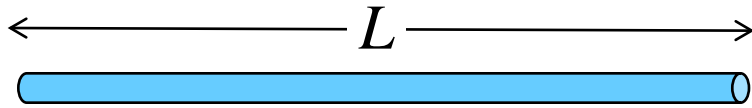


ECE-656: Fall 2011

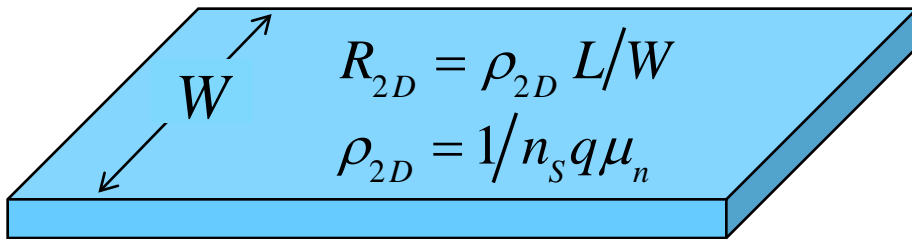
**Lecture 8:
More about Resistance**

Professor Mark Lundstrom
Electrical and Computer Engineering
Purdue University, West Lafayette, IN USA

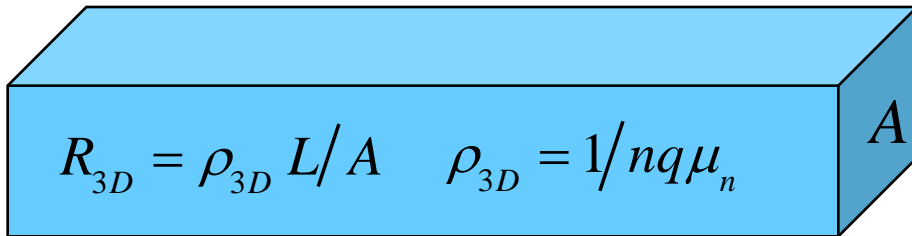
resistors



$$R_{1D} = \rho_{1D} L \quad \rho_{1D} = 1/n_L q \mu_n$$



$$R_{2D} = \rho_{2D} L/W$$
$$\rho_{2D} = 1/n_S q \mu_n$$



$$R_{3D} = \rho_{3D} L/A \quad \rho_{3D} = 1/n q \mu_n$$

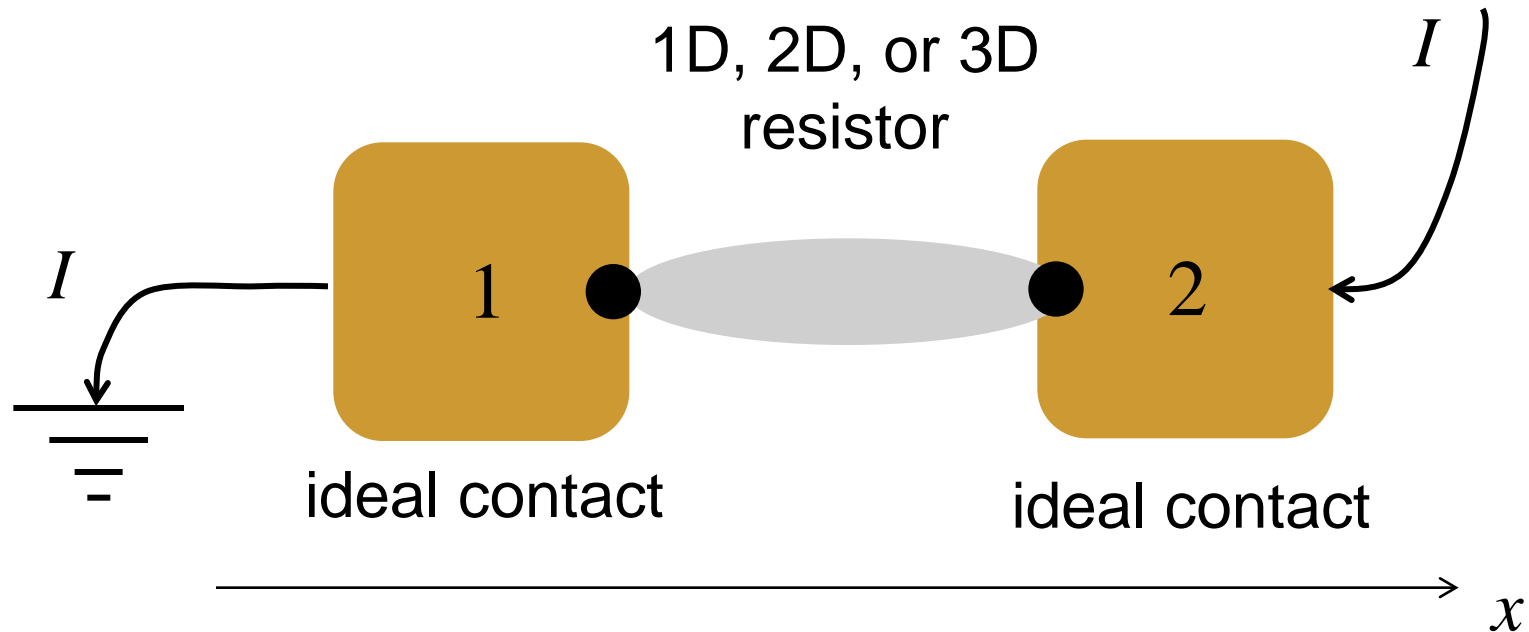
General (ballistic to diffusive):

$$G = \frac{2q^2}{h} \langle\langle T \rangle\rangle \langle M \rangle$$

Diffusive:

$$G = \frac{2q^2}{h} \frac{\langle\langle \lambda \rangle\rangle}{L} \langle M \rangle$$

definitions of $\langle M \rangle$ and $\langle\langle \lambda \rangle\rangle$



Current is positive when it flows **into** contact 2 (i.e. positive current flows in the $-x$ direction).

$$G = \frac{2q^2}{h} \int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \equiv \frac{2q^2}{h} \langle\langle T(E) \rangle\rangle \langle M(E) \rangle$$

outline

- 1) Review
- 2) **Discussion**
 - power dissipation
 - voltage drop
 - n-type vs. p-type
 - “apparent” mobility
- 3) 1D and 3D resistors
- 4) Graphene: A case study
- 5) Summary



