ECE-656: Fall 2011

Lecture 8: More about Resistance

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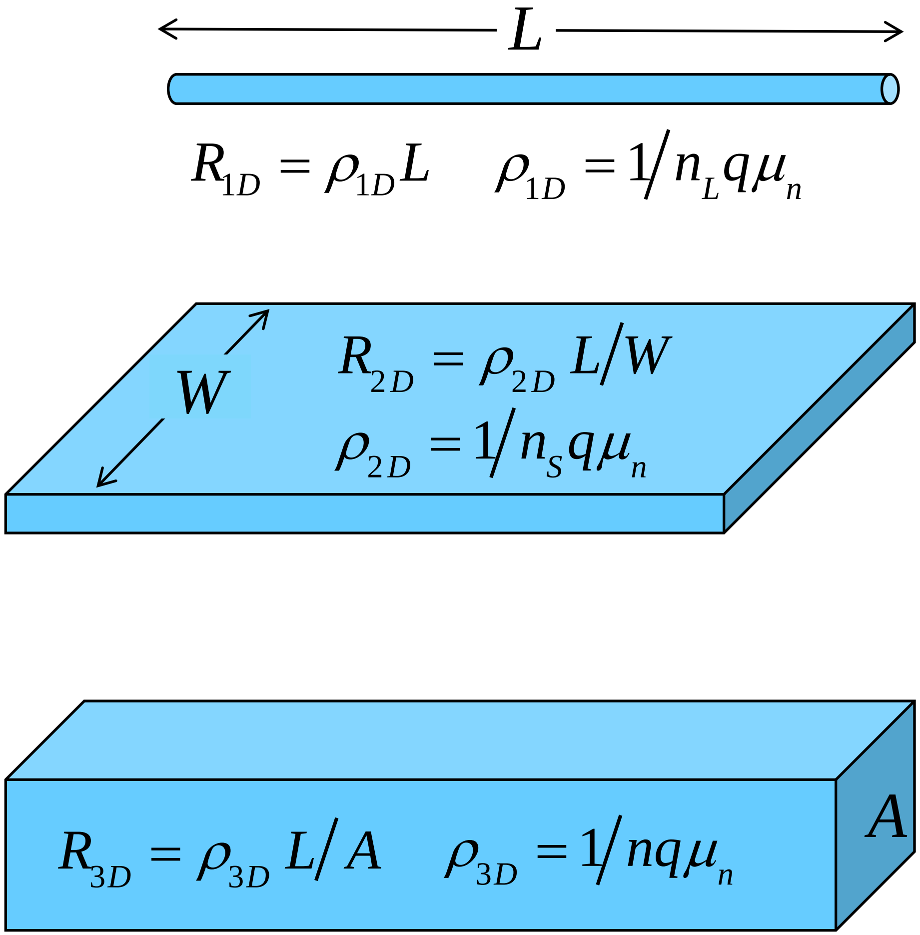
resistors



$$R_{1D} = \rho_{1D} L \qquad \rho_{1D} = 1 / n_L q \mu_n$$

$$R_{2D} = \rho_{2D}\, L / W$$

$$\rho_{2D} = 1 / n_S q \mu_n$$

$$R_{3D} = \rho_{3D}\, L / A \qquad \rho_{3D} = 1 / n q \mu_n$$

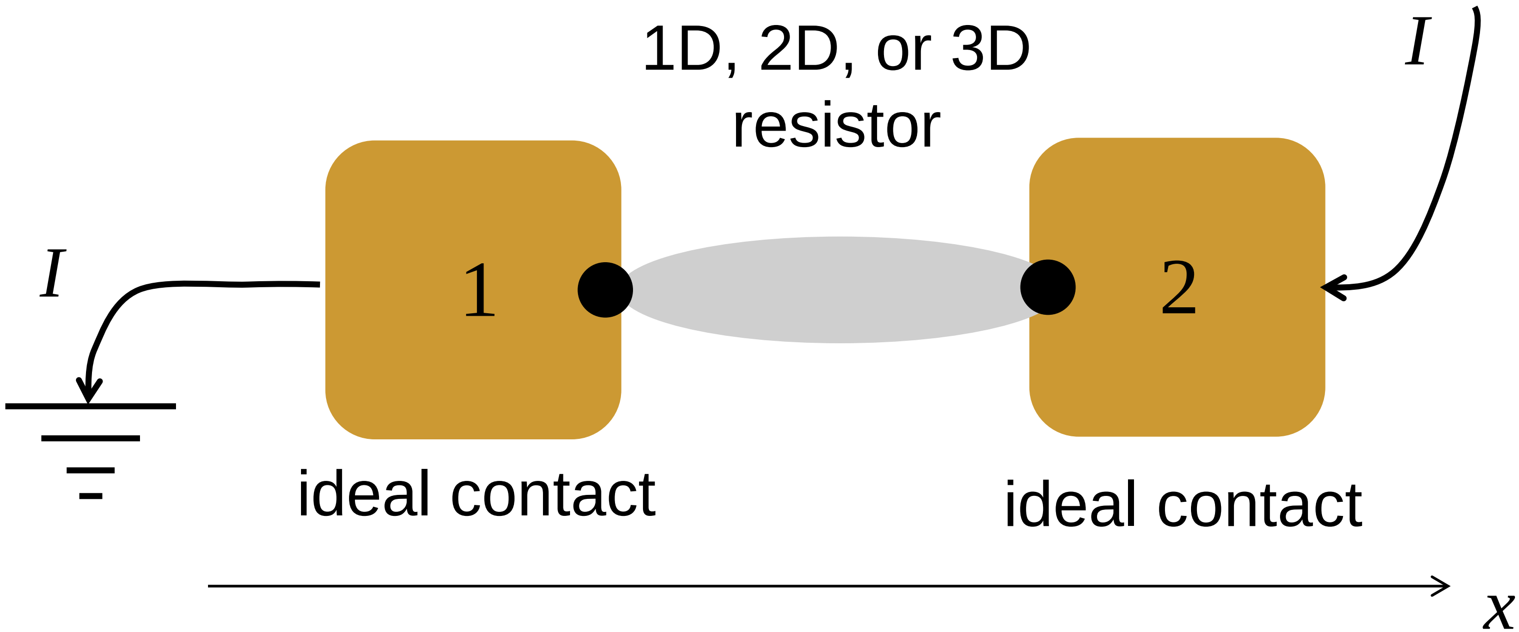
General (ballistic to diffusive):

$$G = \frac{2q^2}{h} \langle\langle T \rangle\rangle \langle M \rangle$$

Diffusive:

$$G = \frac{2q^2}{h} \frac{\langle\langle \lambda \rangle\rangle}{L} \langle M \rangle$$

definitions of <M> and <<λ>>



Current is positive when it flows **into** contact 2 (i.e. positive current flows in the -*x* direction).

$$G = \frac{2q^2}{h}\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \equiv \frac{2q^2}{h}\langle\langle T(E)\rangle\rangle\langle M(E)\rangle$$

outline

1) Review
2) **Discussion**
   - -power dissipation
   - -voltage drop
   - -n-type vs. p-type
   - -“apparent” mobility
3) 1D and 3D resistors
4) Graphene: A case study
5) Summary

