Homework 3

Due in class on September 15, 2011

1. (20%) A linearly polarized light is normally incident on a polarizer that rotates with angular frequency w. (The axis of rotation is parallel to the direction of light propagation.)
   a) Write down the expression for the output intensity versus time.
   b) A second polarizer is placed after the first; it rotates with angular frequency w'. Write down the final output vs. time.
   c) The two polarizers are now fixed with their axes at 90 degrees with respect to each other, and the first is oriented parallel with the incident field. What is the output intensity?
   d) Continuing from c), a third polarizer is placed in between the previous two, oriented at 45 degrees with respect to each of them. What is the final output intensity?
   e) Generalize the result in d) for a large number N of polarizers that are placed in between the two crossed polarizers and incrementally rotated by 90^o/N.

2. (30%) A light pulse has the following electric field distribution in time

$$E(t) = \exp[-\frac{t^2}{2\tau^2}]\exp(-i\omega_0 t)\,,$$

where τ is a constant and ω_0 the central optical frequency. This pulse is incident on a piece of glass whose *transfer function* can be approximated as $\tilde{h}(\omega) = \exp\left[i\beta\omega^2\right]$, with β a constant that defines glass dispersion.

   a) What is the electric field after passing through the glass?
   b) Compare the pulse duration before and after the glass (calculate the pulse duration as the full-width half maximum of the intensity, $I(t) = \left|E(t)\right|^2$).
   c) Compare the field autocorrelation before and after the glass
   d) Compare the power spectrum of the field before and after the glass
   e) Cross-correlate the field before with the one after the glass.

3. (30%) The electric field transmitted by a particle can be approximated by

$$E(x, y) = \exp(-\frac{(x^2 + y^2)}{2a^2}\exp[\frac{i(x^2 + y^2)}{2b^2}]$$

The particle is imaged by a microscope with a *transfer function* $\tilde{h}(u, v) = \exp\left[i\beta(u^2 + v^2)\right]$, with (u, v) the conjugate variables to (x, y). Note that here β is a measure of the particle is out-of-focus

   a) What is the expression of the electric field outputted by the microscope?
   b) Compare the sizes of the particle and that of its image (taken as the full width half maximum of the intensity profile $I(x) = \left|E(x)\right|^2$).
   c) Compare the field autocorrelation of before and after the microscope
   d) Compare the (spatial) power spectrum of the field before and after the microscope
   e) Cross-correlate the field before with the one after the microscope.

4. (10%) Discuss the similarity between problems 2 and 3.

5. (10%) Use Fermat's principle to prove Snell's law