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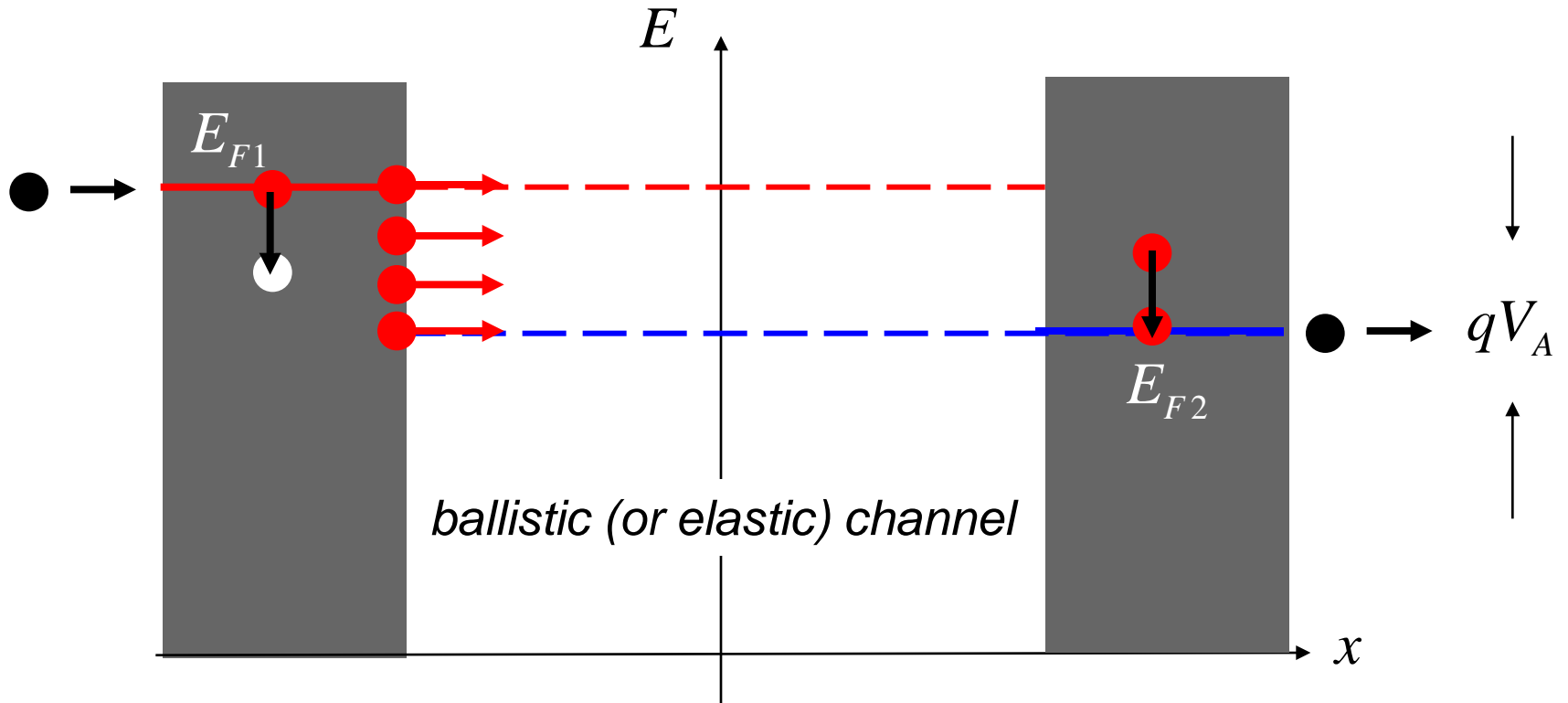
power dissipation in a ballistic resistor

$$P_D = IV = GV^2 = V^2/R$$

Where is the power dissipated in a ballistic resistor?

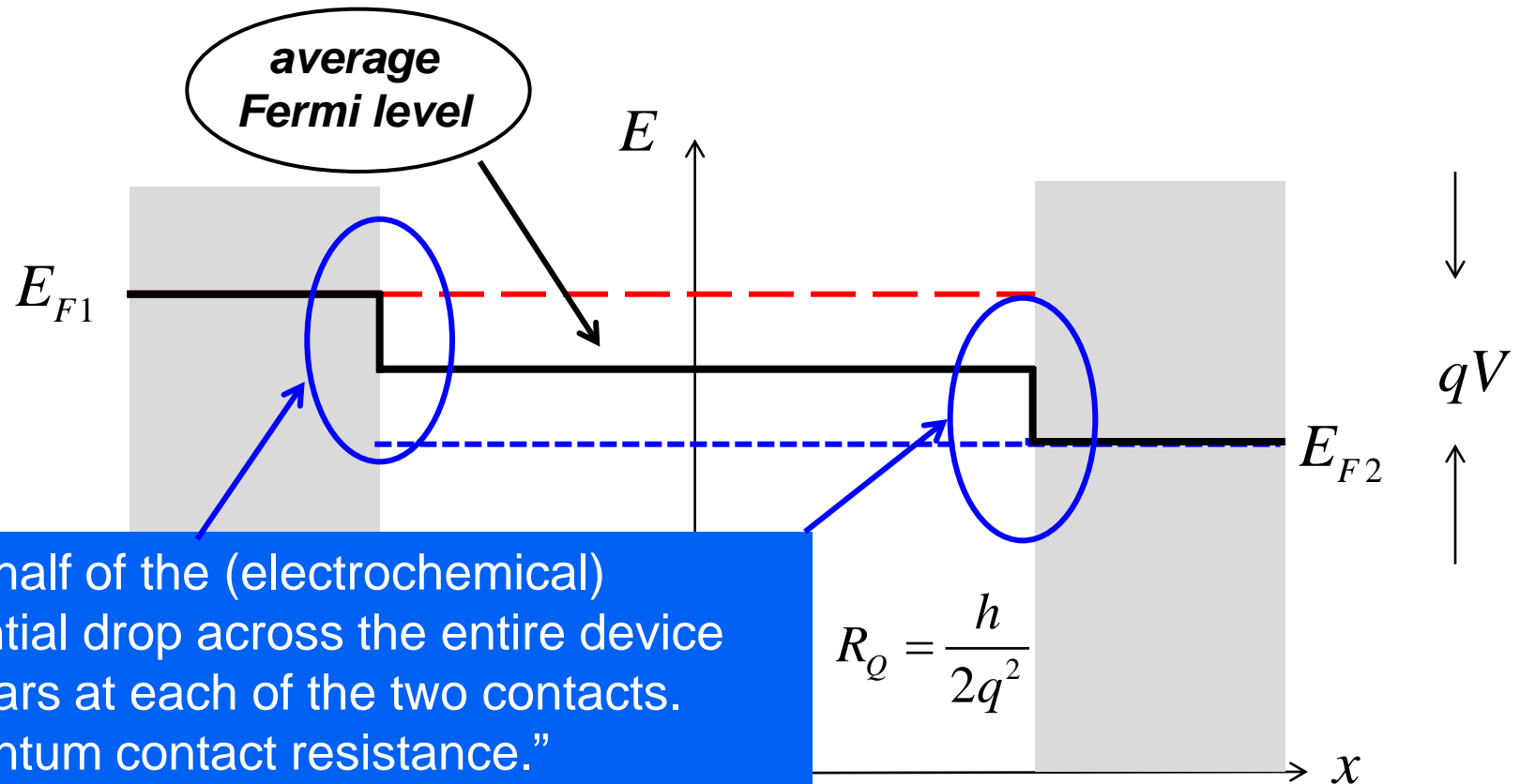
Answer: In the two contacts.