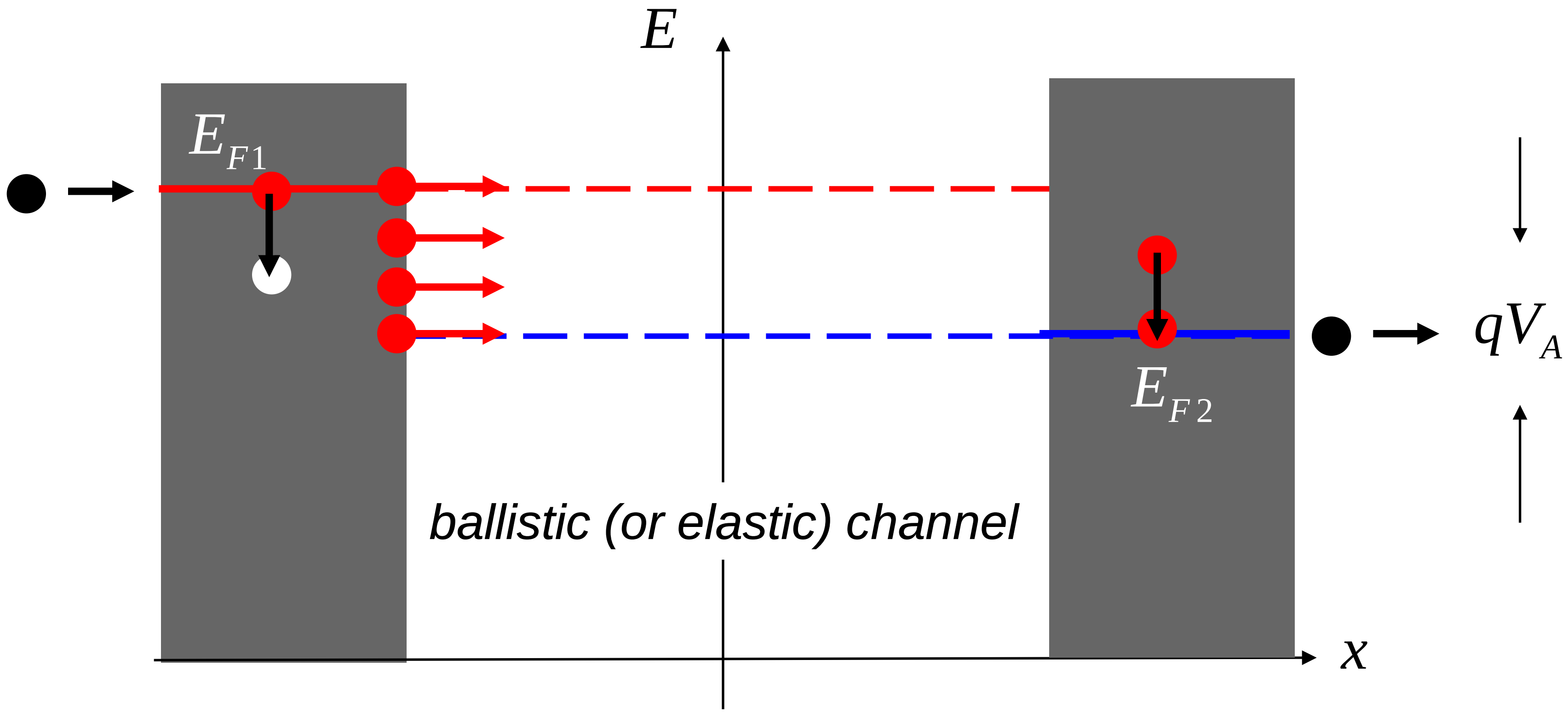
power dissipation in a ballistic resistor

$$P_D = IV = GV^2 = V^2 / R$$

Where is the power dissipated in a ballistic resistor?

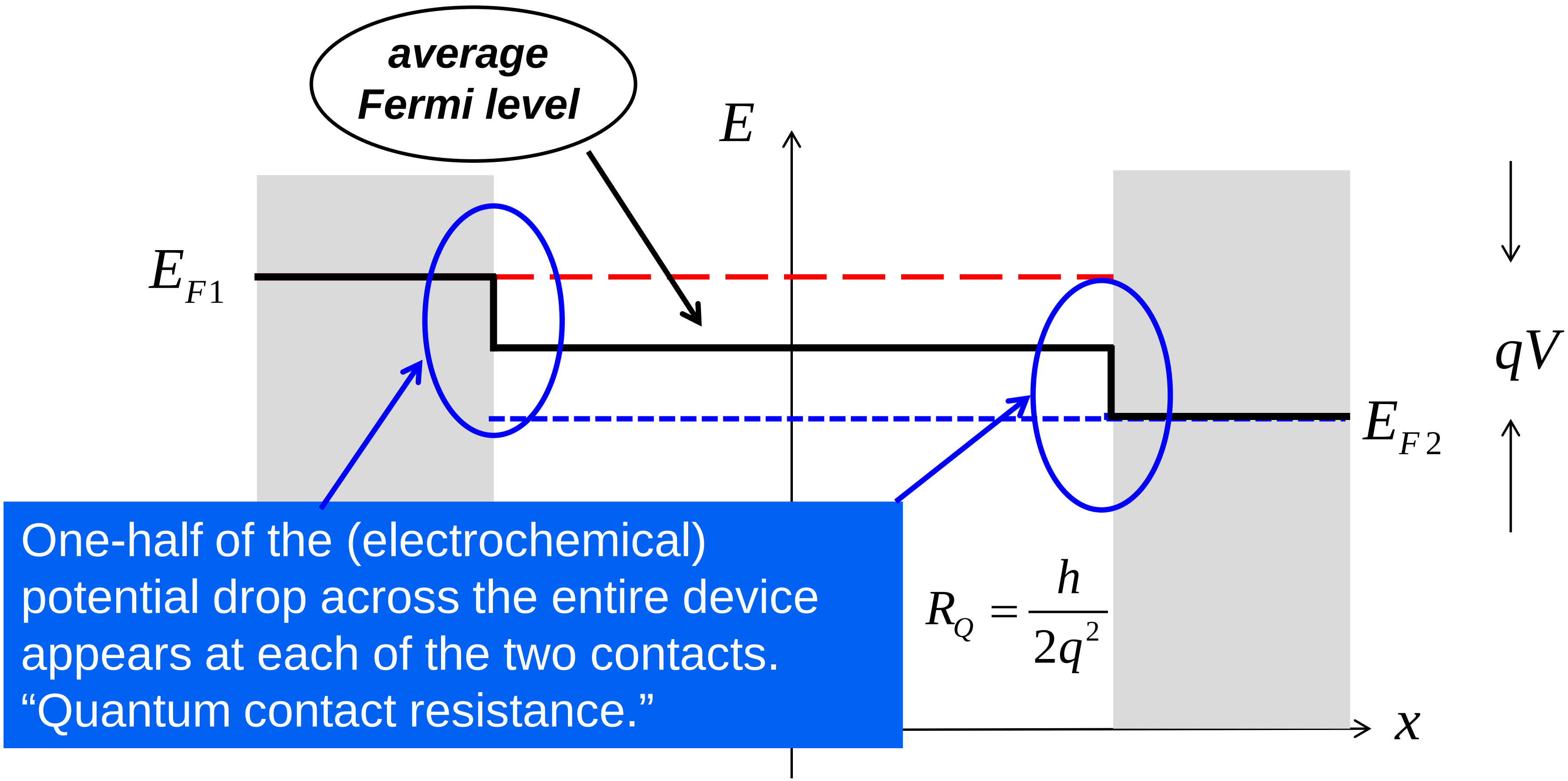
Answer: In the two contacts.

