important in practical, room temperature devices.

4) Transport from the ballistic to diffusive limit is easily treated by using the transmission.

the key point

When calculating the resistance of a new material, begin with:

$$G = \frac{2q^2}{h} \int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

not: $G = nq\mu_n$

questions

1) Review

2) 2D ballistic resistors

3) 2D diffusive resistors

4) Discussion

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