power dissipation in a ballistic resistor



dissipation occurs in the contacts

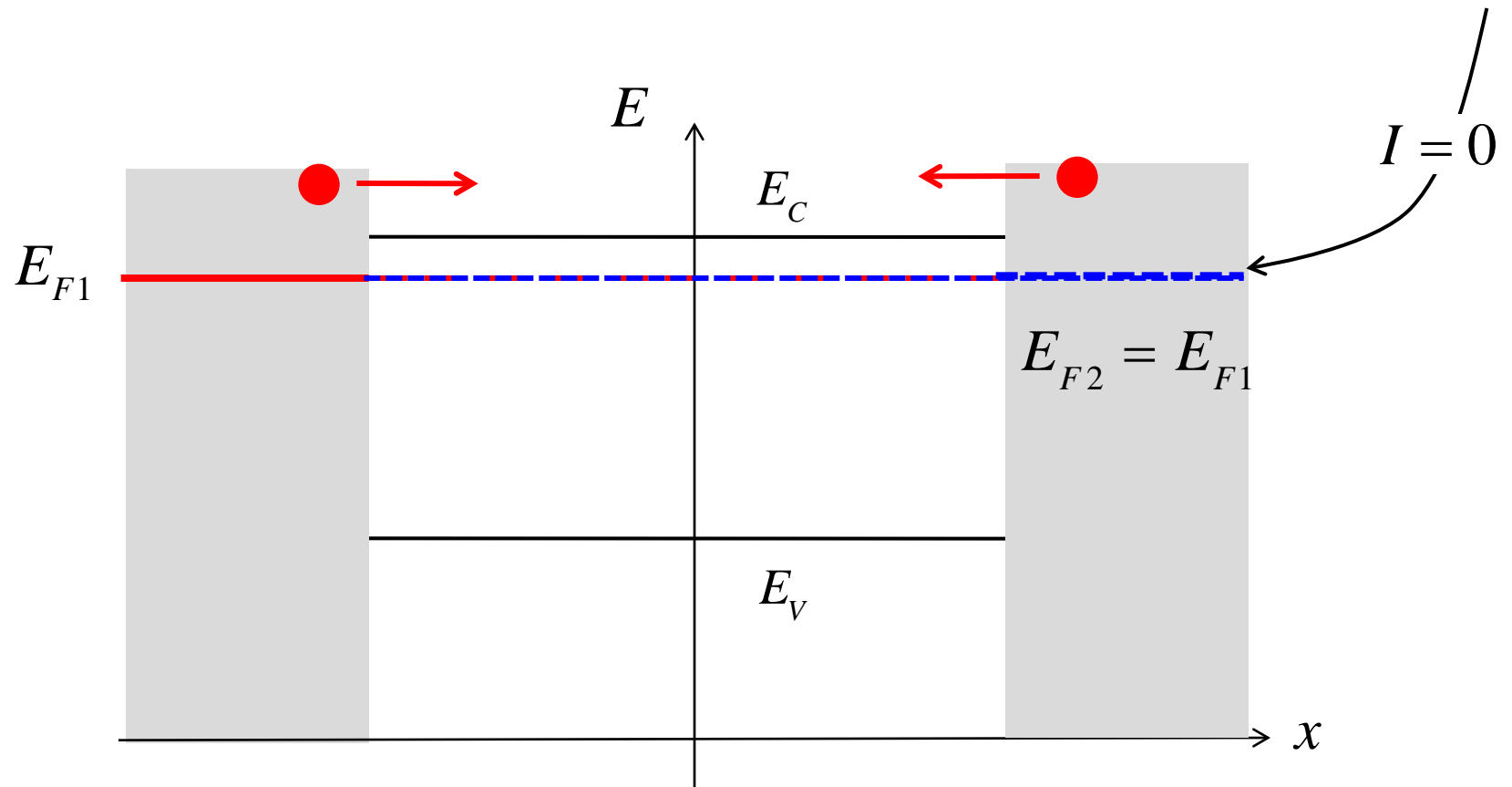
voltage drop in a ballistic resistor



S. Datta, *Electronic Conduction in Mesoscopic Systems*, Chpt. 2, Cambridge, 1995

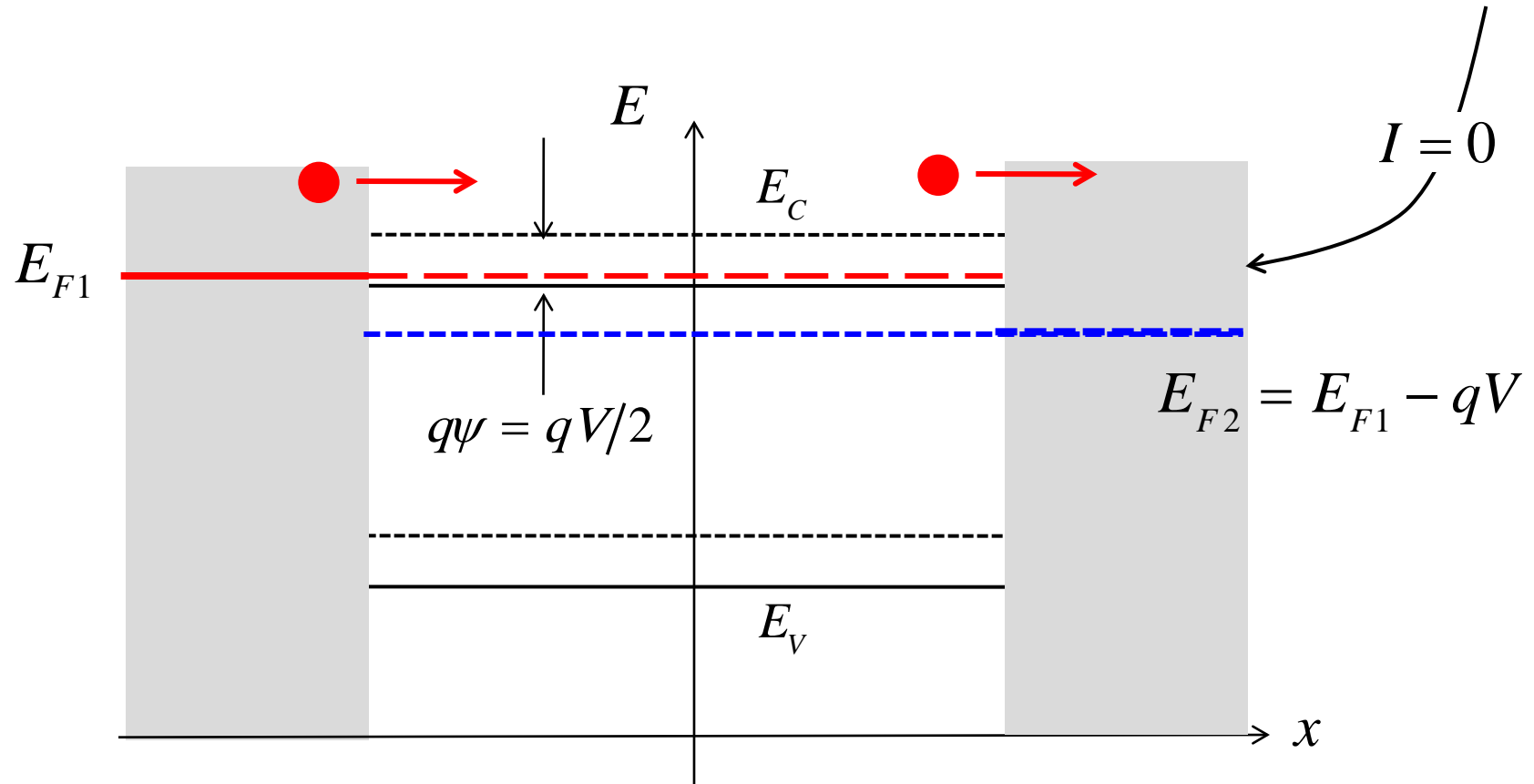
Lundstrom 2011

another view



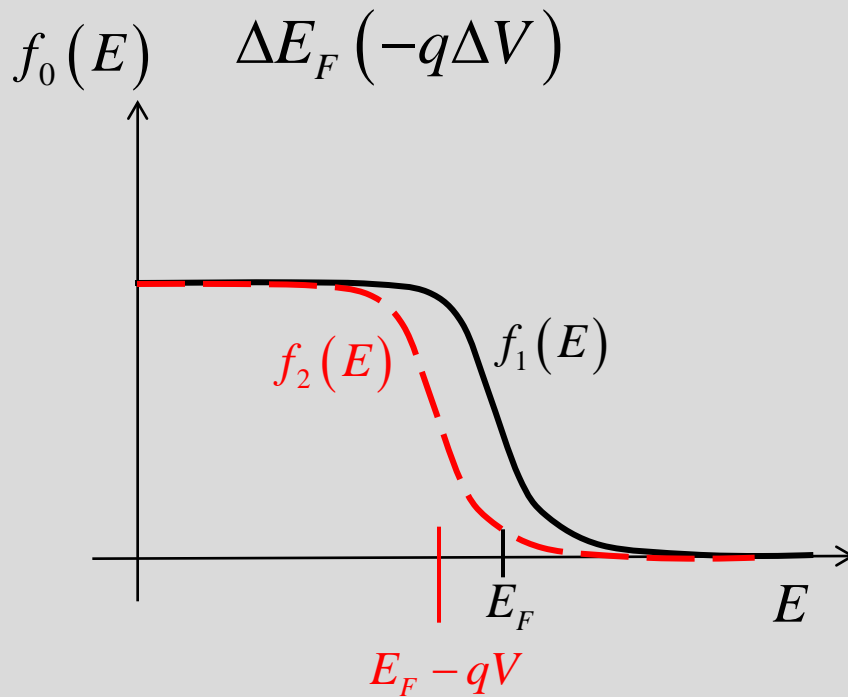
$$N_0 = \frac{D}{2}(E_{F1} - E_C^0) + \frac{D}{2}(E_{F1} - E_C^0)$$

another view



$$N = N_0 = \frac{D}{2} (E_{F1} - E_C^0 + q\psi) + \frac{D}{2} (E_{F1} - qV - E_C^0 + q\psi)$$