power dissipation in a ballistic resistor



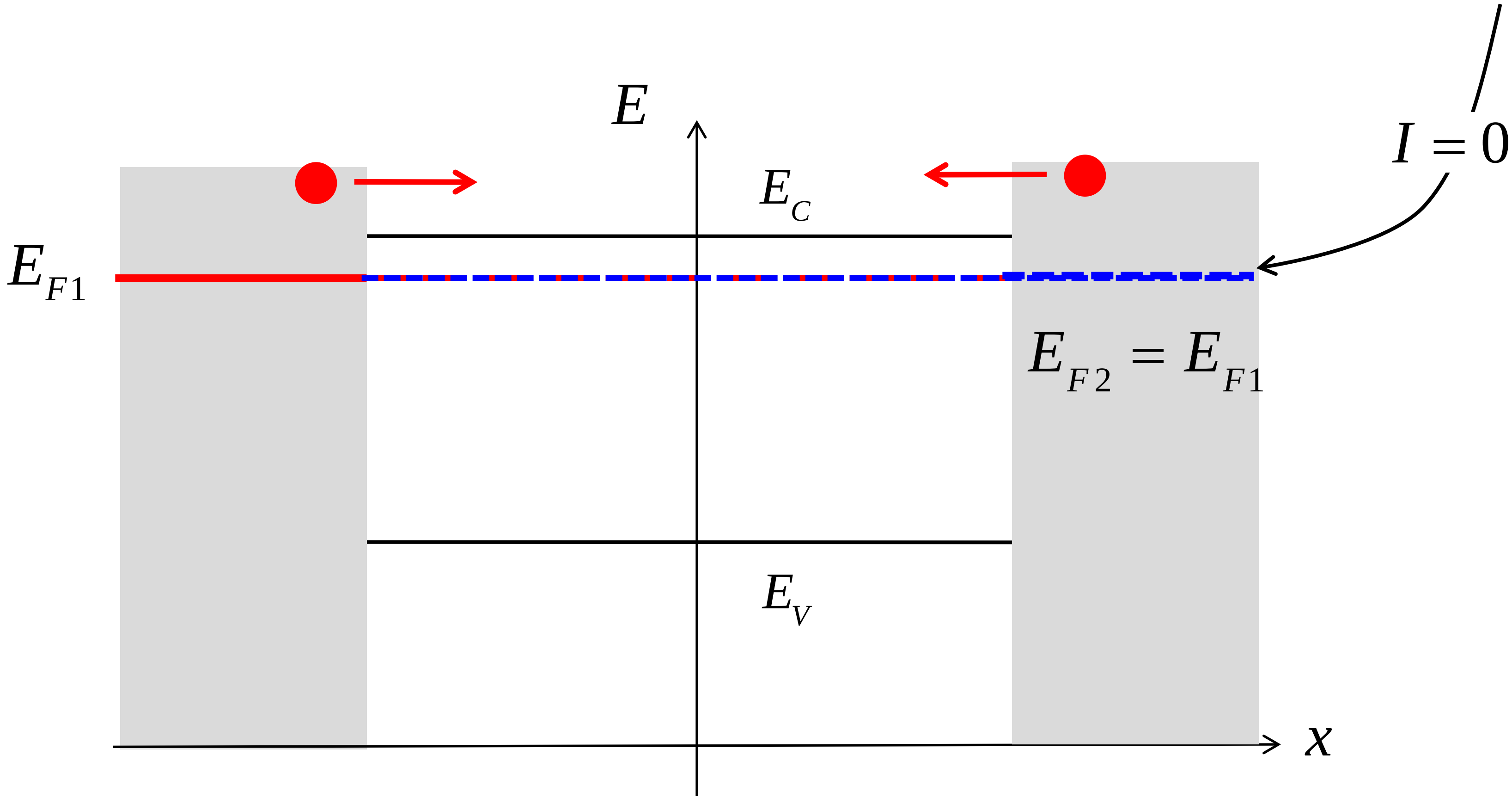
dissipation occurs in the contacts

voltage drop in a ballistic resistor



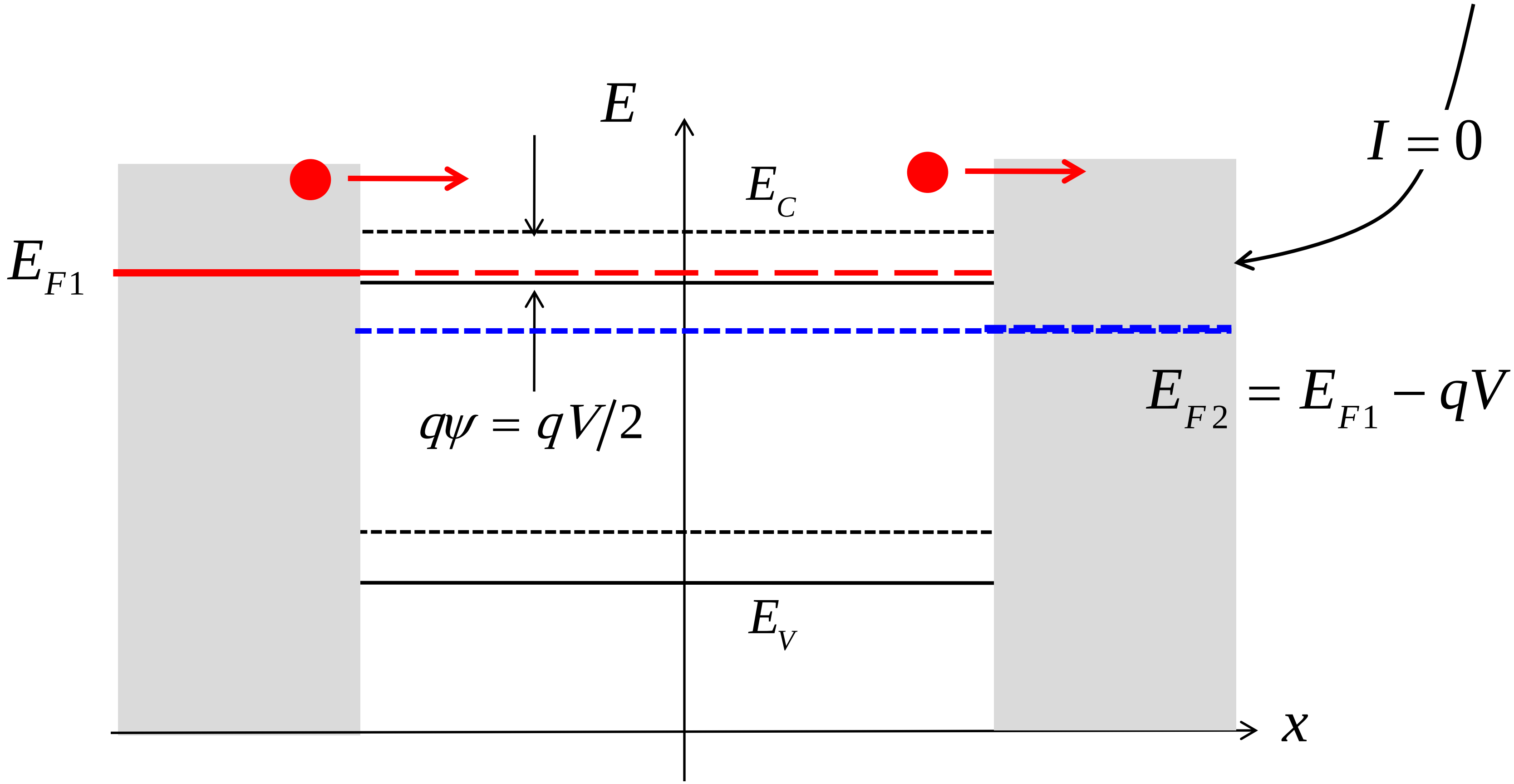
S. Datta, *Electronic Conduction in Mesoscopic Systems*, Chpt. 2, Cambridge, 1995

