Problem 1

a) \[ \frac{1}{7} + \frac{1}{x'} = \frac{1}{5}, \quad x' = 17.5\text{cm} \]

b) \[ \frac{1}{3} + \frac{1}{x'} = \frac{1}{5}, \quad x' = -7.5\text{cm} \]
c) 
\[
\frac{1}{-3} + \frac{1}{x'} = \frac{1}{5}, x' = 1.875 \text{cm}
\]

d) 
\[
\frac{1}{3} + \frac{1}{x'} = \frac{1}{5}, x' = -7.5 \text{cm}
\]
1.2-7)

a) See last homework. Fermat’s principle claims that light will travel in the minimum path between two points. Here is one way to interpret this physically. If light can move faster in one media, then it will move a further distance in the fast media before exiting into the slow media. Once in the slow media, the light will take the most direct path to the target. For different contrast of refractive index, different angles will be taken upon exit from the fast to the slow media. Deriving Snell’s law from Fermat’s principle clearly shows that the angle of incidence and exit are the same from a slab of uniform refractive index:

\[ n_i \sin(\theta_i) = n_2 \sin(\theta_2) = K = n_n \sin(\theta_n), \]  
so if \( n_i = n_n \), then \( \theta_i = \theta_n \)

b) Once again... \( n_1 \sin(\theta_1) = n_2 \sin(\theta_2) = K = n_\text{exit} \sin(\theta_\text{exit}) \), so if \( n_1 = n_n \), then \( \theta_1 = \theta_\text{exit} \). Since the tangential electric field component is conserved along the boundary, only the first and last media matter. In other words, what is in between the entrance and exit points does not affect the exit angle.

1.2-8)

For a thick lens:

\[ \frac{-1}{f} = \frac{n_2 - n_1}{n_1} \left( \frac{1}{R_1} - \frac{1}{R_2} + \frac{n_1 - n_2}{n_2 R_1 R_2} d \right) \]

a) \( f = 24 \text{ cm} \)

b) \( f = 96 \text{ cm} \)

1.2-9)

NA for an optical fiber with \( n_{\text{core}} > n_{\text{cladding}} > n_{\text{incident}} \) is

\[ NA = n_1 \sin(\theta_1) = \sqrt{(n_{\text{core}})^2 - (n_{\text{cladding}})^2} \]

a) \( NA = 1.064 \), so the acceptance angle is greater than 90 degrees. At the greatest acceptance angle of 90 degrees, the refracted ray will be at the “critical angle” inside of the fiber. Since the since the critical angle is below 45 degrees, 90 degrees minus the critical angle will always exceed the critical angle at the fiber edge, leading to TIR for all acceptance angles between 0 and 90 degrees.

1.2-10)

Part 1

\[ 1 \times \sin \theta_1 = 1.8 \times \sin \theta_2, \sin \theta_1 = 0.7, \theta_2 = 22.8854^\circ \]
\[ 1.8 \times \sin \theta_2 = 1 \times \sin \theta_3, \sin \theta_3 = 0.7, \theta_1 = \theta_3 = 44.427^\circ \]
\( \theta_3 = 180 - \theta_1 - (180 - \theta_2 - \theta_3) = 2\theta_2 - \theta_1 = 1.34^\circ \)
\( \theta_6 = 180 - \theta_3 - (90 - \theta_3) = 90 + 2(\theta_2 - \theta_1) = 47^\circ \)
\( x = 1 \cdot \sin(\theta_3) \)
\( f = x \cdot \tan(\theta_6) - (1 - 1 \cdot \cos(\theta_3)) = 0.0248\text{mm} \)