n-type conduction vs. p-type

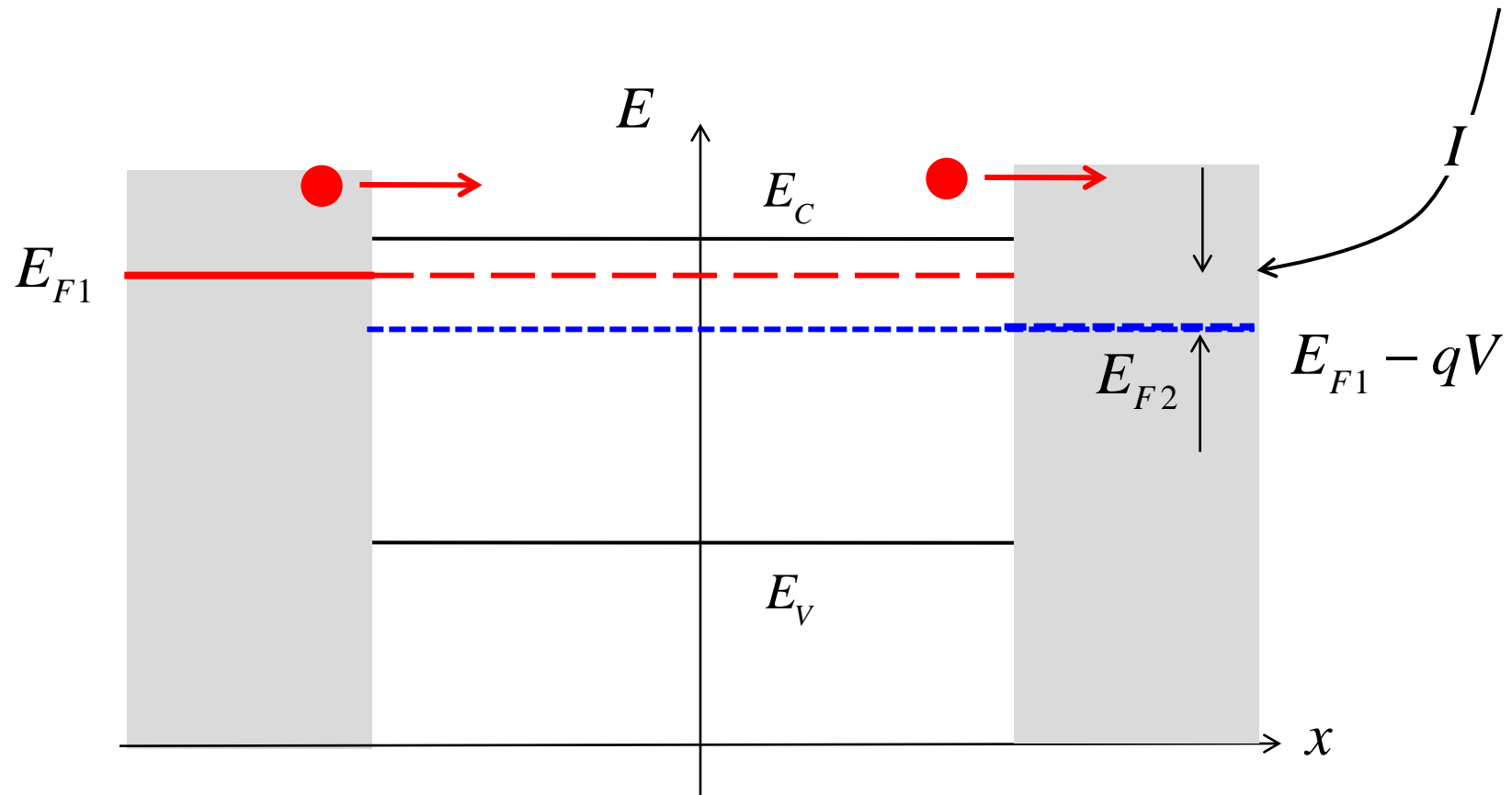


$$I = \frac{2q}{h} \int T(E) M(E) (f_1 - f_2) dE$$

$$V > 0 \Rightarrow f_1 > f_2 \Rightarrow I > 0$$

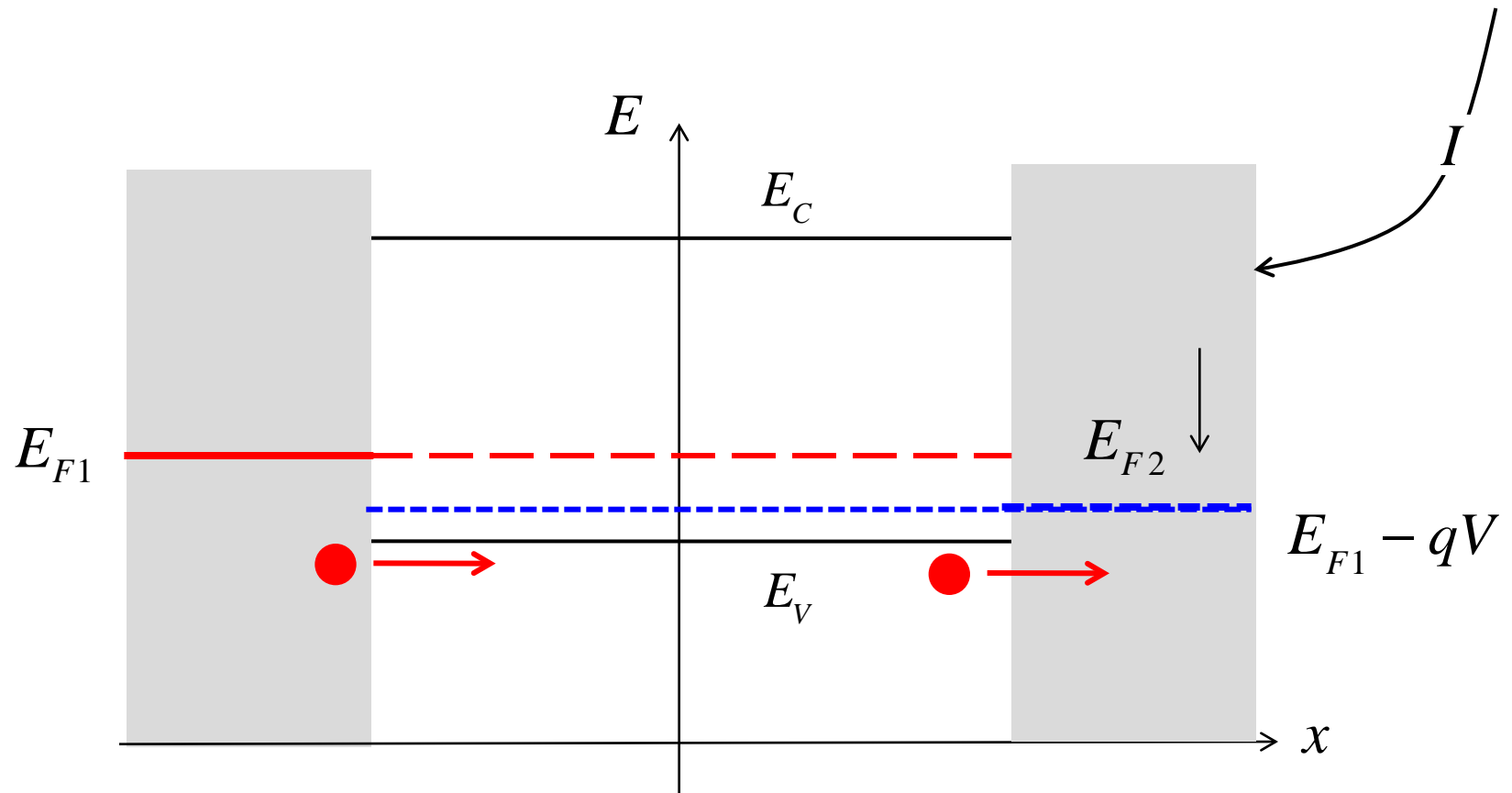
for either *n*-type or *p*-type

n-type



current is due to electrons flowing in the conduction band

p-type



current is due to electrons flowing in the valence band

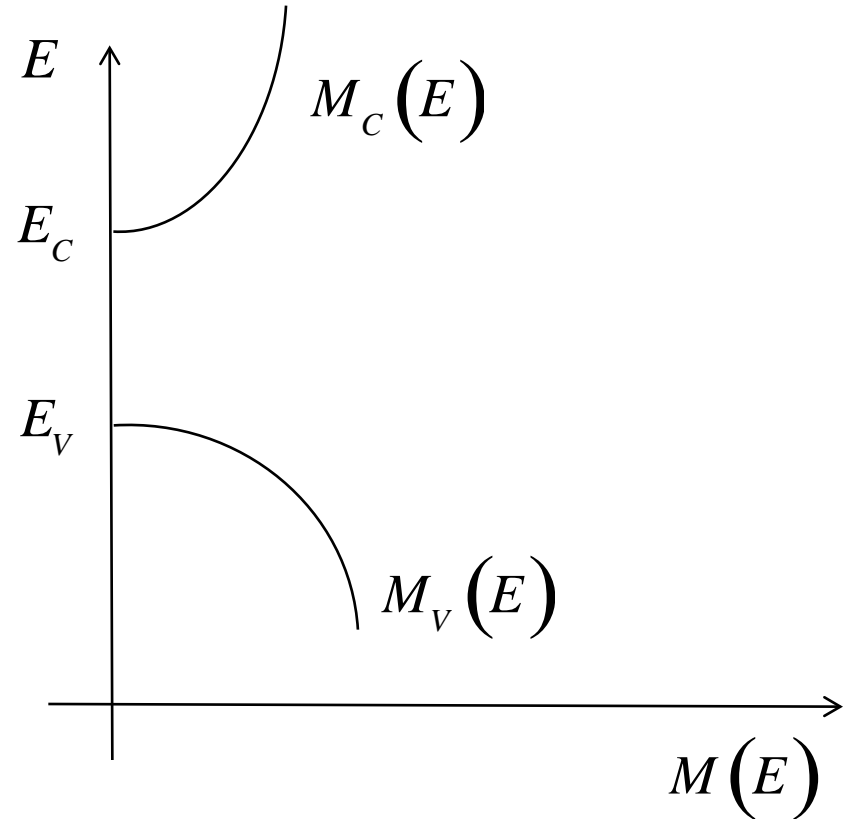
bipolar conduction

$$I = \frac{2q}{h} \int T(E) M(E) (f_1 - f_2) dE$$

$$M(E) = M_V(E_V - E) + M_C(E - E_C)$$

$$M_C(E) = g_V W \frac{\sqrt{2m^*(E - E_C)}}{\pi \hbar}$$

$$M_V(E) = g_V W \frac{\sqrt{2m^*(E_V - E)}}{\pi \hbar}$$



discussion

-power dissipation

-voltage drop

-n-type vs. p-type

-“apparent” mobility

mobility

We usually like to think of mobility as a material dependent quantity.