another view



$$N_0 = \frac{D}{2}\left(E_{F1} - E_C^0\right) + \frac{D}{2}\left(E_{F1} - E_C^0\right)$$

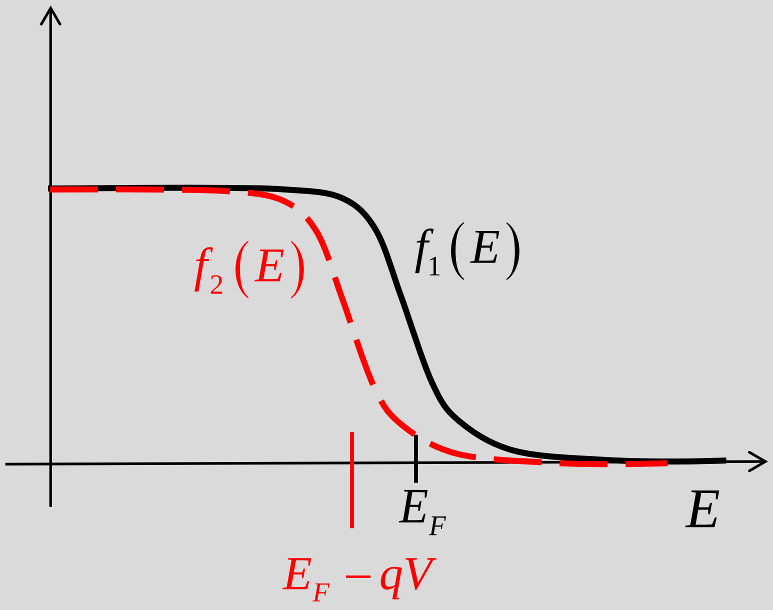
another view



$$N = N_0 = \frac{D}{2}\left(E_{F1} - E_C^0 + q\psi\right) + \frac{D}{2}\left(E_{F1} - qV - E_C^0 + q\psi\right)$$

