ECE616 Homework #3
Out: Thursday, September 29, 2011
Due: Thursday, October 13, 2011

Topic: Pulse Measurement
Text: Ultrafast Optics
Chapter 3, Sections 3.1 – 3.5

1) Text 3.1 Note: illustrate your solution with plots as appropriate.
2) See attached
3) Text 3.2
For part (b), comment on the relationship of your plots to those from part (a).
4) Text 3.3
5) Text 3.7
6) See attached

2) Although the full-width half-maximum (FWHM) pulse duration is a useful way to characterize well-behaved, clean pulses, it is less useful for characterization of pulses with low intensity wings or substructure. In this problem we will compare four different pulse width metrics:

Δt_{FWHM} – the intensity full-width half-maximum
Δt_{rms} – rms pulse width, defined in text eq. (3.5)
$\Delta t_{intensity}$ – defined as

$$\Delta t_{intensity} = \frac{\int |a(t)|^2 \, dt}{\max\left\{|a|^2\right\}}$$

That is, $\Delta t_{intensity}$ = (pulse energy)/(peak intensity)
In this definition, a rectangular pulse of duration $\Delta t_{intensity}$ and the same peak intensity as the real pulse will give the correct pulse energy.

Δt_{SHG} - defined as $\Delta t_{SHG} = \dfrac{\left[\int |a(t)|^2 \, dt\right]^2}{\int |a(t)|^4 \, dt}$

In this definition, the numerator is the square of the pulse energy, while the denominator gives the energy at the second harmonic frequency that would be generated in an ideal second harmonic generation experiment (constants governing the efficiency of the process are normalized out).

In this definition, a rectangular pulse of duration Δt_{SHG} and the same energy as the real pulse will give the correct second harmonic energy.

Please compute (note: a numerical approach is suggested) and compare the values of these four pulse width metrics for:

(a) a bandwidth-limited Gaussian – see equations (3.60). This is an example of a single pulse.
(b) more complicated pulses – here I can suggest to use fields with Gaussian spectra and cubic spectral phase – see eq. (3.67) and Fig. 3.16. Vary the cubic spectral phase coefficient to vary the strength of the oscillating pulse tail.
(c) Optionally, you may also look at pulses that you constructed in problem (1).

Based on your results, please comment on which pulse width metric (if any) you think would be the best to describe pulses that may have complicated intensity profiles.

Also (optionally), if you have any creative ideas for pulse width metrics of your own, please describe!

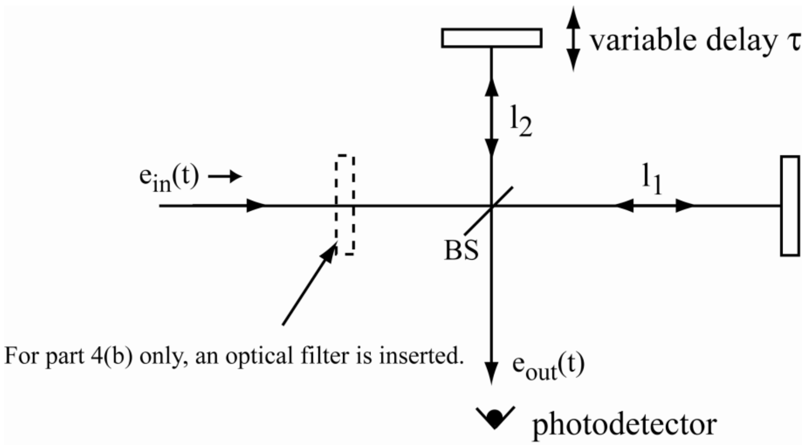
6) Consider a laser that produces a multi-mode output spectrum of the form:

$$A(\tilde{\omega}) = \exp\left[-\tilde{\omega}^2 / \Delta\Omega^2\right] \times \sum_n \delta(\tilde{\omega} - n\Delta\omega) e^{j\psi_n}$$

where $\tilde{\omega} = \omega - \omega_o$ and ω_o is the center frequency. Assume that

$$\Delta\Omega = 2\pi \times 10^{12} \text{ Hz}$$
$$\Delta\omega = 2\pi \times 10^{10} \text{ Hz}$$
$$\omega_o = 2\pi \times \left(5 \cdot 10^{14}\right) \text{Hz}$$

(a) Assume that the laser is perfectly modelocked, so that the spectral phases are all the same ($\psi_n = 0$ for all n). The laser output is directed into an interferometer (see figure), and one of the outputs of the interferometer is detected by a simple power meter. **Sketch or compute the signal recorded by the power meter as a function of the relative delay between the two arms of the interferometer. Be sure to give values (at least approximately) for the time scales corresponding to ALL the important features in your sketch.** Note: You are NOT required to determine the exact mathematical form of this output, so no detailed calculations should be needed.



(b) A bandpass filter with ~100 GHz bandwidth is inserted prior to the interferometer. Consider 2 cases:

(1) The filter has flat phase; its filter function $H_1(\omega)$ is written:

$$H_1(\omega) = \exp\left[-\tilde{\omega}^2 / B^2\right], \text{ where } B = 2\pi \times 10^{11} \text{ Hz}$$

(2) The filter has quadratic spectral phase; its filter function $H_2(\omega)$ is written

$$H_2(\omega) = \exp\left[-\tilde{\omega}^2 / B^2\right] \exp\left(j\tilde{\omega}^2 / C^2\right), \text{ where } B = 2\pi \times 10^{11} \text{ Hz and } C = \pi \times 10^{11} \text{ Hz}.$$

Once again the output of the interferometer is recorded by a power meter as a function of delay. For both the flat-phase filter $\left(H_1(\omega)\right)$ and the quadratic-phase filter $\left(H_2(\omega)\right)$, sketch the signal recorded by the power meter as a function of the relative delay. Compare/contrast the form of each of the output traces relative to that of part (a) and to each other.