In bulk materials with a large electric field, we generalize this to a field-dependent quantity.

More generally, “mobility” can be device dependent.

how should we define mobility?

$$G_{2D} = \frac{2q^2}{h} \int T(E) W M_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \quad G_{diff} = n_S q \mu_n \frac{W}{L}$$

(ballistic to diffusive) (diffusive)

definition of “apparent” mobility:

$$\mu_{app} \equiv \frac{1}{n_S} \frac{2q}{h} \int T(E) L M_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

not:

$$\mu_n = \frac{q\tau_m}{m^*}$$

in the diffusive limit

definition of mobility:

$$\mu_n \equiv \frac{1}{n_S} \frac{2q}{h} \int T(E) L M_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$T(E) = \frac{\lambda(E)}{\lambda(E) + L} \rightarrow \frac{\lambda(E)}{L}$$

$$\mu_n \equiv \frac{1}{n_S} \frac{2q}{h} \int \lambda(E) M_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$\mu_n = \frac{q\tau_m}{m^*}$$

in the ballistic limit

definition of mobility:

$$\mu_n \equiv \frac{1}{n_S} \frac{2q}{h} \int T(E) LM_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$T(E) = 1$$

$$\mu_{ball} \equiv \frac{1}{n_S} \frac{2q}{h} \int LM_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$\mu_{ball} \neq \frac{q\tau_m}{m^*}$$

ballistic to diffusive...

definition of **apparent** mobility:

$$\mu_{app} \equiv \frac{1}{n_S} \frac{2q}{h} \int T(E) LM_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$T(E)L = \frac{\lambda(E)L}{\lambda(E)+L} \rightarrow \lambda_{app}(E) \qquad \frac{1}{\lambda_{app}(E)} \equiv \frac{1}{\lambda(E)} + \frac{1}{L}$$

$$\mu_{app} \equiv \frac{1}{n_S} \frac{2q}{h} \int \lambda_{app}(E) M_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$\mu_n = \frac{q\tau_m}{m^*}$$

$T_L = 0\text{K}$ apparent mobility

$$\mu_{app} \equiv \frac{1}{n_S} \frac{2q}{h} \int \lambda_{app}(E) M_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$\mu_{app}(T_L = 0\text{K}) = \frac{1}{n_S} \frac{2q}{h} \lambda_{app}(E_F) M_{2D}(E_F)$$

$$\frac{1}{\mu_{app}} = \frac{1}{\mu_n} + \frac{1}{\mu_{ball}}$$

$$\frac{1}{\lambda_{app}(E)} = \frac{1}{\lambda(E)} + \frac{1}{L}$$

$$M_{2D} = \frac{h}{4} \langle v_x^+ \rangle D_{2D} \quad \langle v_x^+ \rangle = \frac{2}{\pi} v_F$$

$$\begin{aligned} n_S &= g_V \frac{m^*}{\pi \hbar^2} (E_F - E_C) \\ &= D_{2D} (E_F - E_C) \end{aligned}$$

“ballistic mobility” in 2D at $T_L = 0\text{K}$

$$\frac{1}{\mu_{app}} = \frac{1}{\mu_n} + \frac{1}{\mu_{ball}}$$

$$\mu_n = \frac{D_n}{(E_F - E_C)/q}$$

$$\mu_{ball} = \frac{D_{ball}}{(E_F - E_C)/q}$$

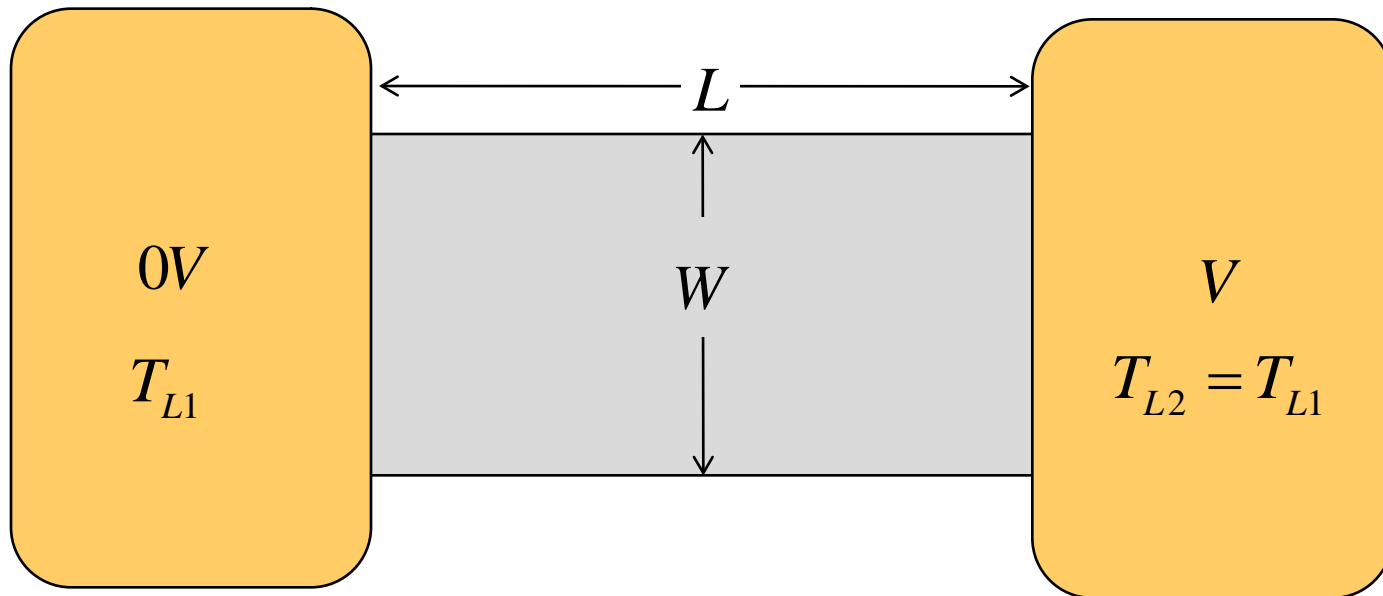
$$D_n = \frac{\langle v_x^+ \rangle \lambda(E_F)}{2}$$

$$D_{ball} = \frac{\langle v_x^+ \rangle L}{2}$$

$$G_{2D} = \frac{2q^2}{h} T(E_F) M_{2D}(E_F) = n_s q \mu_{app} \frac{W}{L}$$

M.S. Shur, “Low Ballistic Mobility in GaAs HEMTs,” *IEEE Electron Dev. Lett.*, **23**, 511-513, 2002

physical interpretation



$$\frac{1}{\lambda_{app}(E)} \equiv \frac{1}{\lambda(E)} + \frac{1}{L}$$

ballistic conductance ($T_L = 0$ K)