n-type conduction vs. p-type

$f_0(E) \quad \Delta E_F\left(-q\Delta V\right)$



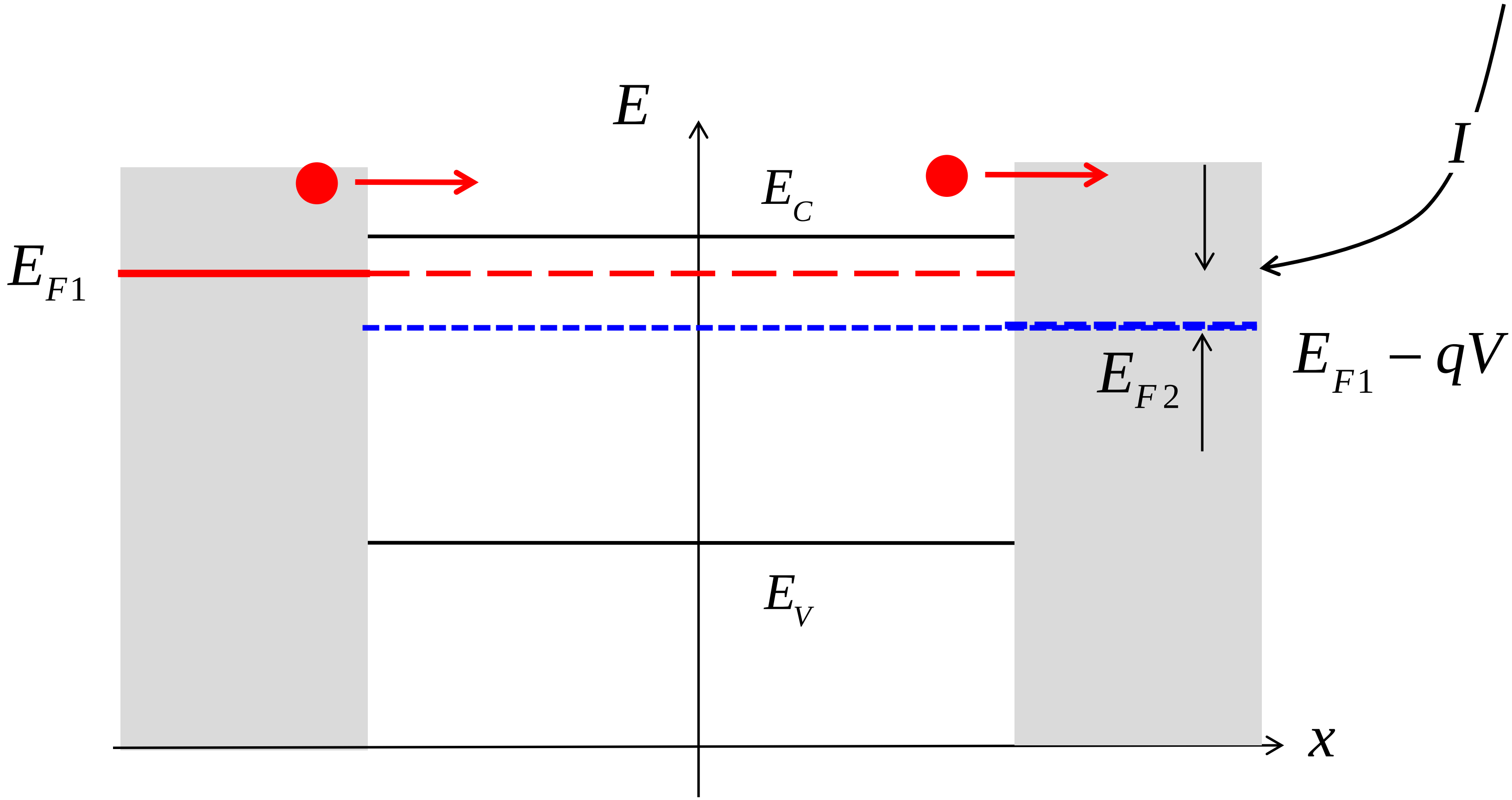
$f_1 > f_2$ and current flows

$$I = \frac{2q}{h}\int T(E)M(E)\left(f_1 - f_2\right)dE$$

$$V > 0 \Rightarrow f_1 > f_2 \Rightarrow I > 0$$

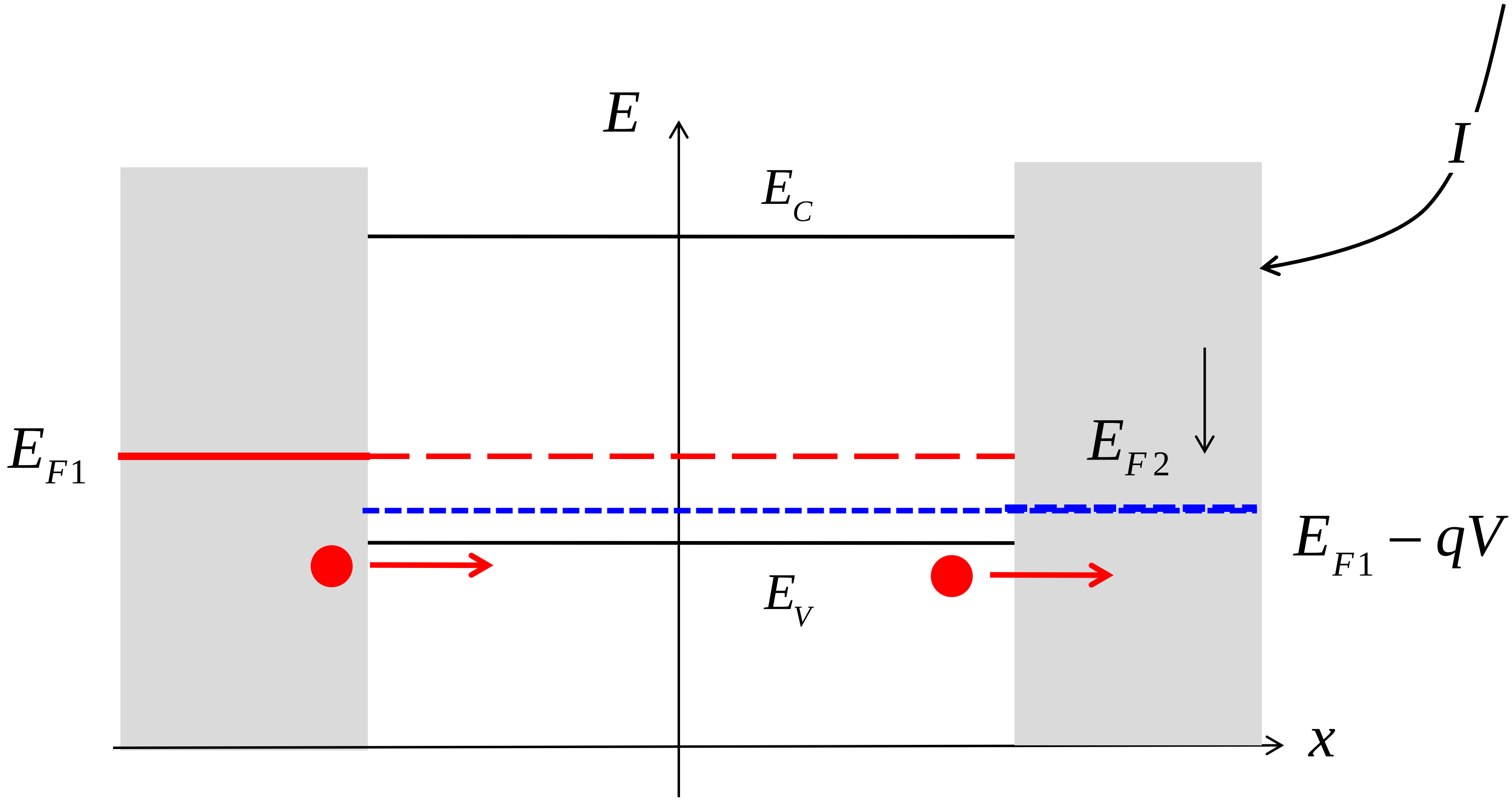
for either *n*-type or *p*-type

n-type



current is due to electrons flowing in the conduction band

p-type



current is due to electrons flowing in the valence band

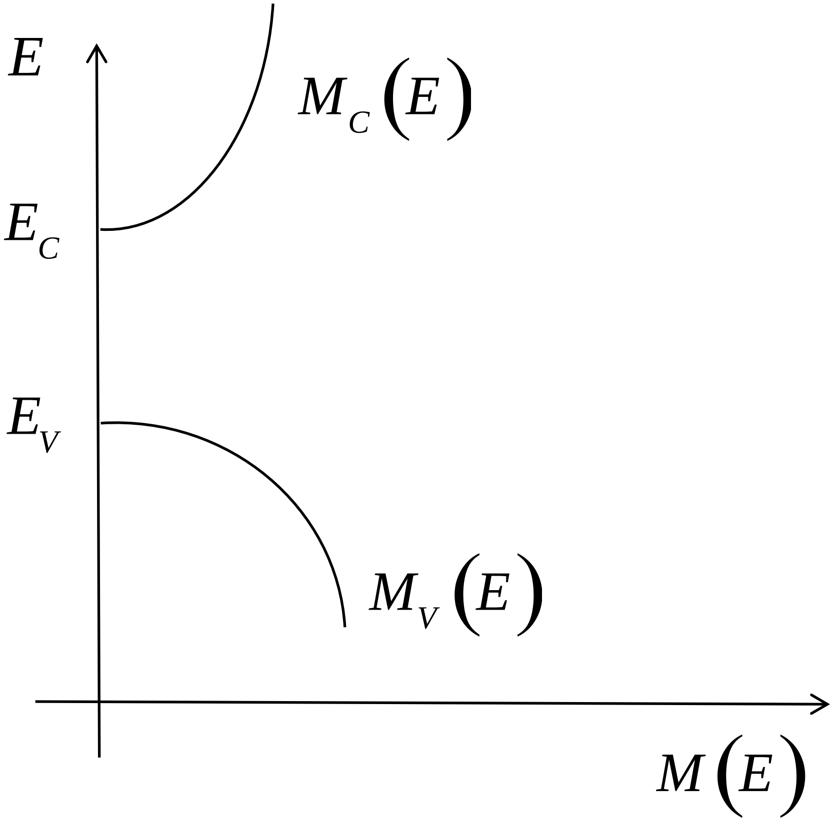
bipolar conduction

$$I = \frac{2q}{h}\int T(E)M(E)(f_1 - f_2)dE$$

$$M(E) = M_V(E_V - E) + M_C(E - E_C)$$

$$M_C(E) = g_V W \frac{\sqrt{2m^*(E - E_C)}}{\pi\hbar}$$

$$M_V(E) = g_V W \frac{\sqrt{2m^*(E_V - E)}}{\pi\hbar}$$



discussion

-power dissipation

-voltage drop

-n-type vs. p-type

-“apparent” mobility

mobility

We usually like to think of mobility as a material dependent quantity.