Part 2

\[
\begin{pmatrix}
\frac{n_0}{n_1} & \frac{n_1}{n_2}
\end{pmatrix}
\begin{pmatrix}
M_{HH'}
\end{pmatrix}
= \frac{1}{H''}
= \begin{pmatrix}
\frac{M_{11}}{M_{21}} & M_{21}
M_{21} & M_{22}
\end{pmatrix}
= \begin{pmatrix}
\frac{M_{11} + M_{21} \cdot \lambda_2}{M_{22}} & \frac{M_{21} \cdot \lambda_2}{M_{22}}
\frac{M_{21} \cdot \lambda_2}{M_{22}} & \frac{M_{22} \cdot \lambda_2}{M_{22}}
\end{pmatrix}
\]

For thick lenses:
\( M_{11} = 1 + \frac{f(n_2 - n_1)}{n_1 R_1}, \quad M_{22} = 1 + \frac{f(n_2 - n_1)}{n_1 R_2} \)
\( M_{12} \cdot \lambda_2 = 1 \Rightarrow \lambda_2 = \frac{1 - M_{11}}{M_{22}} \)
\( M_{21} \cdot \lambda_1 = 1 \Rightarrow \lambda_1 = \frac{1 - M_{22}}{M_{22}} \)
\( f = \frac{n_2 - n_1 \cdot (-2 - 2 \cdot \frac{n_1 - n_2}{n_2})}{2 \cdot (n_2 - n_1)(n_2 + n_1 - n_2)} \)

Plugging in \( f \) into the equation for the principal plane, we see that all terms cancel out, leaving us with \( h_i = 1\text{mm} \) from the edge of the sphere. The principal planes are therefore at the center of the sphere for both the air and water scenario. You could similarly draw principal planes for thick lenses as the thickness of the lens approaches \( 2^\ast\text{radius of curvature} \). You will see that the planes approach the center of the lens and overlap at the midpoint of the sphere.

1.4-8)

Transfer matrix:
\[
M = \begin{pmatrix}
1 & 0 \\
\frac{1}{f} & 1
\end{pmatrix}
\begin{pmatrix}
1 & f \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
-\frac{1}{f} & 1
\end{pmatrix}
= \begin{pmatrix}
0 & f \\
-\frac{1}{f} & 2
\end{pmatrix}
\]

Imaging property: For an object at \( d_1 \) being imaged at \( d_2 \) we get the following:
Using the superposition principle of lenses we can form an intermediary image with the first converging lens. The real intermediary image is formed beyond the focal length \( f \) of the converging lens. Assuming the thin lens approximation, the image is therefore formed on the far side of the diverging lens. The diverging lens then forms a virtual image on the far side of the lens system. Setting \( M_{12} = 0 \) gives you the imaging condition for an image at \( d_i \) and focal distance \( f \). i.e.: \( f = 1 \text{mm}, \ d_i = 6 \text{mm}, \ d_2 = \frac{1}{4} \).

\[
\begin{pmatrix}
1 & d_2 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & f \\
1 & f
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
-\frac{1}{f} & 1
\end{pmatrix}
\begin{pmatrix}
1 & d_i \\
0 & 1
\end{pmatrix}
= \begin{pmatrix}
-d_2 f & f + 2d_2 - \frac{d_i d_2}{f} \\
-\frac{1}{f} & 2 - \frac{d_i}{f}
\end{pmatrix}
\]

1.4-9)

For a graded refractive index film with optical axis along \( z \)-direction:

\[
\frac{\partial^2 y}{\partial z^2} = \frac{1}{n(y)} \frac{dn(y)}{dy} \frac{\alpha^2 * y * n_o}{n_o (1 - 0.5 \alpha^2 * y^2)} = \alpha^2 y(z).
\]

The solution to the pde is of the form of sine and cosine. Using \( y(z) = y_o \cos(\alpha z) + \theta_o \frac{\sin(\alpha z)}{\alpha} \) and \( \theta(z) = -y_o \alpha \sin(\alpha z) + \theta_o \frac{\cos(\alpha z)}{\alpha} \), we find a transfer matrix:

\[
\begin{pmatrix}
\cos(\alpha z) & \sin(\alpha z) \\
-\alpha \sin(\alpha z) & \cos(\alpha z)
\end{pmatrix}
\]