$$G_{2D} = n_s q \mu_{app} \frac{W}{L}$$

$$G_{2D} = n_s q \mu_{ball} \frac{W}{L}$$

$$G_{2D} = \frac{2q^2}{h} M(E_F)$$

$$\frac{1}{\lambda_{app}(E_F)} = \frac{1}{\lambda(E_F)} + \frac{1}{L}$$

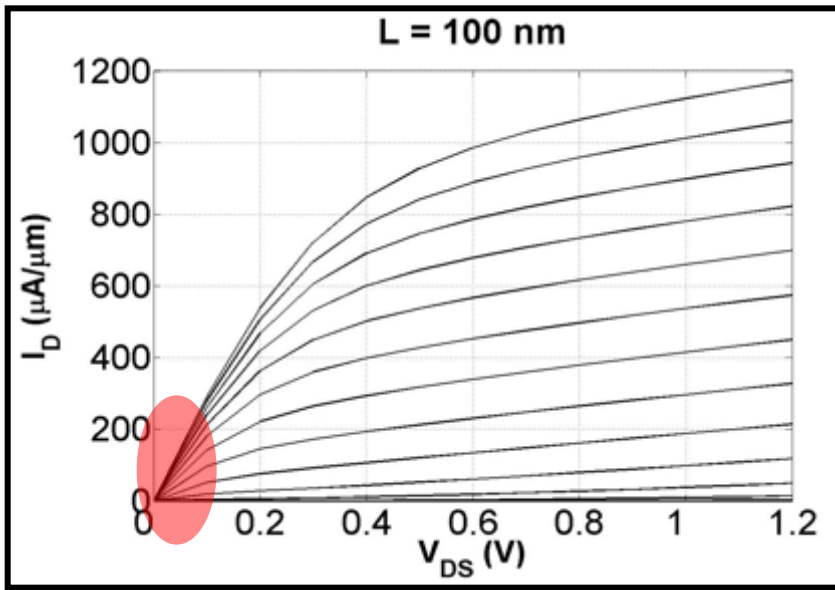
$$M = W \frac{h}{4} \langle v_x^+ \rangle D_{2D}$$

$$n_s = D_{2D} (E_F - E_C)$$

$$\mu_{ball} = \frac{D_{ball}}{(E_F - E_C)/q}$$

$$D_{ball} = \frac{\langle v_x^+ \rangle L}{2}$$

exercise: nanoscale FETs



(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

$$L \approx 60 \text{ nm}$$

$$\mu_n \approx 260 \text{ cm}^2/\text{V-s}$$

$$n_S \approx 1 \times 10^{13} \text{ cm}^{-2}$$

$$\mu_{ball} \approx \frac{v_T L}{2} \frac{1}{k_B T / q} \approx 1400 \text{ cm}^2/\text{V-s}$$

$$\mu_{eff} = \left(\frac{1}{\mu_n} + \frac{1}{\mu_{ball}} \right)^{-1} \approx \mu_n$$

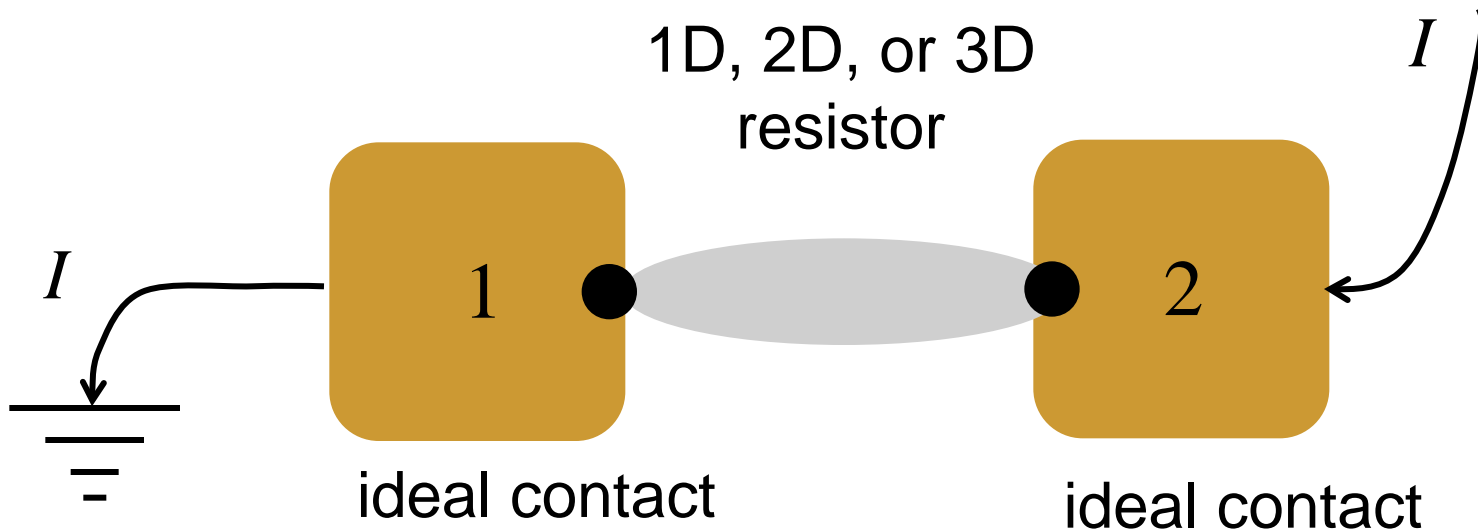
outline

- 1) Review
- 2) Discussion
- 3) 1D and 3D resistors**
- 4) Graphene: A case study
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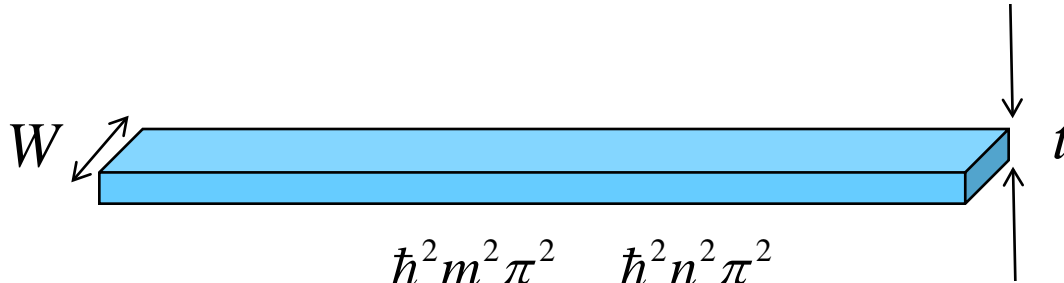
1D and 3D resistors



$$G = \frac{2q^2}{h} \int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

This expression describes 1D, 2D, or 3D resistors.

1D



$$\varepsilon_{m,n} = \frac{\hbar^2 m^2 \pi^2}{2m^* W^2} + \frac{\hbar^2 n^2 \pi^2}{2m^* t^2}$$

1) If W and t are small, all subbands are widely spaced, and we just count them.