In bulk materials with a large electric field, we generalize this to a field-dependent quantity.

More generally, "mobility" can be device dependent.

how should we define mobility?

$$G_{2D} = \frac{2q^2}{h}\int T(E)\,W\,M_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \qquad G_{diff} = n_S q \mu_n \frac{W}{L}$$

(ballistic to diffusive) (diffusive)

definition of "apparent" mobility:

$$\mu_{app} \equiv \frac{1}{n_S}\frac{2q}{h}\int T(E)\,L M_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

not:

$$\mu_n = \frac{q\tau_m}{m^*}$$

in the diffusive limit

definition of mobility:

$$\mu_n \equiv \frac{1}{n_S}\frac{2q}{h}\int T(E)\,LM_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$T(E) = \frac{\lambda(E)}{\lambda(E)+L} \rightarrow \frac{\lambda(E)}{L}$$

$$\mu_n \equiv \frac{1}{n_S}\frac{2q}{h}\int \lambda(E)M_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$\mu_n = \frac{q\tau_m}{m^*}$$

in the ballistic limit

definition of mobility:

$$\mu_n \equiv \frac{1}{n_S}\frac{2q}{h}\int T(E)\,LM_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$T(E) = 1$$

$$\mu_{ball} \equiv \frac{1}{n_S}\frac{2q}{h}\int LM_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$\mu_{ball} \neq \frac{q\tau_m}{m^*}$$

ballistic to diffusive…

definition of **apparent** mobility:

$$\mu_{app} \equiv \frac{1}{n_S}\frac{2q}{h}\int T(E)\,LM_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$T(E)L = \frac{\lambda(E)L}{\lambda(E)+L} \rightarrow \lambda_{app}(E) \qquad \frac{1}{\lambda_{app}(E)} \equiv \frac{1}{\lambda(E)} + \frac{1}{L}$$

$$\mu_{app} \equiv \frac{1}{n_S}\frac{2q}{h}\int \lambda_{app}(E)\,M_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$\mu_n = \frac{q\tau_m}{m^*}$$

T_L = 0K apparent mobility

$$\mu_{app} \equiv \frac{1}{n_S}\frac{2q}{h}\int \lambda_{app}(E) M_{2D}(E)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$\mu_{app}\left(T_L = 0\,\mathrm{K}\right) = \frac{1}{n_S}\frac{2q}{h}\lambda_{app}(E_F) M_{2D}(E_F)$$

$$\frac{1}{\lambda_{app}(E)} = \frac{1}{\lambda(E)} + \frac{1}{L}$$

$$M_{2D} = \frac{h}{4}\left\langle \upsilon_x^+ \right\rangle D_{2D} \qquad \left\langle \upsilon_x^+ \right\rangle = \frac{2}{\pi}\upsilon_F$$

$$n_S = g_V \frac{m^*}{\pi\hbar^2}\left(E_F - E_C\right)$$
$$= D_{2D}\left(E_F - E_C\right)$$

$$\frac{1}{\mu_{app}} = \frac{1}{\mu_n} + \frac{1}{\mu_{ball}}$$

“ballistic mobility” in 2D at T_L = 0K

$$\frac{1}{\mu_{app}} = \frac{1}{\mu_n} + \frac{1}{\mu_{ball}}$$

$$\mu_n = \frac{D_n}{\left(E_F - E_C\right)/q}$$

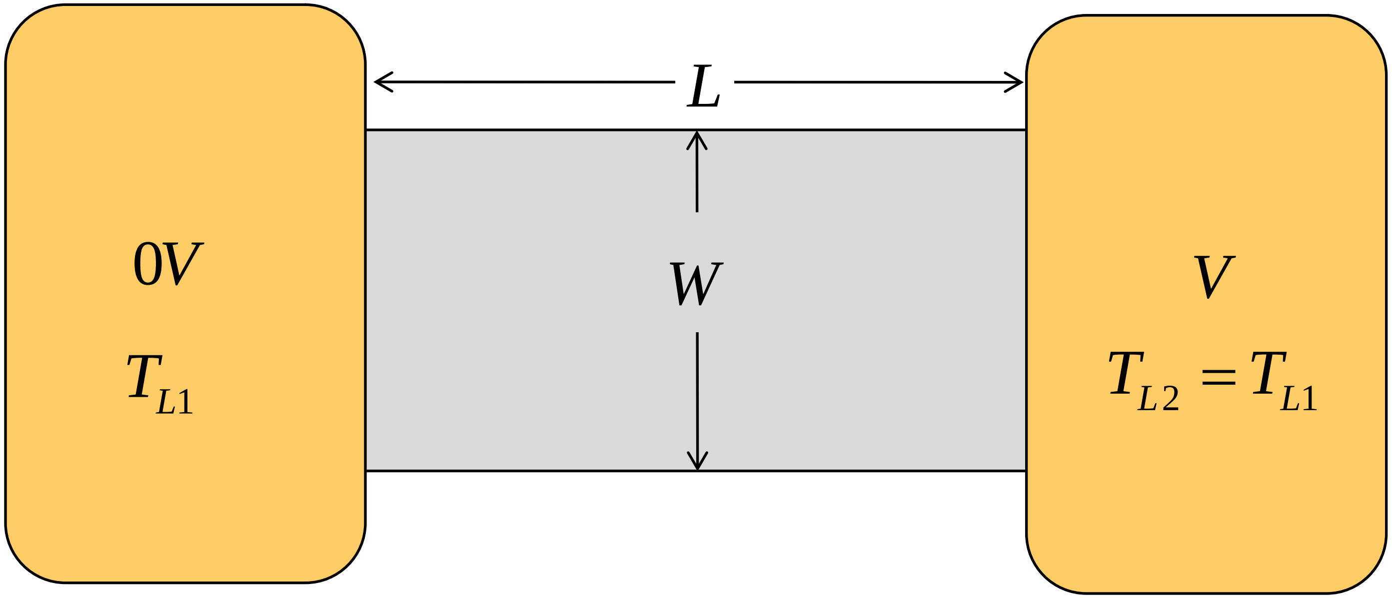
$$\mu_{ball} = \frac{D_{ball}}{\left(E_F - E_C\right)/q}$$

$$D_n = \frac{\left\langle \upsilon_x^+ \right\rangle \lambda\left(E_F\right)}{2} \qquad D_{ball} = \frac{\left\langle \upsilon_x^+ \right\rangle L}{2}$$

$$G_{2D} = \frac{2q^2}{h} T\left(E_F\right) M_{2D}\left(E_F\right) = n_S q \mu_{app} \frac{W}{L}$$

M.S. Shur, “Low Ballistic Mobility in GaAs HEMTs,” *IEEE Electron Dev. Lett.*, **23**, 511-513, 2002

physical interpretation



$$\frac{1}{\lambda_{app}(E)} \equiv \frac{1}{\lambda(E)} + \frac{1}{L}$$

ballistic conductance (T_L = 0 K)

$$G_{2D} = n_S q \mu_{app} \frac{W}{L}$$

$$G_{2D} = n_S q \mu_{ball} \frac{W}{L}$$

$$G_{2D} = \frac{2q^2}{h} M\left(E_F\right)$$

$$\frac{1}{\lambda_{app}\left(E_F\right)} = \frac{1}{\lambda\left(E_F\right)} + \frac{1}{L}$$

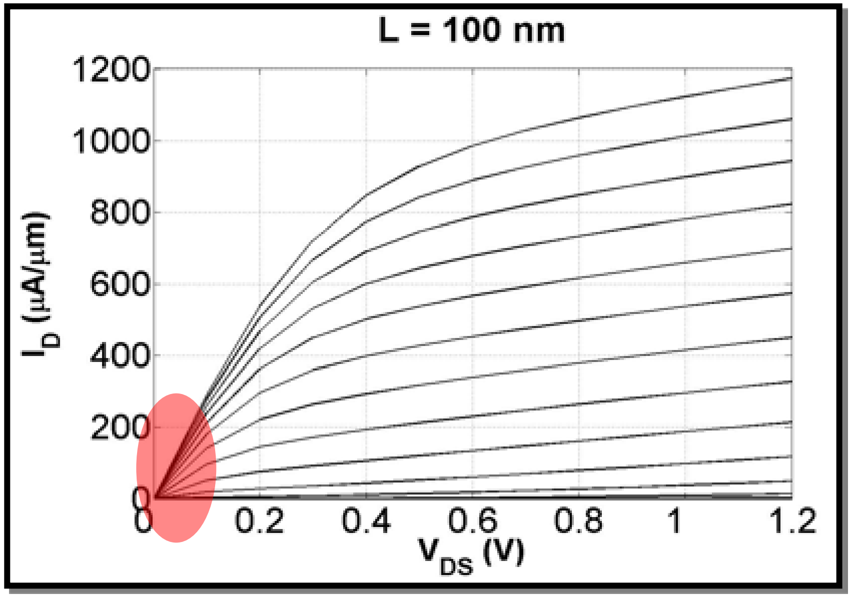
$$M = W\frac{h}{4}\left\langle \upsilon_x^+ \right\rangle D_{2D}$$

$$n_S = D_{2D}\left(E_F - E_C\right)$$

$$\mu_{ball} = \frac{D_{ball}}{\left(E_F - E_C\right)/q}$$

$$D_{ball} = \frac{\left\langle \upsilon_x^+ \right\rangle L}{2}$$

exercise: nanoscale FETs



(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

$$L \approx 60 \text{ nm}$$

$$\mu_n \approx 260 \text{ cm}^2/\text{V-s}$$

$$n_S \approx 1 \times 10^{13} \text{ cm}^{-2}$$

$$\mu_{ball} \approx \frac{\upsilon_T L}{2} \frac{1}{k_B T/q} \approx 1400 \text{ cm}^2/\text{V-s}$$

$$\mu_{eff} = \left(\frac{1}{\mu_n} + \frac{1}{\mu_{ball}} \right)^{-1} \approx \mu_n$$

outline

1) Review

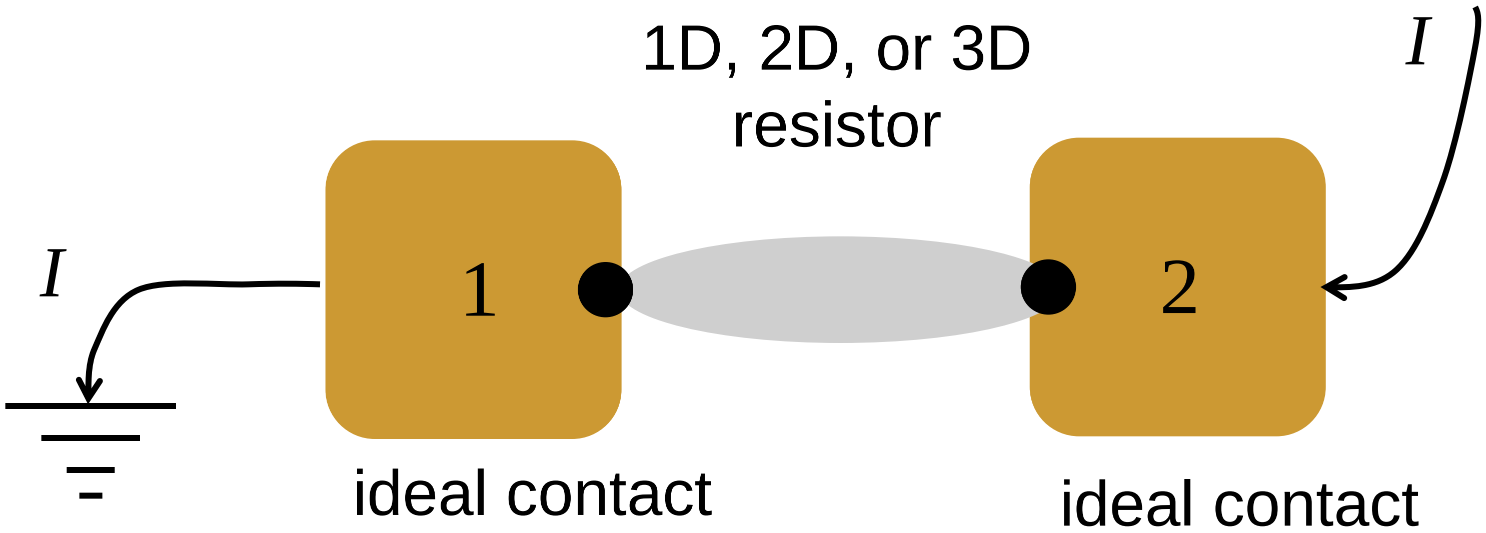
2) Discussion

3) 1D and 3D resistors

4) Graphene: A case study

5) Summary

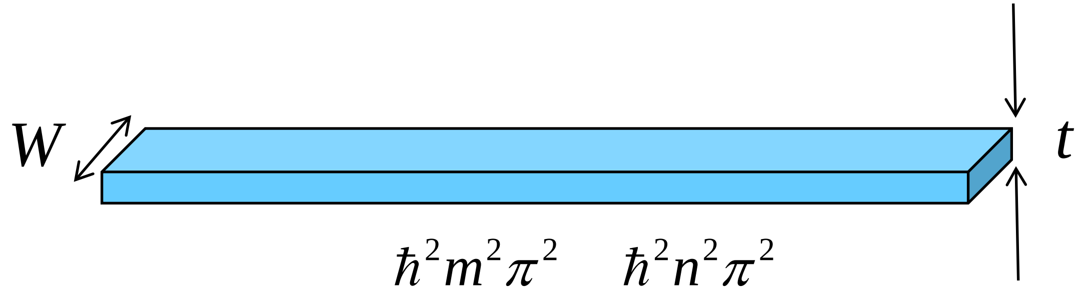
1D and 3D resistors



$$G = \frac{2q^2}{h} \int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

This expression describes 1D, 2D, or 3D resistors.

1D



$$\varepsilon_{m,n} = \frac{\hbar^2 m^2 \pi^2}{2m^* W^2} + \frac{\hbar^2 n^2 \pi^2}{2m^* t^2}$$

1) If *W* and *t* are small, all subbands are widely spaced, and we just count them.