2) For a 1D resistor: $M(E)$ = no. of subbands at energy, E

$$G_{1D} = \frac{2q^2}{h} \langle M \rangle$$

3D



1) If W and t are large, all of the subbands are closely spaced.

2) For a 3D resistor: $M(E) = A \frac{m^*}{2\pi\hbar^2} (E - E_C)$

$$G_{3D} = \frac{2q^2}{h} \langle M \rangle$$

1D, 2D, and 3D resistors

$$G = \frac{2q^2}{h} \int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

1) In 1D (quantum confinement in two dimensions):

$$M(E) = \text{no. of subbands at energy, } E$$

2) In 2D (quantum confinement in one dimension):

$$M(E) = \sum_{n=1}^N W g_v \sqrt{2m^* (E - \varepsilon_n)} / \pi \hbar$$

3) In 3D (no quantum confinement):

$$M(E) = A g_v \frac{m^*}{2\pi \hbar^2} (E - E_c)$$

1D, 2D, and 3D resistors

$$G = \frac{2q^2}{h} \int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$G = \frac{2q^2}{h} \langle\langle T \rangle\rangle \langle M \rangle$$

$$\langle M \rangle = \int M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$\langle\langle T \rangle\rangle = \frac{\int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE}{\int M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE}$$

For a constant mfp:

$$\langle\langle T \rangle\rangle = \frac{\lambda_0}{\lambda_0 + L}$$

1D resistors

$$G = \frac{2q^2}{h} \frac{\lambda_0}{\lambda_0 + L} \langle M \rangle$$

$$\langle M \rangle = \sum_i \mathcal{F}_{-1}(\eta_{Fi})$$

$$\eta_{Fi} = \frac{E_F - \varepsilon_i}{k_B T_L}$$

(parabolic energy bands)
(constant mfp)

$T_L = 0$ K:

$$\langle M \rangle = M(E_F)$$

MB statistics:

$$\langle M \rangle \propto n_L$$

Diffusive:

$$G \propto \frac{1}{L}$$

2D resistors

$$G = \frac{2q^2}{h} \frac{\lambda_0}{\lambda_0 + L} \langle M \rangle$$

$$\langle M \rangle = \frac{\sqrt{\pi}}{2} W M_{2D}(k_B T_L) \mathcal{F}_{-1/2}(\eta_{F1})$$

$$M_{2D}(k_B T_L) = g_V \frac{\sqrt{2m^* k_B T_L}}{\pi \hbar}$$

$$\eta_{F1} = \frac{E_F - \varepsilon_1}{k_B T_L}$$

(parabolic energy bands)
(constant mfp)

$T_L = 0$ K:

$$\langle M \rangle = M(E_F)$$

MB statistics:

$$\langle M \rangle \propto n_S$$

Diffusive:

$$G \propto \frac{W}{L}$$

3D resistors

$$G = \frac{2q^2}{h} \frac{\lambda_0}{\lambda_0 + L} \langle M \rangle$$

$$\langle M \rangle = A M_{3D}(k_B T_L) \mathcal{F}_0(\eta_F)$$

$$M_{3D}(k_B T_L) = g_V \frac{m^* k_B T_L}{2\pi \hbar^2}$$

$$\eta_F = \frac{E_F - E_C}{k_B T_L}$$

(parabolic energy bands)
(constant mfp)

$T_L = 0$ K:

$$\langle M \rangle = M(E_F)$$

MB statistics:

$$\langle M \rangle \propto n$$

Diffusive:

$$G \propto \frac{A}{L}$$

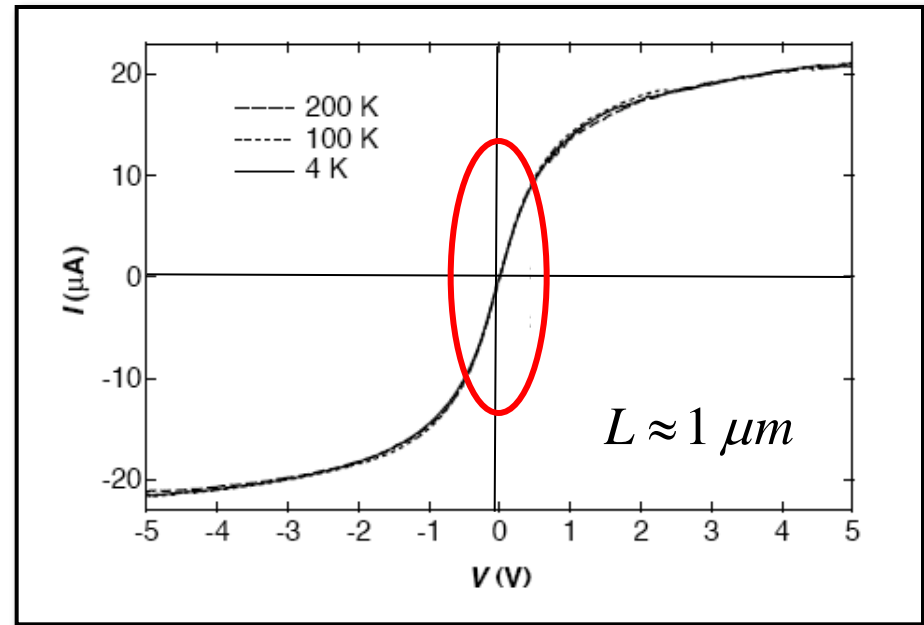
1D: example: low-field transport in metallic CNTs

$$G_{1D} = \frac{\Delta I}{\Delta V} = \frac{22 \mu A}{1.0 V} = 22 \mu S$$

$$G_B = \frac{4q^2}{h} = 154 \mu S \quad (g_V = 2)$$

$$G_{1D} = \frac{4q^2}{h} \frac{\lambda_0}{\lambda_0 + L}$$

$$\lambda_0 \approx 167 \text{ nm} \ll L$$



Zhen Yao, Charles L. Kane, and Cees Dekker, "High-Field Electrical Transport in Single-Wall Carbon Nanotubes," *Phys. Rev. Lett.*, **84**, 2941-2944, 2000.

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graphene

We now have all the tools and concepts needed to analyze transport in a wide variety of materials. To see how to apply these concepts to graphene, see Lecture 10 of “Near-Equilibrium Transport: Fundamentals and Applications” by M. Lundstrom.

or view an online lecture at:

<http://nanohub.org/resources/11873>

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key points

1) To understand conductance, we begin with:

$$G = \frac{2q^2}{h} \int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

which can be written as:

$$G = \frac{2q^2}{h} \langle\langle T(E) \rangle\rangle \langle M(E) \rangle$$

$$\langle M \rangle \equiv \int M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \quad \langle\langle T \rangle\rangle \equiv \frac{\int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE}{\int M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE}$$

key points

2) To relate conductance to mobility, we define

$$G_{2D} = nq\mu_{app} \frac{W}{L} = \frac{2q^2}{h} \int T(E)M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

and solve for the apparent mobility.

questions

- 1) Review
- 2) Discussion
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