2) For a 1D resistor: $M(E) = \text{no. of subbands at energy}, E$

$$G_{1D} = \frac{2q^2}{h}\langle M \rangle$$

3D

1) Is *W* and *t* are large, all of the subbands are closely spaced.

2) For a 3D resistor: $M(E) = A\dfrac{m^*}{2\pi\hbar^2}(E - E_C)$

$$G_{3D} = \frac{2q^2}{h}\langle M \rangle$$

1D, 2D, and 3D resistors

$$G = \frac{2q^2}{h}\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

1) In 1D (quantum confinement in two dimensions):

$$M(E) = \text{no. of subbands at energy}, E$$

2) In 2D (quantum confinement in one dimension):

$$M(E) = \sum_{n=1}^{N} W g_V \sqrt{2m^*(E-\varepsilon_n)} \Big/ \pi\hbar$$

3) In 3D (no quantum confinement):

$$M(E) = A g_V \frac{m^*}{2\pi\hbar^2}(E - E_C)$$

1D, 2D, and 3D resistors

$$G = \frac{2q^2}{h}\int T(E)\,M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$G = \frac{2q^2}{h}\langle\langle T\rangle\rangle\langle M\rangle$$

$$\langle M\rangle = \int M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$\langle\langle T\rangle\rangle = \frac{\int T(E)\,M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE}{\int M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE}$$

For a constant mfp:

$$\langle\langle T\rangle\rangle = \frac{\lambda_0}{\lambda_0 + L}$$

1D resistors

$$G = \frac{2q^2}{h} \frac{\lambda_0}{\lambda_0 + L} \langle M \rangle$$

$$\langle M \rangle = \sum_i \mathcal{F}_{-1}(\eta_{Fi})$$

$$\eta_{Fi} = \frac{E_F - \varepsilon_i}{k_B T_L}$$

(parabolic energy bands)
(constant mfp)

T_L = 0 K:

$$\langle M \rangle = M(E_F)$$

MB statistics:

$$\langle M \rangle \propto n_L$$

Diffusive:

$$G \propto \frac{1}{L}$$

2D resistors

$$G = \frac{2q^2}{h}\frac{\lambda_0}{\lambda_0 + L}\langle M \rangle$$

$$\langle M \rangle = \frac{\sqrt{\pi}}{2} W M_{2D}\left(k_B T_L\right)\mathcal{F}_{-1/2}\left(\eta_{F1}\right)$$

$$M_{2D}\left(k_B T_L\right) = g_V \frac{\sqrt{2m^* k_B T_L}}{\pi\hbar}$$

$$\eta_{F1} = \frac{E_F - \varepsilon_1}{k_B T_L}$$

(parabolic energy bands)

(constant mfp)

***T_L* = 0 K:**

$$\langle M \rangle = M\left(E_F\right)$$

MB statistics:

$$\langle M \rangle \propto n_S$$

Diffusive:

$$G \propto \frac{W}{L}$$

3D resistors

$$G = \frac{2q^2}{h} \frac{\lambda_0}{\lambda_0 + L} \langle M \rangle$$

$$\langle M \rangle = A M_{3D}\left(k_B T_L\right) \mathcal{F}_0\left(\eta_F\right)$$

$$M_{3D}\left(k_B T_L\right) = g_V \frac{m^* k_B T_L}{2\pi\hbar^2}$$

$$\eta_F = \frac{E_F - E_C}{k_B T_L}$$

(parabolic energy bands)

(constant mfp)

T_L = 0 K:

$$\langle M \rangle = M\left(E_F\right)$$

MB statistics:

$$\langle M \rangle \propto n$$

Diffusive:

$$G \propto \frac{A}{L}$$

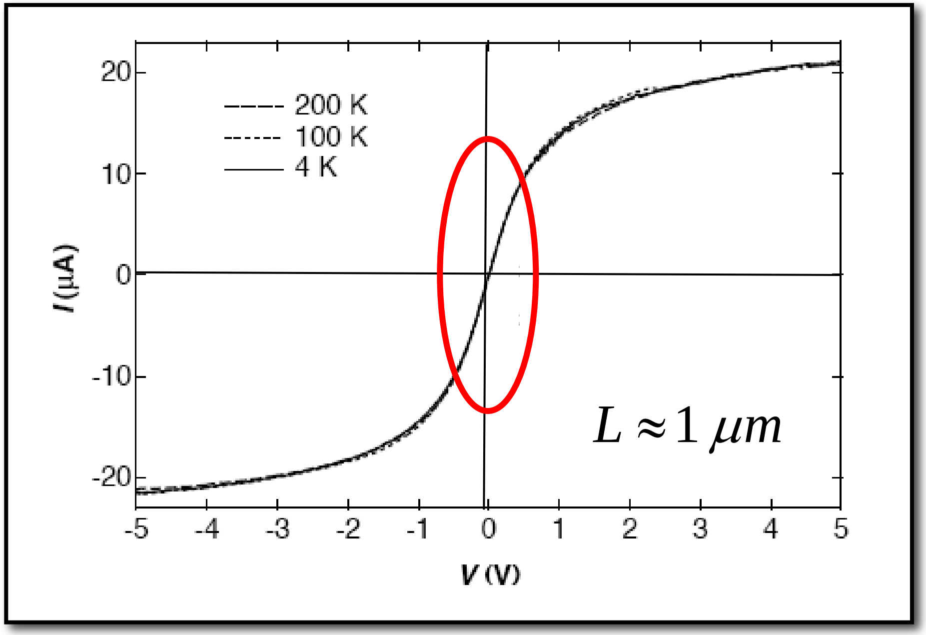
1D: example: low-field transport in metallic CNTs

$$G_{1D} = \frac{\Delta I}{\Delta V} = \frac{22\ \mu A}{1.0\ V} = 22\ \mu S$$

$$G_B = \frac{4q^2}{h} = 154\ \mu S \qquad (g_V = 2)$$

$$G_{1D} = \frac{4q^2}{h}\frac{\lambda_0}{\lambda_0 + L}$$

$$\lambda_0 \approx 167 \text{ nm} << L$$



Zhen Yao, Charles L. Kane, and Cees Dekker, "High-Field Electrical Transport in Single-Wall Carbon Nanotubes," *Phys. Rev. Lett*., **84**, 2941-2944, 2000.

outline

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graphene

We now have all the tools and concepts needed to analyze transport in a wide variety of materials. To see how to apply these concepts to graphene, see Lecture 10 of Near-Equilibrium Transport: Fundamentals and Applications" by M. Lundstrom.

or view an online lecture at:

http://nanohub.org/resources/11873

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key points

1) To understand conductance, we begin with:

$$G = \frac{2q^2}{h} \int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

which can be written as:

$$G = \frac{2q^2}{h} \langle\langle T(E) \rangle\rangle \langle M(E) \rangle$$

$$\langle M \rangle \equiv \int M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \qquad \langle\langle T \rangle\rangle \equiv \frac{\int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE}{\int M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE}$$

key points

2) To relate conductance to mobility, we define

$$G_{2D} = nq\mu_{app}\frac{W}{L} = \frac{2q^2}{h}\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

and solve for the apparent mobility.

questions

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